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Research Article

Geometric thinking levels among college of education students

Nader Hilf[1], and Muhammad Abu-Naja[2]

Kaye Academic College of Education, Israel

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Abstract

Geometry is a key area of math. Reviewing the curriculum of primary and secondary school indicates that geometry is one of the major academic subjects, and it is consider one of the most difficult areas of mathematics to pupils. Quite a few studies conducted in recent decades reported the difficulties encountered by pupils that learning geometry. One of the main reasons for these difficulties is the gap between the level of teaching and learning abilities to the level of pupils understanding. The pupils are low-leveled geometric thinking, while the teachers are trying to provide them their high-leveled knowledge. Students that received in the mathematics department at academic college specialize elementary and junior high School curriculums are committed to studying various courses in geometry. Our experience at college of education, meet us with students that have difficulty at learning geometry. In order to make teaching more effective and efficient, we conducted a study that examining the level of geometric thinking of the students who want to be math teachers and come to learn in college of education. To this end, a questionnaire was comprised of 15 questions that examine the first three levels of geometric thinking by Van Hiele theory. The questionnaire was given to students who specialize in mathematics program primary and secondary school (N=84). The conclusion obtained from the study is that a significant proportion of the students received in the mathematics department at academic college control only at the lowest level. In order to qualify students to the third level, at least, we need to teach them geometric during the first semester of learning.

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Introduction

Quite a few studies conducted in recent decades reported the difficulties encountered by students in learning geometry (Senk, 1984; Usiskin, 1982). One of the main reasons for these difficulties is the gap between the level of teaching and learning abilities to the level of pupils understanding. The pupils are low-level geometric thinking, while the teachers are trying to provide them the high-level of knowledge of their own (Patkin, 1994). The teachers usually teach the higher-level that is not appropriate level of the pupil's abilities and understanding.

Many of the research on geometric thinking focused more on theory of Van Hiele (see Section 2.2.1). A series of studies on the subject were shown that, there are difficulties in learning geometry in Primary school for young pupils (Vinner & Hershkowitz, 1983). These difficulties are reflected in the low geometric thinking of pupils on the basis of the theory of Van Hiele. Usiskin (1982) found that the majority of high school pupils and students at college of education do not control all the levels of thinking of Van Hiele, but until the third level (Ordering), and that is even after learning geometry. Burger and Shaughnessy (1986) found that, a year after completing the study of geometry,

[1] Corresponding author, Dr., Kaye Academic College of Education, Israel. e-Mail: naderh1973@gmail.com ORCID: 0000-0001-5897-0075
[2] Mathematics education at Kaye Academic College of Education, Israel. E-mail: muhamadnja@gmail.com

pupils and students could withdraw from their Van Hiele level. Hershkovitz (1991; 1992) discusses the role played by visualization at acquisition of geometric concepts according to the Van Hiele theory and Piaget's theory.

Tepper (1986) founds that, the use of computer learner integrate effectively, sufficiently the means of stimulation such as color, sound, and animation, promotes the pupils greatly about geometric thinking levels. Patkin (1994) found that, over the time the level of thinking of the pupils who studied with computer-aided (by illustrating the visually sides) is higher than those who studied using worksheets.

Hershkowitz and Vinner (1984) reported about teachers in eighth grades who have difficulty understanding the geometric concepts as much as their pupils do. In another study (Hershkovitz, 1987), that was made about the perception of simple geometric concepts in three different groups: pupils in fifth grade to eighth grade, student in college of education and seminars, and teachers in primary school, were found similar definitions of geometric shapes in the three populations. Barbash (2003) argues that the lack of theoretical knowledge based among primary school teachers is one of the problems in teaching geometry. If one of the goals of teaching it is creating rich mental structures, such base is needed. She (ibid.) adds that enabled the construction of theoretical knowledge relevant for future teachers, and enables a comprehensive view of the subject. Euclidean geometry theoretical mathematical fields of knowledge can be a great didactic, if building the appropriate course by trying to achieve this goal. As part of the discussions on the role of knowledge-disciplinary training for teachers to teach math in primary school, emphasizes, Barbash (2003) that, creating structures mental wealthy is at least one of the goals of mathematics teaching and studying it in all age groups, it is impossible to be satisfied with an empirical-instrumental approach teaching geometry, even in primary school. Since that is, a developmental continuum of learning geometry that begins to emerge steps of its deductive structure. Preparation of these steps involves proper mental ripening of pupils, which requires teachers to be trained to be able to provide them that.

