



Farklı Gölgeleyen Kanallar Üzerinden Enerji-Verimli Veri İletimi için Geçmiş-temelli Su-Doldurma Algoritması

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Öz

Bu çalışmada, kablosuz ağlarda çoklu gölgeleyen kanallar üzerinden bir radio kaynak tahsisi problem ele alınmaktadır. Bu problem iki yönlü incelenmektedir. İlk olarak, problemi tüm gölgeleyen kanalları düşünerek ele alınmaktadır. Çevrimdışı su-doldurma algoritmasını düşünerek bu problemin en iyi çözümünü sunulmuştur. Daha sonra bu probleme geçmiş-temelli çevrimiçi su doldurma algoritmaları önerilmiştir. Bu çevrimiçi algoritma, geçmişin bir kısmına bağlı bir su-doldurma seviyesine karar vermek amacıyla geçmiş kısmı olarak kullanılmaktadır. Daha sonra, bu çevrimiçi algoritma problemin zaman ufkuunda veri iletmek için bu geçmiş-temelli su-doldurma seviyesini uygulamaktadır. Çevrimiçi ve çevrimdışı politikaların görelî performansını, çeşitli tiplerde (Rayleigh, Rician, Nakagami, Weibull) gölgeleyen kanallar için çeşitli zaman ufuklarında değerlendirilmektedir. Sayısal sonuçlar, özellikle daha uzun zaman ufukları için ve daha uzun geçmişin daha uzun kısımlarını kullanıldığında, bu çevrimiçi su doldurma algoritmalarının çevrimdışı su doldurma algoritmalarına yakın performansı olduğunu göstermektedir.

Anahtar Kelimeler: Su-Doldurma, Takviyeli Öğrenme, Çoklu Erişim Haberleşmesi, Çevrimiçi Politika.

Performance of History-based Water-Filling Algorithm for Energy-Efficient Data Transmission over Different Fading Channels

Abstract

In this paper, we tackle a resource allocation problem over multiple fading channels in wireless networks. This problem is investigated in two ways. First, we consider the problem over the whole multiple fading channels altogether with no power constraint. We look for an optimal solution for this problem by considering an offline waterfilling algorithm. Then, we also propose history-based online waterfilling algorithms for this problem. This online algorithm uses the history partially in order to determine a waterfilling level based on that part of history. Then, the online policy applies this history-based determined waterfilling level to transmit data over the time horizon of the problem. The relative performance of the online and offline policies is evaluated for various types of fading channels (Rayleigh, Rician, Nakagami, Weibull) over various time horizons. The numerical results demonstrate these online waterfilling algorithms shows close performance to offline waterfilling algorithms especially for longer time horizons and by using larger portions of history.

Keywords: Water-Filling, Reinforcement Learning, Multi-access Communications, Online Policy.

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1. Introduction

1.1. Motivation and Related Work

With negligible latency, superior reliability, and high speeds, the fifth generation (5G) communication is expected to expand mobile ecosystems into new directions. It impacts most of the industries, by providing digitized logistics, remote healthcare, precision agriculture and safer transportation as explained in Qualcomm's website (last reached in 2022). One of the key technologies for 5G is direct communications. In mobile strategies, developing period transportation and pervasive mobile services result in consuming considerably much energy under energy constraints, which prevents D2D communications. Due to small range, data transmission will enable new mobile applications and industry models as in Nazir et. Al (2021).

Water-filling algorithms are very essential for optimizing data transmission in communication systems. They affect design of wireless communications systems, network optimization, resource allocation, so on, which can be seen in Boyd et. Al (2004), Cover et. al (2006), Goldsmith et. Al. (2005), Goldsmith et al. (1996), Tse et. al (2005), Teletar et. Al. (1995), Yang et al. (1994), Dai et. Al. (2014), Gai et. Al. (2012). They are generally needed for resource allocation for multidimensional communications systems. Different physical constraints and performance requirements emerges with the evolution of wireless systems over past decade. This results in various waterfilling policies from single to multi waterlevel solutions, from perfect channel state information(CSI)-based to robust solutions Xing et. Al (2020).

The work authored by Qi et. Al. (2012) presents a low-complexity waterfilling algorithm to allocate power at OFDM-based cognitive radio systems by using power-increment or power-decrement waterfilling process in order to overcome power limitations emerged with cognitive radio, which cannot be dealt with basic waterfilling algorithms. Numerical results demonstrate they can succeed in allocating power optimally and more quickly than iterative waterfilling algorithms.

The work authored by He et. Al. (2013) proposes a low-complexity geometric waterfilling approach for weighted and unweighted radio resource allocation problems by solving a nonlinear system from Karush-Kuhn-Tucker conditions of these problems. This work proves optimality of proposed waterfilling algorithm. Numerical results demonstrate proposed approach is robust, insight-seeing and easy to follow.

The work authored by Ajitsinh et. Al. (2017) investigates power allocation algorithms like uniform, waterfilling, suboptimal and optimal algorithms for OFDM-based cognitive radio systems in simulation environment. Numerical results demonstrate the proposed algorithms can achieve higher transmission than uniform and classical waterfilling algorithms.

The paper authored by Hu et. Al. (2017) proposes a power allocation technique for achieving optimum energy efficiency in Massive MIMO. It develops a new technique for computing optimum power allocation technique based on that simplified expression and it was compared with the conventional scheme. For maximizing energy and spectral efficiency, the paper proposes and embeds an improved waterfilling technique in the power allocation algorithm. Simulation results demonstrates that

compared with conventional schemes, energy efficiency and spectral efficiency are improved much at downlink transmission.

