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PAIR DIFFERENCE CORDIAL LABELING OF CERTAIN BROKEN WHEEL GRAPHS

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Abstract. Let $G = (V, E)$ be a (p, q) graph.
Define

$$\rho = \begin{cases} \frac{p}{2}, & \text{if } p \text{ is even} \\ \frac{p-1}{2}, & \text{if } p \text{ is odd} \end{cases}$$

and $L = \{\pm 1, \pm 2, \pm 3, \cdots, \pm \rho\}$ called the set of labels.
Consider a mapping $f : V \longrightarrow L$ by assigning different labels in L to the different elements of V when p is even and different labels in L to p-1 elements of V and repeating a label for the remaining one vertex when p is odd.The labeling as defined above is said to be a pair difference cordial labeling if for each edge uv of G there exists a labeling $|f(u) - f(v)|$ such that $\left|\Delta_{f_1} - \Delta_{f_1^c}\right| \leq 1$, where Δ_{f_1} and $\Delta_{f_1^c}$ respectively denote the number of edges labeled with 1 and number of edges not labeled with 1. A graph G for which there exists a pair difference cordial labeling is called a pair difference cordial graph. In this paper we investigate the pair difference cordial labeling behavior of Certain broken wheel graphs.

1. Introduction

In this paper we consider only finite, undirected and simple graphs. The origin of graph labeling is graceful labeling [17]. In 1980, Cahit introduced the cordial labeling of graphs in [1]. In the sequal several cordial related labeling was studied in [13,14,15,16,18,19,20,21,22]. Ponraj et al introduced the pair difference cordial labeling in [4]. Also we have investigated various graphs [4,5,6,7,8,9,10] for pair difference cordial labeling. In this paper we investigate pair difference cordial labeling behavior of Certain broken wheel graphs.

2. Preliminaries

Definition 2.1. [2] Let $W_3 = C_3 + K_1$ where C_3 is abca and $V(K_1) = \{u\}$. The broken wheel $W(l, m, n)$ is obtained from the wheel W_3 with $V(W(l, m, n)) =$

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$V(W_3) \cup \{u_i : 1 \leq i \leq l-1\} \cup \{v_i : 1 \leq i \leq m-1\} \cup \{w_i : 1 \leq i \leq n-1\}$ and $E(W(l,m,n)) = E(W_3) \cup \{u_i u_{i+1} : 2 \leq i \leq l-2\} \cup \{v_i v_{i+1} : 2 \leq i \leq m-2\} \cup \{w_i w_{i+1} : 2 \leq i \leq n-1\} \cup \{au_1, u_{l-1}, bv_1, v_{m-1}c, aw_1, cw_{n-1}\}$.

3. Pair difference cordial labeling

Definition 3.1. [4] Let $G = (V, E)$ be a (p, q) graph.
Define

$$\rho = \begin{cases} \frac{p}{2}, & \text{if } p \text{ is even} \\ \frac{p-1}{2}, & \text{if } p \text{ is odd} \end{cases}$$

and $L = \{\pm 1, \pm 2, \pm 3, \cdots, \pm \rho\}$ called the set of labels.
Consider a mapping $f : V \longrightarrow L$ by assigning different labels in L to the different elements of V when p is even and different labels in L to p-1 elements of V and repeating a label for the remaining one vertex when p is odd.The labeling as defined above is said to be a pair difference cordial labeling if for each edge uv of G there exists a labeling $|f(u) - f(v)|$ such that $\left|\Delta_{f_1} - \Delta_{f_1^c}\right| \leq 1$, where Δ_{f_1} and $\Delta_{f_1^c}$ respectively denote the number of edges labeled with 1 and number of edges not labeled with 1.A graph G for which there exists a pair difference cordial labeling is called a pair difference cordial graph.

Theorem 3.2. [4] *The wheel W_n is pair difference cordial if and only if n is even.*

4. Main Results

Theorem 4.1. *The broken wheel $W(n, n, n)$ is pair difference cordial for all values of $n \geq 2$.*

Proof. Take the vertex and edges set from the definition 2.1. Now there are four cases arises.
Case(i). $n \equiv 0 \pmod 4$

Fix the label $1, -1$ to the vertex u_1, v_1 respectively and assign the labels $2, 4$ to the vertices u_2, u_3 respectively and assign the labels $3, 5$ respectively to the vertices u_4, u_5. Next assign the labels $6, 8$ to the vertices u_6, u_7 respectively and assign the labels $7, 9$ respectively to the vertices u_8, u_9. Proceeding like this until we reach the vertex u_{n-1}. Note that in this process the vertex u_{n-1} gets the label $n-1$.