This brings us to the next question (Barabash, 2003): "What is the desired level of geometric knowledge among future teachers of mathematics in primary school?" When it comes to theoretical knowledge in the field of Euclidean geometry, there is no intention of teaching the entire system of axioms and theorems. This is for two main reasons: one is the student population that comes to be math teachers, is not ripe for studying such a system, at least not to the knowledge base upon, which to construct the didactics of teaching geometry; The second reason is familiarity with a number of axioms and sentences generally does not guarantee quality teaching and quality of knowledge.

Vinitzky and Reis (2003), also describe a study of the perception of concepts in geometry among teachers, that in the teaching of courses for student at college of education who specialize in teaching mathematics in primary school, had difficulty resulting from prior knowledge content related to the curriculum of the primary school. Main conclusion from the results of this study was that the students have only a partial view of the scope and space concepts.

David (2007) argues that, the mathematical knowledge that we want to instill in pupils is based on concepts, definitions, axioms, and sentences. According to her argues, learner achieves the concepts through examples or definitions, he builds himself the concept of image that usually relies on a number of typical examples, using different representations and connects between the concept and other concepts.

Vinner (1991) referred to the distinction between image and concept definition. He describes the definition of the concept as a formal representation of the concept as it appears in the definition. Image representation concept is in the learner mind as a typical example, visual representation, collection of attributes, relationships with other concepts, associations.

We hypothesize that many difficulties which student encountered in was as a result of the level of geometric thinking which they have. Therefore, this background is a good base to carry out this study and analysis of results.

Theory of Van Hiele – levels of Thinking

From Rise, Van Dormoln-Brahmi and Patkin (1997): "To do math, Van Hiele theory and teaching geometry": a pair of Dutch mathematicians – Dina and Pierre Van Hiele, developed the theory of Van Hiele. The theory attempted to explain the fact that many students have difficulty in cognitive processes the highest order, especially when they have to deal with the provision of evidence. According to this theory, the development of thinking in mathematics, especially geometry, arrange on hierarchical spindle of five levels.

Level 1 – Recognition

At this level, the student can learn a set of geometric shapes. He can recognize and distinguish between different forms. The shape is perceived as a whole (no attention to its components) as it seems. The student's reasons for acting at this level rely on the classification of the forms by the general shape. At this point, the student does not yet know the features of the same geometric shape. If the student is asked why he reads the image rectangle, he might answer: "because it looks like a rectangle. It is similar to the window or door" (the use of visual features).

Level 2 – Analysis

At this level, the student can identify and analyze the characteristics of forms. The student knows and is familiar with the properties of geometric shapes he sees, but he does not know and understand each feature separately, does not know the relationship between various features, and cannot explain how one derives from the other feature. That is, he still does not know and does not understand the relationship between features. The arguments of the children at this level rely on the analysis characteristics of the geometric shape. If a student is asked why the picture is a rectangle, he can say: "opposite sides parallel, opposite sides equal, it has four right angles".

Level 3 – Ordering

The student understands the logical arrangement of shapes, the relationship between shapes and their properties, and the importance of precise definitions. The student still does not grasp the significance of deductive structure as a whole. He is able to understand how one trait arises from the other, but cannot prove the properties of geometric shapes. Example: The student will understand why a square is a rectangle, but may not be able to explain why the diagonals of a rectangle are equal.

Level 4 – Deduction

The student understands the significance of deduction as a means to develop a geometric theory, he understands the role of basic terms, definitions, axioms, theorems, and proofs (link in the chain of deductive structure). At this point, he can use discounts to prove theorems, and understand the meaning of necessary and sufficient conditions. A student at this level can give reasons and explanations proof steps. However, the student still does not understand the importance of accuracy. He does not understand the formal aspect of quantifiers. At this level, for example, a student can use the trials to prove theorems overlap the rectangle.

Level 5 – Rigor

The student understands the importance of accuracy. When dealing with different structures, he is able to perform abstract deduction, while he understood the formal shift of deduction. This level can exploring the consequences of replacing the system of axioms second. He knows and can compare different strategies of proof. He can "discover" new law and methods of proof, and can think about the problem of identifying a broader context, in which a sentence may be applicable. At this level a student understands, for example, how the parallel postulate (Euclidean) is related to the existence of a rectangle, and Non-Euclidean geometry there are other axioms, and therefore no bricks is exists".

