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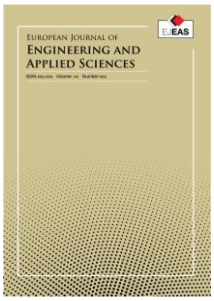

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Research Article

A Two-capacitor Problem with a Memcapacitor and a Conformal Fractional-Order Capacitor Put Together

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Abstract: Fractional-order capacitors and memcapacitors have become a major research area in recent decades. Analog applications of both circuit elements are getting more common. In literature, the conformal fractional derivative (CFD) is getting lots of interest due to its easiness to use and to comprehend. Some supercapacitors have already been modeled with the conformal fractional derivative. Two-capacitor problem is an important problem in physics. Recently, a two-capacitor problem with a CFD capacitor and a linear time-invariant (LTI) capacitor has been examined. To the best of our knowledge, a circuit, which is made of a CFD capacitor and a memcapacitor, has not been analyzed in the literature yet. In this study, a two-capacitor problem, a circuit, which consists of a CFD capacitor and a memcapacitor, has been examined using simulations for the first time in literature. It is found that the circuit is in ever transient state.

Keywords: *Memcapacitor, Conformable Fractional Order Capacitor, Two-capacitor Problem, Circuit Analysis, Circuit Theory.*

Memkapasitör ve Konformal Fraksiyonel Dereceli Kondansatörün Bir Araya Getirildiği İki Kapasitör Problemi

Öz: Kesirli mertebeli kondansatörler ve memkapasitörler son yıllarda önemli bir araştırma alanı haline geldi. Her iki devre elemanının analog uygulamaları giderek yaygınlaşmaktadır. Literatürde, uyumlu kesirli türev (UKT), kullanımı ve anlaşılması kolay olması nedeniyle çok ilgi görmektedir. Bazı süperkondansatörler zaten uyumlu kesirli türev ile modellenmiştir. İki kondansatör problemi fizikte önemli bir problemdir. Son zamanlarda, bir UKT kondansatörü ve bir lineer zamanla değişmeyen (LZD) kondansatörü ile bir iki kondansatör problemi incelenmiştir. Bildiğimiz kadarıyla, literatürde henüz bir UKT kondansatör ve bir memkapasitörden oluşan bir devre incelenmemiştir. Bu çalışmada, literatürde ilk kez simülasyonlar kullanılarak bir UKT kondansatörü ve bir memkapasitörden oluşan bir devre, bir - iki kondansatörlü bir problem incelenmiştir. Devrenin sürekli geçici rejimde olduğu görülmüştür.

Anahtar Kelimeler: *Memkapasitör, Kesir Dereceli Kondansatör, İki Kondansatör Problemi, Devre Analizi, Devre Teorisi.*

1. Introduction

Fractional order derivatives have become a hot research area especially in the recent years [1-3]. Their application areas have been increasing continuously [1-4]. The search to find new fractional derivatives continues since their initial prediction [1, 2]. The conformal fractional derivative (CFD) is one of the last found fractional derivatives [5]. It is easier to understand than the other types of the fractional derivatives [5, 6]. In addition to that, it has a physical interpretation which the others lack [7]. A review on using fractional derivatives in

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electrical circuits can be found in [8]. Supercapacitors have already been modeled using fractional order derivatives in [9-12]. The CFD has become very popular and been used to model supercapacitors [13-15]. The CFD capacitors and analysis of their usage with other circuit elements have emerged as a study area [13-15].

Following the discovery of the nonlinear circuit element memristor [16], existence of the memcapacitive and meminductive systems have also been predicted [17, 18]. The memcapacitor is a subbranch of the memcapacitive systems [17]. A memcapacitor has a flux-dependent capacitance and a zero-crossing hysteresis curve when it is excited with an AC signal [17, 19]. Its usage with other circuit elements and its emulators has also become a hot topic [20-23]. The search for different types of memcapacitors continues [24-33]. They have already been employed in chaotic circuits [34-37]. There are also memcapacitor-based oscillator circuits examined in the literature [38-39]. The stored energy of a memcapacitor is examined in [40-41]. Two capacitor problem is a well-known physical problem, it can be found even in undergrad textbooks [42], and it is still studied [43-48]. A memcapacitor-an LTI capacitor problem is examined in [48]. Recently, a CFD capacitor- an LTI capacitor problem is examined in [15]. To the best of our knowledge, a two-capacitor problem which consists of a memcapacitor and a CFD capacitor have not been examined in literature yet. In this study, the problem has been examined using the memcapacitor model used in [39, 40, 48]. Such an analysis may be needed to model capacitive behavior of living tissues [4]. Simulations have been done to analyze the circuit's behavior using Simulink™ toolbox of MATLAB.

