Avrupa Bilim ve Teknoloji Dergisi
Özel Sayı 36, S. 67-72, Mayıs 2022

Araştırma Makalesi

www.ejosat.com ISSN:2148-2683

European Journal of Science and Technology
Special Issue 36, pp. 67-72, May 2022

Research Article

Comparing Bearing Capacity Factor N_γ of Finite Element Method with Analytical Methods for Sandy Soils

Emrah Dağlı[1*]

[1*] Zonguldak Bulent Ecevit University, Faculty of Engineering, Departmant of Civil Engineering, Zonguldak, Turkey, (ORCID: 0000-0002-5744-8151), emrahdagli@beun.edu.tr

(1st International Conference on Engineering and Applied Natural Sciences ICEANS 2022, May 10-13, 2022)

(**DOI:** 10.31590/ejosat.1102440)

ATIF/REFERENCE: Dağlı, E. (2022). Comparing Bearing Capacity Factor Nγ of Finite Element Method with Analytical Methods for Sandy Soils. *European Journal of Science and Technology*, (36), 67-72.

Abstract

Shallow strip footings are essential to carry loads from structures. Load bearing capacity factors can be calculated both by the field tests (plate load test) and numerically. Bearing capacity factors are the main parameters that affect the bearing capacity of any foundations. N_γ, one of these factors, have significant impact. Increase in internal frictional angle (ϕ), causes enhance the N_γ value. However, after ϕ value reaches 30°, dramatic increase is observed. This make the bearing capacity values complicated. In this study, strip foundation on surface resting on sandy soils were designed with a Geostudio 2012 software. Various foundation width (1, 1.25, 1.5, 1.75 and 2 m) and internal friction angle (29°, 31°, 33°, 35°, 37°, 39°, and 41°) was selected. Bearing capacity values were calculated with both numerical (software) and analytical methods. After, N_γ values of analytical methods were compared to results obtained from software. Results indicate that, Biarez 1961 has the average N_γ values while Terzaghi (1943) and Michalowski (1997) have the maximum. N_γ value obtained from numerical analysis (finite element method) increased with an increase in foundation width also. Values from finite element method is average of other analytical methods when B = 1.25 m, while N_γ values of numerical methods are the biggest when B = 2 m.

Keywords: Bearing capacity, Numerical analysis, Internal friction angle, N_γ value, Shallow strip footing

Kumlu Zeminler için Sonlu Elemanlar Yöntemi ve Analitik Yöntemlerle Bulunan Taşıma Gücü Faktörü N_γ Karşılaştırılması

Öz

Sığ şerit temeller, yapıdan gelen yükleri taşımak için önemlidir. Yük taşıma kapasitesi hem arazi testleri (plaka yük testi) ile hem de sayısal olarak hesaplanabilir. Taşıma kapasitesi (gücü) faktörleri, herhangi bir temelin taşıma gücünü etkileyen ana parametrelerdir. Bu faktörlerden biri olan N_γ önemli bir etkiye sahiptir. İçsel sürtünme açısındaki (ϕ) artış, N_γ değerinin artmasına neden olur. Ancak ϕ değeri 30°'ye ulaştıktan sonra dramatik bir artış gözlenmektedir. Bu, taşıma kapasitesi değerlerini karmaşık hale getirir. Bu çalışmada, Geostudio 2012 yazılımı ile kumlu zeminler üzerine oturan yüzeyde (derinliksiz) şerit temel tasarlanmıştır. Çeşitli temel genişliği (1 m, 1.25 m, 1.5 m, 1.75 m ve 2 m) ve içsel sürtünme açısı (29°, 31°, 33°, 35°, 37°, 39° ve 41°) seçilmiştir. Taşıma kapasitesi değerleri hem sayısal (yazılım) hem de analitik yöntemlerle hesaplanmıştır. Daha sonra, analitik yöntemlerin N_γ değerleri yazılımdan elde edilen sonuçlarla karşılaştırılmıştır. Sonuçlar, Biarez 1961'in ortalama N_γ değerlerine sahip olduğunu, Terzaghi (1943) ve Michalowski (1997)'nin ise maksimum değerlere sahip olduğunu göstermektedir. Sayısal analizden (sonlu elemanlar yöntemi) elde edilen N_γ değeri de temel genişliğinin artmasıyla artmıştır. Sonlu elemanlar yönteminden elde edilen değerler, B = 1.25 m olduğunda diğer analitik yöntemlerin ortalaması olmuştur. Ancak sayısal yöntemlerden elde edilen N_γ değerleri B = 2 m olduğunda en büyüktür.