Many recent works propose water-filling policies for channel resource allocation problems. As an example study, Shi et al. (2016) proposes an improved power allocation waterfilling policy, which avoids iterative calculations in conventional waterfilling policy, simplifies the policy effectively so decreases calculations. Chaeriah et al. (2017) proposes an OFDM-based power allocation method. Then, it applies water flooding technique over fading channels in cognitive radio network. This policy can boost channel performance of the network effectively. Moreover, waterfilling policy can be applied for resource allocation problems in multicarrier uplink NOMA systems in the work authored by Zeng et. Al (2019).

The paper authored by Elgarhy et. Al. (2018) investigates a highly-complex weighted throughput maximization problem which can be solved by MLFP-bAsed PowEr aLlocation (MAPEL) algorithm presented in the work authored by Qian et. Al. (2009). The paper uses several methods to reduce computational times of MAPEL, especially by considering the constraint of minimum rate in that optimisation problem. Moreover, the water filling principle is introduced in the optimization for respecting maximum available power for each user if in more resource blocks, the allocation need to be made.

The work authored by Kim et. Al. (2018) proposes a principled technique to decide optimal nonuniform bitline swings by formulating convex optimisation problem. Under the constraints on mean squared error of retrieved words, it considers criterion for maximizing speed, minimizing energy, and energy-delay product. The studied optimization problems in this paper can be interpreted as classical waterfilling, groundflattening&waterfilling, and sandpouring& waterfilling. Greedy algorithms are proposed for obtaining optimized discrete swings by leveraging these interpretations.

The work authored by Gursadani et. Al. (2021) analyzes bit error rate performance of STBC, Orthogonal-STBC and channel capacity enhancement frameworks of MIMO systems by applying waterfilling algorithms with equalization techniques of ZF, ML and MMSE.

The paper authored by Song et. Al. (2021) proposes an improved waterfilling algorithm where the signum function is used for optimizing temperature limit of transmission interference. This advances flexibility of channel screening and optimises the communication network capacity effectively.

In the related literature, many different waterfilling policies are proposed for different problems in different systems whereas these policies have generally high computationally complexity. Therefore, more robust and efficient waterfilling algorithms are needed for this problem especially in the practical manner. To the best of our knowledge, the effect of using partial history to decide future waterlevel for data transmission has not studied yet.

1.2. Main Contributions

In this paper, we tackle a resource allocation problem over multiple fading channels in wireless networks. This problem is tackled in two ways. First, we consider the problem over the whole multiple fading channels altogether in offline manner. We look for an optimal solution for this problem over each fading channel by considering an offline waterfilling algorithm. Then, we consider the problem over the multiple fading channels in

online manner. We propose a history-based online waterfilling policy which benefits from partial history information.

1.3. Organization

The rest of the paper is organized as follows. In Section II, we gave our system model and then defined our problem. In Section III, I gave a brief background on waterfilling approach. Section IV, we proposed optimal offline waterfilling algorithms and then we proposed history-based online algorithms for the problem at hand. In Section V, we presented the numerical results under different fading scenarios for various time horizons. In Section VI, we concluded our work.

2. System Model and Problem Formulation

2.1. System Model

In this paper, we consider energy-efficient data transmission problem over multiple fading channels in wireless networks.

In this system, a transmitter with K channels transmits data to K single-channel receivers. We assume that channels are orthogonal to each other, so interference is neglected.

Figure 1 shows an example scenario with 4 fading channels.

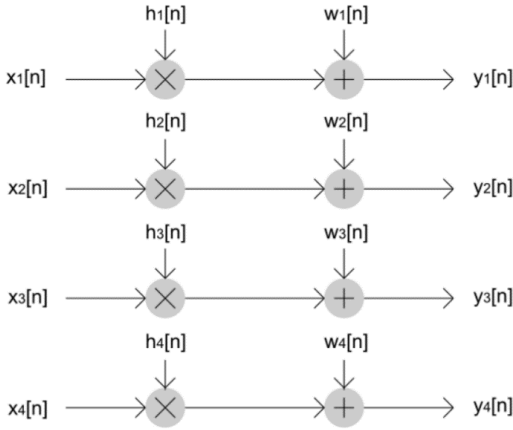


Figure 1. System model with $K=4$ fading channel scenario
(Şekil 1. $K=4$ gölgelenen kanallı senaryo ile Sistem Modeli)

The data transmission over each channel is modelled as,

$$y_k[n] = h_k[n] \cdot x_k[n] + w_k[n], k = 1, \dots, K; n = 1, \dots, N \quad (1)$$

where $y_k[n]$, $x_k[n]$, and $w_k[n]$ are output, input and noise signals in the k^{th} subchannel in n^{th} time slot, respectively. $h_k[n]$ is channel gain for each subchannel.

Assuming the transmit power in each subchannel is $P_k[n]$, the maximum rate of reliable communication over OFDM channel is

$$C_k[n] = B \cdot \log \left(1 + \frac{P_k[n] \cdot |h_k[n]|^2}{N_0} \right) \text{ bit/symbol} \quad (2)$$

where N_0 is power density of noise.

By allocating transmission powers $P_k[n]$ optimally, we aim to maximize total transmitted data over the time horizon.

