Available online at www.dergipark.gov.tr/beuscitech

Journal of Science and Technology

E-ISSN 2146-7706

New Soliton Solutions Arising in Some NLEEs

Ugur BAYRAKCI [a,*], Seyma TULUCE DEMIRAY [a], Vehpi YILDIRIM [b]

[a] Osmaniye Korkut Ata University, Department of Mathematics, TR-80000, Osmaniye, Türkiye

[b] Erzurum Technical University, Department of Mathematics, TR-25000, Erzurum, Türkiye

ARTICLE INFO

Article history:
Received 11 May 2022
Received in revised form 25 June 2022
Accepted 27 June 2022

Keywords:
Generalized Kudryashov method
(2+1)-dimensional dissipative long wave system
(2+1)-dimensional Date-Jimbo-Kashiwara-Miwa equation
Soliton solutions

ABSTRACT

We have worked on (2+1)-dimensional dissipative long wave system (DLWS) and (2+1)-dimensional Date-Jimbo-Kashiwara-Miwa (DJKM) equation. We have applied GKM, which has been obtained by generalizing the Kudryashov method, to the (2+1)- dimensional DLWS and (2+1)-dimensional DJKM equation. Thus, we have got some new soliton solutions of handled system and equation. We have plotted 2D and 3D surfaces of these acquired results by using Wolfram Mathematica 12. Then, we have shown the validity of the acquired solutions.

1. Introduction

Nonlinear evolution equations (NLEEs) have very significant applications in areas such as mathematical physics, biology, economy, mathematical chemistry, hydrodynamics, fluid dynamics, geochemistry, control theory, meteorology, optics, mechanics, chemical kinematics, biophysics, biogenetics, and so on. Particularly nonlinear partial differential equations, researching chemical reactions occurring in various scientific environments, changes in living populations, heat dissipation on metals, determination of charge and current in electrical circuits, plate and wire vibrations; It is widely used in the study and interpretation of important physical phenomena such as sea, lake, stream and tidal waves, decay of a radioactive object. Various studies are carried out by many scientists to find the solutions of equations, which have such extremely important and widespread areas of use (Ali et al., 2021; Yokus et al., 2020; Gao et al., 2020; Cinar et al., 2021; Rani et al., 2021; Manafian et al., 2020; Yaslan and Girgin, 2019; Ghanbari and Inc, 2018; Dusunceli et al., 2021; Kumar and Kaplan, 2018; Mirzazadeh et al., 2021). (2+1)-dimensional DLWS is a famous system of equations used in physical applications, nonlinear science and nonlinear wave theory (Chang et al., 2020). (2+1)-dimensional DLWS is given as (Yang and Feng, 2021):

$$\begin{aligned} &u_t - u_{xx} - 2u_x v - 2uv_x = 0, \\ &v_{ty} + v_{xxy} - 2u_{xx} - 2v_x v_y - 2vv_{xy} = 0. \end{aligned} \tag{1}$$

Chang et al. used Lie symmetry analysis and dynamical system method for system (Chang et al., 2020). Yang and Feng obtained the solutions of this system by applying the variable separation method and exp $(-\Phi(\xi))$ -expansion method (Yang and Feng, 2021).

(2+1)-dimensional DJKM equation is given as (Yuan et al.,

* *Corresponding author.* Tel.: +0 328 827 1000; fax: +0 000 000 0000
E-mail address: *ubayrakci42@gmail.com*
ORCID : *0000-0002-1765-2318 (U..Bayrakci), 0000-0002-8027-7290 (S..Tuluce Demiray), 0000-0003-3837-4756 (V. Yildirim)*

2017; Pu and Hu, 2019; Ismael et al., 2020):

$$u_{xxxxy}+4u_{xxy}u_x+2u_{xxx}u_y+6u_{xy}u_{xx}+u_{yyy}-2u_{xxt}=0, \tag{2}$$

Pu and Hu examined solitary wave solutions of the equation with the sine-Gordon expansion method (Pu and Hu, 2019). Ismael et al. applied Hirota bilinear method to the equation (Ismael et al., 2020). Yuan et al. obtained solutions of the equation with Hirota method and auxiliary variable (Yuan et al., 2017). Adem et al. performed extended transformed rational function algorithm to the equation (Adem et al., 2019). Singh and Gupta applied the Pickering's algorithm to the equation (Singh and Gupta, 2018). Guo and Lin used the direct ansatz method for the equation (Guo and Lin, 2019).

In this study, GKM, which is one of the solution methods of NLEEs, is discussed (Tuluce Demiray, 2020; Tuluce Demiray and Bayrakci, 2021; Pandır and Eren, 2021; Kaplan and Akbulut, 2021; Gurefe, 2020; Islam et al. 2019). Firstly, the structure of the method is introduced. Afterwards, some soliton solutions of (2+1)-dimensional DJKM and (2+1)-dimensional DLWS were obtained by applying GKM.

2. Structure of GKM

We take into account a general nonlinear partial differential equation (NLPDE) for a function v of three different variables in the following form:

$$R\left(v,v_t,v_y,v_x,v_{xx},\ldots\right)=0. \tag{3}$$

Step1: Firstly, we regard the travelling wave transform as following form;

$$v\left(x,y,t\right)=v\left(\eta\right),\quad \eta=x+y-mt. \tag{4}$$

Eq. (3) is turned into ordinary differential equation by Eq. (4):

$$L\left(t,y,x,v,v',v'',\ldots\right)=0, \tag{5}$$

where superscripts demonstrate ordinary derivatives according η.