Anahtar Kelimeler: Taşıma kapasitesi, Sayısal analiz, İçsel sürtünme açısı, N_γ değeri, Sığ şerit temel

[*] Corresponding Author: emrahdagli@beun.edu.tr

1. Introduction

Foundations are designed to carry the structural loads. These are the combination of dead, live and other loads (wind, earthquake, etc.). Shallow or deep foundations are requested to have sufficient bearing capacity for loads acting on them.

Bearing capacity values mainly depend on the bearing capacity factors of N_c, N_q and N_γ which are the functions of internal friction angle (ϕ). N_c, and N_q values give similar results for the weightless soil [1]. However, N_γ gave different values when compared to all equations created by the researches.

Numerous researchers [2-17] focused on the bearing capacity factor N_γ. [10] and [17] used upper bound limit analysis were used to evaluate the bearing capacity factor N_γ. [11] also used numerical analysis and found the relationship between ϕ−N_γ. [14], researched the effect of dilatation angle to evaluate the N_γ for rough strip ring footing. [15], proposed failure mechanism to centrally loaded strip footings. [16], used method of characteristics to find out bearing capacity factor of soils containing both cohesion and friction angle.

Equations created by some researchers are presented in Table 1. In this table, when compared to N_γ relationship of [6] and [7]. It is clearly seen that there is a significant difference.

Table 1. N_γ equations [18]

Researchers	Relationship
Terzaghi (1943)	$N_\gamma = 0.5 \tan\phi\ (K_{p\gamma} \tan\phi - 1)$
Biarez (1961)	$N_\gamma = 1.8(N_q - 1)\tan\phi$
Meyerhof (1963)	$N_\gamma = (N_q - 1)\tan 1.4\phi$
Booker (1969)	$N_\gamma = 0.1045e^{9.6\phi}$
Hansen (1970)	$N_\gamma = 1.5(N_q - 1)\tan\phi$
Vesic (1973)	$N_\gamma = 2(N_q + 1)\tan\phi$
Michalowski (1997)	$N_\gamma = e^{(0.66+5.1\tan\phi)}\tan\phi$
Hjiaj et al. (2005)	$N_\gamma = e^{(1/6(\pi+3\pi^2\tan\phi)}\ (\tan\phi)^{2\pi/5}$
Martin (2005)	$N_\gamma = (N_q - 1)\tan 1.32\phi$

In this study, shallow strip surface footing resting on sand soils were modelled with Geostudio 2012 software by changing internal friction angle (29°, 31°, 33°, 35°, 37°, 39° and 41°) were and foundation width (1, 1.25, 1.5, 1.75 and 2 m). Since there is no depth, no load inclination factor and no cohesion, bearing capacity equations are simplified to 0.5γBNγ. In this manner, from displacement load curve from software, 0.5γBNγ value is calculated. Therefore, N_γ value is obtained and compared to analytical methods.

2. Material and Method

2.1. Bearing Capacity Theories

There are numerous bearing capacity theories. Most widely used are [2,4,7].

