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New approaches in choosing a suitable growth model: Mean Curvature and Arc Length Values

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Abstract

Logistic, Gompertz and Bertalanffy sigmoid growth models are widely used to study the growth dynamics of populations such as living plants, animals and bacteria. Appropriate model selection and parameter estimation are very important as these models will be used to make practical inferences. Because different growth models are modeled biologically, regardless of whether the parameters are definable or not. Applications that do not take into account parameter identifiability can lead to unreliable parameter estimates and misleading interpretations. Therefore, first the most suitable model should be determined and then the parameters should be defined. In this study, two new suitable model determination criteria such as mean curvature and arc length are proposed. For this, firstly, the definition of curvature was given. Then, the mean curvature and arc length values of the data belonging to two different species (kangal dog growth and eucalyptus plant growth) were calculated. For this purpose, a comparison was made with model selection criteria available in the literature such as coefficient of determination, error sum of squares and Akaike information criterion (AIC). It has been determined that the results obtained from the mean curvature and arc length values are in accordance with the existing criteria. In the two datasets, it was seen that the fit model ranking for both the existing criteria and the criteria we proposed was the same. For this reason, it is thought that the mean curvature and arc length values can be accepted as suitable model selection criteria.

Keywords: Mean curvature values; arc length values; model selection criteria; Akaike information criterion; growth model

En uygun model seçimi için yeni yaklaşımlar: Ortalama Eğrilik ve Yay Uzunluğu Değerleri

Öz

Lojistik, Gompertz ve Bertalanffy sigmoid büyüme modelleri, canlı bitki, hayvan ve bakteri gibi populasyonların büyüme dinamiklerini incelemek için yaygın olarak kullanılmaktadır. Bu modeller pratik çıkarımlar yapmak için kullanılacağından, uygun model seçimi ve parametre tahmini çok önemlidir. Çünkü farklı büyüme modelleri, parametrelerin tanımlanabilir olup olmadığı dikkate alınmadan biyolojik olarak modellenmiştir. Parametre tanımlanabilirliğini dikkate almayan uygulamalar, güvenilir olmayan parametre tahminlerine ve yanıltıcı yorumlara yol açabilir. Bu yüzden, öncelikle en uygun model belirlenmeli ve sonrasında parametreler tanımlanmalıdır. Bu çalışmada ortalama eğrilik ve yay uzunluğu gibi iki yeni uygun model belirleme kriteri önerilmiştir. Bunun için öncelikle eğriliğin tanımı verildi. Daha sonra iki farklı canlı türüne ait verilerin ortalama eğrilik ve yay uzunluğu değerleri hesaplandı. Bu amaçla belirleme katsayısı, hata kareler toplamı ve Akaike bilgi kriteri (AIC) gibi literatürde mevcut olan model seçim kriterleri ile karşılaştırma yapıldı. Ortalama eğrilik ve yay uzunluğu değerlerinden elde edilen sonuçların mevcut kriterlere uygun olduğu tespit edilmiştir. İki veri setinde, hem mevcut kriterler, hem de önerdiğimiz kriterler için uygun model sıralamasının aynı olduğu görüldü. Bu nedenle ortalama eğrilik ve yay uzunluğu değerlerinin uygun model seçim kriterleri olarak kabul edilebileceği düşünülmektedir.

Anahtar Kelimeler: Ortalama eğrilik değerleri; yay uzunluğu değerleri; model seçim kriteri; Akaike bilgi kriteri; büyüme modeli

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1. Introduction

In general terms, the curvature of a function is expressed as the rate of rotation of that curve. Since the tangent line shows the direction of the curve, we can say that the curvature is the rotational speed of the tangent line or velocity vector. Two revisions are necessary to this definition. First, the rate of rotation of a tangent line of a curve depends on the velocity of the tangent line along the curve. The second is; Curvature is a geometric property and should not change with movement. Therefore, curvature was defined as the absolute value of the rotational speed of the tangent line moving along the curve at one unit per second (Nutbourne et al., 1972).