2.2. Problem Formulation

The problem at hand is investigated in both offline and online manner in the paper. First, the problem is given as follows.

$$C(K, N) = \sum_{n=1}^N \sum_{k=1}^K C_k[n] \quad (3)$$

From Equation (2), the power allocation is the solution to the optimization problem:

$$C(K, N) = \max_{\{P_k[n]\}_{n=1}^N} \sum_{n=1}^N \sum_{k=1}^K B \cdot \log \left(1 + \frac{P_k[n] \cdot |h_k[n]|^2}{N_0} \right) \quad (4)$$

s.t. $\sum_{n=1}^N \sum_{k=1}^K P_k[n] \leq E$ where E is total transmission energy; and $P_k[n] \geq 0, k = 1, \dots, K, n = 1, \dots, N$.

3. Brief Background on Waterfilling Policy

In this section, we give a brief background on waterfilling policy which is optimal for the problem in Equation (4). In the waterfilling scheme, all optimal power levels can be obtained as:

$$P_k[n] = \left[\frac{1}{K \cdot N} \left(E + \sum_{n=1}^N \sum_{k=1}^K \frac{1}{|h_k[n]|^2} \right) - |h_k[n]|^2 \right]^+ \quad (5)$$

where $[A]^+ = A$ if $A > 0$. Otherwise, $[A]^+ = 0$ if $A \leq 0$.

As single-channel examples, Figure 2a and Figure 2b illustrate two different waterfilling schemes with $E=50$ units and $E=25$ units, respectively. In Figure 2a, data is transmitted in all TSs whereas data is not transmitted in TS 6 in Figure 2b.

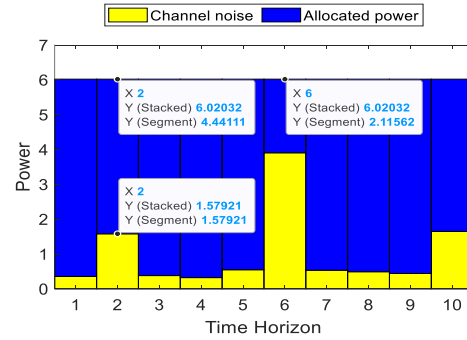


Figure 2a. Waterfilling with $E=50$ on $K=10$ fading channel (Şekil 2a. $K=10$ gölgelenen kanal üzerine $P=50$ ile Su-Doldurma)

In Figure 2a, X, Y(Segment) and Y(Staked) represent the time, the channel gain level and the allocated transmission power times channel gain, respectively. In Figure 2b, Y(Staked) equals to Y(Segment) in TS 6 when there is no transmission.

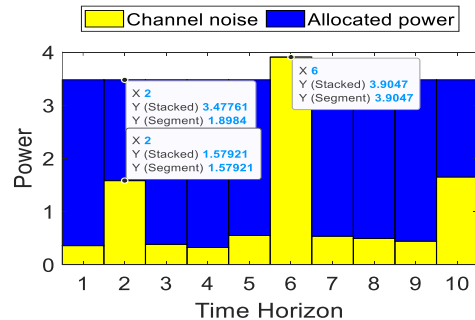


Figure 2b. Waterfilling with $E=25$ on $K=10$ fading channel (Şekil 2b. $K=10$ gölgelenen kanal üstüne $P=25$ ile Su-Doldurma)

4. Proposed Waterfilling Algorithms

In this section, we consider the problem in both offline and online manner. Firstly, an optimal solution is proposed for this problem by considering an offline waterfilling algorithm. Then, we propose a history-based online waterfilling algorithm.

4.1. Offline Waterfilling Algorithm

In this subsection, we consider the problem in the offline manner. We present the conventional offline waterfilling algorithm which is omniscient and so optimal by using bisection method for the problem as an optimal solution (Please see Figure 3). It will be used in the section of numerical results as a benchmark policy with which the online policy is compared.

Algorithm 1 Offline Waterfilling Policy

Input: $D(K, N)$ is the number of data packets to be transmitted via all K channels over a time horizon N .

$h_k[n]$ is the channel gain k in the time slot n .

Initialize: WFL_{max} , WFL_{min} and ϵ are maximum waterfilling level, minimum waterfilling level and the tolerance, respectively.

1) Let's initiate $K \times N$ Water-Filling Level Matrix as $WFL \leftarrow \left(\frac{WFL_{max} + WFL_{min}}{2} \right) .ones(K, N)$

2) Let's initiate $K \times N$ Channel Gain Matrix G such that $G(k, n) = \frac{N_0}{|h_k[n]|^2} \quad \forall k \in \{1, \dots, K\}, \forall n \in \{1, \dots, N\}$

3) Define $K \times N$ data rate matrix from channel capacity theorem $C(K, N) \triangleq B \cdot \log_2(1 + 10^{(WFL - G)})$ by Eq. (2)

Procedure:

Comment: The loop is terminated only if the difference between channel capacity and the number of data packets to be transmitted is very little (less than the ϵ value).

while $(|C(K, N) - D(K, N)| > \epsilon)$ do

if $C(K, N) < D(K, N)$ then

$WFL_{min} \leftarrow \frac{WFL_{max} + WFL_{min}}{2}$

else

$WFL_{max} \leftarrow \frac{WFL_{max} + WFL_{min}}{2}$

end if

$WFL \leftarrow \left(\frac{WFL_{max} + WFL_{min}}{2} \right) .ones(K, N)$

Comment: We calculate channel capacity value for each channel in each time slot by checking whether the difference between waterfilling level and channel gain is positive or not $C(K, N) \leftarrow B \cdot \log_2(1 + 10^{\min(WFL - G, 0)})$

endwhile

Output: Return WFL

Figure 3. Omniscient, Offline Waterfilling Algorithm (Şekil 3. Herşeyi bilen, Çevrimdışı Su Doldurma Algoritması)

4.2. History-based Online Waterfilling Algorithm

In this subsection, we suggest a history-based online waterfilling policy by using the history for the problem. Figure 4 shows three different 10-TS scenarios with 1-TS, 2-TS, 3-TS long histories.