2.1.1. Terzaghi Bearing Capacity Theory

[2] bearing capacity is the main theory of all those theories mentioned. According to this theory ultimate bearing capacity of shallow strip footing (q_u) can be calculated with the following equation

$$q_u = cN_c + qN_q + 0.5\gamma BN_\gamma \quad (1)$$

N_c, N_q, and N_γ : bearing capacity factors

B : foundation width

c : cohesion

γ : unit weight of soil

q : surcharge load (γD_f)

2.1.2. Meyerhof Bearing Capacity Theory

[4] modifies the bearing capacity equation of [2] by adding shape, depth and inclination factors and equation 2 is obtained.

$$q_u = cN_cF_{cs}F_{cd}F_{ci}+qN_qF_{qs}F_{qd}F_{qi}+0.5\gamma BN_\gamma F_{\gamma s}F_{\gamma d}F_{\gamma i} \quad (2)$$

2.1.3. Vesic Bearing Capacity Theory

[7] used the same capacity equation of [2] except by changing load inclination factors with compressibility factor.

$$q_u = cN_cF_{cs}F_{cd}F_{cc}+qN_qF_{qs}F_{qd}F_{qc}+0.5\gamma BN_\gamma F_{\gamma s}F_{\gamma d}F_{\gamma c} \quad (3)$$

In this study, since the cohesion (c) value is equal to zero and foundation rest on the surface (q = 0), first two terms of equation 1 is neglected. Furthermore, strip footing were used (B/L = 0), foundation depth (D_f = 0) and no inclination load conditions make all shape (F_{cs}, F_{qs} and $F_{\gamma s}$), depth (F_{cd}, F_{qd} and $F_{\gamma d}$), compressibility (F_{cc}, F_{qc} and $F_{\gamma c}$) and load inclination factors (F_{ci}, F_{qi}, and $F_{\gamma i}$), are equal to 1 for the equation 2. Therefore, there is only third term ($0.5\gamma BN_\gamma$) for all theories [2, 4, 7] remaining for the bearing capacity value.

2.2. Finite Element Method

In Finite element models were created with [19] program as shown in Figure 1. Problem boundaries depend on the width of the foundation. Boundaries are selected as 5B x 5B for vertical and horizontal sides. Half of the model were used since the symmetry condition exists. 8-noded quadrilateral mesh elements were used. Mesh densifications were applied areas close to foundation. Displacements were restricted both horizontally and vertically for the right and bottom side of the model while they were restricted only horizontally for the left side of the model Displacement velocity were used as 0.0025 mm per step.

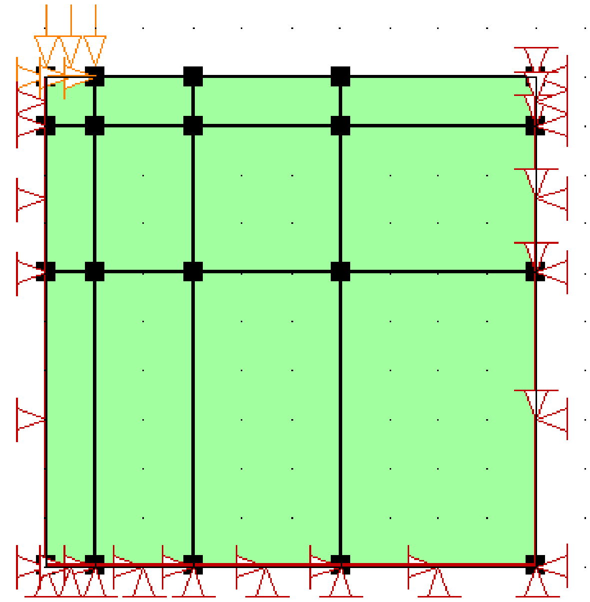

Fig. 1 Finite element model of the problem

Load−displacement of curve for the sample example (B=1m and ϕ = 29°) obtained from the finite element analysis is shown in Figure 2. This load values are transformed to stress values to evaluate the bearing capacity value for the numerical method.

