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Wave Solution Analysis of a Nonlinear Mathematical Model on Fluid Mechanics

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Abstract

This study obtains some wave solutions of the B-type Kadomtsev Petviashvili equation by applying the modified exponential function method (MEFM). Thanks to this method, the exact solutions of the non-linear partial differential equations will be obtained and there will be an opportunity to examine the physical structure of these solutions. Due to the nature of MEFM, two different cases are presented here that have been analyzed to obtain more solutions in this structure. More wave solutions can be obtained by analyzing different situations. When the resulting solutions are analyzed, hyperbolic, trigonometric, and rational functions are observed. It has been checked whether the solution functions found with Wolfram Mathematica software provide the B type Kadomtsev Petviashvili equation and graphs simulating the wave solution behavior with the determined appropriate parameters are presented.

Keywords: Modified Exponential Function Method (MEFM); B-type Kadomtsev Petviashvili Equation; Wave Solutions.

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Akışkanlar Mekaniği Üzerine Doğrusal Olmayan Bir Matematiksel Modelin Dalga Çözümü Analizi

Öz

Bu çalışmada, modifiye edilmiş üstel fonksiyon metodu uygulanarak B tipi Kadomtsev Petviashvili denkleminin bazı dalga çözümleri elde edilmiştir. Modifiye edilmiş üstel fonksiyon yönteminin doğası gereği, bu yapıdaki çözümlerden daha fazla elde etmek için incelenilmiş olan iki farklı durum burada sunulmuştur. Farklı durumlar da incelenerek daha fazla dalga çözümü elde edilebilir. Ortaya çıkan çözümler analiz edildiğinde hiperbolik, trigonometrik ve rasyonel fonksiyonlar gözlemlenmiştir. Wolfram Mathematica yazılımı ile bulunan çözüm fonksiyonlarının B tipi Kadomtsev Petviashvili denklemini sağlayıp sağlamadığı kontrol edilmiş ve belirlenen uygun parametrelerle dalga çözümünün üç boyutlu kontur, yoğunluk ve iki boyutlu grafiklerin analizi sunulmuştur.

Anahtar Kelimeler: Geliştirilmiş Üstel Fonksiyon Metodu (GÜFM); B tipi Kadomtsev Petviashvili; Dalga Çözümleri.

1. Introduction

All events encountered in natural and applied sciences such as physics, engineering, health, etc., are represented by mathematical models. These models are generally stated in nonlinear partial differential equations (NPDE). Therefore, it is important to obtain the solutions to such equations. There are various methods to investigate the solutions of such equations in scientific studies in the literature. Some of these methods are the modified extended tanh-function method [1], the generalized tanh function method [2], the trial equation method [3-5], the generalized Bernoulli sub-equation function method [6-8], the first integral method [9], the quintic B-spline collocation method [10-12], the modified exponential function method (MEFM) [13-17],

In this paper, the B type Kadomtsev Petviashvili equation [18-24] which marine scientists are using for oceanic investigation is considered as follows,

$$u_{xxxy} + a(u_x u_y)_x + (u_x + u_y + u_z)_t - (u_{xx} + u_{zz}) = 0, \tag{1}$$

where a is a real non-zero parameter. A mathematical model given as Eqn. (1) is encountered in fluid mechanics, a branch of physics [18-29].

In the second part of this study, method is introduced and detailed information is given about the steps of the process. NPDEs have been reduced to nonlinear ordinary differential equations (NODEs) to implement this method. In the third chapter, the solutions obtained by applying the determined method B type Kadomtsev Petviashvili Equation graphs simulating of these results are presented for two cases. In the conclusion part of the study, the obtained results are given.

2. Modified Exponential Function Method (MEFM)

In this section, the general form of the nonlinear mathematical model is as follows;

$$P\left(U,U_x,U_y,U_z,U_t,U_{xx},U_{xt},U_{yy},U_{xxxz},\cdots\right)=0, \tag{2}$$

where $U=U(x,y,z,t)$ is the function that is thought to provide the nonlinear mathematical model.

Step 1. Taking the independent variables given in Eqn. (1) into consideration, the wave transformation given below is considered,

$$U(x,y,z,t)=U(\xi),\ \xi=k\left(x+y+z-ct\right), \tag{3}$$

c represents the frequency of the wave, and k the height of the wave. If the necessary derivative terms in Eqn. (2) are arranged according to Eqn. (1) are obtained by using wave Eqn. (3) and written instead,

$$N\left(U,U',\left(U'\right)^2,U'',U''',\cdots\right)=0. \tag{4}$$

Step 2: According to MEFM, the default solution function of Eqn. (1) is as follows;

$$U(\xi)=\frac{\sum_{i=0}^{n}A_i\left[\exp\left(-\Omega(\xi)\right)\right]^i}{\sum_{j=0}^{m}B_j\left[\exp\left(-\Omega(\xi)\right)\right]^j}=\frac{A_0+A_1\exp\left(-\Omega\right)+\cdots+A_n\exp\left(n\left(-\Omega\right)\right)}{B_0+B_1\exp\left(-\Omega\right)+\cdots+B_m\exp\left(m\left(-\Omega\right)\right)}, \tag{5}$$

where $A_i,B_j,\left(0\le i\le n,0\le j\le m\right)$ are constants. The balance procedure determines the relationship between m and n, which are the upper bounds of the sum symbols in the method's solution Eqn. (5). By applying the balance procedure to Eqn. (4), the relationship between the constants m and n, which ensures the equivalence of the term containing the highest order derivative and the nonlinear term, is determined, and the general structure of the solution function is formed by giving values to these constants.

$$\Omega'(\xi)=\exp\left(-\Omega(\xi)\right)+\mu\exp\left(\Omega(\xi)\right)+\lambda, \tag{6}$$

when the Eqn. (6) is solved, the following families are obtained by He et al [13]:

Condition 1: If $\mu\neq 0,\ \lambda^2-4\mu>0,$

$$\Omega(\xi)=\ln\left(\frac{-\sqrt{\lambda^2-4\mu}}{2\mu}\tanh\left(\frac{\sqrt{\lambda^2-4\mu}}{2}(\xi+E)\right)-\frac{\lambda}{2\mu}\right). \tag{7}$$