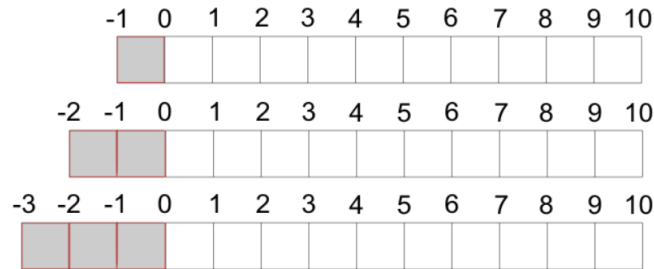


Figure 4. In $T=10$ TS time horizon, white-color TSs show online scenarios while dark-colored ones show the histories. (Şekil 4. $T=10$ TS zaman ufkunda açık renkli zaman dilimleri çevrimiçi senaryoları gösterirken koyu renkli olanlar geçmişleri gösterir.)

The history-based online algorithm uses the history partially in order to determine a waterfilling level based on that part of history. Then, the online policy applies this history-based determined waterfilling level to transmit data over the time horizon of the problem. Thus, the online policy aims to make as efficient data transmission as the offline policy by making inference from the channel conditions in the past time slots.

Algorithm 2 History-based Online Waterfilling Policy

Input: $D(K, N)$ is the number of data packets to be transmitted via all K channels over a time horizon of N TSs.

$h_k[n]$ is the channel gain k in the time slot n .

Initialize: WFL_{max} , WFL_{min} and ϵ are maximum waterfilling level, minimum waterfilling level and the tolerance, respectively.

1) Let's initiate $K \times M$ Water-Filling Level Matrix as

$WFL \leftarrow \left(\frac{WFL_{max} + WFL_{min}}{2} \right) .ones(K, M)$

2) Let's initiate $K \times N$ Channel Gain Matrix G such that $G(k, n) = \frac{N_0}{|h_k[n]|^2} \quad \forall k \in \{1, \dots, K\}, \forall n \in \{1, \dots, N\}$

3) Define $K \times N$ data rate matrix from channel capacity theorem $C(K, N) \triangleq B \cdot \log_2(1 + 10^{(WFL - G)})$ by Eq. (2)

Procedure:

Comment: The loop is terminated only if the difference between channel capacity and the number of data packets to be transmitted is very little (less than the ϵ value).

while $(|C(K, M) - D(K, N)| > \epsilon)$ do

if $C(K, M) < D(K, N) * \frac{M}{N}$ then

$WFL_{min} \leftarrow \frac{WFL_{max} + WFL_{min}}{2}$

else

$WFL_{max} \leftarrow \frac{WFL_{max} + WFL_{min}}{2}$

end if

$WFL \leftarrow \left(\frac{WFL_{max} + WFL_{min}}{2} \right) .ones(K, M)$

Comment: We calculate channel capacity value for each channel in each time slot by checking whether the difference between waterfilling level and channel gain is positive or not $C(K, M) \leftarrow B \cdot \log_2(1 + 10^{\min(WFL - G, 0)})$

endwhile

Comment: For obtaining a robust algorithm, we need to consider a history of the past M time slots instead of waiting for all N time slots ($M \leq N$). Therefore, the WFL obtained from all time slots will be applied more comfortably.

Output: Return WFL

Figure 5. History-based Online Waterfilling Algorithm (Şekil 5. Geçmiş-temelli Çevrimiçi Su Doldurma Algoritması)

5. Numerical Results

In this paper, we consider energy-efficient data transmission problem over multiple fading channels in a communications system [7]. In this section, the relative performance of the history-based online waterfilling policy to offline waterfilling policy is evaluated in two ways: 1) Various Time Horizons, namely, $T = 1000$ TSs and $T = 10000$ TSs. 2) Different fading channels, namely, Rayleigh fading, Nakagami fading with $m = 2$, Rician fading, Weibull fading with $k = 1.5$.

We investigate the ratio of total transmission energy by History-based Online Waterfilling Policy to that of Optimal Offline Waterfilling Policy.

The history of Online Waterfilling Policy is given in terms of time slices each of which lasts 10% of time horizon. For example, for $T=1000$ TS, 4 time-slice history has a length of $4 \cdot 10\% \cdot 1000$ TSs= 400 TSs whereas 4 time-slice history has a length of $4 \cdot 10\% \cdot 10000$ TSs= 4000 TSs for $T=10000$ TSs.

5.1. Time Horizon

In this subsection, the relative performance of the online and offline policies is evaluated for various fading channels over the time horizons of $T = 1000$ TSs and $T = 10000$ TSs.

