RESEARCH ON EDUCATION AND PSYCHOLOGY (REP)

Received: May 12, 2022
Accepted: June 30, 2022
http://dergipark.org.tr/rep

e-ISSN: 2602-3733

June 2022 ◆ 6(1) ◆ 46-67

Research Article

Doi: 10.54535/rep.1115808

General Understanding Regarding to the Fraction and Rational Number Concepts*

Berna Yıldızhan[1]
Necmettin Erbakan University

Erhan Ertekin[2]
Necmettin Erbakan University

Abstract

The aim of the current research is to examine the studies carried out to determine the understanding of the concepts of fraction and rational number of students, teachers and prospective teachers and to reveal the general understanding by synthesizing the findings of these studies. For this purpose, meta-synthesis research method, in which qualitative studies are again interpreted and synthesized with a qualitative understanding, was used in the study. Within the scope of the study, a total of 14 studies conducted in Turkey within the framework of the qualitative research paradigm were examined in order to reveal the understanding regarding to the concepts of fraction and rational number. Content analysis was used in the analysis of the obtained data. As a result of the research; It has been determined that students, teachers and prospective teachers have various understandings regarding to the concepts of fraction and rational number. One of the remarkable results is that the prospective teachers have an understanding that the numerator and denominator of the fraction are real numbers, while the teachers have an understanding that the numerator and denominator can be integer, natural number or positive integer. Also, it was found that students associate the rational number with negativity while associating the fraction with positivity.

Key Words

Fraction • Rational number • Understanding • Comprehension • Meta-synthesis

* This study was conducted by 5th International Symposium of Turkish Computer and Mathematics Education (TÜRKBİLMAT-5) (28-30 Oct 2021, Alanya, Turkey) are presented as oral presentations.

[1] **Correspondence to:** Necmettin Erbakan University, Ahmet Kelesoglu Education Faculty, Department of Mathematics Education, Konya, Turkey. **E-mail**: berna.yldzhnn@hotmail.com **ORCID:** 0000-0001-7233-9992
[2] Prof. Dr., Necmettin Erbakan University, Ahmet Kelesoglu Education Faculty, Department of Mathematics Education, Konya, Turkey. **E-mail:** eertekin@erbakan.edu.tr **ORCID:** 0000-0002-6466-8996

Citation: Yıldızhan, B., & Ertekin, E. (2022). General understanding regarding to the fraction and rational number concepts. *Research on Education and Psychology, 6*(1), 46-67.

Concepts

The concept of fraction is the basis of such important topics as rational number, ratio and proportion in mathematics, but it is also directly or indirectly related to many concepts and topics included in the curriculum. For this reason, the teaching of the concept of fraction, which has an important place in all stages of the education process, starts with the first grade (MEB, 2018). After the students become able to perform abstract operations with the concept of fraction, the concept of rational number, which covers fractions, is taught in the 7th grade. From the different definitions encountered in the literature on the concepts of fraction and rational number in the later years of school mathematics, definitions made in accordance with the fact that the rational number includes fractions are included in the content of mathematics courses in parallel with the curriculum. In this sense, fraction knowledge is seen important for the development of the rational number concept (Carraher, 1993).

Fraction has five different meanings as part-whole, quotient (division), operator, ratio and measure (Lamon, 1999). The part-whole meaning includes a meaning beyond scanning a region, showing the relationship between the parts of continuous or discontinuous objects and themselves. Considered from this framework, the fraction represents a certain part of a set divided into equal sized pieces (Jones, 2012). The quotient (division) meaning of fraction is the quotient obtained by dividing one number by another number (a: numerator; b: denominator; $\frac{a}{b}$: quotient). This meaning is mostly seen in situations where some multiplicity is shared to certain people in daily life. The operator meaning of fraction emerges in cases of a certain amount is enlarged or reduced (Lamon, 1999). Fraction in the sense of ratio expresses the relationship between the quantities to be compared (Cramer & Post, 1995). According to the meaning of measure, fraction; represents measurement quantities such as length, area, and volume. While the fraction measures the distance of the point on the number line from zero in one-dimensional space, it measures the area in a two-dimensional space (Lamon, 1999). As previously stated, different opinions have been put forward by mathematics educators about the the definitions of fraction and rational number concepts, which are given as a continuation of each other in our curriculum. For example, Musser et al., (2014) states that the values of a and b that make up the fraction $\frac{a}{b}$ can take non-negative integer ($b\neq0$) values. Niven (1961), on the other hand, uses the concepts of rational number and rational fraction as synonymously; he defines a rational number (or a rational fraction) as a number that can be expressed in the form $\frac{a}{b}$, where a and b are integers and b are different from zero. According to him, when the concept of fraction is used alone, it expresses an algebraic representation consisting of numerator and denominator; as in the examples $\frac{\sqrt{5}}{3}$, $\frac{x}{2}$, $\frac{x^2+1}{x-1}$. According to Bennett et al. (2016), the numerator and denominator of a fraction can be any number as long as the denominator is not zero. For example; $\frac{3}{8}$, $\frac{7}{16}$, $\frac{1,2}{7}$, $\frac{1}{\sqrt{3}}$ ... as such. Based on this expression, the mathematical definition of the fraction concept can be made as $K = \{\frac{a}{b}: a,b\in R \text{ and } b\neq0\}$. If the numerator and denominator of a fraction take integer values, this fraction is called a rational number. For example; $\frac{1}{9}$, $\frac{-3}{7}$, $\frac{10}{3}$ … Thus, the mathematical definition of the rational number concept can be expressed as follows:

$$R = \{\frac{a}{b}: a,b\in Z \text{ and } b\neq0\}.$$

Kieren (1976) expresses rational numbers as the representative of the equivalence class formed by fractions. Rational numbers are the equivalence class of $\frac{a}{b}$ fractions, with b≠0 under the ≈ relation. If the following equality exists between the (a,b) and (c,d) ordered pairs, the (a,b) fraction is said to be equivalent to the (c,d) fraction.

$$\frac{a}{b} \approx \frac{c}{d} \leftrightarrow a.d=b.c$$

Depending on the differences encountered in the definitions of fraction and rational number concepts, there are also many different opinions in the literature about the relationship of these concepts to each other. Some of these opinions include:

- Fractions do not have to be rational number (Niven, 1961).
- Rational numbers are fractions on which arithmetic operations can be performed (Kieren, 1976).
- The simplest form of fractions is rational numbers (Çelik, 2006).
- Fractions are a subset of rational numbers. That is, every fraction is a rational number, but not every rational number may express a fraction (Lamon, 2007).
- Fraction and a rational number are separate concepts from each other. Although a fraction is not a rational number, a rational number is also not a fraction (Argün et al., 2014).

Rationale and Purpose of the Study

There are many different definitions of the concepts of fraction and rational number in the current literature. Due to the diversity in the literature, it is thought that it is difficult for students, teachers and adults to make sense of these concepts. Many studies have been carried out in Turkey in order to determine the meanings that students, teachers and prospective teachers attribute to the concepts and how they define the concepts. However, since each study is conducted with a limited number of participants and a specific audience, there is no study in which studies are evaluated together and comprehensive inferences are made. There is a need for a large-scale research on how students, teachers and prospective teachers define concepts. For this reason, in the present study, it is aimed to examine the studies carried out to determine the understanding about the concepts of fraction and rational number of students, teachers and prospective teachers and to reveal the general understanding by synthesizing the findings of these studies. Therefore, within the scope of the research, the answer to the question "How are the detected understandings about these concepts as a result of the studies carried out in Turkey before May 2021 on Fractions and Rational Numbers?" is being sought. The sub-problems of the research question are as follows:

a) What are the understandings of students, teachers and prospective teachers about the concept of fraction?
b) What are the understandings of students, teachers and prospective teachers about the concept of rational number?
c) What are the understandings of students, teachers and prospective teachers towards the relationship between fraction and rational number concepts?

Method

Research Design

In the present study, the Meta-Synthesis (Thematic Content Analysis) research method was used due to the fact that a systematic review of the findings obtained from qualitative research carried out to reveal the understanding of students, teachers and prospective teachers about fraction and rational numbers in Turkey before May 2021 has been carried out. The meta-synthesis method includes the synthesis and interpretation of qualitative studies carried out a specific topic, again, within the framework of a qualitative understanding (Gümüş, 2018). With this method, beyond the individual findings of the studies, new meanings and explanations are revealed as a result of comparing the key concepts obtained from each study with each other (Thomas and Harden, 2008).

Research Sample/Study Group/Participants

Within the scope of the research, qualitative studies/documents conducted to determine the understanding, image and schemas of individuals related to the concepts of fraction and rational number were used. The studies included and excluded from the research are explained in detail below:

Inclusion Criteria

The theses and articles considered in the present research should meet the following criteria;

- ✓ Must have been published before May 2021,
- ✓ The sample subject to the study should be within the borders of Turkey,
- ✓ Should have been made to determine the understanding of individuals regarding the concepts of fraction and rational number,
- ✓ Should use one of the qualitative research designs or, if quantitative, use qualitative data collection methods,
- ✓ Must have been published in a peer-reviewed journal (for articles only), and
- ✓ The full text of the study should be reached.

Exclusion Criteria

Contrary to the items mentioned above; Studies published after May 2021, studied on a foreign sample, conducted with a quantitative understanding and using quantitative data collection methods, containing incomplete text, not published in a peer-reviewed journal, and not aimed at determining the conceptual understanding of individuals for the concepts of fraction and rational number were excluded from the scope.