Methodology

This quantitative study was conducted to examine geometric reasoning levels among the students from the first year in mathematics department at academic college of education in Israel. In this section of the paper, the participants, instruments, procedures and data analysis of the study were explained.

Participants:

Participants of the study were all of the students from the first year in mathematics department at one selected academic college who learn in the first semester of the academic year. 9.5% of the sample were male (N=8) and 90.5% female (N=76). Moreover the sample consisted from two nationalities; 41.7% were Jewish (N=35) and 58.3% were Arabs (N=49). Table 1 shows the distributions of the subjects according to study's variables.

Table 1. The Distributions of the Subjects according to Study's Variables

z	The categories of the variables	Frequencies	Percentages
Educational Path	Primary	39	46.4
	Above Primary	45	53.6
Begrut Mathematics	Five Units	18	21.4
	Four Units	41	48.8
	Three Units	25	29.8
Nationality	Jewish	35	41.7
	Arabs	49	58.3
Gender	Male	8	9.5
	Female	76	90.5
Total		84	100

Instruments and Procedures:

The scale was administered to the students from the first year in mathematics department at one selected academic college. The instrument for the data collection was a test that developed by the researcher based on Van Hiele levels of geometric thought (Usiskin, 1982). The Van Hiele test designed as part of the Cognitive Development and Achievement in Secondary School Geometry project (ibid.), to test the ability of Van Hiele theory to describe and predict performance in geometry. The test has been widely used for both diagnostic and research purposes to test subjects of various ages (Usiskin & Senk, 1990).

The test consists of fifteen multiple-choice questions. The instrument was divided into three groups each of which contains five questions. Each group of five questions corresponds to a Van Hiele level. Scoring was done according to the following criteria:

- A Van Hiele level was considered attained if either "3 out of 5" or "4 out of 5" questions are answered correctly (Usiskin, 1982, p. 24).
- If a participant met the criterion for passing each level up to and including level N and failed to meet the criterion for all levels above, then the participant was assigned to level N.
- If a participant passed a higher level (N+1), but failed to pass the preceding lower level (N), this participant would not be assigned Van Hiele level (N+1). This participant would be assigned level according to rule 2 (Usiskin, 1982, pp. 22-26).

Although the test was originally administered as a paper-and-pencil test, participants were not allowed to draw or write to aid their thinking process while answering questions. In this study participants were asked to answer questions by selecting a multiple choice response; however, just as in the original test, participants were not allowed to do any writing or drawing to aid their thinking process.

Data Analysis

Means and standard divisions of upper 27% (N=22) and lower 27% scores and P value and t-tests between items' means of upper 27% and lower 27% points in item analysis of the scale were calculated in order to validity of the test items. Table 2 presents means, standard divisions, P value and t-tests between items' means of upper 27% and lower 27% points in item analysis of the test. As seen in table 2, the t-test results showed significant differences between each item's means of upper 27% and lower 27% points. According to this result, all items in the test is appropriate to measure students' geometric reasoning.

Table 2. Students' Geometric Reasoning

Item No	Upper		Lower		T value	P value
	Means	SD	Means	SD		
.1	0.91	0.29	0.61	0.49	2.34	0.024
.2	0.82	0.39	0.25	0.44	4.38	0.000
.3	0.86	0.35	0.09	0.29	7.91	0.001
.4	0.95	0.21	0.54	0.50	3.47	0.002
.5	0.68	0.47	0.22	0.42	3.32	0.000
.6	1.00	0.00	0.36	0.49	6.06	0.000
.7	1.00	0.00	0.31	0.47	6.70	0.000
.8	0.91	0.29	0.45	0.50	3.62	0.001
.9	1.00	0.00	0.50	0.51	4.58	0.000
.10	0.86	0.35	0.36	0.49	3.87	0.000
.11	0.91	0.29	0.61	0.49	2.34	0.024
.12	0.95	0.21	0.20	0.41	7.57	0.000
.13	0.54	0.50	0.10	0.30	3.38	0.002
.14	0.82	0.39	0.25	0.44	4.38	0.000
.15	0.68	0.47	0.25	0.44	3.02	0.004

Moreover, the researcher calculated the test reliability using the Kuder-Richardson-20 coefficient to determine internal consistency, which was 0.75.