5.1.1. Rayleigh Fading Channel

In this subsection, the relative performance of the online and offline policies is evaluated for Rayleigh fading channels over various time horizons, namely, $T = 1000$ TSs and $T = 10000$ TSs.

Figure 6 represents the general trends of ratio of total transmission energy by History-based Online Waterfilling Policy to that of Optimal Offline Waterfilling Policy for Rayleigh fading channel. Table 1 shows more precise results than Figure 6.

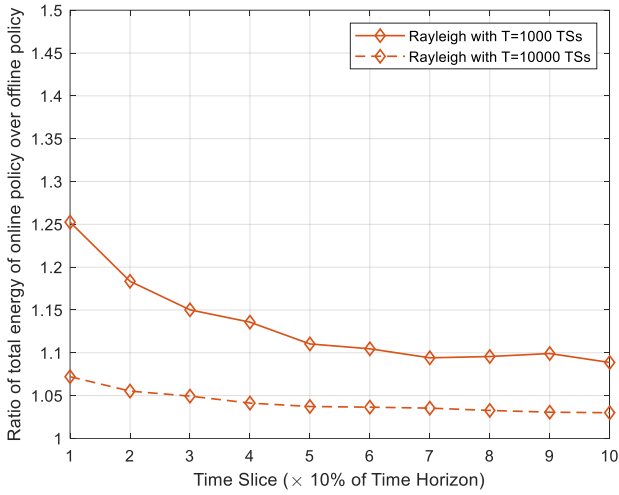


Figure 6. Ratio of total transmission energy by History-based Online Waterfilling Policy to Offline Waterfilling Policy for Rayleigh Fading Channel. (Şekil 6. Rayleigh Gölgeleli kanal için Geçmiş-temelli Çevrimiçi Su-doldurma Politikası'nın toplam iletim enerjisinin Çevrimdışı Su-doldurma Politikası'na oranı.)

From Figure 6, it can be observed that the difference between total energy consumption of history-based online waterfilling policy from offline waterfilling policy becomes nearly 3 times more in case of $T = 1000$ TS than the case of $T = 10000$ TS.

Table 1. This table presents ratio of total transmission energy by History-based Online Waterfilling Policy to that of Optimal Offline Waterfilling Policy for Rayleigh fading channel. $C=1$ shows $T=1000$ TS case whereas $C=2$ shows $T=10000$ TS case. (Tablo 1. Bu tablo, Rayleigh gölgeleli kanal için Geçmiş-temelli Çevrimiçi Su-doldurma Politikası'nın toplam iletim enerjisinin Çevrimdışı Su-doldurma Politikası'ninkine oranı. $C=1$, $T=1000$ TS durumunu gösterirken; $C=2$, $T=10000$ TS durumunu gösterir).

C	0.1T	0.2T	0.3T	0.4T	0.5T	0.6T	0.7T	0.8T	0.9T	1.0T
1	1.25	1.18	1.15	1.14	1.11	1.10	1.10	1.10	1.10	1.09
2	1.07	1.06	1.05	1.04	1.04	1.04	1.04	1.03	1.03	1.03

From Table 1, by using a history of 1 time slice, (10% of time horizon, i.e., 0.1 T), the online policy consumes 25% more energy than the offline policy in $T = 1000$ TS whereas it consumes just 7% more energy than the offline algorithm in $T = 10000$ TS.

By using a history of 5 time slices, (half of time horizon, i.e., 0.5 T), the online policy consumes 11% more energy than the offline policy in $T = 1000$ TS whereas it consumes just 4% more energy than the offline algorithm in $T = 10000$ TS.

Using a history of larger than 6 time slices does not make a big difference (just 1%) for neither the online and offline policies.

5.1.2. Nakagami Fading Channel

In this subsection, the relative performance of the online and offline policies is evaluated for Nakagami fading channels with $m = 2$ over various time horizons, namely, $T = 1000$ TSs and $T = 10000$ TSs.

Figure 7 represents the general trends of ratio of total transmission energy by History-based Online Waterfilling Policy to that of Optimal Offline Waterfilling Policy for Nakagami fading channel. Table 2 shows more precise results than Figure 7.

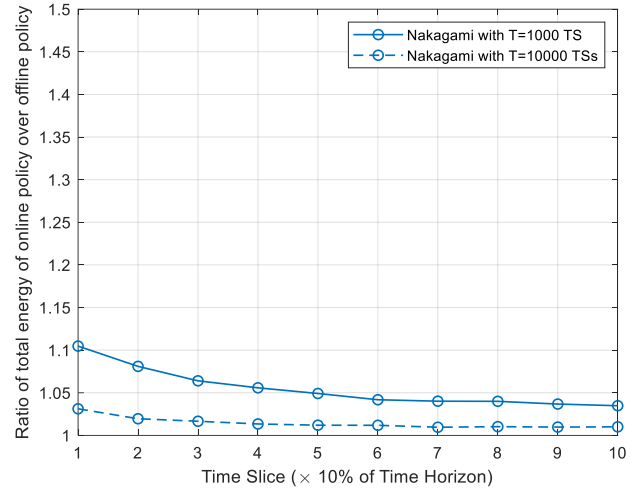


Figure 7. Ratio of total transmission energy by History-based Online Waterfilling Policy to Offline Waterfilling Policy for Nakagami Fading Channel (Şekil 7. Nakagami Gölgeleli kanal için Geçmiş-temelli Çevrimiçi Su-doldurma Politikası'nın toplam iletim enerjisinin Çevrimdışı Su-doldurma Politikası'na oranı.)