If ϕ is the angle between the tangent line and the x-axis, then the curvature K is defined by

$$K=\left|\frac{d\phi}{ds}\right| \tag{1}$$

where s is arc length. By using the chain rule,

$$\frac{d\phi}{ds}\frac{ds}{dx}=\frac{d\phi}{dx} \tag{2}$$

and putting

$$v=\frac{ds}{dx} \tag{3}$$

we get

$$\left|\frac{d\phi}{ds}\right|=\frac{\left|\frac{d\phi}{dx}\right|}{v}. \tag{4}$$

Thus for getting the curvature, it suffices to find $\frac{d\phi}{dx}$ and v .

Clearly $s=\int v\,dx$. If the formula for arc length is remembered, we can find

$$v=\sqrt{1+\left(\frac{dy}{dx}\right)^2}. \tag{5}$$

The intuitive significance of v is that it is the speed at which a point travels along the curve when its x-coordinate increases at a rate of one unit/second. So the formula $s=\int v\,dx$ says that the speed is integrated to compute the distance.

Since ϕ is the angle between the direction in which the point on the curve is moving and the direction of the x-axis (i. e. horizontal), it can be seen that $v = \sec\phi$.

Since ($\tan\phi=\frac{dy}{dx}$) is the slope of the curve, we get

$$v^2 = sec^2\phi = 1 + tan^2\phi = 1 + (\frac{dy}{dx})^2 , \quad (6)$$

which is the required formula for ν anticipated above.(Note that by the definition ϕ is an acute angle, so sec ϕ ≥ 0)

The formula for the curvature of the graph of a function in the plane is now easy to obtain. Since ϕ is the angle of the tangent line, it is known that tanϕ is the slope of the curve at a given point, i. e.

$$\tan \phi(x) = \frac{dy}{dx} \quad (7)$$

By differentiating with respect to x the following equation yields:

$$sec^2\phi \frac{d\phi}{dx} = \frac{d^2y}{dx^2} , \quad (8)$$

and so we get:

$$\frac{d\phi}{dx} = \frac{y''}{sec^2\phi} = \frac{y''}{v^2} = \frac{y''}{1+(\frac{dy}{dx})^2} , \quad (9)$$

and then

$$K=\left|\frac{d\phi}{ds}\right| = \frac{|y''|}{v^2}\frac{1}{v} = \frac{|y''|}{v^3} = \frac{|y''|}{[1+(y')^2]^{\frac{3}{2}}} \quad \text{(Curvature, E. L. Lady).} \quad (10)$$

In order to test the suitability of the proposed model in this study, the weights of the Kangal Dogs for the first eight weeks were examined. Weight gain is low in the early stages of growth. Then it gradually rises, reaches its highest level and decreases as the adult age approaches. The fastest growth in animals is at the young age, and the growth rate decreases as they approach physical maturity. Therefore, the growth curve first rises steeply, then gradually flattens and finally stops. Therefore, the weights of dogs (Kangal Dogs) in the growth phase are similar to sigmoidal curves (Çoban et al., 2011). In one study, a linear model was used to determine the growth and some body size characteristics of Kangal Dogs. In another study, the variation of body weights and heights of dogs of different sizes over time was investigated using a logistic model (Posada et al., 2014). In another study, growth differences and relationships between healthy dogs and diseased dogs were modeled (Salt et al., 2020). When the current literature is examined, the growth curves are examined by classical methods (gompertz, logistics, bertalanffy and etc.) (Senol et al., 2020). In the mentioned studies, the coefficient of determination (R^2) and/or AIC , Bayesian information criterion were used as the performance criteria of the models. Different types of modeling, such as sigmoidal growth curves, have been used quite frequently in previous studies in the literature (Kara and Şenol 2022; Bianco et al., 2022, Şenol et al., 2022, Kara, 2019). However, in no previous study, the mean curves and arc lengths of the models were used as model performance criteria.

Accordingly, our hypothesis in the study is that it can be selected as the most suitable model with the lowest mean curvature and arc length values.