Results

In which level of the geometric reasoning the students in the mathematics department at academic college are categorized?

To answer this question, the researcher computed the frequencies and percentages for the three levels, then testing the differences among these levels using χ^2-test to discover how the students are distributed in these levels. As seen in table 3, about 54% of students are assigned in level 3, about 23% are assigned in level 2, and about 24% are assigned in level 1. The χ^2-test results showed significant differences among these levels in benefit to level 3.

Table 3. Frequencies and Percentages for Geometric Reasoning Levels and χ^2 Value

The Levels	Frequency	Percentage	χ^2 Value	df	p
Level 1	20	23.8	.15.50	2	0.000
Level 2	19	22.6			
Level 3	45	53.6			
Level 4	84	100			

Is there a significant association between the educational paths and geometric reasoning levels?

To answer this question, cross-tabulation and chi-square test were calculated to investigate if there was a significant association between educational path and geometric reasoning levels. As seen in table 4 the χ^2-test results showed insignificant association between educational path and geometric reasoning levels. Moreover to compute correlation coefficient between educational path and geometric reasoning levels, the researcher used Spearman test (r=0.09).

Table 4. Frequencies and Percentages according to Educational Paths and Geometric Reasoning Levels and χ^2 Value

		Educational Paths		Total	χ^2 Value	df	P Value
		Primary	secondary				
The levels	Level 1	12 (14.3%)	8 (9.5%)	20 (23.8%)	2.25	2	0.324
	Level 2	7 (8.3%)	12 (14.3%)	19 (22.6%)			
	Level 3	20 (23.8%)	25 (29.8%)	45 (53.6%)	**Correlation coefficient**		**P Value**
Total		39 (46.4%)	45 (53.6%)	84 (100%)	0.09		0.423

Is there a significant association between the Bagrut mathematics (number of units) and geometric reasoning levels?

To answer this question, cross-tabulation and chi-square test were calculated to investigate if there was a significant association between Bagrut mathematics (number of units) and geometric reasoning levels. As seen in table 5 the χ^2-test results showed significant association between Bagrut mathematics (number of units) and geometric reasoning levels, which means the students who studied more than three units were likely to be categorized in the third level of geometric reasoning. Moreover to compute correlation coefficient between Bagrut mathematics (number of units) and geometric reasoning levels, the researcher used Spearman test (r=0.37),

Table 5. Frequencies and Percentages According to Bagrut Mathematics (Number of Units) and Geometric Reasoning Levels and X^2 Value

		Begrut Mathematics (number of units)			Total			
		Five Units	Four Units	Three Units		χ^2 Value	df	P Value
The levels	Level 1	1 (1.2%)	6 (7.1%)	13 (15.5%)	20 (23.8%)	18.75	4	0.001
	Level 2	3 (3.6%)	13 (15.5%)	3 (3.6%)	19 (22.6%)			
	Level 3	14 (16.7%)	22 (26.2%)	9 (10.7%)	45 (53.6%)	**Correlation coefficient**		**P Value**
Total		18 (21.4%)	41 (48.8%)	25 (29.8%)	84 (100%)	0.37		0.001

Is there a significant association between nationality and geometric reasoning levels?

To answer this question, cross-tabulation and chi-square test were calculated to investigate if there was a significant association between nationality and geometric reasoning levels. As seen in table 6 the χ^2-test results showed significant association between nationality and geometric reasoning levels in benefit to Jewish students, which means the Jewish students were likely to be categorized in the third level of geometric reasoning compared with Arab students. Moreover to compute correlation coefficient between nationality and geometric reasoning levels, the researcher used Rank-Biserial correlation coefficient (r=0.37).

Table 6. Frequencies and Percentages according to Nationality and Geometric Reasoning Levels and X^2 Value

		Nationality		Total			
		Jewish	Arabs		χ^2 Value	df	P Value
The levels	Level 1	6 (7.1%)	14 (16.7%)	20 (23.8%)	8.01	2	0.018
	Level 2	4 (4.8%)	15 (17.9%)	19 (22.6%)			
	Level 3	25 (29.8%)	20 (23.8%)	45 (53.6%)	**Correlation coefficient**		**P Value**
Total		35 (41.7%)	49 (58.3%)	84 (100%)	0.27		0.013

Is there a significant association between gender and geometric reasoning levels?