Condition 2: If $\mu \neq 0,\ \lambda^2-4\mu<0,$

$$\Omega(\xi)=\ln\left(\frac{\sqrt{-\lambda^2+4\mu}}{2\mu}\tan\left(\frac{\sqrt{-\lambda^2+4\mu}}{2}(\xi+E)\right)-\frac{\lambda}{2\mu}\right). \tag{8}$$

Condition 3: If $\mu=0,\ \lambda\neq 0$ and $\lambda^2-4\mu>0,$

$$\Omega(\xi)=-\ln\left(\frac{\lambda}{\exp\left(\lambda(\xi+E)\right)-1}\right). \tag{9}$$

Condition 4: If $\mu\neq 0,\ \lambda\neq 0$ and $\lambda^2-4\mu=0,$

$$\Omega(\xi)=\ln\left(-\frac{2\lambda(\xi+E)+4}{\lambda^2(\xi+E)}\right). \tag{10}$$

Condition 5: If $\mu=0,\ \lambda=0$ and $\lambda^2-4\mu=0,$

$$\Omega(\xi)=\ln\left(\xi+E\right). \tag{11}$$

Step 3: After Eqn. (6) is solved, when Eqn. (5) is written in its place, an algebraic equation system consisting of coefficients is obtained. After this system of equations is solved with the Mathematica program, the relations between the coefficients of the solution function satisfying the Eqn. (4) are obtained. Therefore, the solution obtained in each case is checked and the traveling wave solution satisfying Eqn. (1) is found.

3. Application

When Eqn. (3) is applied to Eqn. (1) and after one integration with respect to ξ, the following nonlinear ordinary differential equation is get,

$$k^2U''' + ak\left(U'\right)^2-(3c+2)U'=0. \tag{12}$$

If $U'=V$ is Eqn. (12),

$$k^2V''+akV^2-(3c+2)V=0. \tag{13}$$

If the balance procedure is applied between the terms V'' and V^2 in Eqn. (13),

$$M+2=N. \tag{14}$$

If $M=1$ then $N=3$ is obtained from the Eqn. (14). Thus, the upper limits of the sum symbols in the sought solution function in Eqn. (5) are determined. Accordingly, the terms required in the Eqn. (15) are given.

$$\begin{aligned} V(\xi) &= \frac{\psi}{\varphi} = \frac{A_0 + A_1 e^{-\Omega(\xi)} + A_2 e^{-2\Omega(\xi)} + A_3 e^{-3\Omega(\xi)}}{B_0 + B_1 e^{-\Omega(\xi)}}, \\ V'(\xi) &= \frac{\psi'\varphi - \psi\varphi'}{\varphi^2}, \\ V''(\xi) &= \frac{\psi''\varphi^3 - \varphi^2\psi'\varphi' - (\psi\varphi'' + \psi'\varphi')\varphi^2 + 2(\psi')^2\psi\varphi}{\varphi^4}. \end{aligned} \tag{15}$$

Case 1:

$$A_0 = -\frac{6B_0 k\mu}{a},\ A_1 = -\frac{6k(B_0\lambda + B_1\mu)}{a},\ A_2 = -\frac{6k(B_1\lambda + B_0)}{a},\ A_3 = -\frac{6B_1 k}{a},$$

$$c = \frac{1}{3}\left(k^2(\lambda^2 - 4\mu) - 2\right).$$

By using these coefficients, solutions of the mathematical model are presented and analyzed by considering the previously mentioned family cases.

Family 1:

$$V_{1,1}(\xi) = \frac{6k\mu(\lambda^2 - 4\mu)}{a\left(\sqrt{\lambda^2 - 4\mu}\sinh\left(\frac{1}{2}\phi\right) + \lambda\cosh\left(\frac{1}{2}\phi\right)\right)^2}, \tag{16}$$

where $\left(\phi = \sqrt{\lambda^2 - 4\mu}\,(E + \xi)\right)$. Integrating Eqn. (16) with respect to ξ,

$$U_{1,1}(\xi) = \frac{3k\left(2\mu\sqrt{\lambda^2 - 4\mu}\sinh\left((\mathrm{E} + \xi)\sqrt{\lambda^2 - 4\mu}\right) + \lambda^3 - 4\lambda\mu\right)}{a\left(2\mu\cosh\left((\mathrm{E} + \xi)\sqrt{\lambda^2 - 4\mu}\right) + \lambda^2 - 2\mu\right)}, \tag{17}$$

is obtained.

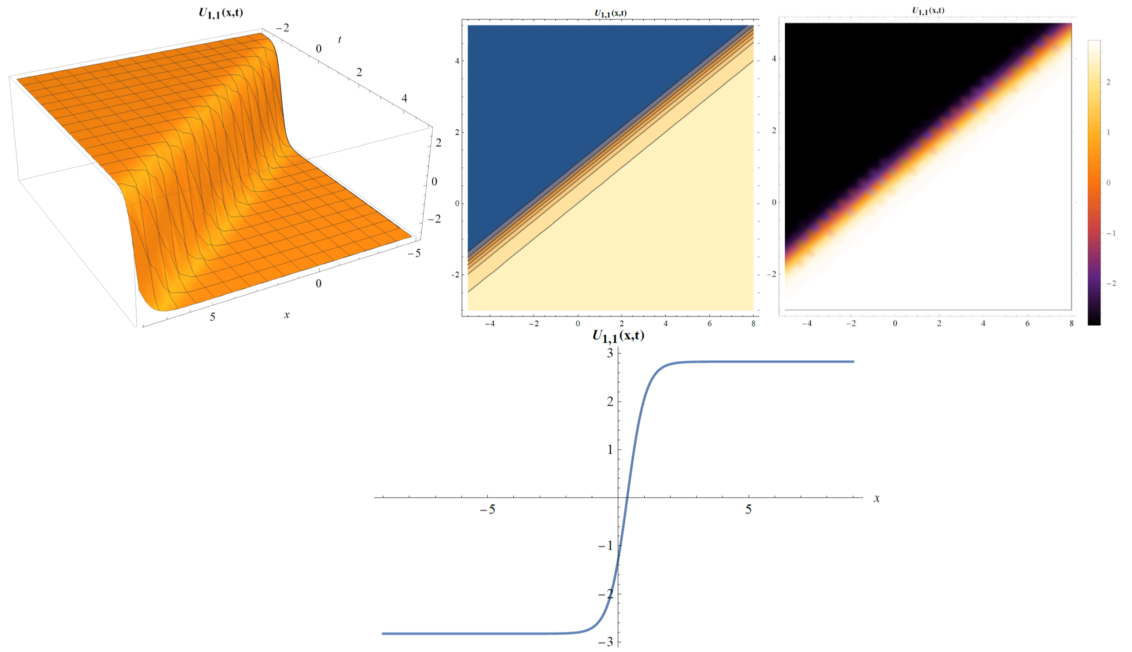


Figure 1: Graphs simulating the behavior of Eqn. (17) for the values $a=3$, $c=2$, $\mathrm{E}=0.82$, $k=1$, $\lambda=2\sqrt{3}$, $\mu=1$, $y=0.01$, $z=0.01$ and two-dimensional graph for $t=1$