Now we assign the labels $-2, -4$ to the vertices v_2, v_3 respectively and assign the labels $-3, -5$ respectively to the vertices v_4, v_5. Next assign the labels $-6, -8$ to the vertices v_6, v_7 respectively and assign the labels $-7, -9$ respectively to the vertices v_8, v_9. Proceeding like this until we reach the vertex v_{n-1}. Here we notice that in this process the vertex v_{n-1} gets the label $-n+1$.

Next we assign the labels $n, n+1$ to the vertices w_{n-1}, w_{n-2} respectively and assign the labels $-n, -(n+1)$ respectively to the vertices w_{n-3}, w_{n-4}. Now we assign the labels $n+2, n+3$ to the vertices w_{n-5}, w_{n-6} respectively and assign the labels $-(n+2), -(n+3)$ respectively to the vertices w_{n-7}, w_{n-8}. Proceeding this process until we reach the vertices w_1, a. Finally assign the labels $\frac{3n}{2}, \frac{3n-2}{2}, -\frac{3n}{2}$

respectively to the vertices u, b, c.

Case(ii). $n \equiv 1 \pmod 4$

First we assign the labels $1, 2, 4, 3$ to the vertices u_1, u_2, u_3, u_4 respectively and assign the labels $5, 6, 8, 7$ respectively to the vertices u_5, u_6, u_7, u_8. Next assign the labels $9, 10, 12, 11$ to the vertices $u_9, u_{10}, u_{11}, u_{12}$ respectively and assign the labels $13, 14, 16, 15$ respectively to the vertices $u_{13}, u_{14}, u_{15}, u_{16}$. Proceeding like this until we reach the vertex u_{n-1}. Note that in this process the vertices $u_{n-4}, u_{n-3}, u_{n-2}$, u_{n-1} gets the label $n-4, n-3, n-1, n-2$.

Now we assign the labels $-1, -2, -4, -3$ to the vertices v_1, v_2, v_3, v_4 respectively and assign the labels $-5, -6, -8, -7$ respectively to the vertices v_5, v_6, v_7, v_8. Next assign the labels $-9, -10, -12, -11$ to the vertices $v_9, v_{10}, v_{11}, v_{12}$ respectively and assign the labels $-13, -14, -16, -15$ respectively to the vertices $v_{13}, v_{14}, v_{15}, v_{16}$. Proceeding like this until we reach the vertex v_{n-4}. We notice that in this process the vertex v_{n-4} gets the label $-n+4$ and assign the labels $-(n-3), -(n-2), -(n-1)$ respectively to the vertices $v_{n-3}, v_{n-2}, v_{n-1}$.

Next we assign the labels $n, n+1$ to the vertices c, w_{n-1} respectively and assign the labels $-n, -(n+1)$ respectively to the vertices w_{n-2}, w_{n-3}. Now we assign the labels $n+2, n+3$ to the vertices w_{n-4}, w_{n-5} respectively and assign the labels $-(n+2), -(n+3)$ respectively to the vertices w_{n-6}, w_{n-7}. Proceeding this process until we reach the vertices w_2. Finally assign the labels $\frac{3n-1}{2}, \frac{3n+1}{2}, -\frac{3n-1}{2}, -\frac{3n+1}{2}$ respectively to the vertices w_1, a, u, c.

Case(iii). $n \equiv 2 \pmod 4$
There are subcases arises.

Subcase(i). $n = 2$

Assign the labels $1, 2, 3, -1, -2, -3, 1$ respectively to the vertices $a, u_1, b, v_1, c, w_1, u$.