Step2: Suppose that we consider the solutions of Eq. (5) as:

$$v\left(\eta\right)=\frac{\sum_{i=0}^{k}a_iQ^i\left(\eta\right)}{\sum_{j=0}^{l}b_jQ^j\left(\eta\right)}=\frac{P\left[Q(\eta)\right]}{S\left[Q(\eta)\right]}, \tag{6}$$

where Q is $\frac{1}{1\pm e^{\eta}}$. We should point out that Q is the solution to the following equation.

$$Q_\eta=Q^2-Q. \tag{7}$$

Step3: The solution of the nonlinear ordinary differential equation given by Eq. (5) is sought according to the GKM as follows:

$$v\left(\eta\right)=\frac{a_0+a_1Q+a_2Q^2+\cdots+a_kQ^k}{b_0+b_1Q+b_2Q^2+\cdots+b_lQ^l}. \tag{8}$$

If we can ascertain values of k and l in Eq. (6) through the homogeneous balance principle. Therefore we balance between the highest order derivative and highest order nonlinear term in Eq. (5).

Step4: We embed Eq. (6) into Eq. (5). Thus we get a polynomial $R\left(Q\right)$ of Q. Thereafter equalizing all coefficients of $R\left(Q\right)$ to zero, we find an algebraic equation system. By solving obtained system, we determine c and coefficients of $a_0,a_1,a_2,\cdots,a_k,b_0,b_1,b_2,\cdots,b_l$. Finally, we can obtain the soliton solutions of Eq. (5).

3. Application of GKM to the equations

Example 1:

For the find the soliton solutions of system (1) we consider the following transformation:

$$u(x,y,t)=u(\eta),v(x,y,t)=v(\eta),\eta=x+y-mt. \tag{9}$$

Putting the Eq. (9) into the system (1), we get the equation $u(x,y,t)=\frac{v'}{2}-\frac{mv}{2}-\frac{v^2}{2}$. By performing the necessary mathematical operations, system (1) is converted to the following ordinary differential equation. Replace Eq. (9) into system (1) and we get the following equation,

$$-m^2v-v''+3mv^2+2v^3=0. \tag{10}$$

By using balance principle in Eq. (9), we obtain,

$$k=l+1, \tag{11}$$

If we give $l=1$ then $k=2$ we find,

$$u\left(\eta\right)=\frac{a_0+a_1Q+a_2Q^2}{b_0+b_1Q}, \tag{12}$$

$$u'\left(\eta\right)=\left(Q^2-Q\right)\times\left[\frac{\left(a_1+2a_2Q\right)\left(b_0+b_1Q\right)-b_1\left(a_0+a_1Q+a_2Q^2\right)}{\left(b_0+b_1Q\right)^2}\right], \tag{13}$$

$$u''(\eta) = \frac{Q^2 - Q}{(b_0 + b_1 Q)^2}(2Q - 1) \times \left[(a_1 + 2a_2 Q)(b_0 + b_1 Q) - b_1(a_0 + a_1 Q + a_2 Q^2)\right] + \frac{(Q^2 - Q)^2}{(b_0 + b_1 Q)^3} \times \left[\begin{array}{l} 2a_2(b_0 + b_1 Q)^2 - 2b_1(a_1 + 2a_2 Q)(b_0 + b_1 Q) \\ +2b_1^2(a_0 + a_1 Q + a_2 Q^2) \end{array}\right]. \tag{14}$$

We obtain the soliton solutions of system (1) in the following different cases;

Case1:

$$a_0 = 0, a_1 = -b_0, a_2 = -b_1, m = 1. \tag{15}$$

Substituting the above values in Eq. (12), we acquire the dark soliton solutions of system (1).

$$u_1(x,y,t) = -\frac{1}{4}\left(-1 + \tanh\left[\frac{x+y-t}{2}\right]\right) \times \left(m + \tanh\left[\frac{x+y-t}{2}\right]\right), \tag{16}$$

$$v_1(x,y,t) = -\frac{1}{2}\left(1 - \tanh\left[\frac{x+y-t}{2}\right]\right). \tag{17}$$

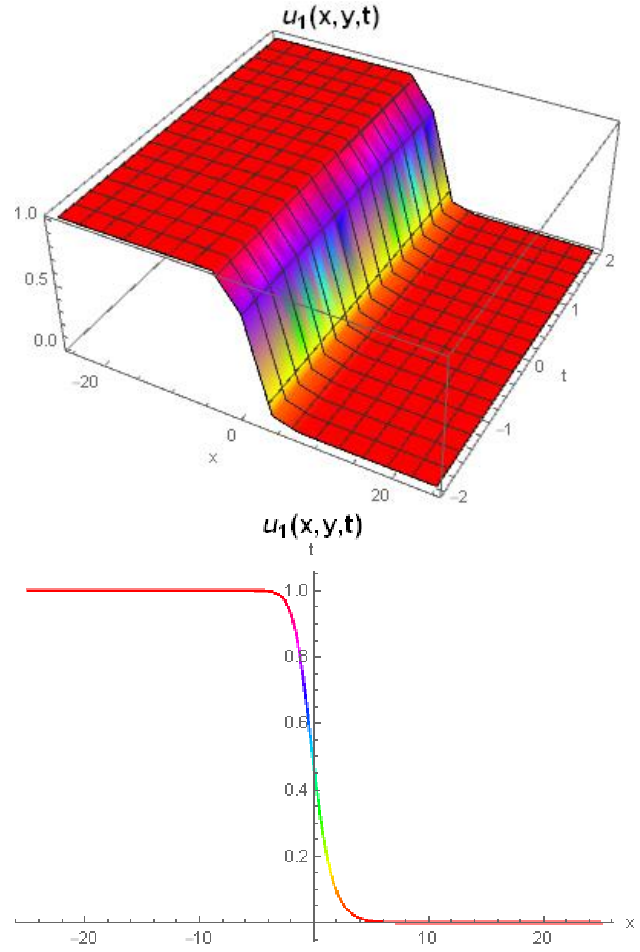

Figure 1. 3D of solution (16) for $y = 2, -25 \le x \le 25$ values with $-2 \le t \le 2$ range and 2D plot of solution for $t = 1$ with these values.