The remainder of the paper is organized as follows. In the second section, Conformal Fractional Derivative and CFD Capacitor Model are briefly told. In the third section, the memcapacitor model used in this paper is given. In the fourth section, the differential equation which describes the two-capacitor problem with a CFD Capacitor and a memcapacitor are derived. The simulations are given in the fifth section. The paper is finished with the conclusion section.

2. The Conformal Fractional Derivative and the CFD Capacitor Model

In the literature, the fractional calculus [2-4] is reported to be able to model fractional order elements. The conformal fractional derivative has been developed in recent decade and it has emerged as a popular fractional order derivative method [5-7]. It is easy to use and to understand compared to the other types of fractional derivatives [5-7]. In [5], the general description of Conformal Fractional Derivative (CFD) is given as

$$\frac{d^{\alpha}f(t)}{dt^{\alpha}} = f^{'}(t)t^{1-\alpha} = \frac{df(t)}{dt}t^{1-\alpha} \quad (1)$$

where $\frac{d^{\alpha}f(t)}{dt^{\alpha}}$is the fractional derivative, and $t^{1-\alpha}$is the time coefficient depending on the fractional order coefficient α.

Some capacitors can be modeled using fractional order derivatives [9-15]. The constitutional law of a capacitor modeled with the conformal fractional derivative is written as

$$i_C(t) = C_{\alpha}\frac{d^{\alpha}V_C(t)}{dt^{\alpha}} = C_{\alpha}V_C^{'}(t)t^{1-\alpha} = C_{\alpha}\frac{dV_C(t)}{dt}t^{1-\alpha} \quad (2)$$

where C_{α} is the CFD capacitor coefficient, $i_C(t)$ is the capacitor current, and $v_C(t)$ is capacitor voltage.

The circuit symbol of a CFD capacitor is shown in Figure 1.

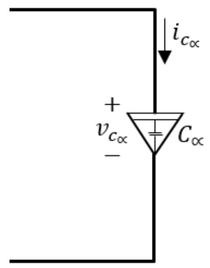

Figure 1. *The CFD capacitor symbol*

3. The Memcapacitor Model

A Memcapacitive systems and memcapacitors have been described in [17]. Memcapacitors have become a hot research topic in recent decade [18]. A voltage-controlled memcapacitor model is given as

$$q_m(t) = C(\lambda(t))V_{mc}(t) \quad (3)$$

$$\frac{d\lambda(t)}{dt} = V_{mc}(t) \quad (4)$$

where *λ(t)* is the memcapacitor flux, which is the integral of memcapacitor voltage by respect to time.

A memcapacitor flux is written as either

$$\lambda(t) = \int_{-\infty}^{t} V_{mc}(\tau)\,d\tau \quad (5)$$

or

$$\lambda(t) = \int_{0}^{t} V_{mc}(\tau)\,d\tau + \lambda_0 \quad (6)$$

where λ_0 is the initial flux of the memcapacitor at t=0.

Literature has different types of memcapacitors [18]. Their emulator circuits have been examined in [20-23]. Often, phenomenological models are used for their study [18, 19]. An explanation for hysteresis curve of the small signal memcapacitor circuits have been given using the model in [19]. The following memcapacitor phenomenological model given in [48] is also used in this study. More about the model and its zero-crossing voltage-charge hysteresis loop can be found in [48]. The memcapacitor symbol is shown in Figure 2. For the polarity shown in Figure 2, the memcapacitance of the memcapacitor is given as

$$C_m(\lambda) = C_0 + K\lambda(t) \quad (7)$$

where C_0 is the minimum memcapacitor memcapacitance and K is the flux coefficient of the memcapacitor.