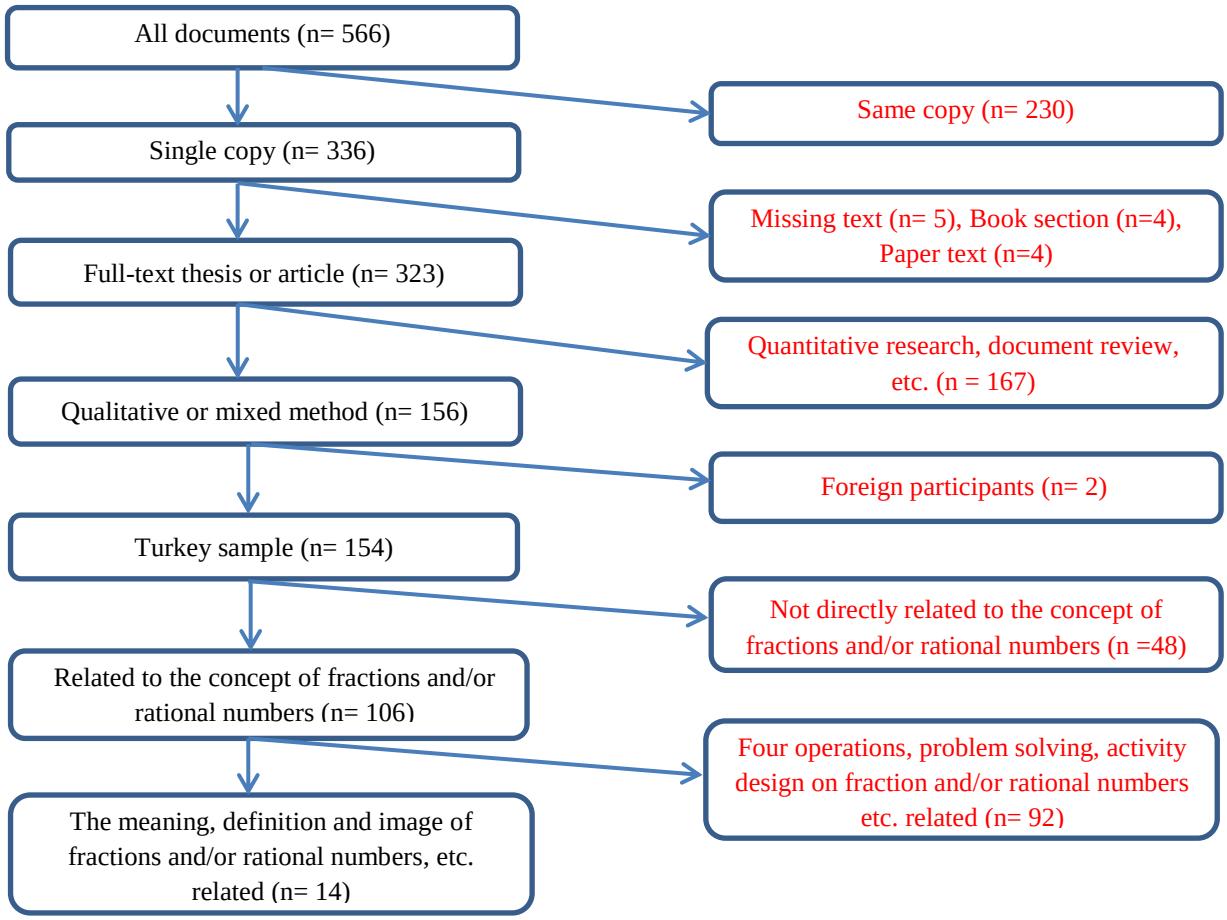
Research Instruments and Processes

In order to collect data, studies on Fractions and Rational Numbers were reached by using Yök Thesis, Yök Academic (article), Google Scholar, ULAKBIM and Web of Science databases. At this stage, ‘kesir’, ‘kesir kavrayışı’, ‘rasyonel sayı’, ‘rasyonel sayı kavrayışı’, ‘fraction’, ‘fraction conception’, ‘fraction understanding’, ‘rational number’, ‘rational number conception’ and ‘rational number understanding’ keywords were scanned in

databases and 566 studies were reached. The shema regarding the inclusion process of the documents that are the subject of the study is given in Figure 1.

Figure 1

Flow Chart of the Inclusion Process of Studies



Firstly in the process of elimination in accordance with the criteria the obtained documents, 230 studies were excluded from the scope, which were determined to be the same copy among 566 documents. Since the research will be carried out with articles and dissertations, documents such as missing text (5), book section (4) and paper text (4) were excluded from the remaining 336 studies. The qualitative findings of the studies were evaluated because it was wanted to reveal the understanding of individuals about fraction and rational number. For this reason, studies carried out with only quantitative research method or studies such as document review are excluded from the scope. As a result, out of 323 studies, 156 studies carried out by qualitative and mixed research methods were determined in accordance with the criteria. Among these studies, 2 studies that were not carried out on a sample from Turkey were excluded from the scope. In addition, the titles and abstracts of the remaining 154 studies were examined in general and studies (48) that were found not to be directly related to the concepts of fraction and/or rational number were excluded from the scope. The abstracts, research problems and methods of 106 studies, which were determined to be directly related to the concepts of fraction and/or rational number, were examined in detail and those related to

individuals' understanding, image or definition of fractions and/or rational numbers were determined (14). The identified 14 studies were subjected to 'quality assessment'. The quality assessment put forward by Pluye et al. (2009) evaluates the methodological quality of qualitative, quantitative and mixed-method primary studies. As a result of the evaluation, it was determined that all studies received 75 points and above. This evaluation can be interpreted that the studies are of high quality in terms of qualitative research.