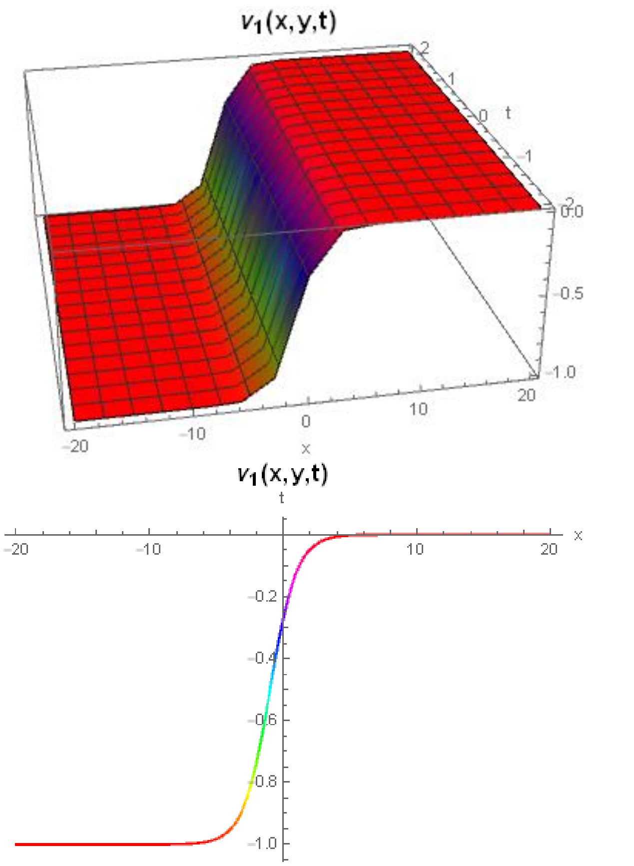

Figure 2. 3D of solution (17) for $y=2, -20 \le x \le 20$ values with $-2 \le t \le 2$ range and 2D plot of solution for $t=1$ with these values.

Case2:

$$a_0 = b_1, a_1 = -2b_1, a_2 = b_1, b_0 = -\frac{b_1}{2}, m = 2. \tag{18}$$

Substituting the above values in Eq. (12), we acquire the dark-bright and dark soliton solutions of system (1).

$$u_2(x,y,t) = \frac{1}{4}\left(1+\coth[x+y-2t]\right)\times \left(4+4\coth[x+y-2t]+\operatorname{csch}[x+y-2t]\right), \tag{19}$$

$$v_2(x,y,t) = -\frac{1}{2}\left(1+\coth\left[\frac{x+y-2t}{2}\right]\right)^2 \times \tanh\left[\frac{x+y-2t}{2}\right]. \tag{20}$$

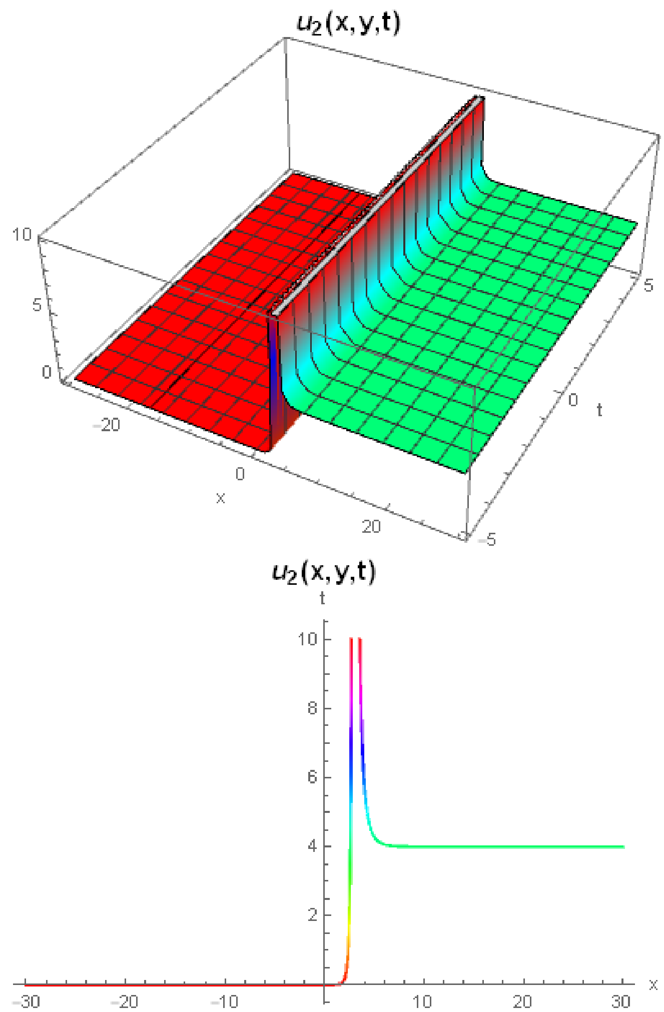

Figure 3. 3D of solution (19) for $y=2, -30 \le x \le 30$ values with $-5 \le t \le 5$ range and 2D plot of solution for $t=4$ with these values.