To answer this question, cross-tabulation and chi-square test were calculated to investigate if there was a significant association between gender and geometric reasoning levels. As seen in table 7 the χ^2-test results showed insignificant association between gender and geometric reasoning levels. Moreover to compute correlation coefficient between gender and geometric reasoning levels, the researcher used Rank-Biserial correlation coefficient (r=0.12).

Table 7. Frequencies and Percentages according to Gender and Geometric Reasoning Levels and X^2 Value

		Gender		Total	χ^2 Value	df	P Value
		Males	Females				
The levels	Level 1	3 (3.6%)	17 (20.2%)	20 (23.8%)	1.145	2	0.564
	Level 2	2 (2.4%)	17 (20.2%)	19 (22.6%)			
	Level 3	3 (3.6%)	42 (50%)	45 (53.6%)	**Correlation coefficient**		**P Value**
Total		8 (9.5%)	76 (90.5%)	84 (100%)	0.12		0.30

Is there a significant association between the Bagrut mathematics (number of units) and nationality?

To answer this question, cross-tabulation and chi-square test were calculated to investigate if there was a significant association between Bagrut mathematics (number of units) and nationality. As seen in table 8 the χ^2-test results showed insignificant association between Bagrut mathematics (number of units) and nationality. Moreover to compute correlation coefficient between Bagrut mathematics (number of units) and nationality, the researcher used Rank-Biserial correlation coefficient (r=0.15).

Table 8. Frequencies and Percentages according to Bagrut Mathematics (Number of Units) and Geometric Reasoning Levels and X^2 Value

		Begrut Mathematics (number of units)			Total	χ^2 Value	df	P Value
		Five Units	Four Units	Three Units				
Nationality	Jewish	8 (22.9%)	12 (34.3%)	15 (42.9%)	35 (100%)	6.107	2	0.052
	Arab	10 (20.4%)	29 (59.2%)	10 (20.4%)	49 (100%)	**Correlation coefficient**		**P Value**
Total		18 (21.4%)	41 (48.8%)	25 (29.8%)	84 (100%)	0.15		0.18

Discussion

This quantitative study was conducted to examine the level of geometric thinking of the students who want to be math teachers and came to learn in colleges of education. The researcher based on Van Hiele levels of geometric thought (Usiskin, 1982). The aim was to test the ability of Van Hiele theory to describe and predict performance in geometry in the first three levels of 84 students at mathematics primary and secondary school teaching department: 35 Jewish students, 49 Arabs students.

The study confirmed the discovery that preceded it in its main findings. That means it has showed that the majority of students learning math teaching at colleges of education do not control all the level of thinking of Van Hiele, but until the third level. As stated, 54% of the tested students in this study are assigned in level 3 – Ordering, about 23% of them are assigned in level 2 – Analysis, and about 24% of them are assigned in level 1 - Recognition. The findings approved the research of Usiskin (1982) that found out, that even after learning geometry, the majority of students do control until the third level of thinking of Van Hiele. In addition, also Vinitzky and Reis (2003) found out in their study, that the students have only a partial view of the scope and space concepts.

Another conclusion of this study is that there is insignificant association between educational path and gender to geometric reasoning levels. There was another insignificant association between Bagrut mathematics (number of units) and nationality.

Despite this, the study has showed significant association between nationality and geometric reasoning levels in benefit to Jewish students. Simultaneously, the study has showed significant association between Bagrut mathematics (number of units) and geometric thinking levels. Which means, that Jewish student managed to study more than three units Bagrut other than Arabs students.

From this study we are highly recommended that secondary school will operate program of enhanced geometrics in particularly and math in generally lessons in order to assist pupils raising their Bagrut units. We also recommended that this program will focus more on Arab society in order to minimize the gap. We also highly recommended that students who come to learn math teaching in colleges of education will be accepted to study only if they have three or more units of math Bagrut. We found it very essential.

At studies math teaching in colleges of education we highly recommended perform visual geometry studies program as early as first year of study, and give the future teachers minded toolboxes to make studies like this in the primary and secondary school. We found that essential to teach them using the computerization in their teaching class to give the pupils visual illustration invention as found out at Tepper (1986) research.

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