From Figure 7, it can be observed that the difference between total energy consumption of history-based online waterfilling policy from offline waterfilling policy becomes nearly 4 times more in case of $T = 1000$ TS than the case of $T = 10000$ TS.

Table 2. This table presents ratio of total transmission energy by History-based Online Waterfilling Policy to that of Optimal Offline Waterfilling Policy for Nakagami Fading Channel. $C=1$ shows $T=1000$ TS case whereas $C=2$ shows $T=10000$ TS case. (Tablo 2. Bu tablo, Nakagami gölgeleli kanal için Geçmiş-temelli Çevrimiçi Su-doldurma Politikası'nın toplam iletim enerjisinin Çevrimdışı Su-doldurma Politikası'ninkine oranı. $C=1$, $T=1000$ TS durumunu gösterirken; $C=2$, $T=10000$ TS durumunu gösterir).

C	0.1T	0.2T	0.3T	0.4T	0.5T	0.6T	0.7T	0.8T	0.9T	1.0T
1	1.10	1.08	1.07	1.06	1.05	1.04	1.04	1.04	1.04	1.03
2	1.03	1.02	1.02	1.01	1.01	1.01	1.01	1.01	1.01	1.01

From Table 2, by using a history of 1 time slice, (10% of time horizon, i.e., 0.1 T), the online policy consumes 10% more energy than the offline policy in $T = 1000$ TS whereas it consumes just 3% more energy than the offline algorithm in $T = 10000$ TS.

By using a history of 5 time slices, (half of time horizon, i.e., 0.5 T), the online policy consumes 5% more energy than the offline policy in $T = 1000$ TS whereas it consumes just 1% more energy than the offline algorithm in $T = 10000$ TS.

Using a history of larger than 6 time slices does not make a big difference (just 1%) for neither the online and offline policies.

5.1.3. Rician Fading Channel

In this subsection, the relative performance of the online and offline policies is evaluated for Rician fading channels with over various time horizons, namely, $T = 1000$ TSs and $T = 10000$ TSs.

Figure 8 represents the general trends of ratio of total transmission energy by History-based Online Waterfilling Policy to that of Optimal Offline Waterfilling Policy for Rician fading channel. Table 3 shows more precise results than Figure 8.

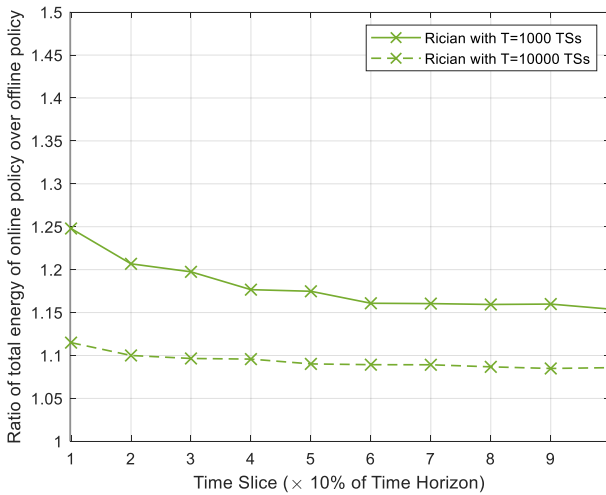


Figure 8. Ratio of total transmission energy by History-based Online Waterfilling Policy to Offline Waterfilling Policy for Rician Fading Channel. (Şekil 8. Rician Gölgele kanal için Geçmiş-temelli Çevrimiçi Su-doldurma Politikası'nun toplam iletim enerjisinin Çevrimdışı Su-doldurma Politikası'na oranı.)

From Figure 8, it can be observed that the difference between total energy consumption of history-based online waterfilling policy from offline waterfilling policy becomes nearly 2 times more in case of $T = 1000$ TS than the case of $T = 10000$ TS.

Table 3. This table presents ratio of total transmission energy by History-based Online Waterfilling Policy to that of Optimal Offline Waterfilling Policy for Rician Fading Channel. $C=1$ shows $T=1000$ TS case whereas $C=2$ shows $T=10000$ TS case. (Tablo 3. Bu tablo, Rician gölgele kanal için Geçmiş-temelli Çevrimiçi Su-doldurma Politikası'nun toplam iletim enerjisinin Çevrimdışı Su-doldurma Politikası'nunkine oranı. $C=1$, $T=1000$ TS durumunu gösterirken; $C=2$, $T=10000$ TS durumunu gösterir).

C	0.1T	0.2T	0.3T	0.4T	0.5T	0.6T	0.7T	0.8T	0.9T	1.0T
1	1.25	1.21	1.20	1.18	1.17	1.16	1.16	1.16	1.16	1.15
2	1.12	1.10	1.10	1.10	1.09	1.09	1.09	1.09	1.08	1.08

From Table 3, by using a history of 1 time slice, (10% of time horizon, i.e., 0.1 T), the online policy consumes 25% more energy than the offline policy in $T = 1000$ TS whereas it consumes just 12% more energy than the offline algorithm in $T = 10000$ TS.

By using a history of 5 time slices, (half of time horizon, i.e., 0.5 T), the online policy consumes 17% more energy than the offline policy in $T = 1000$ TS whereas it consumes just 9% more energy than the offline algorithm in $T = 10000$ TS.