In this study firstly; the existence of the curvature formula was explained. Then some applications were made with numerical data. Time dependent curvature values were calculated. Therefore, total curvature and mean curvature values are calculated by taking the absolute values. In this study Gompertz (Winsor, 1932), Logistic (Ricker, 1979) and Bertalanffy (Von Bertalanffy, 1957) growth models were used for comparing the results.

2. Material and Methods

Two different data sets were used in this study. The first one is the weight of eight weeks for female and male Kangal dogs. The second one is the length of the eucalyptus plant. Calculations were made with the MAPLE package program. The vşalues are given in tables and then interpreted. In this study, the data from the male and female Kangal dogs were used for the growth model in Table 1. The data set was taken from the study of Çoban et al. (2011).

Table 1. Observed live weights(kg) of the Kangal dogs according to the gender

Weeks	1	2	3	4	5	6	7	8
Male	0.803	1.269	2.145	3.046	3.700	4.408	5.285	5.962
Female	0.813	1.275	2.275	2.933	3.600	4.308	5.092	5.975

In this study, the data taken from the tree, E. Camaldulensis Dehn. were used for the growth model in Table 2. The data set was taken from the study of Yıldızbakan (2005).

Table 2. The height growth value of the trees (E. Camaldulensis Dehn) according to year

Planting Age (year)	0	1	2	3	4	5	6	7	8	9
Height Growth (m)	0.41	3.23	7.45	11.41	14.83	18.11	18.95	19.69	21.50	23.40

To determine a compatible model, the following statistical indicators were determined and compared: the coefficient of determination (R^2), error sum of squares (SSE) (Draper and Smith, 2014), the second-order AIC test (Akaike, 1974).

$$\mathrm{SSE} = \sum_{k=1}^{N}(y_i - \widehat{y_i})^2 \tag{11}$$

$$\mathrm{AIC} = \begin{cases} N\ln\left(\frac{RSS}{N}\right) + 2K, & when\ \frac{N}{K} \geq 40 \\ N\ln\left(\frac{RSS}{N}\right) + 2K + \frac{2K(K+1)}{N-K-1}, & when\ \frac{N}{K} < 40 \end{cases} \tag{12}$$

where, y_i : measured values, $\hat{y}_i$: estimated values, N : number of data point, K: number of model parameters.

3. Results and Discussion

In this study, in order to test the performance of the newly proposed model selection criteria, the growth amounts of the eucalyptus plant were analyzed in addition to the 8-week growth of male and female kangal dogs. By using the Tables 1 and 2 of these two sets of data, firstly, the parameters of Gompertz, Logistic and Bertalanffy growth models are calculated in Table 3 and Table 6, respectively and then the values of time dependent curvature were calculated in Table 4 and Table 7, respectively.

For two data sets, the values of the total curvature, mean curvature, arc length, error sum of squares (SSE), coefficient of determination (R^2) and AIC are calculated in Table 5 and Table 8 respectively. To make a comparison; error sum of squares, coefficient of determination and AIC (Akaike, 1970; Ucal, 2006) are used as known model selection criteria. All calculation results made are shown in the tables. Tables 3,4 and 5 were calculated according to the values in Table 1. In addition, Table 3 is taken from Oda's master thesis (Oda, 2017).

Table 3. Model parameters calculated according to the gender

	Models	**Equations**	**a**	**b**	**c**
Male	Gompertz	$y=ae^{-e^{(b-ct)}}$	8.659	1.098	0.257
	Logistic	$y=\frac{a}{1+e^{(b-ct)}}$	7.008	2.326	0.495
	Bertalanffy	$y=a[1-be^{-ct}]^3$	10.279	0.681	0.175
Female	Gompertz	$y=ae^{-e^{(b-ct)}}$	9.700	1.104	0.224
	Logistic	$y=\frac{a}{1+e^{(b-ct)}}$	7.373	2.343	0.460
	Bertalanffy	$y=a[1-be^{-ct}]^3$	12.296	0.684	0.143

Growth curves of live weights of male and female Kangal dogs by the models used are given in Figure 1 and 2.

The curvature values calculated according to live weights are given in Table 4. In both genders, it is seen that Bertalanffy model has the lowest curvature values (**bold font**) compared to the other models used in this study.