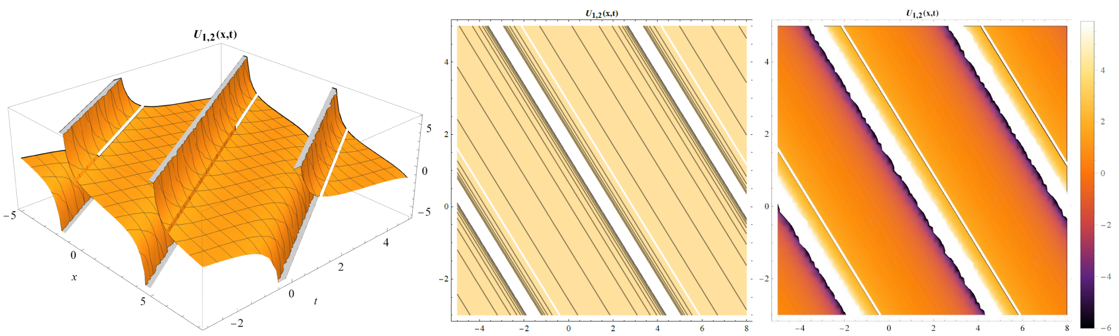
Family 2:

$$V_{1,2}(\xi)=\frac{6k\mu\left(\lambda^2-4\mu\right)}{a\left(\lambda\cos\left(\frac{1}{2}\phi\right)-\sqrt{4\mu-\lambda^2}\sin\left(\frac{1}{2}\phi\right)\right)^2}, \tag{18}$$

where $\left(\phi=\sqrt{-\lambda^2+4\mu}\left(E+\xi\right)\right)$. Integrating Eqn. (18) with respect to ξ,

$$U_{1,2}(\xi)=\frac{3k\left(-2\mu\sqrt{4\mu-\lambda^2}\sin\left((\mathrm{E}+\xi)\sqrt{4\mu-\lambda^2}\right)+\lambda^3-4\lambda\mu\right)}{a\left(2\mu\cos\left((\mathrm{E}+\xi)\sqrt{4\mu-\lambda^2}\right)+\lambda^2-2\mu\right)}, \tag{19}$$

is obtained.



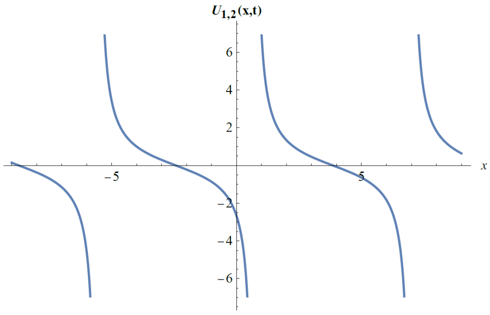


Figure 2: Graphs simulating the behavior of Eqn. (19) for the values $a=3$, $c=-1$, $\mathrm{E}=0.82$, $k=1$, $\lambda=\sqrt{11}$, $\mu=3$, $y=0.01$, $z=0.01$, and two-dimensional graph for $t=1$

Family 3:

$$V_{1,3}(\xi)=\frac{3k\lambda^2}{a-a\cosh(\lambda(\mathrm{E}+\xi))}. \tag{20}$$

Integrating Eqn. (20) with respect to ξ,

$$U_{1,3}(\xi)=\frac{3k\lambda\coth\left(\frac{1}{2}\lambda(\mathrm{E}+\xi)\right)}{a}. \tag{21}$$

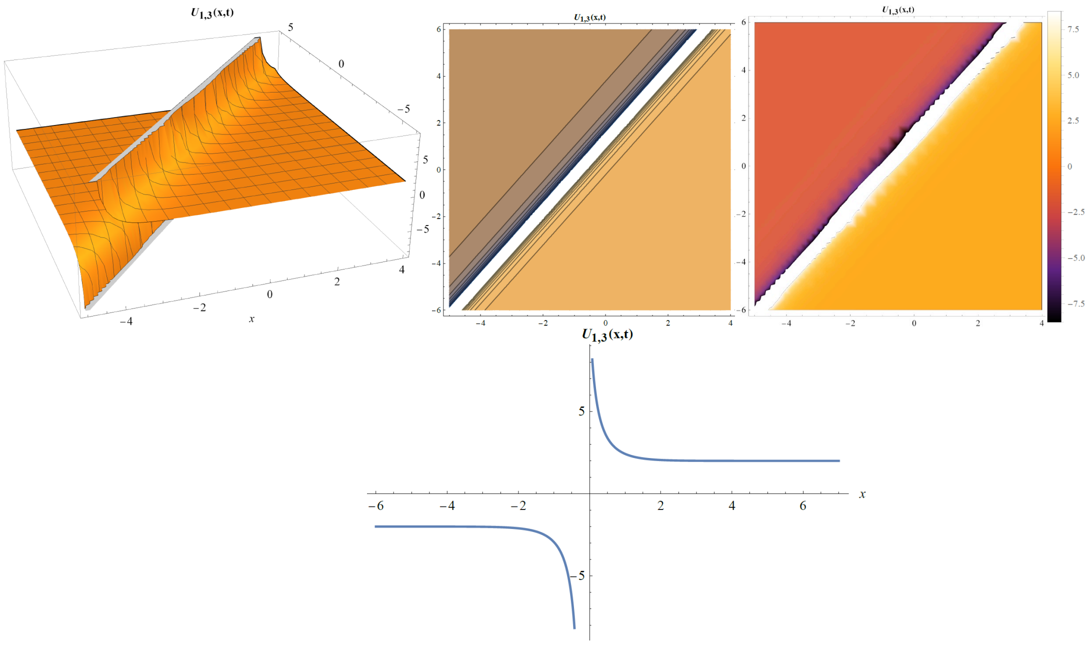


Figure 3: Graphs simulating the behavior of Eqn. (21) for the values $a=3$, $c=0.\bar{6}$, $\mathrm{E}=0.82$, $k=1$, $\lambda=2$, $\mu=0$, $y=0.01$, $z=0.01$ and two-dimensional graph for $t=1$