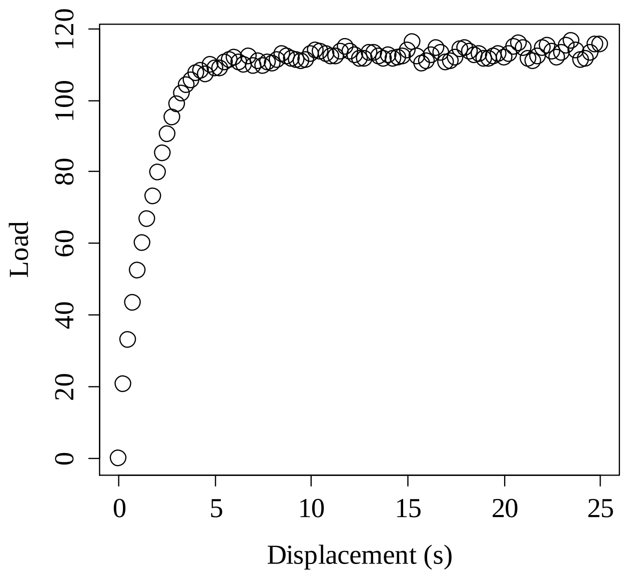

Fig.2 Load-displacement curve for B = 1m and ϕ = 29°

Shallow strip footing resting on cohesionless sandy soils are used. Geostudio 2012 program [19] were used for this purpose. Mohr-Coulomb material model was used. Modulus of elasticity (E), Poisson's ratio (ν), unit weight of soil (γ), internal friction angle (ϕ) and dilatation angle (ψ) used in the analyses are given in Table 2.

Table 2. Material parameters for the study (cohesion = 0)

γ kN/m³	ϕ °	ψ °	B (m)	E (kPa)	ν
16	29	0	1, 1.25, 1.5, 1.75, 2	15000	0.20
16.25	31	1	1, 1.25, 1.5, 1.75, 2	20000	0.24
16.5	33	3	1, 1.25, 1.5, 1.75, 2	25000	0.28
16.75	35	5	1, 1.25, 1.5, 1.75, 2	30000	0.32
17	37	7	1, 1.25, 1.5, 1.75, 2	35000	0.36
17.25	39	9	1, 1.25, 1.5, 1.75, 2	40000	0.40
17.5	41	11	1, 1.25, 1.5, 1.75, 2	45000	0.44

2.3. De Beer Method

Load-displacement curve from Fig 2 is transformed to stress-displacement curve as seen in Fig 3 which describes the [20] method. According to this method, linear portion of the elastic region (2) and linear portion (1) of the plastic regions are extended and intercepted as seen in Fig 3. The y-coodinate of this point is defined as the bearing capacity.

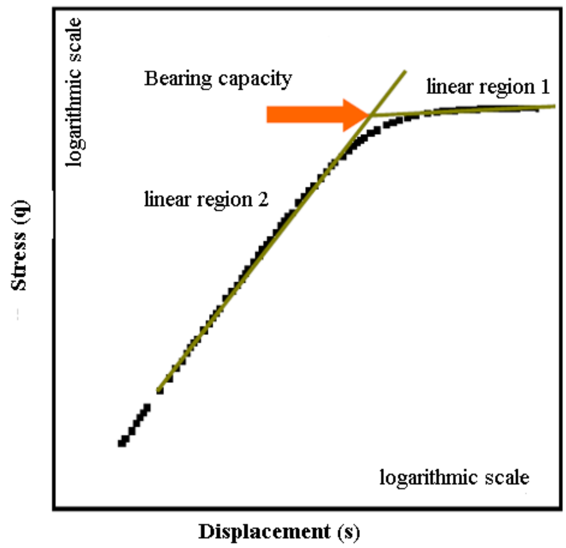

Fig. 3 De Beer method for bearing capacity [21]

3. Results and Discussion

3.1. Results

After applying finite element modelling, displacement arrows are developed as shown in Fig 4.