Using a history of larger than 6 time slices does not make a big difference (just 1%) for neither the online and offline policies.

5.1.4. Weibull Fading Channel

In this subsection, the relative performance of the online and offline policies is evaluated for Weibull fading channels with $k = 1.5$ over various time horizons, namely, $T = 1000$ TSs and $T = 10000$ TSs.

Figure 9 represents the general trends of ratio of total transmission energy by History-based Online Waterfilling Policy to that of Optimal Offline Waterfilling Policy for Rayleigh fading channel. Table 4 shows more precise result than Figure 9.

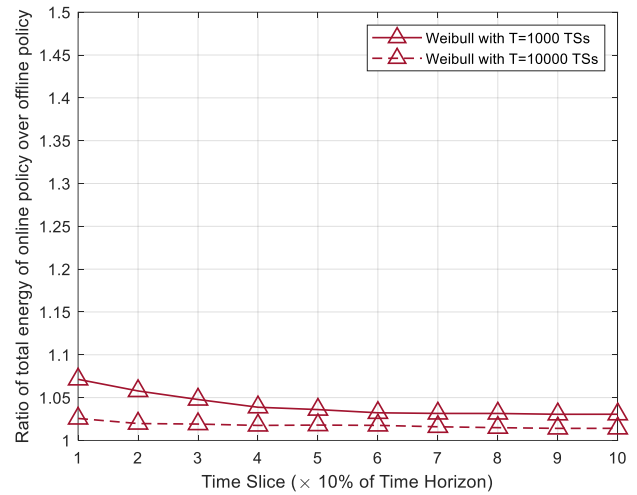


Figure 9. Ratio of total transmission energy by History-based Online Waterfilling Policy to Offline Waterfilling Policy for Weibull Fading Channel. (Şekil 9. Weibull Gölgele kanal için Geçmiş-temelli Çevrimiçi Su-doldurma Politikası'nun toplam iletim enerjisinin Çevrimdışı Su-doldurma Politikası'na oranı.)

From Figure 9, it can be observed that the difference between total energy consumption of history-based online waterfilling policy from offline waterfilling policy becomes nearly 2 times more in case of $T = 1000$ TS than the case of $T = 10000$ TS.

Table 4. This table presents ratio of total transmission energy by History-based Online Waterfilling Policy to that of Optimal Offline Waterfilling Policy for Weibull Fading Channel. $C=1$ shows $T=1000$ TS case whereas $C=2$ shows $T=10000$ TS case. (Tablo 4. Bu tablo, Weibull gölgele kanal için Geçmiş-temelli Çevrimiçi Su-doldurma Politikası'nun toplam iletim enerjisinin Çevrimdışı Su-doldurma Politikası'nunkine oranı. $C=1$, $T=1000$ TS durumunu gösterirken; $C=2$, $T=10000$ TS durumunu gösterir).

C	0.1T	0.2T	0.3T	0.4T	0.5T	0.6T	0.7T	0.8T	0.9T	1.0T
1	1.07	1.06	1.05	1.04	1.04	1.03	1.03	1.03	1.03	1.03
2	1.03	1.02	1.02	1.02	1.02	1.02	1.02	1.01	1.01	1.01

From Table 4, by using a history of 1 time slice, (10% of time horizon, i.e., 0.1 T), the online policy consumes 7% more energy than the offline policy in $T = 1000$ TS whereas it consumes just 3% more energy than the offline algorithm in $T = 10000$ TS.

By using a history of 5 time slices, (half of time horizon, i.e., 0.5 T), the online policy consumes 4% more energy than the offline policy in $T = 1000$ TS whereas it consumes just 2% more energy than the offline algorithm in $T = 10000$ TS.

Using a history of larger than 6 time slices does not make a big difference (just 1%) for neither the online and offline policies.

5.2. Different Fading Channels

In this subsection, the relative performance of the online and offline policies is evaluated over the time horizons of $T = 1000$ TSs and $T = 10000$ TSs under various fading channels, namely, Rayleigh fading, Nakagami fading with $m = 2$, Rician fading, Weibull fading with $k = 1.5$.

5.2.1. $T=1000$ TS Time Horizon

In this subsection, the relative performance of the online and offline policies is evaluated for $T = 1000$ TSs time horizon under Rayleigh, Nakagami with $m = 2$, Rician, Weibull with $k = 1.5$ fading channels.

Figure 10 represents the general trends of ratio of total transmission energy by History-based Online Waterfilling Policy to that of Optimal Offline Waterfilling Policy over the time horizons of $T = 1000$ TSs.

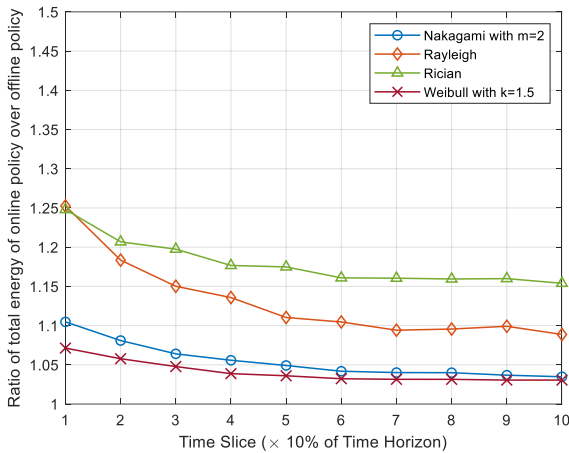


Figure 10. Ratio of total transmission energy by History-based Online Waterfilling Policy to that of Optimal Offline Waterfilling Policy for $T=1000$ TS. (Şekil 10. $T=1000$ TS için Geçmiş-temelli Çevrimiçi Su-doldurma Politikası'nın toplam iletim enerjisinin Çevrimdışı Su-doldurma Politikası'ninkine oranı.)