Family 4:

$$V_{1,4}(\xi)=\frac{3k\left(\lambda^2\left(1-\dfrac{4}{(\lambda(\mathrm{E}+\xi)+2)^2}\right)-4\mu\right)}{2a}. \tag{22}$$

Integrating Eqn. (22) with respect to ξ,

$$U_{1,4}(\xi)=\frac{3k\left(\lambda(\lambda(\mathrm{E}+\xi)+2)+\dfrac{4\lambda}{\lambda(\mathrm{E}+\xi)+2}-4\mu\xi\right)}{2a}. \tag{23}$$

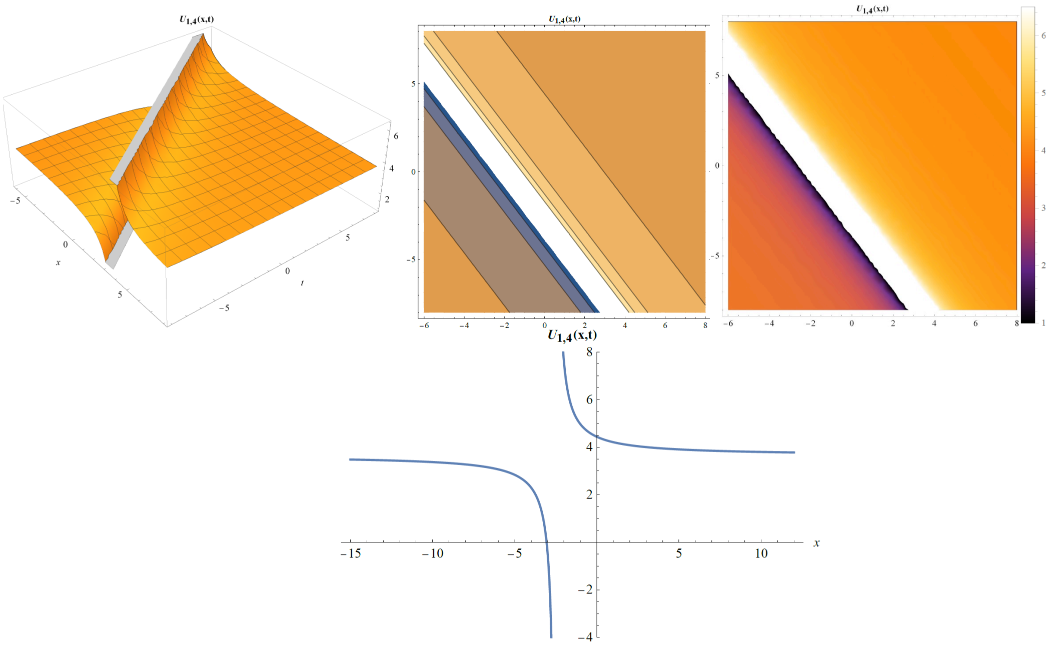


Figure 4: Graphs simulating the behavior of Eqn. (23) for the values $a=3$, $c=-0.\bar{6}$, $\mathrm{E}=0.82$, $\lambda=2$, $k=1$, $\mu=1$, $y=0.01$, $z=0.01$ and two-dimensional graph for $t=1$

Family 5:

$$V_{1,5}(\xi)=-\frac{6k}{a(\mathrm{E}+\xi)^2}. \tag{24}$$

Integrating Eqn. (24) with respect to ξ,

$$U_{1,5}(\xi)=\frac{6k}{a(\mathrm{E}+\xi)}, \tag{25}$$

is obtained.

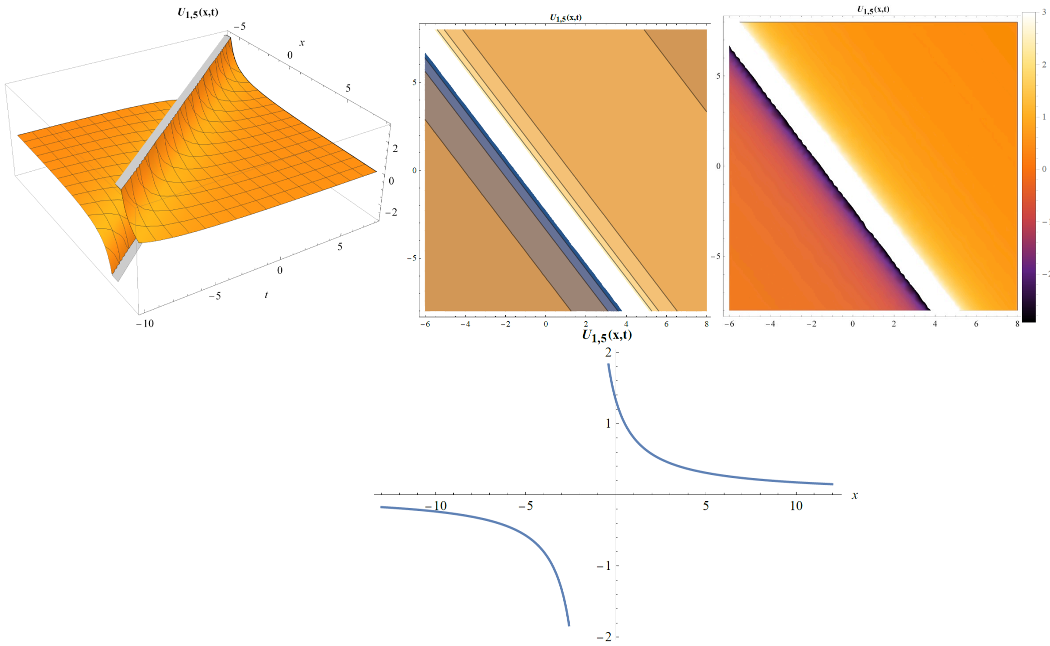


Figure 5: Graphs simulating the behavior of Eqn. (25) for the values $a=3$, $c=-0.\bar{6}$, $\mathrm{E}=0.82$, $\lambda=0$, $k=1$, $\mu=0$, $y=0.01$, $z=0.01$ and two-dimensional graph for $t=1$

Case 2:

$$A_0=-\frac{B_0\left(3c+3k^2\lambda^2+2\right)}{2ak},\ A_1=-\frac{B_1\left(3c+3k^2\lambda^2+2\right)}{2ak}-\frac{6B_0k\lambda}{a},$$

$$A_2=-\frac{6k\left(B_1\lambda+B_0\right)}{a},\quad A_3=-\frac{6B_1k}{a},\quad \mu=\frac{3c+k^2\lambda^2+2}{4k^2}.$$

By using these coefficients, the traveling wave solutions of the nonlinear differential equation are presented and analyzed by considering the previously mentioned family cases.