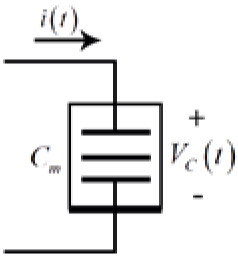

Figure 2. *The Memcapacitor symbol*

The minimum memcapacitor memcapacitance is calculated at $\lambda = 0$

$$C(0) = C_0 \tag{8}$$

The memcapacitor has been assumed to have a saturation mechanism in [57]. Therefore,

$$0 \leq \lambda \leq \lambda_{SAT} \tag{9}$$

where λ_{SAT} is the maximum value of the memcapacitor flux.

The maximum memcapacitance occurs at $\lambda = \lambda_{SAT}$ and given as:

$$C_{SAT} = C_0 + K\lambda_{SAT} \tag{10}$$

Therefore, the memcapacitance ranges from C_0 to C_{SAT}:

$$C_0 \leq C(\lambda(t)) \leq C_{SAT} \tag{11}$$

The memcapacitor current can be found as

$$i_m(t) = \frac{dq_m(t)}{dt} = \frac{d}{dt}((C_0 + K\lambda(t))V_{mc}(t))$$

$$i_m(t) = C_0\frac{dV_{mc}(t)}{dt} + K\lambda'(t)V_{mc}(t) + K\lambda(t)\frac{dV_{mc}(t)}{dt} \tag{12}$$

Since $\frac{d\lambda(t)}{dt} = V_{mc}(t)$, the memcapacitor current turns into: (13)

$$i_m(t) = (C_0 + K\lambda(t))\frac{dV_{mc}(t)}{dt} + K{V_{mc}}^2(t) \tag{14}$$

4. Examination of the Memcapacitor- the CFD Capacitor Circuit

Recently, two new two-capacitor problems have been examined [15, 48]. A memcapacitor with an LTI capacitor, whose circuit is shown in Figure 3, has been examined in [48]. A CFD with an LTI capacitor, whose circuit is shown in Figure 4, has been examined in [15]. A circuit, which consists of a memcapacitor and a CFD capacitor, is shown in Figure 5. The circuit is lossless since it has no resistance. Perhaps, such a circuit can also be used to model memcapacitive and fractional order behavior of the living tissue when they coexist [4].

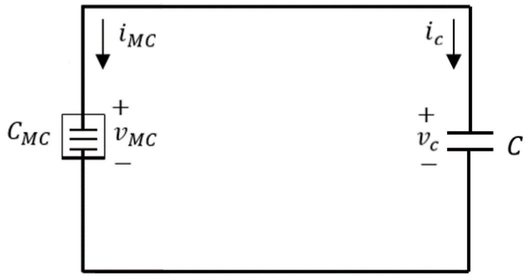

Figure 3. *The memcapacitor-the LTI capacitor circuit examined in [48].*

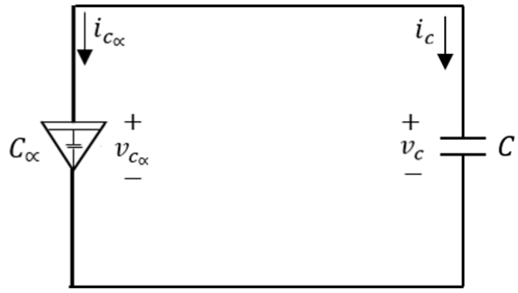

Figure 4. *The CFD capacitor-the LTI capacitor circuit examined in [15].*

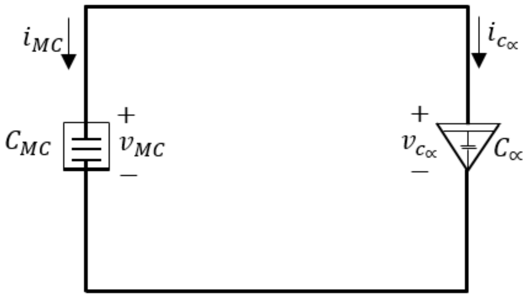

Figure 5. *The CFD capacitor-the memcapacitor circuit. Using Kirchhoff's current law:*