Subcase(i). $n > 2$

Assign the labels to the vertices u_i, v_i , $(1 \le i \le n-1)$ as in technique of case (i). Next we assign the labels $-n, -(n+1)$ to the vertices c, w_{n-1} respectively and assign the labels $n, (n+1)$ respectively to the vertices w_{n-2}, w_{n-3}. Now we assign the labels $-(n+2), -(n+3)$ to the vertices w_{n-4}, w_{n-5} respectively and assign the labels $(n+2), (n+3)$ respectively to the vertices w_{n-6}, w_{n-7}. Proceeding this process until we reach the vertices w_1, a.Note that the vertices w_4, w_3 gets the labels $\frac{3n-6}{2}, \frac{3n-4}{2}$. Lastly assign the labels $-\frac{3n}{2}, -\frac{3n-2}{2}, \frac{3n-2}{2}, \frac{3n}{2}, \frac{3n-2}{2}$ respectively to the vertices w_2, w_1, a, u, b.

Case(iv). $n \equiv 3 \pmod 4$

There are subcases arises.

Subcase(i). $n = 3$

Assign the labels $1, 2, 3, 4, -1, -2, -3, -4, 5, -5$ respectively to the vertices $a, u_1, u_2, b, v_1, v_2, c, w_1, w_2, u$.

Subcase(i). $n > 2$

Assign the labels to the vertices u_i, v_i , $(1 \leq i \leq n-1)$ as in technique of case (i). Next we assign the labels $-n, -(n+1)$ to the vertices c, w_{n-1} respectively and assign the labels $n, (n+1)$ respectively to the vertices w_{n-2}, w_{n-3}. Now we assign the labels $-(n+2), -(n+3)$ to the vertices w_{n-4}, w_{n-5} respectively and assign the labels $(n+2), (n+3)$ respectively to the vertices w_{n-6}, w_{n-7}. Proceeding this process until we reach the vertices w_3. Note that the vertices w_1, a gets the labels $\frac{3n-3}{2}, \frac{3n-1}{2}$ Finally assign the labels $\frac{3n+1}{2}, -\frac{3n+1}{2}$ respectively to the vertices u, b.

The Table 1 given below establish that this vertex labeling f is a pair difference cordial of $W(n, n, n)$.

Nature of n	Δ_{f_1}	$\Delta_{f_1^c}$
$n \equiv 0 \pmod 4$	$\frac{3n+2}{2}$	$\frac{3n+4}{2}$
$n \equiv 1 \pmod 4$	$\frac{3n+3}{2}$	$\frac{3n+3}{2}$
$n \equiv 2 \pmod 4$	$\frac{3n+2}{2}$	$\frac{3n+4}{2}$
$n \equiv 3 \pmod 4$	$\frac{3n+3}{2}$	$\frac{3n+3}{2}$

TABLE 1

□

Theorem 4.2. *The broken wheel $W(l, m, m)$ is pair difference cordial for all values of $l, m \geq 2$.*

Proof. Take the vertex and edges set from the definition 2.1. Now there are four cases arises.
Case(1). $l \equiv 0 \pmod 4$
Here four subcases arises.
Sub Case(i). $m \equiv 0 \pmod 4$

First we assign the labels $1, 2, 4, 3$ to the vertices v_1, v_2, v_3, v_4 respectively and assign the labels $5, 6, 8, 7$ respectively to the vertices v_5, v_6, v_7, v_8. Next assign the labels $9, 10, 12, 11$ to the vertices $v_9, v_{10}, v_{11}, v_{12}$ respectively and assign the labels $13, 14, 16, 15$ respectively to the vertices $v_{13}, v_{14}, v_{15}, v_{16}$. Proceeding like this until we reach the vertex v_{m-1}. Note that in this process the vertices $v_{m-4}, v_{m-3}, v_{m-2}, v_{m-1}$ gets the label $m-4, m-3, m-1, m-2$.

Now we assign the labels $-1, -2, -4, -3$ to the vertices $w_{m-1}, w_{m-2}, w_{m-3}, w_{m-4}$ respectively and assign the labels $-5, -6, -8, -7$ respectively to the vertices $w_{m-5}, w_{m-6}, w_{m-7}, w_{m-8}$. Next assign the labels $-9, -10, -12, -11$ to the vertices $w_{m-9}, w_{m-10}, w_{m-11}, w_{m-12}$ respectively and assign the labels $-13, -14, -16, -15$ respectively to the vertices $w_{m-13}, w_{m-14}, w_{m-15}, w_{m-16}$. Proceeding like this until we reach the vertex w_1. We notice that in this process the vertex w_1 gets the label $-n+1$.