Family 1:

$$V_{2,1}(\xi)=-\frac{3c+\dfrac{3k^2\left(\lambda\sqrt{\lambda^2-4\mu}\tanh\left(\frac{1}{2}(\mathrm{E}+\xi)\sqrt{\lambda^2-4\mu}\right)+\lambda^2-4\mu\right)^2}{\left(\sqrt{\lambda^2-4\mu}\tanh\left(\frac{1}{2}(\mathrm{E}+\xi)\sqrt{\lambda^2-4\mu}\right)+\lambda\right)^2}+2}{2ak},\tag{26}$$

where $\left(\beta=\sqrt{\lambda^2-4\mu}\ ,\tau=\tanh\left(\frac{1}{2}\beta\left(E+\xi\right)\right)\right)$. Integrating Eqn. (26) with respect to ξ,

$$U_{2,1}(\xi)=-\frac{\beta\left(2(3c+2)\mu\xi+3\lambda k^2\left(\lambda^2-8\mu\right)\right)\tau+2(3c+2)\lambda\mu\xi+12k^2\mu\tanh^{-1}\left(\tau\right)\left(\beta^2\tau+\lambda\beta\right)+3k^2\beta^4}{4ak\mu\left(\beta\tau+\lambda\right)},\tag{27}$$

is obtained.

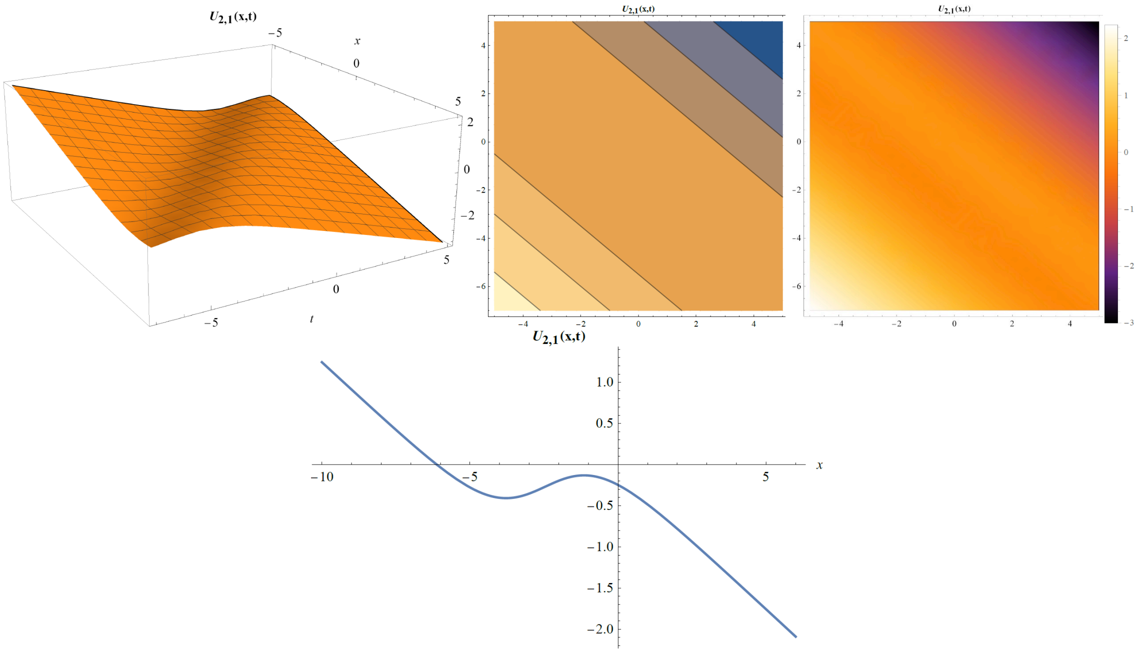


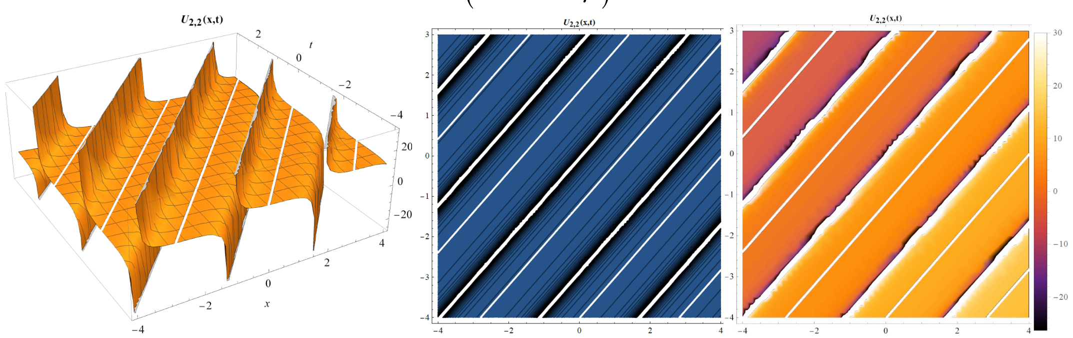
Figure 6: Graphs simulating the behavior of Eqn. (27) for the values $a=3$, $c=-1$, $\mathrm{E}=0.75$, $\lambda=3$, $k=1$, $\mu=2$, $y=0.01$, $z=0.01$ and two-dimensional graph for $t=1$