$$i_{MC} = -i_{C_\alpha} \tag{15}$$

Using Kirchhoff's voltage law:

$$V_{C_\alpha}(t) = V_{MC}(t) \tag{16}$$

Eq. (17) is written with using Eq. (2) and Eq. (12)

$$C_\alpha\frac{dV_{C_\alpha}(t)}{dt}t^{1-\alpha} + \frac{d}{dt}((C_0 + K\lambda(t))V_{MC}(t)) = 0 \tag{17}$$

$$C_\alpha\frac{dV_{C_\alpha}(t)}{dt}t^{1-\alpha} + \frac{d}{dt}((C_0 + K\lambda(t))V_{C_\alpha}(t)) = 0 \tag{18}$$

$$C_\alpha\frac{dV_{C_\alpha}(t)}{dt}t^{1-\alpha} + C_0\frac{dV_{C_\alpha}(t)}{dt} + K\lambda(t)\frac{dV_{C_\alpha}(t)}{dt} + K(V_{C_\alpha}(t))^2 = 0$$

$$\frac{dV_{C_\alpha}(t)}{dt}(C_\alpha t^{1-\alpha} + C_0 + K\lambda(t)) + K(V_{C_\alpha}(t))^2 = 0 \tag{19}$$

When $V_{C_\alpha}(t) = \lambda'(t)$ and $\frac{dV_{C_\alpha}(t)}{dt} = \lambda''(t)$ are used, the simplified equation is found as:

$$\lambda''(t)(C_\alpha t^{1-\alpha} + C_0 + K\lambda(t)) + K(\lambda'(t))^2 = 0 \qquad (20)$$

$$C_\alpha(\lambda''(t))t^{1-\alpha} + C_0(\lambda''(t)) + K\lambda(t)(\lambda''(t)) + K(\lambda'(t))^2 = 0$$

$$(\lambda''(t))(C_\alpha t^{1-\alpha} + C_0 + K\lambda(t)) + K(\lambda'(t))^2 = 0 \qquad (21)$$

The differential equation does not have any analytical solution. That's why a simulation program is employed for its solution in the next section.

5. Simulations of the Memcapacitor-the CFD Capacitor Circuit

Since a numerical solution for Eq. (21) could not be found, Simulink is used to analyze the two-capacitor problem. The Simulink block diagram of the circuit which is shown in Figure 6 made using Eq.s (5), (15), and (19) and the parameters given in Table 1. A Matlab function is used to obtain the term of $t^{1-\alpha}$ needed for the CFD. The voltage of the CFD capacitor or the memcapacitor, the current of the CFD capacitor, the current of the memcapacitor, the charge of the memcapacitor, and flux of the memcapacitor are obtained with the Simulink simulation and given in Figures 7-11.

After a sufficiently long time, the memcapacitor gets saturated and its memcapacitance value becomes equal to C_{SAT}. In this case, the circuit turns into a two-capacitor problem, which can be considered to consist of a constant capacitor, whose capacitance is equal to C_{SAT}, and the CFD capacitor.

The circuit waveforms are simulated for $\alpha = 0.3$, 0.6, and 1 The simulation parameters are given in Table 1.

Table 1 The circuit parameters and the memcapacitor initial conditions.

Parameter	Value
K	0.5 F / Wb
C_0	0.5 mF
C_{SAT}	2.5 mF
C_α	0.05 F
$V_{C_\alpha}(0)$	1 V
$V_{MC}(0)$	1 V