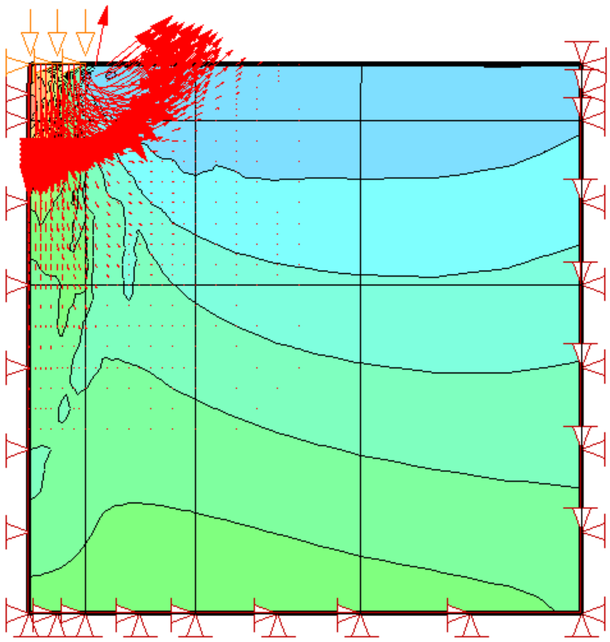

Fig. 4 Displacement arrows developed after analysis

After applying finite element modelling, displacement arro Arrows in Fig 4, indicates the movement of displacement. Arrows move from left to right since just half of the problem is modelled.ws are developed as shown in Fig 4.

Internal friction angle (ϕ) and bearing capacity factor (N_γ) relationships for B = 1, 1.25, 1.5, 1.75 and 2 are shown in Fig 5, 6, 7, 8 and 9 respectively. Values of bearing capacity factor (N_γ) increase exponentially with an increase in internal friction angle (ϕ). N_γ values of program were obtained by calculating the "qu/0.5γB".