Next we assign the labels $m, m+1, -m, -(m+1)$ to the vertices a, u_1, u_2, u_3 respectively and assign the labels $m+2, m+3, -(m+2), -(m+3)$ respectively to the vertices u_4, u_5, u_6, u_7. Now we assign the labels $m+4, m+5, -(m+4), -(m+5)$ to the vertices u_8, u_9, u_{10}, u_{11} respectively and assign the labels $m+6, m+7, -(m+6), -(m+7)$ respectively to the vertices $u_{12}, u_{13}, u_{14}, u_{15}$. Proceeding this process until we reach the vertices u_{l-1}.Note that in this process the vertices $u_{l-4}, u_{l-3}, u_{l-2}, u_{l-1}$ gets the label $\frac{l+2m-4}{2}, \frac{l+2m-2}{2}, -\frac{l+2m-4}{2}, -\frac{l+2m-2}{2}$ respectively . Finally assign the labels $-\frac{l+2m}{2}, \frac{l+2m}{2}, \frac{l+2m}{2}$ respectively to the vertices b, u, c.

Sub Case(ii). $m \equiv 1 \pmod 4$
Assign the labels to the vertices v_i , $(1 \leq i \leq m-1)$, u_i , $(1 \leq i \leq l-1)$ and w_i , $(1 \leq i \leq m-5)$ as in technique of case (i). Next we assign the labels $-(m-4), -(m-3), -(m-2), -(m-1)$ to the vertices $w_{m-4}, w_{m-3}, w_{m-2}, w_{m-1}$ respectively and assign the labels $\frac{l+2m}{2}, -\frac{l+2m}{2}, -\frac{l+2m-2}{2}$ respectively to the vertices b, u, c.

Sub Case(iii). $m \equiv 2 \pmod 4$
Assign the labels to the vertices v_i, w_i , $(1 \leq i \leq m-1)$ as in technique of case (i). Next we assign the labels $-n, -(n+1), n, n+1$ to the vertices a, u_1, u_2, u_3 respectively and assign the labels $-(n+2), -(n+3), n+2, n+3$ respectively to the vertices u_4, u_5, u_6, u_7. Next assign the labels $-(n+4), -(n+5), n+4, n+5$ respectively to the vertices u_8, u_9, u_{10}, u_{11}. Proceeding this process until we reach the vertices u_{l-1}.Note that in this process the vertices $u_{l-4}, u_{l-3}, u_{l-2}, u_{l-1}$ gets the label $-\frac{l+2m-4}{2}, -\frac{l+2m-2}{2}, \frac{l+2m-4}{2}, \frac{l+2m-2}{2}$ respectively . Finally assign the labels $\frac{l+2m}{2}, -\frac{l+2m}{2}, -\frac{l+2m-2}{2}$ respectively to the vertices b, u, c.

Sub Case(iv). $m \equiv 3 \pmod 4$
Assign the labels to the vertices v_i, w_i , $(1 \leq i \leq m-1)$, u_i , $(1 \leq i \leq l-1)$ and a, b, c, u as in technique of case (i).

The Table 2 given below establish that this vertex labeling f is a pair difference cordial of $W(l, m, m)$.

Nature of n	Δ_{f_1}	$\Delta_{f_1^c}$
$n \equiv 0 \pmod 4$	$\frac{l+2m+2}{2}$	$\frac{l+2m+4}{2}$
$n \equiv 1 \pmod 4$	$\frac{l+2m+2}{2}$	$\frac{l+2m+4}{2}$
$n \equiv 2 \pmod 4$	$\frac{l+2m+2}{2}$	$\frac{l+2m+4}{2}$
$n \equiv 3 \pmod 4$	$\frac{l+2m+4}{2}$	$\frac{l+2m+2}{2}$

TABLE 2

Case(2). $l \equiv 1 \pmod 4$
Here four subcases arises.