Family 2:

$$V_{2,2}(\xi)=-\frac{3c+\dfrac{3k^2\left(-\lambda\sqrt{4\mu-\lambda^2}\tan\left(\frac{1}{2}(\mathrm{E}+\xi)\sqrt{4\mu-\lambda^2}\right)+\lambda^2-4\mu\right)^2}{\left(\lambda-\sqrt{4\mu-\lambda^2}\tan\left(\frac{1}{2}(\mathrm{E}+\xi)\sqrt{4\mu-\lambda^2}\right)\right)^2}+2}{2ak}, \tag{28}$$

where $\left(\varsigma=\mu\tau\sin(q\tau),\ \omega=2\mu\cos(q\tau),\ \tau=\sqrt{-\lambda^2+4\mu},\ q=(\mathrm{E}+\xi)\right)$. Integrating Eqn. (28) with respect to ξ,

$$U_{2,2}(\xi)=-\frac{\omega\left((3c+2)\xi+3k^2q\left(-\tau^2\right)\right)+(3c+2)\xi\left(\lambda^2-2\mu\right)+3k^2\left(-\tau^2\right)\left(\lambda^2q-2\mu q-2\lambda\right)+12k^2\varsigma}{2ak\left(\omega+\lambda^2-2\mu\right)}. \tag{29}$$



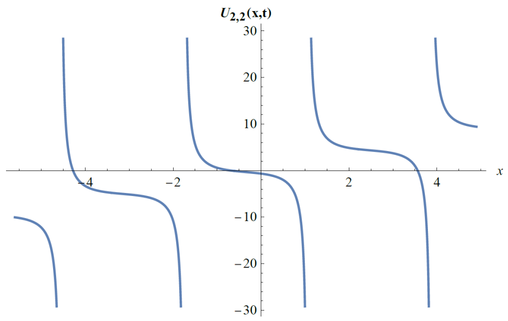


Figure 7: Graphs simulating the behavior of Eqn. (29) for the values $a=3$, $c=1$, $\mathrm{E}=0.75$, $\lambda=3$, $k=1$, $\mu=3.5$, $y=0.01$, $z=0.01$ and two-dimensional graph for $t=1$

Family 3:

$$V_{2,3}(\xi)=-\frac{3c+3k^2\lambda^2\coth^2\left(\frac{1}{2}\lambda(\mathrm{E}+\xi)\right)+2}{2ak}. \tag{30}$$

Integrating Eqn. (30) with respect to ξ,

$$U_{2,3}(\xi)=\frac{6k^2\lambda\left(\coth\left(\frac{1}{2}\lambda(\mathrm{E}+\xi)\right)-\left(\frac{1}{2}\lambda(\mathrm{E}+\xi)\right)\right)-(3c+2)\xi}{2ak}. \tag{31}$$

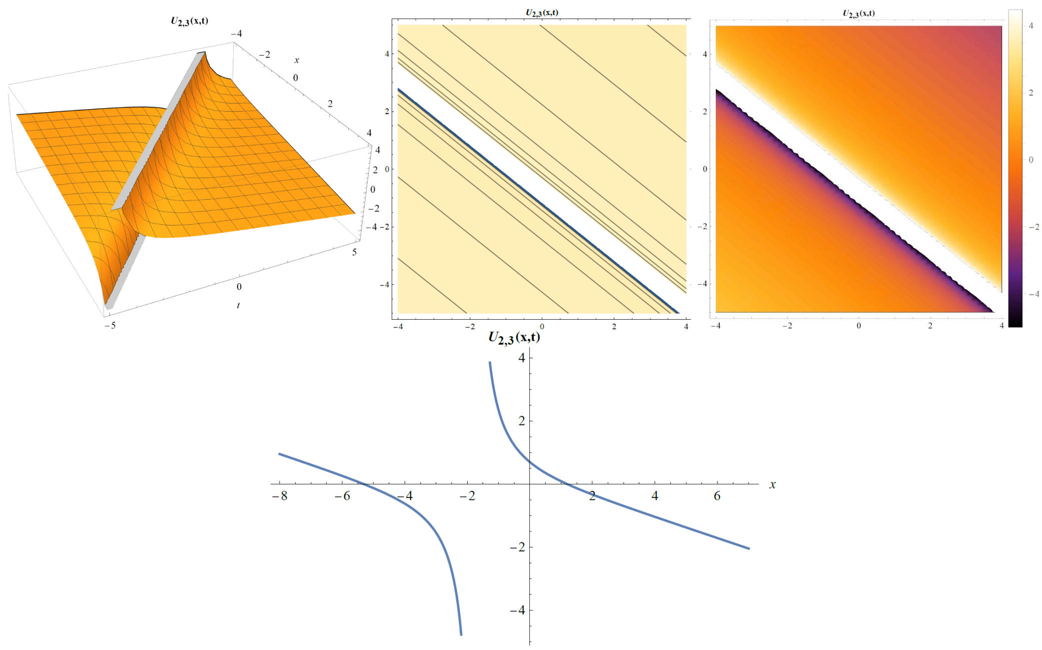


Figure 8: Graphs simulating the behavior of Eqn. (31) for the values $a=3$, $c=-1$, $\mathrm{E}=0.75$, $\lambda=1$, $k=1$, $\mu=0$, $y=0.01$, $z=0.01$ and two-dimensional graph for $t=1$

Family 4:

$$V_{2,4}(\xi) = -\frac{3c + \dfrac{12k^2\lambda^2}{(\lambda(\mathrm{E}+\xi)+2)^2} + 2}{2ak}. \tag{32}$$

Integrating Eqn. (32) with respect to ξ,

$$U_{2,4}(\xi) = \frac{6k\lambda}{a(\lambda(\mathrm{E}+\xi)+2)} - \frac{(3c+2)\xi}{2ak}. \tag{33}$$