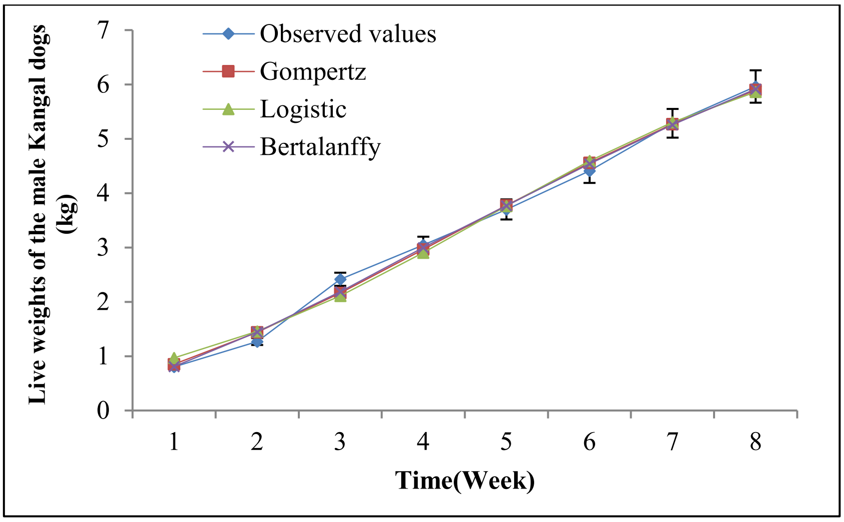

Figure 1. Growth curves of male Kangal dogs by the models used

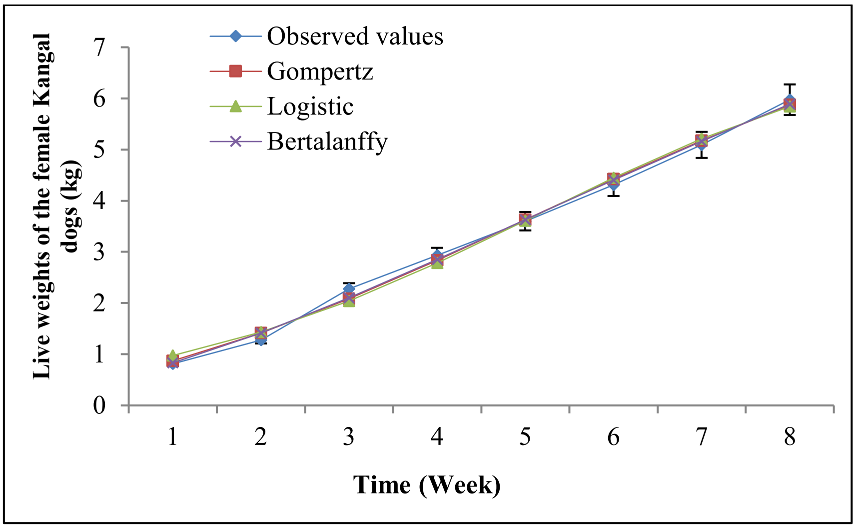

Figure 2. Growth curves of female Kangal dogs by the models used

Table 4. Weekly calculated curvature values of the models according to the gender

	Gompertz		**Logistic**		**Bertalanffy**	
Weeks	**Male**	**Female**	**Male**	**Female**	**Male**	**Female**
1	0.122	0.110	0.117	0.107	**0.109**	**0.099**
2	0.078	0.047	0.076	0.079	**0.023**	**0.033**
3	0.038	0.047	0.076	0.079	**0.023**	**0.033**
4	0.007	0.020	0.032	0.043	**0.001**	**0.014**
5	0.017	0.001	0.014	0.004	**0.014**	**0.000**
6	0.035	0.018	0.059	0.036	**0.025**	**0.010**
7	0.050	0.032	0.097	0.073	**0.034**	**0.018**
8	0.060	0.043	0.117	0.100	**0.040**	**0.024**

By using the first data set, the values of the total curvature, mean curvature, arc length, error sum of squares, coefficient of determination and AIC of the models according to genders are shown in Table 5.