Data Analysis

The data obtained from the documents were analyzed with the content analysis method. Content analysis is a technique in which inferences are made for the objective and systematic recognition of certain characteristics of a message (Büyüköztürk et al., 2017). The main purpose of using this analysis method is to reach the concepts and relationships that can explain the obtained data (Yıldırım and Şimşek, 2018).

After the data analysis was performed, part of the documents was examined by a second expert independent of the study in order to ensure the reliability of the encoder. As a result of expert evaluation, the Miles Huberman coefficient was found to be 84%.

$$\frac{Number\ of\ Subject/Terms\ with\ Consensus}{Total\ Number\ of\ Views} * 100$$

In order to ensure internal consistency, it is necessary to have at least 80% consensus among encoders (Patton, 2014). Therefore, the coefficient obtained in the context of our research provides the specified criterion.

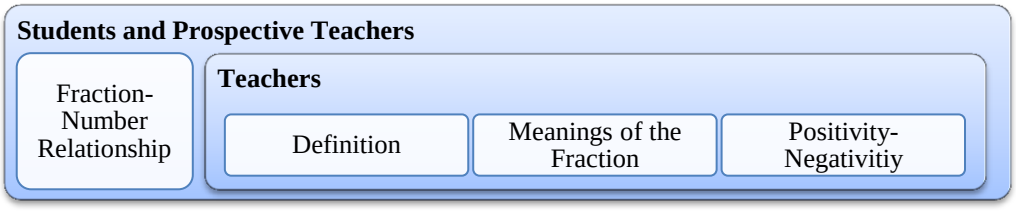
Results

Findings about Fractions

As a result of data analysis, 4 basic categories related to fractions have emerged (see Figure 2). In the studies conducted with students and prospective teachers, opinions belonging to the categories of *Definition*, *Meanings of the Fraction*, *Fraction-Number Relationship* and *Positivity-Negativity* are included, while in the studies conducted with teachers, opinions belonging to the categories of *Definition*, *Meanings of the Fraction* and *Positivity-Negativity* are included.

Figure 2

The Main Categories That Arise in Relation to Fractions



The resulting sub-categories related to fractions are presented in Table 1.

Table 1

Sub-Categories Related to the Concept of Fraction

Categories	Sub-categories					
	Student		Prospective Teacher		Teacher	
Definition	Fractional notation	$\left\{\frac{a}{b}: \text{a and b relatively prime}\right\}$	Fractional notation	$\left\{\frac{a}{b}: \text{a, b } \in \text{R}\right\}$	Fractional notation	$\left\{\frac{a}{b}: \text{a, b } \in \text{Z and b} \neq 0\right\}$
				$\left\{\frac{a}{b}: \text{a, b } \in Z^{+} \text{ and b} \neq 0\right\}$		$\left\{\frac{a}{b}: \text{a, b } \in Z^{+} \text{ and b} \neq 0\right\}$
				$\left\{\frac{a}{b}: \text{a, b } \in \text{N and b} \neq 0\right\}$		$\left\{\frac{a}{b}: \text{a, b } \in \text{N and b} \neq 0\right\}$
						$\left\{\frac{a}{b}: \text{a, b } \in \text{N}\right\}$
	Simple verbal expression		Simple verbal expression		Simple verbal expression	
	Exact divisible number		Exact divisible number		Ordered pair	
	Expression over fraction types		Exact non-divisible number			
Meanings of the Fraction	Quotient (Division)		Quotient (Division)		Quotient (Division)	
	Operator		Operator		Operator	
	Ratio		Ratio		Ratio	
					Ratio and fraction are different things	
	Measure		Measure		Measure	
	Part-Whole	Equal parts	Part-Whole	Equal parts	Part-Whole	Equal parts
		No equal parts requirement		No equal parts requirement		No equal parts requirement
Fraction-Number Relationship	Cyclic expressions are not fractions		Each number is a fraction			
			Irrational numbers are not fractions			
	Irrational numbers are not fractions		Fractions do not cover integers			
			Fractions cover integers			
	Decimal number		Positive integers are fractions			
			Decimal number			
Positivity-Negativitiy	Positive		Positive		Positive	
			Positive or negative		Positive or negative	

The first category that emerges in relation to fractions is the category "*Definition*". Within the scope of this category, it is revealed how students, prospective teachers and teachers define the fraction. Accordingly, students define the fraction as *fractional notation*, *simple verbal expression*, *exact divisible number* and *expression over fraction types*. The definition that emerges in the fractional notation sub-category is $\{\frac{a}{b}$: a and b relatively prime$\}$. In other words, students who expressed fraction as fractional notation stated that a and b should be prime between them. The definition that emerges in the category of simple verbal expression is 'the structure consisting of numerator, denominator and fraction line'. Students in the category of numbers that are exactly divisible, on the other hand, define a fraction as an expression that is exactly divisible. In addition to these, students whose definition is in the category of 'expression over fraction types' define fractions as 'simple, compound and integer fractions'.