Sub Case(i). $m \equiv 0 \pmod 4$
Assign the labels to the vertices v_i, w_i , $(1 \leq i \leq m-1)$, u_i , $(1 \leq i \leq l-2)$ and a as in technique of subcase (i) in case 1. Finally assign the labels $\frac{l+2m-1}{2}, \frac{l+2m+1}{2}, -\frac{l+2m-1}{2}, -\frac{l+2m+1}{2}$ respectively to the vertices u_{n-1}, b, u, c.

Sub Case(ii). $m \equiv 1 \pmod 4$
Assign the labels to the vertices v_i, w_i , $(1 \leq i \leq m-1)$ as in technique of subcase (i) in case 1. Next we assign the labels $-m, -(m+1), m, m+1$ to the vertices a, u_1, u_2, u_3 respectively and assign the labels $-(m+2), -(m+3), m+2, m+3$ respectively to the vertices u_4, u_5, u_6, u_7. Next assign the labels $-(m+4), -(m+5), m+4, m+5$ respectively to the vertices u_8, u_9, u_{10}, u_{11}. Proceeding this process until we reach the vertices u_{l-2}.Note that in this process the vertices $u_{l-5}, u_{l-4}, u_{l-3}, u_{l-2}$ gets the label $\frac{l+2m-5}{2}, \frac{l+2m-3}{2}, -\frac{l+2m-5}{2}, -\frac{l+2m-3}{2}$ respectively . Finally assign the labels $\frac{l+2m-1}{2}, \frac{l+2m+1}{2}, -\frac{l+2m-1}{2}, -\frac{l+2m+1}{2}$ respectively to the vertices u_{n-1}, b, u, c.

Sub Case(iii). $m \equiv 2 \pmod 4$
Assign the labels to the vertices v_i, w_i , $(1 \leq i \leq m-1)$, u_i , $(1 \leq i \leq l-1)$ and a, b, c, u as in technique of subcase (ii) in case 2.

Sub Case(iv). $m \equiv 3 \pmod 4$
Assign the labels to the vertices v_i, w_i , $(1 \leq i \leq m-1)$, u_i , $(1 \leq i \leq l-2)$ and a as in technique of subcase (ii) in case 2. Finally assign the labels $-\frac{l+2m-1}{2}, -\frac{l+2m+1}{2}, \frac{l+2m-1}{2}, \frac{l+2m+1}{2}$ respectively to the vertices u_{l-1}, b, u, c.

The Table 3 given below establish that this vertex labeling f is a pair difference cordial of $W(l, m, m)$.

Nature of n	Δ_{f_1}	$\Delta_{f_1^c}$
$n \equiv 0 \pmod 4$	$\frac{l+2m+3}{2}$	$\frac{l+2m+3}{2}$
$n \equiv 1 \pmod 4$	$\frac{l+2m+3}{2}$	$\frac{l+2m+3}{2}$
$n \equiv 2 \pmod 4$	$\frac{l+2m+3}{2}$	$\frac{l+2m+3}{2}$
$n \equiv 3 \pmod 4$	$\frac{l+2m+3}{2}$	$\frac{l+2m+3}{2}$

TABLE 3

Case(3). $l \equiv 2 \pmod 4$
Here four subcases arises.

Sub Case(i). $m \equiv 0 \pmod 4$
Assign the labels to the vertices v_i, w_i , $(1 \leq i \leq m-1)$, u_i , $(1 \leq i \leq l-1)$ and a as in technique of subcase (i) in case 1. Finally assign the labels $-\frac{l+2m-2}{2}, -\frac{l+2m}{2}, -\frac{l+2m-2}{2}$ respectively to the vertices b, u, c.

Sub Case(ii). $m \equiv 1 \pmod 4$
Assign the labels to the vertices v_i, w_i , $(1 \leq i \leq m-1)$, u_i , $(1 \leq i \leq l-1)$ and a, b, c, u as in technique of subcase (ii) in case 3.

Sub Case(iii). $m \equiv 2 \pmod 4$
Assign the labels to the vertices v_i, w_i , $(1 \leq i \leq m-1)$, u_i , $(1 \leq i \leq l-1)$ and a as in technique of subcase (i) in case 1. Finally assign the labels $\frac{l+2m-2}{2}, \frac{l+2m}{2}, \frac{l+2m-2}{2}$ respectively to the vertices b, u, c.