Table 5. The values of total curvature, mean curvature, arc length, error sum of squares, coefficient of determination and AIC of the models according to the gender

Models	**Gompertz**		**Logistic**		**Bertalanffy**	
	Male	**Female**	**Male**	**Female**	**Male**	**Female**
Total Curvature Values	0.407	0.350	0.620	0.545	**0.303**	**0.259**
Mean Curvature Values	0.051	0.044	0.078	0.068	**0.038**	**0.032**
Arc Length Values	8.644	8.616	8.578	8.560	**8.248**	**8.234**
Error Sum of Squares	0.135	0.101	0.224	0.181	**0.110**	**0.080**
Coefficient of Determination	0.994	0.996	0.990	0.992	**0.995**	**0.997**
AIC	-11.322	-13.643	-7.271	-8.976	**-12.960**	**-15.508**

It can be seen that for the values of the total curvature, mean curvature, arc length, error sum of squares and AIC, Bertalanffy model has the lowest values (**bold font**) and also for the values of coefficient of determination, Bertalanffy model has the highest values (**bold font**). For that reason, we can say that for the data from Table 1 according to the criteria above, the best appropriate model is Bertalanffy model compared to the other models used in this study.

Table 2 shows the growth values of the eucalyptus plant as a whole. The model parameters calculated according to these values are given in Table 6.

Table 6. Model parameters calculated

Models	**Equations**	**a**	**b**	**c**
Gompertz	$y=ae^{-e^{(b-ct)}}$	23.071	1.137	0.488
Logistic	$y=\frac{a}{1+e^{(b-ct)}}$	21.887	2.369	0.781
Bertalanffy	$y=a[1-be^{-ct}]^3$	23.897	0.711	0.392

The time dependent curvature values of the models are shown in Table 7.

Table 7. Yearly curvature values calculated

Year	**Gompertz**	**Logistic**	**Bertalanffy**
0	0.254	0.186	0.348
1	0.038	0.070	0.022
2	0.005	0.020	0.000
3	0.008	0.000	0.011
4	0.023	0.018	0.024
5	0.051	0.065	0.045
6	0.098	0.177	0.077
7	0.152	0.275	0.114
8	0.176	0.216	0.139
9	0.154	0.117	0.140

Tables 6,7, 8 are calculated according to the values in Table 2. The curvature values calculated according to height lengths are given in Table 7. For the second data set, the values of total curvature, mean curvature, arc length, error sum of squares, R^2 and AIC of the models used in this study are shown in Table 8.

Table 8. The values of total curvature, mean curvature, arc length, error sum of squares, coefficient of determination and AIC of the models

Models	Gompertz	Logistic	Bertalanffy
Total Curvature Values	0.959	1.144	**0.920**
Mean Curvature Values	0.096	0.114	**0.092**
Arc Length Values	23.610	22.710	**21.835**
Error Sum of Squares	3.919	8.713	**2.876**
Coefficient of Determination	0.993	0.984	**0.995**
AIC	6.633	14.622	**3.538**

Bertalanffy model was the lowest values of total curvature, mean curvature, arc lengths, error sum of squares and AIC. It was seen that Bertalanffy model had the lowest values (bold font) for total curvature, mean curvature, arc length, error sum of squares and AIC values. In addition, when the R^2 values of the model evaluation criteria are examined, it is seen that the most suitable model is Bertalanffy. For that reason, we can say that for the data from Table 2 according to the criteria above, the best appropriate model is Bertalanffy model compared to the other models used in this study. There are limited applications of the modified Bertalanffy model, which is an alternative sigmoidal model, in the literature. Şenol et al.(2020), modeled the bacterial growth curve with Logistics and Bertalanffy equations in a study he conducted, and mostly the R^2 values of Bertalanffy model were higher than the Logistic model. Therefore, despite limited applications, Bertalanffy model can be used as an alternative and can perform better than the more commonly used Logistic and Gompertz models, as in this study (Oda et al., 2016; Şenol et al., 2020).