According to the categories obtained from the research findings carried out on the prospective teachers, it is seen that the prospective teachers define the fraction as *fractional notation*, *simple verbal expression*, *exact divisible number* and *exact non-divisible number*. It has been determined that there are 3 different definitions in the fractional notation sub-category. These are;

$$\{\frac{a}{b}: a,b \in R\},$$

$$\{\frac{a}{b}: a,b \in Z^{+} \text{ and } b\neq 0\} \text{ and}$$

$$\{\frac{a}{b}: a,b \in N \text{ and } b\neq 0\}.$$

Accordingly, it is seen that the prospective teachers also have the idea that numbers a and b can take real number values other than integer and natural numbers in fraction representation. In addition to fractional notation, there are also prospective teachers who define fraction as numerator, denominator and fraction line (simple verbal expression), exact divisible numbers and exact non-divisible numbers (i.e. the result of divisions is not equal to an integer).

Teachers define the fraction as *fractional notation*, *simple verbal expression* and *ordered pair*. When the studies with teachers were examined, 4 different definitions emerged in the fractional notation sub-category. These are;

$$\{\frac{a}{b}: a,b \in Z \text{ and } b\neq 0\},$$

$$\{\frac{a}{b}: a,b \in Z^{+} \text{ and } b\neq 0\},$$

$$\{\frac{a}{b}: a,b \in N \text{ and } b\neq 0\}, \text{ and}$$

$$\{\frac{a}{b}: a,b \in N\}.$$

In addition to fractional notation, there are also teachers who define fractions as numerator, denominator and fraction line (simple verbal expression) and ordered pairs (a,b).

Another category that has emerged regarding fractions is the category of "*Meanings of the Fractions*". Within the scope of this category, the meanings attributed to the concept of fraction by students, prospective teachers and teachers are revealed. When the categories are examined, it is seen that the meanings attributed to fractions by

students and prospective teachers are similar. Students and prospective teachers attribute quotient (division), operator, ratio, measure and part-whole meanings to fractions. When the part-whole meaning is examined in detail, it is seen that there are those who think that the whole should consist of equal parts and there are those who think that there is no obligation to divide them into equal parts. It can be said that the findings obtained from student and prospective teachers are similar for teachers. However, in the studies conducted on teachers, it has been determined that there is also the opinion that ratio and fraction are different concepts from each other.

One of the emerging categories related to fractions is the category "*Fraction-Number Relationship*". Within the scope of this category, opinions are expressed about the relationship of fraction with numbers such as integers and natural numbers. There are no opinions of teachers about this category. That is why the opinions of prospective students and teachers are explained. It was determined that the students expressed three different views that cyclic expressions are not fractions, irrational numbers are not fractions, and decimal numbers are fractions. On the other hand, it has been determined that the prospective teachers have different opinions that each number represents a fraction, irrational numbers are not fractions, fractions do not cover integers, fractions cover integers, positive integers are fractions, and decimal numbers express fractions.

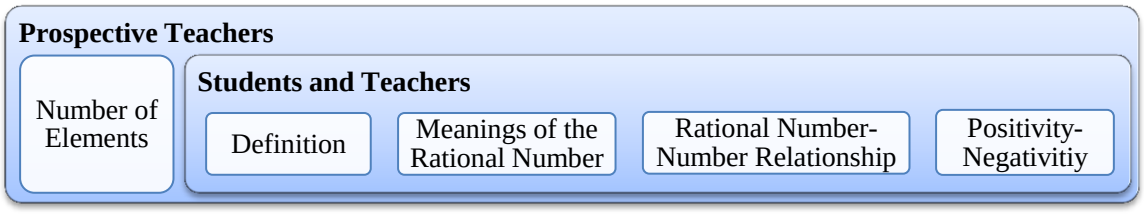
The last category that emerged about fractions is the category of "*Positivity-Negativity*". Within the scope of this category, opinions about the fraction sign are presented. While the students thought that the fraction could only be positive, two different opinions were found among the prospective teachers and teachers. One group has the opinion that the fraction can only be positive, while the other group has the opinion that the fraction can be both positive and negative.

Findings about Rational Numbers

As a result of the research, 5 basic categories related to rational numbers emerged (see Figure 3). In the studies conducted with prospective teachers, opinions belonging to *Definition*, *Meanings of the Rational Number*, *Rational Number-Number Relationship*, *Positivity-Negativity* and *Number of Elements* categories are included, while opinions belonging to *Definition*, *Meanings of the Rational Number*, *Rational Number-Number Relationship* and *Positivity-Negativity* categories are included in the studies conducted with students and teachers.