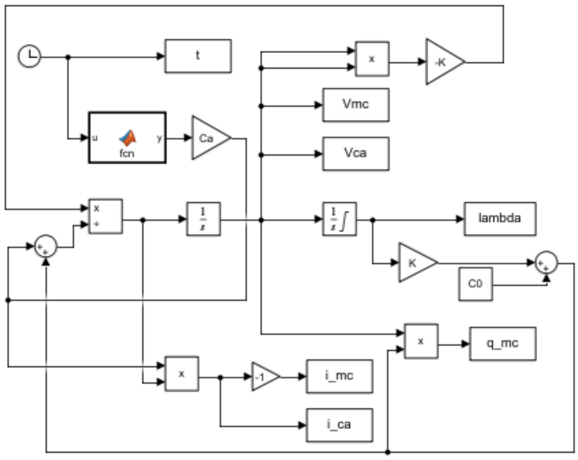

Figure 6. *Simulink block diagram*

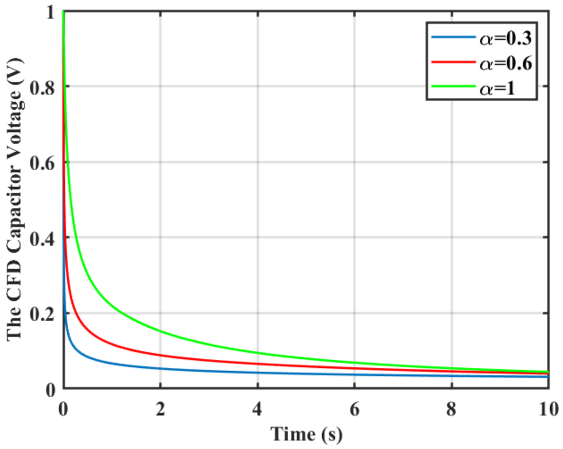

Figure 7. *The voltage of the CFD capacitor or memcapacitor for three different α values*

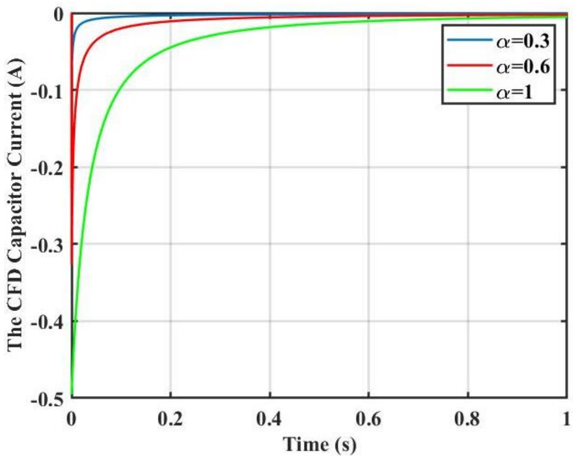

Figure 8. *The current of the CFD capacitor for three different α values*

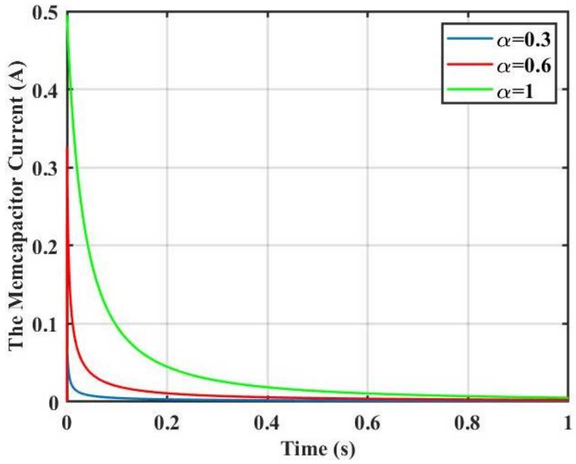

Figure 9. *The current of the memcapacitor for three different α values*

As the memcapacitor capacitor charges, the CFD discharges as shown in Figures 8 and 9. The current of the memcapacitor capacitor reaches its maximum value while the CFD current reaches its minimum value after a sufficiently enough time since their polarities are opposite. The charging and discharging curves look like exponential functions even though they are not. When α increases, the curves discharges quicklier. The quickliest discharge curve is obtained for$\alpha = 1$. Furthermore, the CFD