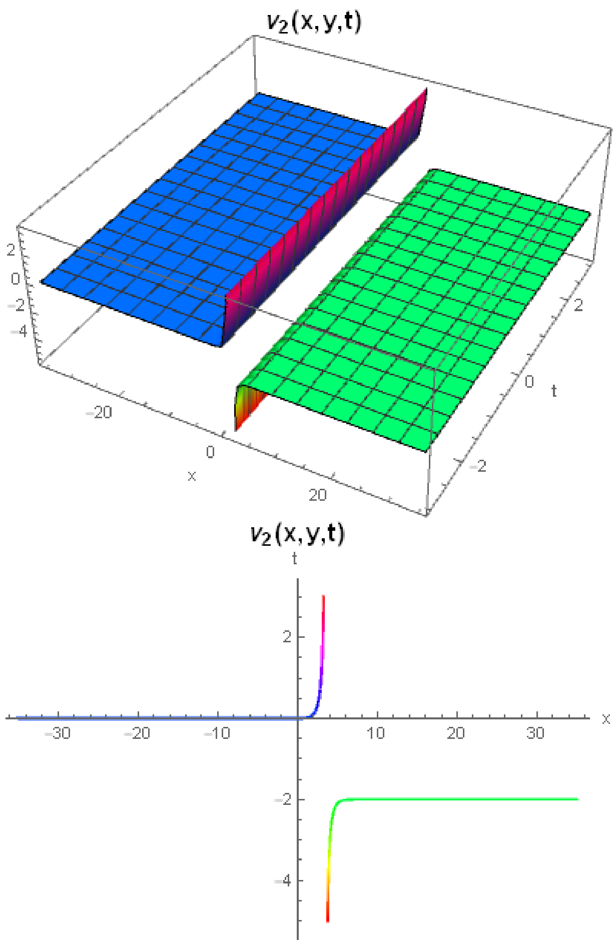

Figure 4. 3D of solution (20) for $y = 0.5, -35 \le x \le 35$ values with $-3 \le t \le 3$ range and 2D plot of solution for $t = 2$ with these values.

Case3:

$$a_0 = \frac{ib_1}{2\sqrt{2}}, a_1 = \left(1 - \frac{i}{\sqrt{2}}\right) b_1, \\ a_2 = -b_1, b_0 = -\frac{b_1}{2}, m = i\sqrt{2}. \tag{21}$$

Substituting the above values in Eq. (12), we acquire the bright soliton solutions of system (1).

$$u_3(x,y,t) = \frac{-1}{4} + \frac{i}{2}\csc[ix + iy + \sqrt{2}t] \times \\ \tanh\left[\frac{x + y - i\sqrt{2}t}{2}\right], \tag{22}$$

$$v_3(x,y,t) = -\frac{i}{\sqrt{2}} - i\csc[ix + iy + \sqrt{2}t]. \tag{23}$$

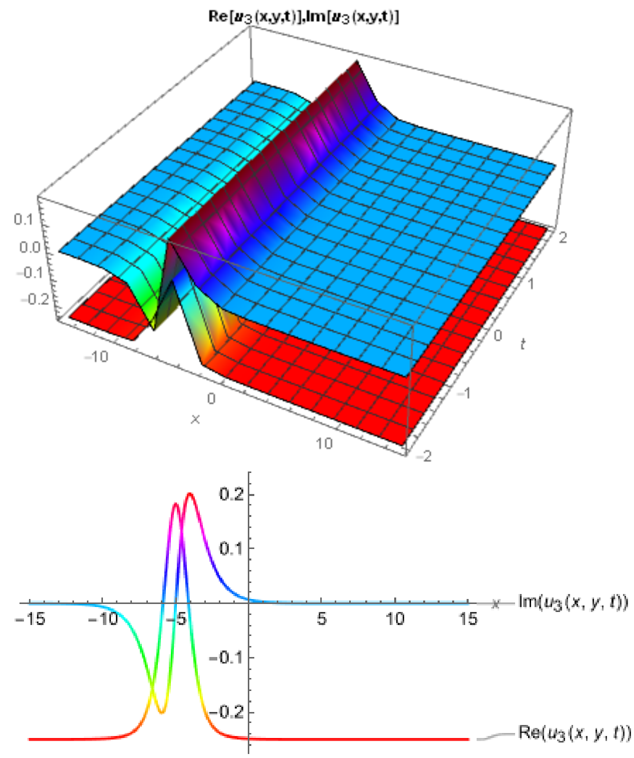

Figure 5. 3D of solution (22) for $y = 5, -15 \le x \le 15$ values with $-2 \le t \le 2$ range and 2D plot of solution for $t = 1$ with these values.

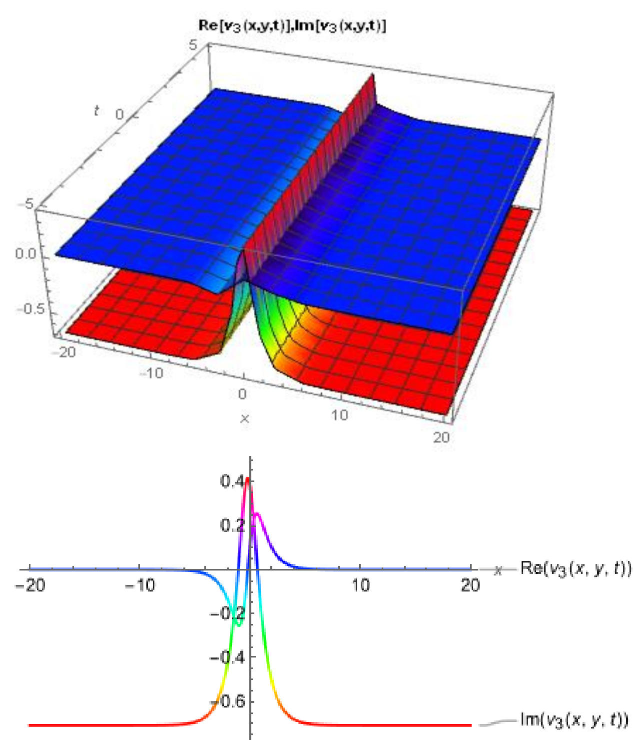

Figure 6. 3D of solution (23) for $y = 0.2, -20 \le x \le 20$ values with $-5 \le t \le 5$ range and 2D plot of solution for $t = 3$ with these values.