Figure 3

The Main Categories that Arise in Relation to Rational Number



The resulting sub-categories related to rational number are presented in Table 2.

Table 2

Sub-Categories Related to the Concept of Rational Number

Categories	**Sub-categories**					
	Student		**Prospective Teacher**		**Teacher**	
Definition	Fractional notation	$\left\{\frac{a}{b}: a, b \in Z\right\}$	Fractional notation	$\left\{\frac{a}{b}: a, b \in Z, a \text{ and } b \text{ relatively prime}\right\}$	Fractional notation	$\left\{\frac{a}{b}: a, b \in Z, a \text{ and } b \text{ relatively prime}\right\}$
				$\left\{\frac{a}{b}: a, b \in Z^{+} \text{ and } b \neq 0\right\}$		$\left\{\frac{a}{b}: a, b \in Z \text{ and } b \neq 0\right\}$
				$\left\{\frac{a}{b}: a, b \in Z \text{ and } b \neq 0\right\}$		$\left\{\frac{a}{b}: a, b \in Z\right\}$
				$\left\{\frac{a}{b}: a, b \in Z\right\}$		
	Simple verbal expression		Simple verbal expression		Simple verbal expression	
	Non-irrational real numbers		Non-irrational real numbers			
	Numbers that can go out of the root		Representative of the equivalence class		Representative of the equivalence class	
	Numbers with a limited or cyclic decimal					
Meanings of the Rational Number	Part-Whole	Equal parts	Part-Whole	No equal parts requirement		
		No equal parts requirement				
			Quotient (Division)			
			Ratio		Ratio	
Rational Number-Number Relationship	Natural Number	Natural numbers are rational numbers	Natural Number	Natural numbers are rational numbers		
		Natural numbers are not rational numbers				
	Zero	Zero is the rational number	Zero	Zero is the rational number	Zero	Zero is the rational number
		Zero is not the rational number				
	Integer	Rational numbers cover integers	Integer	Rational numbers cover integers	Integer	Rational numbers cover integers
		Rational numbers do not cover integers		Rational numbers do not cover integers		
		Integers are not rational numbers				
	Comma	Numbers with commas are	Comma	Numbers with commas are rational		

	expression	rational numbers	expression	numbers	
		Cyclic or decimal expressions are not rational numbers		Cyclic or decimal expressions are not rational numbers	
	Other	Every number is a rational number			
		There are rational numbers that are not real numbers			
		Every irrational number is a rational number			
Positivity-Negativity			Positive		
	Negative		Negative		
	Positive or negative		Positive or negative		Positive or negative
Number of Elements			Rational numbers are infinite		
			Rational numbers are countably infinite		
			Rational numbers are uncountably infinite		
			Rational numbers are finite		

The first category that emerges in relation to rational numbers is the category "*Definition*". Within the scope of this category, it is revealed how students, prospective teachers and teachers define the rational number. When the studies are examined, it is seen that students define a rational number as *fractional notation*, *simple verbal expression*, *non-irrational real numbers*, *numbers that can go out of the root*, and *numbers with a limited or cyclic decimal*. The definition that emerges in the fractional notation sub-category is $\{\frac{a}{b}: a,b \in Z\}$. In other words, students who express rational numbers as fractional notation have the understanding that a and b must be integers. The definition that emerges in the category of simple verbal expression is 'the structure consisting of numerator, denominator and fraction line'. In addition, it has been determined that some students try to define rational numbers through irrational, rooted or decimal notation. These definitions are as follows; 'Non-irrational real numbers are called rational numbers', 'Numbers that can go out of the root are called rational numbers' and 'Numbers with a limited or cyclic decimal'.

In the studies examined, it is seen that prospective teachers define rational numbers as *fractional notation*, *simple verbal expression*, *non-irrational real numbers* and *representative of the equivalence class*. It has been determined that there are 3 different definitions in the fractional notation sub-category. In the subcategory of fractional notation, 4 different definitions appear. These are;

$$\{\frac{a}{b}: a,b \in Z, \text{ a and b relatively prime}\},$$

$$\{\frac{a}{b}: a,b \in Z^{+} \text{ and } b\neq 0\},$$

$$\{\frac{a}{b}: a,b \in Z \text{ and } b\neq 0\}, \text{ and}$$

$$\{\frac{a}{b}: a,b \in Z\}.$$

In addition to these, there are also prospective teacher who stated that the rational number consists of numerator, denominator and fraction line (simple verbal expression), non-irrational numbers are called rational numbers, and the expression representing equivalent fractions (representative of the equivalence class) is a rational number.

Teachers, on the other hand, define rational numbers as *fractional notation*, *simple verbal expression* and *representative of the equivalence class*. In studies conducted with teachers, it has been determined that there are 3 different definitions in the fractional notation category. These are;

$$\{\frac{a}{b}: a,b \in Z, \text{ a and b relatively prime}\},$$

$$\{\frac{a}{b}: a,b \in Z \text{ and } b\neq 0\}, \text{ and}$$

$$\{\frac{a}{b}: a,b \in Z\}.$$

Besides these definitions, there are also teachers who state that the rational number consists of numerator, denominator and fraction line (simple verbal expression) and that it is an expression representing equivalent fractions (representative of the equivalence class).