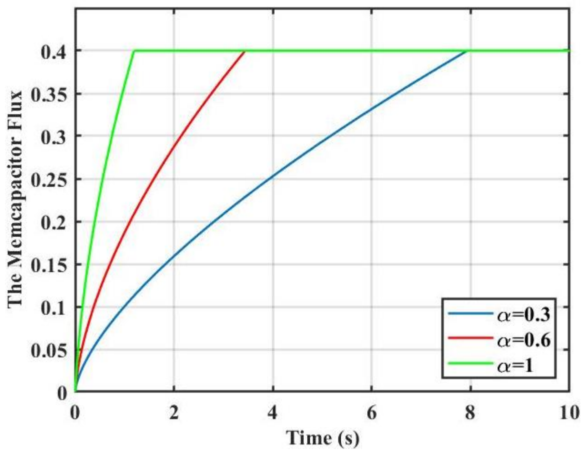

Figure 10. *The flux of the memcapacitor for three different α values*

capacitor behaves as an LTI capacitor when $\alpha = 1$ applied because the time value equals 1 at this condition. Thus, the outputs of the designed capacitor give the same outputs as an LTI capacitor. Also, for $\alpha = 1$, the problem turns into the memcapacitor-the capacitor problem given in [48].

The flux of the memcapacitor for three different α values is shown in Figure 10. The flux of the memcapacitor shows that the memcapacitor saturates if a sufficiently long time passes, the element flux remains constant during saturation, and this saturation is dependent on α as shown in Figure 10. The saturation time decreases when α increases. It becomes the lowest for $\alpha = 1$.

The memcapacitor charge has interesting dynamics as shown in Figure 11. First, it increases, and it starts decreasing after the memcapacitor saturates and the dynamics continuous since the equivalent capacitance starts varying due to the CFD capacitor's time dependent term. The circuit is in ever transient state due to the CFD capacitor. After the saturation occurs, the circuit can be regarded as consisting of a CFD capacitor and an LTI capacitor as examined in [15]. After saturation, the tangent, and the peak value of the memcapacitor charge is dependent on α as shown in Figure 11. The peak value of the memcapacitor charge is the highest for $\alpha = 1$. The memcapacitor charge has the highest descent for $\alpha = 1$. The memcapacitor charge curves appears to start merging when time increases.

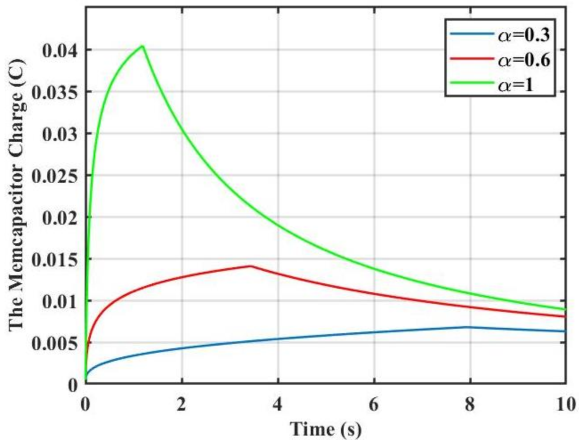

Figure 11. *The charge of the memcapacitor for three different α values*

6. Conclusion

Memcapacitors and CFD capacitors have been becoming a prevalent research area for years. Both capacitor models cannot be described using the constitutive law of an LTI capacitor. The two-capacitor problem examined here is a nonlinear problem and it has no analytical solution even though a simple phenomenological memcapacitor model is used. That's why the Simulink toolbox of MATLAB is used to simulate the derived circuit equation. It has been found that a time dependent charge sharing dynamics exists between two capacitors. The dynamics also depend on the power exponent of the CFD capacitor. If a sufficient time passes, the memcapacitor flux and capacitance become constant and the problem turns into an LTI capacitor and a CFD capacitor problem. The circuit can be thought as in ever transient state due to the CFD capacitor's time dependent capacitance.

New kinds of circuit elements like memcapacitors, CFD capacitors, and the combinations of these circuit elements need to be analyzed in detail to understand their behaviors well and to use them efficiently. The results reported in this study may find usage in the biological circuits where the memcapacitor and the CFD capacitors may exist together.

Author Contribution

Data curation – None (N); Formal analysis Utku Palaz (UP), Reşat Mutlu (RM); Investigation – RM; Experimental Performance - N; Data Collection – UP; Processing - UP; Literature review – UP, RM; Writing – UP, RM; Review and editing – UP, RM;

Declaration of Competing Interest

The authors declared no conflicts of interest with respect to the research, authorship, and/or publication of this article.

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