Example 2:
For the find the soliton solutions of Eq. (2) we consider following equalities:

$$u(x,y,t)=u(\eta),\eta=n\left(x+ay-ht\right). \tag{24}$$

Replace Eq. (24) into Eq. (2) and we get the following equation,

$$an^2u'''+3an\left(u'\right)^2+\left(a^3+2h\right)u'=0. \tag{25}$$

By making the $u'=g,$ transformation in Eq. (25), we find the following equation,

$$an^2g''+3ang^2+\left(a^3+2h\right)g=0. \tag{26}$$

By using balance principle in Eq. (26), we obtain

$$k=l+2. \tag{27}$$

If we give $l=1$ then $k=3$ we find the following equations,

$$u\left(\eta\right)=\frac{a_0+a_1Q+a_2Q^2+a_3Q^3}{b_0+b_1Q}, \tag{28}$$

$$u'\left(\eta\right)=\left(Q^2-Q\right)\times\left[\frac{\left(a_1+2a_2Q+3a_3Q^2\right)\left(b_0+b_1Q\right)-b_1\left(a_0+a_1Q+a_2Q^2+a_3Q^3\right)}{\left(b_0+b_1Q\right)^2}\right], \tag{29}$$

$$u''\left(\eta\right)=\frac{Q^2-Q}{\left(b_0+b_1Q\right)^2}\left(2Q-1\right)\times\left[\begin{array}{l}\left(a_1+2a_2Q+3a_3Q^2\right)\left(b_0+b_1Q\right)\\-b_1\left(a_0+a_1Q+a_2Q^2+a_3Q^3\right)\end{array}\right]+\frac{\left(Q^2-Q\right)^2}{\left(b_0+b_1Q\right)^3}\left[\begin{array}{l}\left(b_0+b_1Q\right)^2\left(2a_2+6a_3Q\right)-\\2b_1\left(b_0+b_1Q\right)\left(a_1+2a_2Q+3a_3Q^2\right)\\+2b_1^2\left(a_0+a_1Q+a_2Q^2+a_3Q^3\right)\end{array}\right]. \tag{30}$$

We get soliton solutions of Eq. (2) in the following different cases;

Case1:

$$a_0=-\frac{nb_0}{6},a_1=nb_0-\frac{nb_1}{6},\quad a_2=n(-b_0+b_1),a_3=-nb_1,\quad a=\frac{3^{1/3}n^2+\left(-9h+\sqrt{81h^2-3n^6}\right)^{2/3}}{3^{2/3}\left(-9h+\sqrt{81h^2-3n^6}\right)^{1/3}}. \tag{31}$$

Substituting the above values in Eq. (28), we acquire dark soliton solution of Eq. (2).

$$u_1(x,y,t)=\frac{-n^2}{6}\left(x+ay-ht\right)+\frac{n}{2}\tanh\left[\frac{n\left(x+ay-ht\right)}{2}\right], \tag{32}$$

where $a=\dfrac{3^{1/3}n^2+\left(-9h+\sqrt{81h^2-3n^6}\right)^{2/3}}{3^{2/3}\left(-9h+\sqrt{81h^2-3n^6}\right)^{1/3}}.$

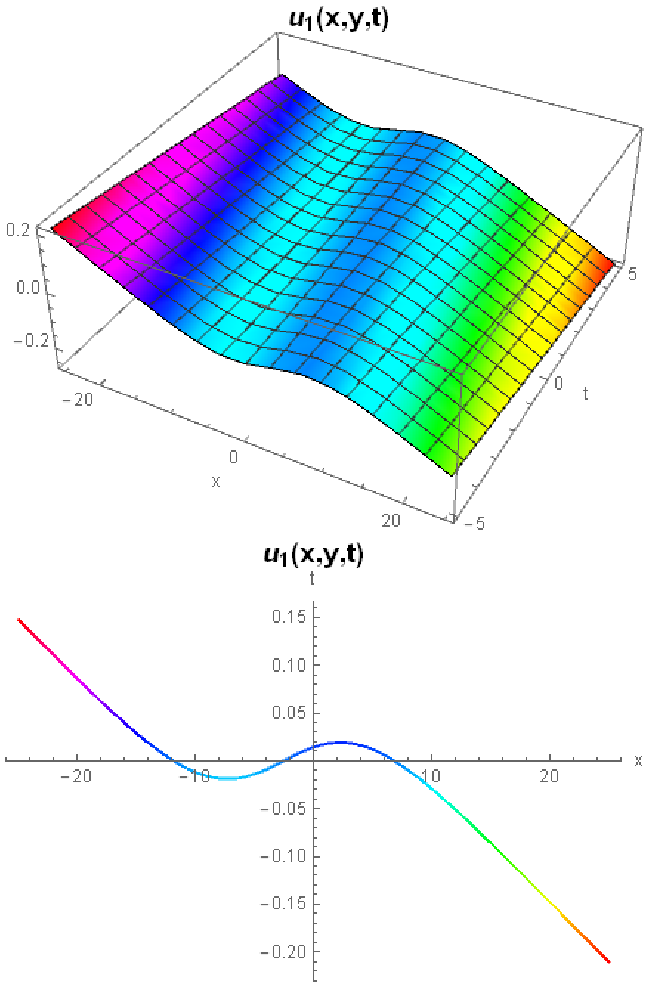

Figure 7. 3D of solution (32) for $n=0.2755, a=4, y=0.5,$ $k=-1, -25 \le x \le 25$ values with $-5 \le t \le 5$ range and 2D plot of solution for $t=0.5$ with these values.