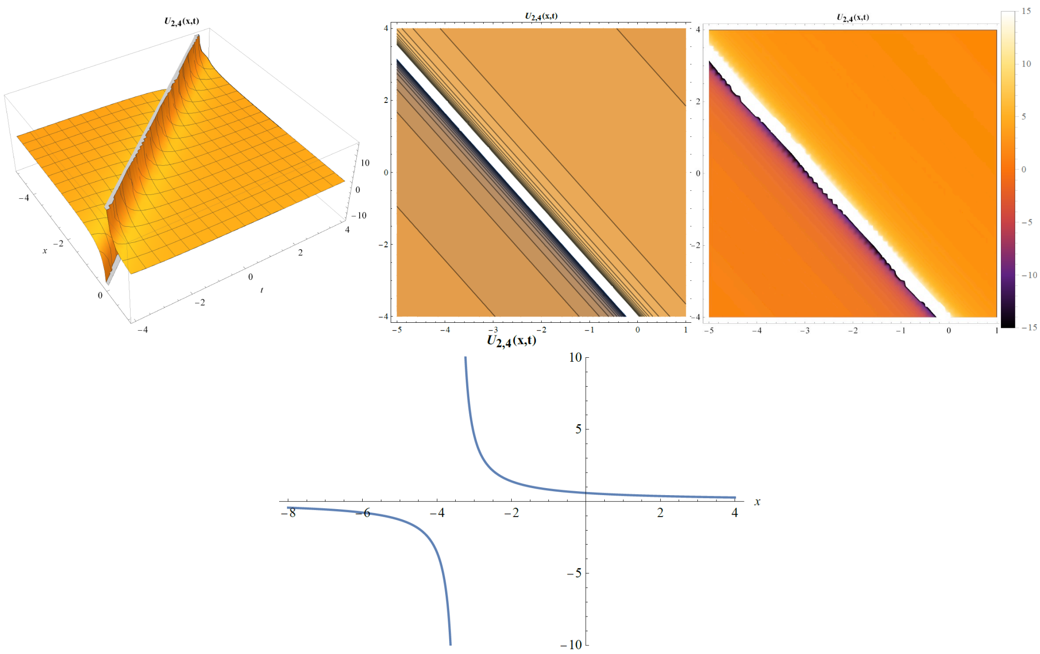


Figure 9: Graphs simulating the behavior of Eqn. (33) for the values $a=3$, $c=-0.\overline{6}$, $\mathrm{E}=0.75$, $\lambda=1$, $k=1$, $\mu=0.25$, $y=0.01$, $z=0.01$ and two-dimensional graph for $t=1$

Family 5:

$$V_{2,5}(\xi) = -\frac{3c + \dfrac{12k^2}{(\mathrm{E}+\xi)^2} + 2}{2ak}. \tag{34}$$

Integrating Eqn. (34) with respect to ξ,

$$U_{2,5}(\xi) = \frac{6k}{a(\mathrm{E}+\xi)} - \frac{(3c+2)\xi}{2ak}, \tag{35}$$

is obtained.

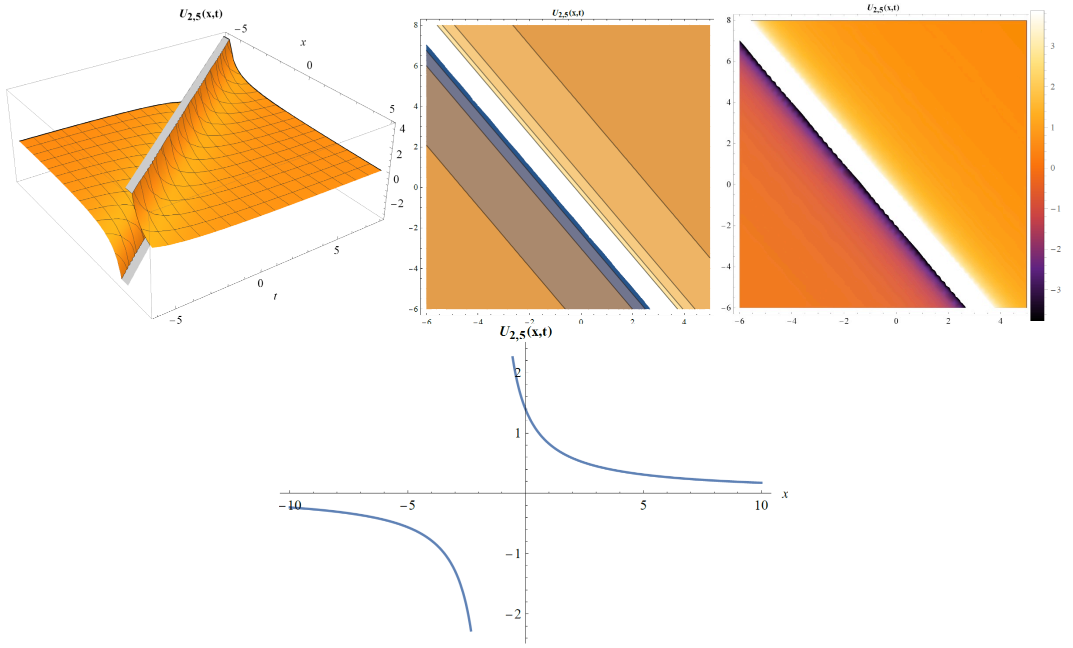


Figure 10: Graphs simulating the behavior of Eqn. (35) for the values $a=3$, $c=-0.\bar{6}$, $\mathrm{E}=0.75$, $\lambda=0$, $k=1$, $\mu=0$, $y=0.01, z=0.01$ and two-dimensional graph for $t=1$

4. Conclusion

This article has been applied to the B-type Kadomtsev Petviashvili equation representing fluid mechanics, which is a nonlinear mathematical model of the developed exponential function method. In this research, it was seen that the analytical solutions found under the conditions were obtained according to the method provided by the Eqn. (1). When the mobile solution functions obtained were evaluated, it was determined that they were soliton, periodic and rational functions. All calculations related to the method and solution function graphs representing the nonlinear mathematical model were made using the Mathematica 12.0.0 software program. It has been observed that two and three-dimensional graphs obtained by determining the appropriate parameters are suitable for the physical behavior of wave solutions. In addition, contour graphics and density graphics were found with the help of software program for analytical solutions. It has been observed that hyperbolic functions are obtained when the solutions of equations with similar structures are investigated with other solution methods in the literature [18-24]. Using this method, different traveling wave solutions can be obtained if more cases are investigated, and different coefficient values are taken. When the obtained solutions are analyzed, it can be stated that MEFM is effective method for finding the traveling wave solutions of nonlinear partial differential equations. The resulting solution functions help us learn more about the physical phenomenon that represented fluid mechanics.

References

[1] Elwakil, S.A., El-Labany, S.K., Zahran, M.A., Sabry, R., *Modified extended tanh-function method for solving nonlinear partial differential equations*, Physics Letters A, 299 (2-3), 179-188, 2002.