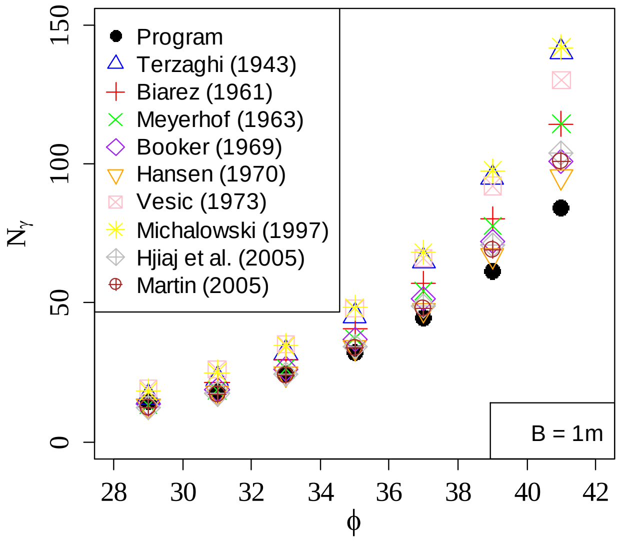

Fig. 5 ϕ-N_γ relationship for B = 1 m

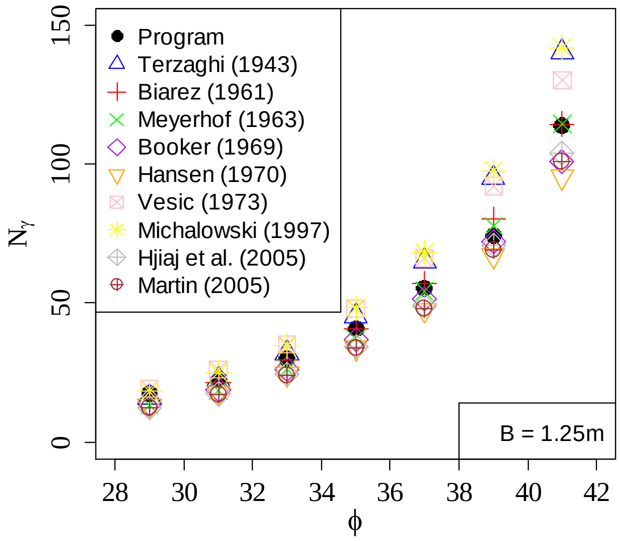

Fig. 6 ϕ-N_γ relationship for B = 1.25 m

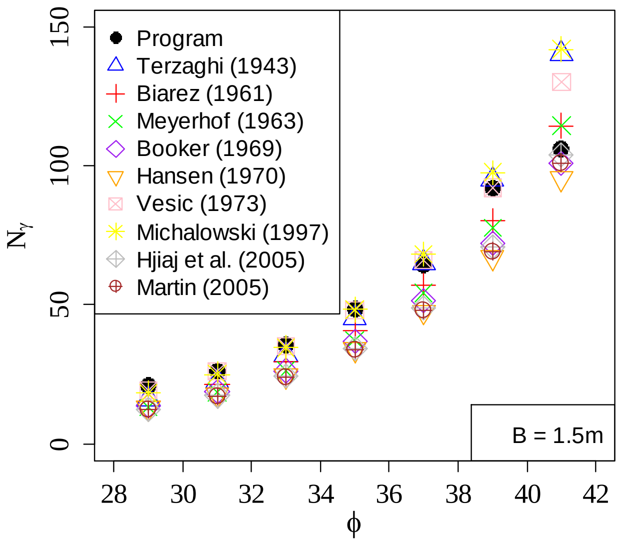

Fig. 7 ϕ-N_γ relationship for B = 1.5 m

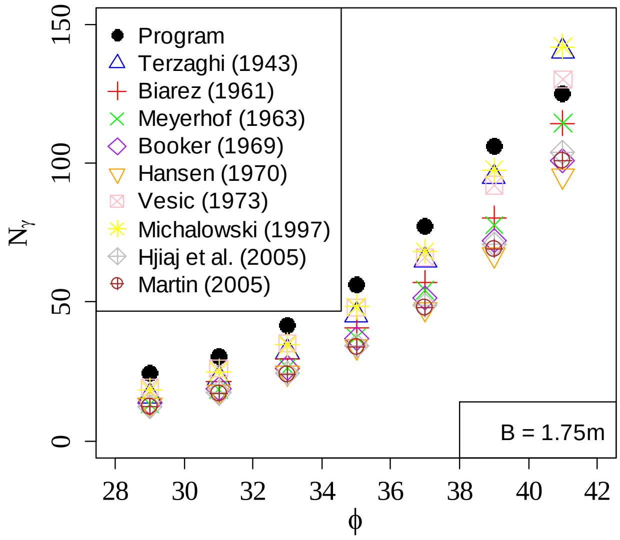

Fig. 8 ϕ-N_γ relationship for B = 1.75 m

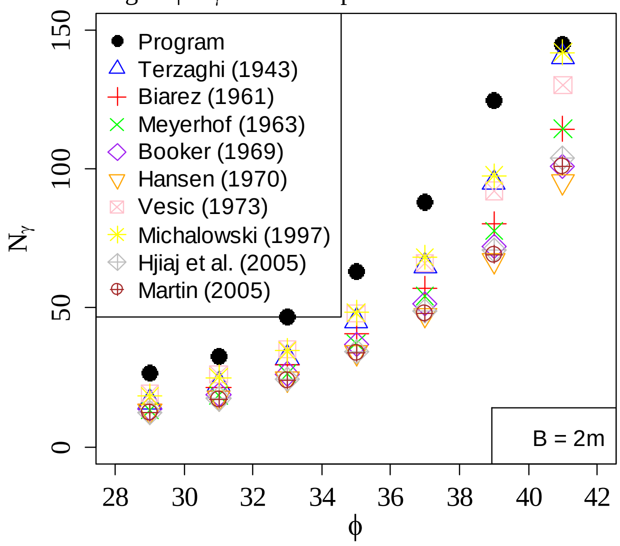

Fig. 9 ϕ-N_γ relationship for B = 2 m

According to Fig 5, N_γ values obtained from program seem to be the minimum compared to other theories. Maximum values were obtained from [2] and [8]. In the Fig 6, N_γ values of Geostudio program began to be somewhere in the middle of other theories for this foundation width (1.25 m). Minimum values of N_γ are obtained from [6] and [12]. In the Fig 7 and 8, N_γ values keep increasing trend line compared to other values. Maximum values of N_γ are the one obtained from the program as shown in Fig 9. [3] seems to be the average value for all foundation width.