Case2:

$$a_0=\frac{\sqrt{a^3+2h}b_0}{6\sqrt{a}}, a_1=\frac{\sqrt{a^3+2h}\left(-6b_0+b_1\right)}{6\sqrt{a}},$$
$$a_2=\frac{\sqrt{a^3+2h}\left(b_0-b_1\right)}{\sqrt{a}}, \tag{33}$$
$$a_3=\frac{\sqrt{a^3+2h}b_1}{\sqrt{a}}, n=-\frac{\sqrt{a^3+2h}}{\sqrt{a}}.$$

Substituting the above values in Eq. (28), we acquire dark soliton solution of Eq. (2).

$$u_2(x,y,t)=\frac{\sqrt{a^3+2h}}{6\sqrt{a}}\left(nx+nay-nht\right)$$
$$-\frac{\sqrt{a^3+2h}}{2\sqrt{a}}\tanh\left[\frac{\left(nx+nay-nht\right)}{2}\right], \tag{34}$$

where $n=-\frac{\sqrt{a^3+2h}}{\sqrt{a}}$.

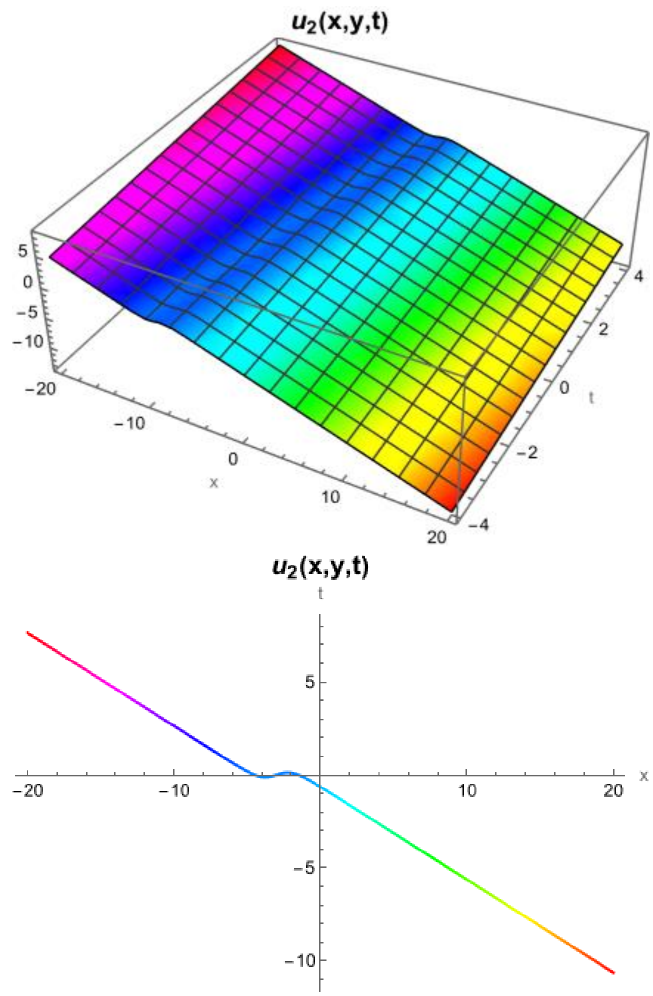

Figure 8. 3D of solution (34) for $n=-\sqrt{3}, a=1, y=5,$ $k=1, -20 \le x \le 20$ values with $-4 \le t \le 4$ range and 2D plot of solution for $t=2$ with these values.

Case3:

$$a_0=0, a_1=0, a_2=-\frac{i\sqrt{a^3+2hb_1}}{\sqrt{a}},$$
$$a_3=\frac{i\sqrt{a^3+2hb_1}}{\sqrt{a}}, b_0=0, n=-\frac{i\sqrt{a^3+2h}}{\sqrt{a}}. \tag{35}$$

Substituting the above values in Eq. (28), we acquire trigonometric function solution of Eq. (2).

$$u_3(x,y,t) = -\frac{\sqrt{a^3+2h}}{2\sqrt{a}} \times \tan\left[\frac{\sqrt{a^3+2h}\left(x+ay-ht\right)}{2\sqrt{a}}\right], \tag{36}$$

where $n = -\frac{i\sqrt{a^3+2h}}{\sqrt{a}}$.