[2] Zheng, X., Chen, Y., Zhang, H., *Generalized extended tanh-function method and its application to (1+1)-dimensional dispersive long wave equation*, Physics Letters A, 311 (2-3), 145-157, 2003.

[3] Liu, C.S., *Trial equation method to nonlinear evolution equations with rank inhomogeneous: mathematical discussions and its applications*, Communications in Theoretical Physics, 45 (2), 219-223, 2006.

[4] Bulut, H., Baskonus, H.M., Pandir Y., *The modified trial equation method for fractional wave equation and time fractional generalized Burgers equation*, In Abstract and Applied Analysis Hindawi, Vol. 2013, 2013.

[5] Gurefe, Y., Misirli, E., Sonmezoglu, A., Ekici, M., *Extended trial equation method to generalized nonlinear partial differential equations*, Applied Mathematics and Computation, 219 (10), 5253-5260, 2013.

[6] Yang, X.F., Deng, Z.C., Wei, Y.A., *Riccati-Bernoulli sub-ODE method for nonlinear partial differential equations and its application*, Advances in Difference Equations, 2015 (1), 1-17, 2015.

[7] Baskonus, H.M., Bulut, H., *Regarding on the prototype solutions for the nonlinear fractional-order biological population model*, In AIP Conference Proceedings AIP Publishing LLC, 1738, 2016.

[8] Abdelrahman, M.A., *A note on Riccati-Bernoulli Sub-ODE method combined with complex transform method applied to fractional differential equations*, Nonlinear Engineering, 7 (4), 279-285, 2018.

[9] Hosseini, K., Gholamin, P., *Feng's first integral method for analytic treatment of two higher dimensional nonlinear partial differential equations*, Differential Equations and Dynamical Systems, 23 (3), 317-325, 2015.

[10] Kutluay, S., Karta, M., Yağmurlu, N.M., *Operator time-splitting techniques combined with quintic B-spline collocation method for the generalized Rosenau–KdV equation*, Numerical Methods for Partial Differential Equations, 35, 2221–2235, 2019.

[11] Yağmurlu, N.M., Karakaş, A.S., *Numerical solutions of the equal width equation by trigonometric cubic B-spline collocation method based on Rubin–Graves type linearization*, Numerical Methods for Partial Differential Equations, 36 (5), 1170-1183, 2020.

[12] Özer, S., Yağmurlu, N.M., *Numerical solutions of nonhomogeneous Rosenau type equations by quintic B-spline collocation method*, Mathematical Methods in the Applied Sciences, 45 (9), 5545–5558, 2022.

[13] He, J.H., Wu, X.H., *Exp-function method for nonlinear wave equations*, Chaos, Solitons & Fractals, 30 (3), 700-708, 2006.

[14] Baskonus, H.M., Askin, M., *Travelling wave simulations to the modified Zakharov-Kuzentsov model arising in plasma physics*, In Litteris et Artibus, Lviv Polytechnic Publishing House. 2016.

[15] Gurefe, Y., Misirli, E., *Exp-function method for solving nonlinear evolution equations with higher order nonlinearity*, Computers & Mathematics with Applications, 61 (8), 2025-2030, 2011.

[16] Misirli, E., Gurefe, Y., *The Exp-function method to solve the generalized Burgers-Fisher equation*, Nonlinear Science Letters A, 323-328, 2010.

[17] Misirli, E., Gurefe, Y., *Exact solutions of the Drinfel'd–Sokolov–Wilson equation using the exp-function method*, Applied Mathematics and Computation, 216 (9), 2623-2627, 2010.

[18] Gao, X., *Bäcklund transformation and shock-wave-type solutions for a generalized (3+1)-dimensional variable-coefficient B-type Kadomtsev–Petviashvili equation in fluid mechanics*, Ocean Engineering, 245–247, 2015.

[19] Cheng, L., Zhang, Y., *Multiple wave solutions and auto-Bäcklund transformation for the (3+1)-dimensional generalized B-type Kadomtsev–Petviashvili equation*, Computers and Mathematics with Applications, 70(5), 765-775, 2015.

[20] Ma, W.X., Zhu, Z.N., *Solving the (3+1)-dimensional generalized KP and BKP equations by the exp-function algorithm*, Applied Mathematics and Computation, 218, 11871–11879, 2012.

[21] Nisar, K.S., Ilhan, O.A., Abdulazeez, S.T., Manafian, J., Mohammed, S.A., Osman, M.S., *Novel multiple soliton solutions for some nonlinear PDEs via multiple Exp-function method*, Results in Physics 21,103769, 2021.

[22] Asaad, M.G., Ma, W.X., *Pfaffian solutions to a (3+1)-dimensional generalized B-type Kadomtsev–Petviashvili equation and its modified counterpart*, Applied Mathematics and Computation 218, 5524–5542, 2012.

[23] Cao, X., *Lump Solutions to the (3+1)-Dimensional Generalized b-type Kadomtsev-Petviashvili Equation*, Advances in Mathematical Physics, 5, 7843498, 2018.

[24] Wazwaz, A.M., *Distinct Kinds of Multiple-Soliton Solutions for a (3+1)-Dimensional Generalized B-type Kadomtsev-Petviashvili Equation*, Physica Scripta, 84, 5, 055006, 2011.

[25] Yokuş, A., Durur, H., Duran, S., *Ample felicitous wave structures for fractional foam drainage equation modeling for fluid-flow mechanism*, Computational and Applied Mathematics, 41, 174, 2022.

[26] Duran, S., Yokuş, A., Durur, H., Kaya, D., *Refraction simulation of internal solitary waves for the fractional Benjamin–Ono equation in fluid dynamics*, Modern Physics Letters B, Vol. 35, 26, 2150363, 2021.

[27] Yokuş, A., *Simulation of bright–dark soliton solutions of the Lonngren wave equation arising the model of transmission lines*, Modern Physics Letters B,Vol. 35, 32, 2150484, 2021.

[28] Duran, S., Kaya, D., *Breaking analysis of solitary waves for the shallow water wave system in fluid dynamics*, The European Physical Journal Plus, 136, 980, 2021.

[29] Yokuş, A., Durur, H., Duran, S., *Simulation and refraction event of complex hyperbolic type solitary wave in plasma and optical fiber for the perturbed Chen-Lee-Liu equation*, Optical and Quantum Electronics 53, 402, 2021.