From Figure 10, it can be observed that the difference of total energy consumption of history-based online waterfilling policy from that of offline waterfilling policy becomes close to each other under Nakagami with $m=2$ and Weibull with $k=1.5$. Except 0.1-T history case, the difference is less than 0.02 (equals to 2% of total energy consumption of offline waterfilling policy). The consumed energy difference of online policy from the offline policy becomes nearly 25% less under Weibull fading than that under Nakagami fading if the history has a length less than $0.4T=400$ TSs. With a $T=1000$ -TS-length history, the difference of the online policy from the offline policy becomes equal to each other (3%) under both Nakagami with $m=2$ and Weibull with $k=1.5$.

Online Waterfilling Policy with a history of 0.1 $T=100$ TSs (10% of the time horizon) achieves the same performance (25% more energy consumption than the optimal offline policy) under Rayleigh and Rician fading channels. On the other hand, the consumed energy difference of online policy from the offline policy becomes nearly 40% less under Rayleigh fading than that under Rician fading if the history has a length greater than $0.5T=500$ TSs (half of the time horizon).

5.2.2. $T=10000$ TS Time Horizon

In this subsection, the relative performance of the online and offline policies is evaluated for $T = 10000$ TSs time horizon under Rayleigh, Nakagami with $m = 2$, Rician, Weibull with $k = 1.5$ fading channels.

Figure 11 represents the general trends of ratio of total transmission energy by History-based Online Waterfilling Policy to that of Optimal Offline Waterfilling Policy over the time horizons of $T = 10000$ TSs.

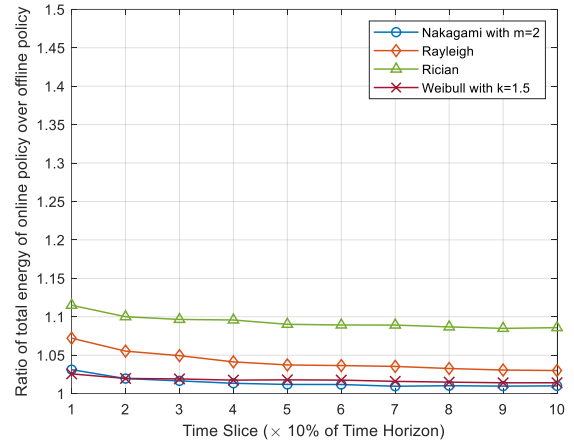


Figure 11. Ratio of total transmission energy by History-based Online Waterfilling Policy to that of Optimal Offline Waterfilling Policy for $T=10000$ TS. (Şekil 11. $T=10000$ TS için Geçmiş-temelli Çevrimiçi Su-doldurma Politikası'nın toplam iletim enerjisinin Çevrimdışı Su-doldurma Politikası'ninkine oranı.)

From Figure 11, it can be observed that the difference of total energy consumption of history-based online waterfilling policy from that of offline waterfilling policy becomes close to each other under Nakagami with $m=2$ and Weibull with $k=1.5$. The difference is less than 0.01 (equals to 1% of total energy consumption of offline waterfilling policy). The consumed energy difference of online policy from the offline policy becomes same under both Weibull fading and Nakagami fading if the history has a length less than $0.4T=4000$ TSs or greater than $0.7T=7000$ TSs. With a history longer than $0.8T=8000$ TSs, the difference of the online policy from the offline policy is subtle (1%) under Nakagami with $m=2$ and Weibull with $k=1.5$.

The consumed energy difference of the Online Waterfilling Policy from the offline policy becomes nearly 50% less under Rayleigh fading than that under Rician fading.

5.3. Further Remarks

The performance of the online waterfilling policy become close to the optimal offline policy under Weibull fading with $k=1.5$. The second best performance of the online policy is achieved under Nakagami Fading. The maximum energy difference of the online policy from the offline policy occurs under Rician fading. The performance difference mainly occurs due to the difference between stochastic distributions of the fading models.

Another remark is that as the time horizon increases, the length of history increases. Even if the ratio of history length to time horizon length is same (for example 0.1 T), the online policy achieves better performance for larger time horizons.

6. Conclusions and Future Works

In this paper, we consider a resource allocation problem over multiple fading channels. This problem is investigated in both offline and online manner. Firstly, we consider the problem over the whole multiple fading channels altogether in offline manner. We search for an optimal solution for this problem by considering an offline waterfilling algorithm. Then, we consider the problem over the whole multiple fading channels altogether in online manner. We look for an optimal solution for this problem over all fading channels. Then, we propose a history-based online waterfilling algorithm.

The numerical results demonstrate the history-based online waterfilling algorithm shows close performance to offline waterfilling algorithm especially for longer time horizons and by using larger portions of history.

In the future work, the problem can be investigated by adding different constraints such as delay, jitter, etc. These variations of the problem will require different perspectives to obtain an efficient and robust solution so different approaches should be applied for these variations of the problem.

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