There are some studies on curvature and arc length values in the literature (Nutbourne et al., 1972; Castro et al., 2016). Of course, the known model selected criteria such as error sum of squares, coefficient of determination and AIC are used in the literature but in this study the proposed model selected criteria total curvature, mean curvature and arc length are also used with some known selected criteria. It is observed that the proposed model selected criteria compliances with the known selected criteria in this study.

The researchers argue that the use of curvature and arc length in finding the best model fit does indeed add a very interesting idea to the world of the researches related to growth. As known that, correlation is a statistical method used to determine whether there is a relationship among numerical measurements and the direction and severity of this relationship if the relationship is present. There are two different tests to determine the type of distribution. One of them is "Kolmogorov-Smirnov" and the other is "Shapiro-Wilk" (Khatun, 2021). The "Shapiro-Wilk" test is more preferred and more used (Hernandez, 2021). In this study "Shapiro-Wilk" test was applied and since "Sig." values are greater than 0.05, it can be said that be a normal distribution of the data.

After estimating the growth of Kangal dogs by gender with sigmoidal models, the statistical difference between the mean values of the predicted values was evaluated with the one-sample T test. First of all, the significance level of the data was determined as 95% (Sig.=0.05), and its conformity to the normal distribution was tested. Kolmogorov-Smirnov and Shapiro-Wilk test results are given in Table 9.

Table 9. Test of normality between models by gender

	Models	Kolmogorov-Smirnov			Shapiro-Wilk		
		Statistic	Df	Sig.	Statistic	Df	Sig.
Male	Gompertz	0.122	8	0.200	0.961	8	**0.820**
	Logistic	0.134	8	0.200	0.950	8	**0.712**
	Bertalanffy	0.117	8	0.200	0.966	8	**0.861**
Female	Gompertz	0.124	8	0.,200	0.963	8	**0.835**
	Logistic	0.134	8	0.200	0.953	8	**0.745**
	Bertalanffy	0.119	8	0.200	0.,966	8	**0.869**

Since the significance levels obtained in the normality tests are more than 0.05, the growth values are in accordance with the normal distribution. A one-sample T test can be applied. The results obtained by determining the significance level as 95% (Sig.=0.05) for the one-sample T test are shown in Table 10.

Table 10: One-Sample T test results

	Models	Test Value=0					
		Statistic	Df	Sig.	Mean Difference	95% Confidence interval of the difference	
						Lower	Upper
Male	Gompertz	5.082	7	0.001	3.330	1.781	4.879
	Logistic	5,131	7	0.001	3.336	1,799	4.873
	Bertalanffy	5.070	7	0.001	3.328	1.776	4.881
	Gompertz	5.167	7	0.001	3.288	1.783	4.792

Female	Logistic	5.219	7	0.001	3.293	1.801	4.786
	Bertalanffy	5.152	7	0.001	3.286	1.778	4.794

According to the significance results in Table 10, the difference between the mean values of the prediction values is statistically significant.

As a result, it is shown that the appropriate model for both genders of data is the Bertalanffy model according to known the model selection criteria. The same results were obtained according to the mean curvature and arc length values. In this study, it is seen that the model with the lower mean curvature and arc length values is suitable model. According to this, it can be judged that total curvature, mean curvature and arc length values can be used as the appropriate model selection criterion.

4. Conclusion

In this study, three different sigmoidal growth models, which are frequently found in scientific studies, have been used. These models have been applied to two different data sets. It has been determined which model is suitable according to the known model selection criteria. Then the mean curvature and arc lengths were calculated. The results were compared to known model selection criteria. It is shown that the results are compatible with each other.

The proposed new model selection criteria total curvature, mean curvature, arc length can be used by applying to animal and plant growth data. It is also recommended to be used in studies in the fields of economics, health, medicine and engineering. Since mean curvature and arc length have not been used as model selection criteria before, we consider them to be a very interesting addition to the literature in their own right.

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Authors' Contributions

Volkan Oda: Experimental study, data analysis and writing.

Mehmet Korkmaz: Experimental study and data analysis.

Halil Şenol: Literature research and writing.

Conflict of Interest Statement

There is no conflict of interest between the authors.

Statement of Research and Publication Ethics

The study is in accordance with research and publication ethics.

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