Another category that has emerged regarding rational numbers is the category of "*Meanings of the Rational Numbers*". Within the scope of this category, the meanings attributed to the concept of rational number by students, prospective teachers and teachers are revealed. In the studies conducted with the students, it was seen that the *part-whole* meaning of rational numbers was revealed. Among the students, there are those who express that the whole should consist of equal parts, as well as those who express that the whole does not have to be divided equally. The meanings that the prospective teachers attribute to the rational number are the meanings of *part-whole, quotient (division)* and *ratio*. The prospective teachers who said that the rational number expresses the part-whole meaning did not state that the whole must be divided into equal parts. In the studies conducted with teachers, it has been seen that only the meaning of *ratio* is loaded into rational numbers.

Another category that has emerged regarding rational numbers is the category of "*Rational Number-Number Relationship*". In the studies conducted with the students, it was seen that the relationship with *natural numbers*, *zero*, *integer*, *comma expression* and *other numbers* of rational numbers was established. Students have both opposite views on whether natural numbers and zero are rational numbers or not. It has been observed that some of the students stated that these expressions were rational numbers, while others stated that they were not rational numbers. There are three different views on integers. These are the views that 'rational numbers cover integers', 'rational numbers do not cover integers' and 'integers are not rational numbers'. There are also two opinions about the relationship of comma expressions with rational numbers in the form of 'Numbers with commas are rational numbers' and 'cyclic or decimal expressions are not rational numbers'. Finally, the opinions contained in the other category are those that 'every number indicates a rational number', 'there are rational numbers that are not real numbers' and 'every irrational number is a rational number'.

In the studies conducted with the prospective teachers, it was seen that the relationship with *natural numbers*, *zero*, *integer* and *comma expression* of rational numbers was established. Prospective teachers are of the opinion that 'natural numbers and zero are rational numbers'. There are two opposite opinions in prospective teacher regarding the integer-rational number relationship as 'rational numbers cover integers' and 'rational numbers don't cover integers'. In addition, there are opinions about the comma expression as 'numbers with commas are rational numbers' and 'cyclic or decimal expressions are not rational numbers'. In the studies conducted with teachers, it was seen that the relationship with *zero* and *integer* of rational numbers was established. Teachers have expressed the opinion that 'zero is a rational number'. As for the integers, it has been understood that there is a common opinion that 'rational numbers cover integers'.

The fourth category that emerged in relation to rational numbers is the "*Positivity-Negativity*" category. Within the scope of this category, the opinions of students, prospective teachers and teachers regarding the sign of the rational number are presented. There are two different opinions among students as 'rational numbers are negative' and 'rational numbers can be positive or negative'. It is revealed that there are three different opinions among prospective teachers. These are 'rational numbers are positive', 'rational numbers are negative' and 'rational numbers can be positive or negative'. In the studies conducted with teachers, it was seen that teachers expressed a

common idea about the sign of rational numbers. Teachers express that 'rational numbers can be positive or negative'.

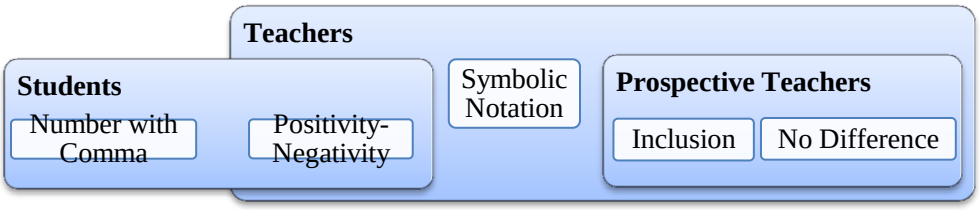
The last category that emerged about rational numbers is the category of "*Number of Elements*". Within the scope of this category, the number of elements of rational numbers is revealed. In the studies conducted, it was observed that students and teachers did not express an opinion about the number of employees. On the other hand, prospective teachers expressed four different opinions. These are; rational numbers are *infinite*, *countably infinite*, *uncountable infinite*, and *finite*.

Findings about Relationship between Fractions and Rational Numbers

As a result of the research, 5 basic categories have emerged regarding the relation between fractions and rational numbers (see Figure 4). Opinions belonging to the categories *Positivity-Negativity* and *Number with Comma* in the studies conducted with students; *Inclusion* and *No Difference* in the studies conducted with prospective teachers; and *Inclusion*, *No Difference*, *Symbolic Notation* and *Positivity-Negativity* in the studies conducted with teachers are included.

Figure 4

The main Categories that Arise for Regarding the Fraction-Rational Number Relationship



The resulting sub-categories related to fraction-rational number relationship are presented in Table 3.