Sub Case(iv). $m \equiv 3 \pmod 4$
Assign the labels to the vertices v_i, w_i , $(1 \leq i \leq m-1)$, u_i , $(1 \leq i \leq l-1)$ and a, b, c, u as in technique of subcase (iii) in case 3.

The Table 4 given below establish that this vertex labeling f is a pair difference cordial of $W(l, m, m)$.

Nature of n	Δ_{f_1}	$\Delta_{f_1^c}$
$n \equiv 0 \pmod 4$	$\frac{l+2m+4}{2}$	$\frac{l+2m+2}{2}$
$n \equiv 1 \pmod 4$	$\frac{l+2m+2}{2}$	$\frac{l+2m+4}{2}$
$n \equiv 2 \pmod 4$	$\frac{l+2m+2}{2}$	$\frac{l+2m+4}{2}$
$n \equiv 3 \pmod 4$	$\frac{l+2m+4}{2}$	$\frac{l+2m+2}{2}$

TABLE 4

Case(4). $l \equiv 3 \pmod 4$
Here four subcases arises.

Sub Case(i). $m \equiv 0 \pmod 4$
Assign the labels to the vertices v_i, w_i , $(1 \leq i \leq m-1)$ as in technique of subcase (i) in case 1. Next we assign the labels $m, m+1, -m, -(m+1)$ to the vertices a, u_1, u_2, u_3 respectively and assign the labels $m+2, m+3, -(m+2), -(m+3)$ respectively to the vertices u_4, u_5, u_6, u_7. Next assign the labels $m+4, m+5, -(m+4), -(m+5)$ respectively to the vertices u_8, u_9, u_{10}, u_{11}. Proceeding this process until we reach the vertices u_{l-1}, b.Note that in this process the vertices $u_{l-3}, u_{l-2}, u_{l-1}, b$ gets the label $\frac{l+2m-3}{2}, \frac{l+2m-1}{2}, -\frac{l+2m-3}{2}, -\frac{l+2m-1}{2}$ respectively . Finally assign the labels $-\frac{l+2m+1}{2}, \frac{l+2m+1}{2}$ respectively to the vertices u, c.

Sub Case(ii). $m \equiv 1 \pmod 4$
Assign the labels to the vertices v_i, w_i , $(1 \leq i \leq m-1)$, u_i , $(1 \leq i \leq l-1)$ and a, b, c, u as in technique of subcase (i) in case 4.

Sub Case(iii). $m \equiv 2 \pmod 4$
Assign the labels to the vertices v_i, w_i , $(1 \leq i \leq m-1)$, u_i , $(1 \leq i \leq l-1)$ and a, b, c, u as in technique of subcase (i) in case 4.

Sub Case(iv). $m \equiv 3 \pmod 4$
Assign the labels to the vertices v_i, w_i , $(1 \leq i \leq m-1)$, u_i , $(1 \leq i \leq l-1)$ and a, b, c, u as in technique of subcase (i) in case 4.

The Table 5 given below establish that this vertex labeling f is a pair difference cordial of $W(l,m,m)$.

Nature of n	Δ_{f_1}	$\Delta_{f_1^c}$
$n \equiv 0 \pmod 4$	$\frac{l+2m+3}{2}$	$\frac{l+2m+3}{2}$
$n \equiv 1 \pmod 4$	$\frac{l+2m+3}{2}$	$\frac{l+2m+3}{2}$
$n \equiv 2 \pmod 4$	$\frac{l+2m+3}{2}$	$\frac{l+2m+3}{2}$
$n \equiv 3 \pmod 4$	$\frac{l+2m+3}{2}$	$\frac{l+2m+3}{2}$

Table 5

□

5. Discussion

The pair sum labeling was introduced by Ponraj and Parthipan in [11]. Recently the difference cordial labeling of graphs was introduced in [12]. Follows from these two concepts, we have defined a new concept of pair difference cordial labeling of graphs [4]. The pair difference cordial labeling behaviour of broken wheel have been investigated in this paper.

6. Conclusion

The pair difference cordial labeling behaviour of some broken wheel graphs have been investigated in this paper. Presently, it is difficult to investigate the pair difference cordial labeling behaviour of generalized web, broken web graphs. The pair difference cordial labeling behaviour of subdivison of broken whell graphs are the open problems.

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The Declaration of Research and Publication Ethics
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