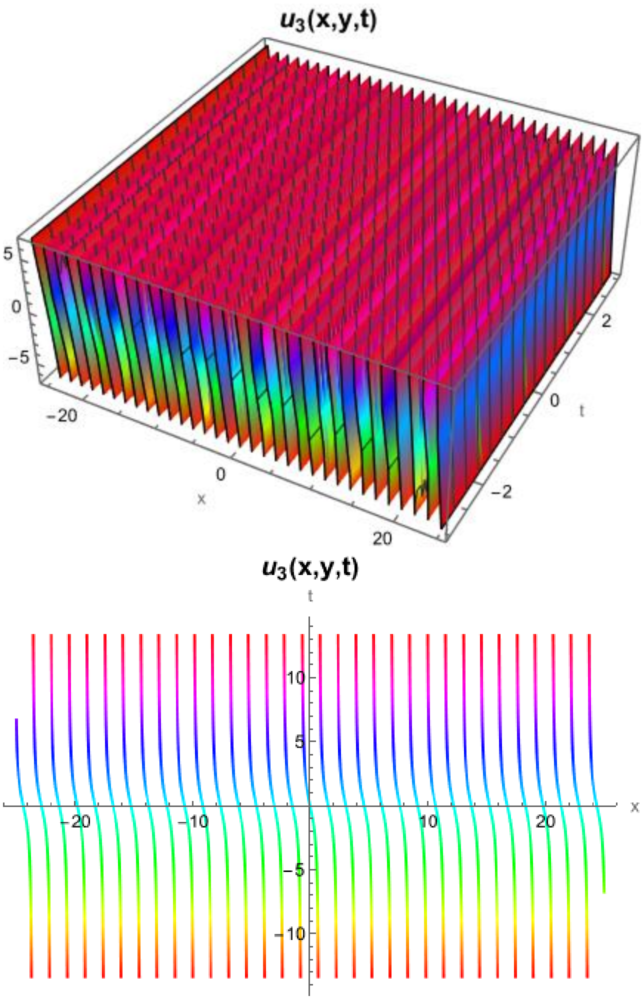

Figure 9. 3D of solution (36) for $a=4, k=2, y=0.5,$ $-25 \le x \le 25$ values with $-3 \le t \le 3$ range and 2D plot of solution for $t=1$ with these values.

Case4:

$$a_0 = -\frac{\sqrt{a^3+2hb_0}}{6\sqrt{a}}, a_3 = -a_2 - \frac{\sqrt{a^3+2hb_0}}{\sqrt{a}},$$
$$a_1 = \frac{1}{6}\left(-a_2 + \frac{5\sqrt{a^3+2hb_0}}{\sqrt{a}}\right), \tag{37}$$
$$b_1 = \frac{\sqrt{a}a_2}{\sqrt{a^3+2h}} + b_0, n = \frac{\sqrt{a^3+2h}}{\sqrt{a}}.$$

Substituting the above values in Eq. (28), we acquire dark soliton solution of Eq. (2).

$$u_4(x,y,t) = \frac{i\sqrt{17}\sqrt{a^3+2h}(x+ay-ht)}{6\sqrt{a}} - \frac{i\sqrt{17}}{2}\tanh\left[\frac{\sqrt{a^3+2h}(x+ay-ht)}{2\sqrt{a}}\right], \tag{38}$$

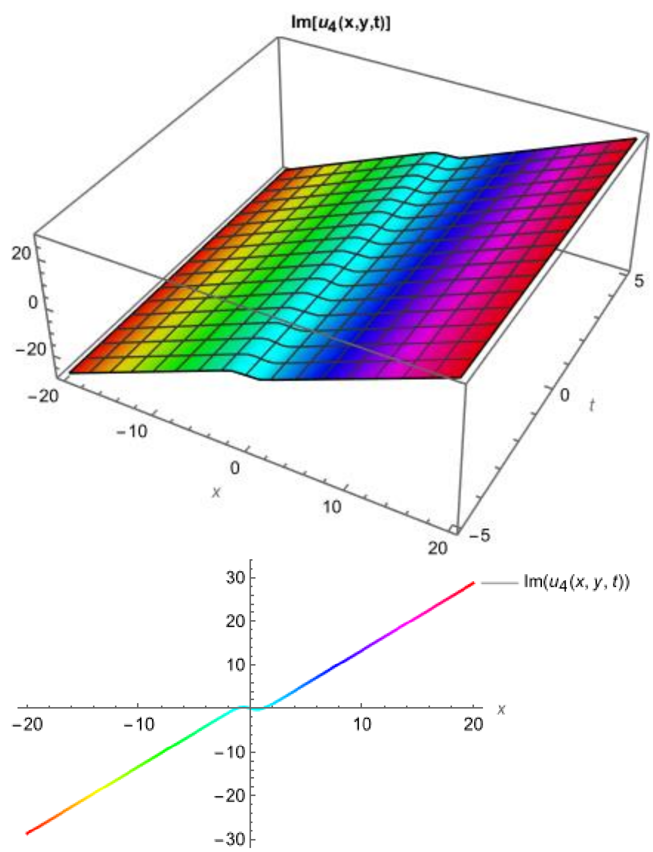

Figure 10. 3D of solution (38) for $a=2, k=1, y=1,$ $-20 \le x \le 20$ values with $-5 \le t \le 5$ range and 2D plot of solution for $t=2$ with these values.

4. Conclusion

In this made study, we discussed GKM, which is one of the NLEEs solution methods. We applied this discussed method to the (2+1)-dimensional DJKM equation and the (2+1)-dimensional DLWS and so we got some soliton solutions of equation and system being studied. At the same time, we drew the drawings of the 2D and 3D graphics of the found soliton solutions by giving certain values. Thus, it has been seen that GKM is a reliable and exact solution method in obtaining NLEEs solutions. In future studies, GKM can be used in research of other NLEEs.

References

Adem, A.R., Yıldırım, Y. and Yaşar, E. 2019. Complexiton solutions and soliton solutions: (2+1)-dimensional Date-Jimbo-Kashiwara-Miwa equation. Pramana Journal Physics, 92 (36), 1-12.