3.2. Discussion

Bearing capacity factor (N_γ) for several researchers found different values since there are various equations created by them as seen in Table 1. Movement of displacement arrows are similar with the other studies [10,11,14,17,22]. N_γ values of [3] seems to be average of all analytical methods [2,4-8,11,12] and program for Fig 5, 6, 7, 8 and 9. N_γ values of finite element model enhances with an increase in foundation width also.

Fig 10. consist of the all results obtained from researchers and finite element model. Total of 98 values (7 different ϕ value * 9 researchers + 7 different ϕ x 5different foundation width). As a result, after drawing trend line for these points following equation is obtained.

$N_\gamma = 0.688\phi 2 - 40.376\phi + 608.7$ (3)

It should be noted that this equation is valid only for the rough strip footings having no depth, inclination or shape factors and internal friction angle between 29° and 41°.

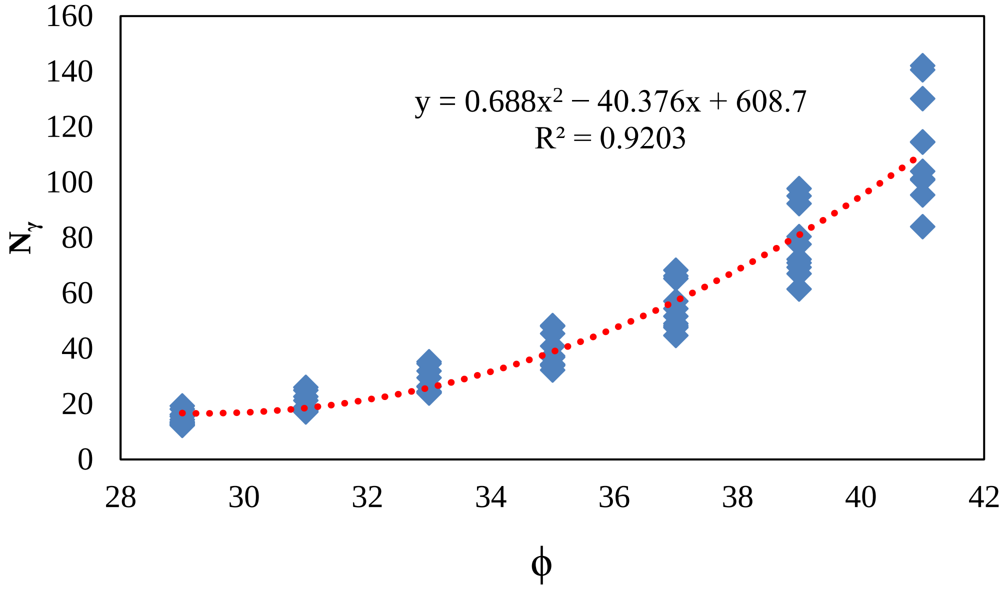

Fig. 10 ϕ-N_γ relationship for all foundation widths and methods together

4. Conclusions and Recommendations

Analytical methods and numerical method were used to evaluate N_γ value. Different foundation widths and friction angle values were chosen for this analysis. Following results are obtained.

1) [2] and [8] have maximum values of Nγ for all foundation widths except B = 2m.
2) N_γ values of software enhances with an increase in foundation width.
3) Differences of N_γ values for the whole methods increase sharply for higher internal friction angles.
4) Displacement arrows for the analyses are consistent with the bearing capacity theory.
5) Equation obtained from these analyses can be used for the soils having internal friction angle satisfies the $29° \leq \phi \leq 41°$.

References

[1] Frydman, S., & Burd, H. J. (1997). Numerical studies of bearing capacity factor. *Journal of Geotechnical and Geoenvironmental Engineering*, 123(1), 20–29.

[2] Terzaghi, K. V. (1943). *Theoretical soil mechanics*. John Wiley & Sons, Inc. New York.

[3] Biarez, J., Burel, M., & Wack, B. (1961). Contribution a l'etude de la Force Portante des Fondations. *Fifth International Conference on Soil Mechanics and Foundation Engineering*, Paris. 1, 603–609.

[4] Meyerhof, G. G. (1963). Some recent research on the bearing capacity of foundations. *Canadian Geotechnical Journal*, 1(1), 16–26.

[5] Booker, J. R. (1969). *Application of theories of plasticity for cohesive frictional soils*. Ph.D. Thesis, University of Sydney, Australia

[6] Hansen, J. B. (1970). *A revised and extended formula for bearing capacity*. Bulletin 28. Danish Geotechnical Institute. Copenhagen.

[7] Vesic, A. S. (1973). Analysis of ultimate loads of shallow foundations. *Journal of Soil Mechanics Foundation Engineering*, 99(1), 45-76.

[8] Michalowski, R. L. (1997). An estimate of the influence of the soil weight on bearing capacity using limit analysis. *Soils and Foundations*, 37(4), 57–64.

[9] Kumar, J. (2003). N_γ for rough strip footing using the method of characteristics. *Canadian Geotechnical Journal*, 40,669-674.

[10] Ukritchon, B., Whittle, A. J., & Klangvijit C. (2003). Calculations of bearing capacity factor N_γ using numerical limit analysis. *Journal of Geotechnical and Geoenvironmental Engineering*, 129 (6), 68-74.

[11] Hjiaj, M., Liyamin, A. V., & Sloan, S. W. (2005). Numerical limit analysis solutions for the bearing capacity factor N_γ. *International Journal of Solids and Structures*, 42, 1681-1704.

[12] Martin, C. M. (2005). Exact bearing capacity calculations using the method of characteristics. *Proceedings Eleventh International Conference of the International Association for Computer Methods and Advances in Geomechanic*, Turin, 4, 441-450.

[13] Jahanandish, M., Veiskarami, M., & Ghahramani, A.(2010). Effect of stress level on the bearing capacity factor, Nγ, by the ZEL method. *KSCE Journal of Civil Engineering*, 40(5), 709-723.

[14] Benmebarek, S., Remadna, M. S., Benmebarek, N., & Belounar, L. (2012). Numerical evaluation of the bearing

capacity factor N_γ of ring footings. *Computers and Geotechnics*, 44, 132-138.

[15] Mrunal, P., Mandal, J. N., & Devaikar, D. M. (2014). Computation of bearing capacity factor N_γ. *International Journal of Geotechnical Engineering*, 8(4), 372-382.

[16] Han, D., Xie, X., & Huang, L. (2016). The bearing capacity factor Nγ of strip footings on c−ϕ−γ soil using the method of characteristics. *SpringerPlus*, 5, 1-17.

[17] Soufi, G. R., Chenari, R. J., & Fard, M. K. (2019). Influence of random heterogeneity of the friction angle on the bearing capacity factor N_γ. *Georisk: Assesment and Management of Risk for Engineered Systems and Geohazards*, 14(1), 69-89.

[18] Das B., & Sivakugan, N. (2019). *Principles of Foundation Engineering* 9E SI ed., R. M. Osgood, Jr., Ed. Boston, USA: Cengage.

[19] Geostudio (2012), Sigma/W 2012 Module, Geo-Slope International, Canada.

[20] De Beer, E. E. (1970). Experimental determination on the shape factors and the bearing capacity of sand. *Geotechnics*, 2(4), 387-411. De Beer, E. E. (1970). Experimental determination on the shape factors and the bearing capacity of sand. *Geotechnics*, 2(4), 387-411.

[21] Elhakim A. F. (2005). *Evaluation of shallow foundation displacements using small-strain stiffness*. Ph.D. Thesis, Georgia Institute of Technology, USA.

[22] Dağlı, E., & Çapar, Ö. F. (2021). Evaluation of the bearing capacity of shallow strip foundations resting on sandy soils with analytical and numerical methods. *Celal Bayar Üniversitesi Fen Bilimleri Dergisi*, 17(1), 91-100.