Ali, K. K., Zahibi, A., Rezazadeh, H., Ansari, R., and Inc, M. 2021. Optical soliton with Kudryashov's equation via sine-Gordon expansion and Kudryashov methods. Optical and Quantum Electronics, 53 (362), 1-15.

Chang, L., Liu, H. and Xin, X. 2020. Lie symmetry analysis, bifurcations and exact solutions for the (2+1)-dimensional dissipative long wave system. Journal of Applied Mathematics and Computing, 64, 807-823.

Cinar, M., Onder, I., Secer, A., Yusuf, A., Sulaiman, T. A., Bayram, M. and Aydin, H. 2021. Soliton Solutions of (2 + 1) Dimensional Heisenberg Ferromagnetic Spin Equation by the Extended Rational sine-cosine and sinh-cosh Method. International Journal of Applied and Computational Mathematics, 7 (135), 1-17.

Dusunceli, F., Celik, E., Askin, M. and Bulut, H. 2021. New exact solutions for the doubly dispersive equation using the improved Bernoulli sub-equation function method. Indian Journal of Physics, 95 (2), 309-314.

Gao, W., Ismael, H. F., Bulut, H. and Baskonus, H. M. 2020. Instability modulation for the (2+1)-dimension paraxial wave equation and its new optical soliton solutions in Kerr media. Physica Scripta, 95 (3), 1-17.

Ghanbari, B. and Inc, M. 2018. A new generalized exponential rational function method to find exact special solutions for the resonance nonlinear Schrödinger equation. The European Physical Journal Plus, 133 (142), 1-18.

Guo, F. and Lin, J. 2019. Interaction solutions between lump and stripe soliton to the (2+1)-dimensional Date-Jimbo-Kashiwara-Miwa equation. Nonlinear Dynamics, 96, 1233-1241.

Gurefe, Y. 2020. The generalized Kudryashov method for the nonlinear fractional partial differential equations with the beta-derivative. Revista Mexicana de Fisica, 66 (6), 771-781.

Islam, N., Khan, K. and Islam, M. H. 2019. Travelling wave solution of Dodd-Bullough-Mikhailov equation: a comparative study between Generalized Kudryashov and improved F-expansion methods. Journal of Physics Communications, 3 (5), 055004.

Ismael, H. F., Bulut, H., Park, C. and Osman, M. S. 2020. M-lump, N-soliton solutions, and the collision phenomena for the $(2+1)$-dimensional Date-Jimbo-Kashiwara-Miwa equation. Results in Physics, 19, 103329.

Kaplan, M. and Akbulut, A. 2021. The analysis of the soliton-type solutions of conformable equations by using generalized Kudryashov method. Optical and Quantum Electronics, 53 (498), 1-21.

Kumar, D. and Kaplan, M. 2018. Application of the modified Kudryashov method to the generalized Schrödinger–Boussinesq equations. Optical and Quantum Electronics, 50 (329), 1-14.

Manafian, J., Ivatloo, B. M. and Abapour M. 2020. Breather wave, periodic, andcross-kink solutions to the generalized Bogoyavlensky-Konopelchenko equation. Mathematical Methods in the Applied Sciences, 43 (4), 1753–1774.

Mirzazadeh, M., Akinyemi, L., Şenol, M. and Hosseini K. 2021. A variety of solitons to the sixth-order dispersive (3+1)-dimensional nonlinear time-fractional Schrödinger equation with cubic-quintic-septic nonlinearities. Optik-International Journal for Light and Electron Optics, 241, 166318.

Pandır, Y. and Eren, S. 2021. Exact Solutions of the Two-Dimensional KdV-Burger Equation by Generalized Kudryashov Method. Journal of the Institute of Science and Technology, 11 (1), 617-624.

Pu, J. C. and Hu, H. C. 2019. Exact solitary wave solutions for two nonlinear systems. Indian Journal of Physics, 93 (2), 229-234.

Rani, A., Zulfiqar, A., Ahmad, J. and Hassan, Q. M. U. 2021. New soliton wave structures of fractional Gilson-Pickering equation using tanh-coth method and their applications. Result in Physics, 29, 104724.

Singh, M. and Gupta, R. K. 2018. On Painlevé analysis, symmetry group and conservation laws of Date-Jimbo-Kashiwara-Miwa equation. Int J Appl Comput Math, 4 (88), 1-15.

Tuluce Demiray, S. 2020. New solutions of Biswas-Arshed equation with beta time derivative. Optik-International Journal for Light and Electron Optics, 222, 165405.

Tuluce Demiray, S. and Bayrakci, U. 2021. Soliton Solutions for space-time fractional Heisenberg ferromagnetic spin chain equation by generalized Kudryashov method and modified exp $\left(-\Omega(\eta)\right)$ -expansion function method. Revista Mexicana de Fisica, 67 (3), 393-402.

Yang, J. and Feng, Q. 2021. The soliton solutions and evolution of the (2+1)-dimensional dissipative long wave equation. Results in Physics, 21, 103974.

Yaslan, H. Ç. and Girgin, A. 2019. Exp-function method for the conformable space-time fractional STO, ZKBBM and coupled Boussinesq equations. Arab Journal of Basic and Applied Sciences, 26 (1), 163-170.

Yokus, A., Durur, H., Ahmad, H. and Yao, S.W. 2020. Construction of Different Types Analytic Solutions for the Zhiber-Shabat Equation. Mathematics, 8 (6), 1-16.

Yuan, Y. Q., Tian, B., Sun, W. R., Chai, J. and Liu, L. 2017. Wronskian and Grammian solutions for a (2+1)-dimensional Date–Jimbo–Kashiwara–Miwa equation. Computers and Mathematics with Applications, 74 (4), 873-879.