Table 3

Sub-Categories Related to Fraction-Rational Number Relationship

Categories	Sub-categories		
	Student	**Prospective Teacher**	**Teacher**
Inclusion		Fractions can contain irrational numbers, but rational numbers can not	Fractions cover rational numbers
		Fractions cover rational numbers	
		Fractions cover integers, rational numbers don't	Rational numbers cover fractions
		Rational numbers cover fractions	
		Rational numbers cover integers, fractions don't	
No Difference		Fraction and rational number are two concepts that have the same meaning	Fraction and rational number are two concepts that have the same meaning
Symbolic Notation			Fraction is one of the symbolic notations of rational numbers
Positivity-Negativity	The number written in $\frac{a}{b}$ format is a fraction if it is positive, and a rational number if it is negative		There is only difference in sign between them
	Fraction can not be negative, rational number can be both negative and positive		Fraction can not be negative, rational number can be both negative and positive
Number with Comma	If it is infinite after the comma, it is rational, if it is finite, it is fraction		

The first category that emerges regarding the fraction-rational number relationship is the category "*Inclusion*". In this category, it is stated that fraction and rational number concepts differ in terms of the number sets they cover. While there was no opinion belonging to this category in the studies conducted with the students, the prospective teachers and teachers gave different opinions. When the opinions of prospective teachers are examined, it is seen that five different views have emerged. These are;

- Fractions can contain irrational numbers, but rational numbers can not.
- Fractions cover rational numbers.
- Fractions cover integers, rational numbers don't.
- Rational numbers cover fractions.
- Rational numbers cover integers, fractions don't.

According to the classifications obtained from the studies conducted with teachers, it has been determined that teachers have two opposite understandings. These are 'Fractions cover rational numbers' and 'Rational numbers cover fractions'.

Another category that has emerged regarding the fraction-rational number relationship is the category of "*No Difference*". In this category, it is stated that there is no difference between the concepts of fractions and rational numbers. In the studies conducted with the students, it was seen that the students did not express their opinions in this category. It has been observed that prospective teachers and teachers have the understanding that 'fraction and rational number are two concepts that have the same meaning'.

The third category that emerged regarding the fraction-rational number relationship is the "*Symbolic Notation*" category. It has been observed that students and prospective teachers did not express their opinions regarding this category. In the teachers under this category, there is an idea that 'fraction is one of the symbolic notations of rational numbers'.

The fourth category that emerged regarding the fraction-rational number relationship is the "*Positivity-Negativity*" category. People who are included in the scope of this category have the opinion that fractions and rational numbers differ due to their signs. In the studies conducted with prospective teachers, there is no opinion expressing that fraction and rational numbers differ due to their signs. In the studies conducted with students, it was determined that there were two opinions about the difference in sign. These are; 'the number written in $\frac{a}{b}$ format is a fraction if it is positive, and a rational number if it is negative' and 'fraction can not be negative, rational number can be both negative and positive'. Two opinions have also emerged in studies conducted with teachers. These are; 'there is only difference in sign between them' and 'fraction can not be negative, rational number can be both negative and positive'.

The last category that emerged regarding the fraction-rational number relationship is the category of "*Number with Comma*". Prospective teachers and teachers did not express an opinion on this category. On the other hand, the students who expressed their opinions in the category of numbers with commas hold the opinion that "if it is infinite after the comma, it is rational, if it is finite, it is fraction".

Discussion, Conclusion & Suggestions

One of the remarkable results that emerged as a result of the synthesis of studies on the concept of fraction is related to the definitions of teachers and prospective teachers under the fractional notation category. While prospective teachers have the opinion that a and b can take real number values in the expression $\frac{a}{b}$, this opinion is not found in the teachers. Teachers are of the opinion that a and b can take integer, natural number or positive integer values in the expression $\frac{a}{b}$. While it is expected that teachers will master higher-level definitions due to factors such as academic equipment and experience, the opposite situation has arisen. The reason why the high-level definition did not appear in teachers may be related to the age level at which they are actively teaching. While the concept of fraction is defined over positive integers at the younger age level, the range of the definition set is expanded as the age level progresses. Therefore, it can be said that teachers have over time moved away from the high-level definition and adopted the definition of the age level at which they teach. It is thought that the reason why prospective teachers make higher-level definitions is that they encounter definitions at this level during their undergraduate courses.

The effect of defining the concept of fraction, defined over positive integers at young age levels, as high level at higher age levels is also seen in the findings related to the sign of the fraction. In the studies carried out on the fraction sign, the students have said that the fraction can only be positive, while the teachers and prospective teachers said that the fraction can only be positive besides can receive both positive and negative values. While it is an expected result that the students think that the fraction should only be positive, the opinion of the teachers and prospective teachers that the fraction can only have a positive value is compatible with the different definitions. This situation is considered as an indication that teachers and prospective teachers have adopted the definition appropriate for the age level they are teaching/will be doing instead of the high-level definition.

One of the interesting results that has emerged regarding the concept of fractions is that some students and prospective teachers define a fraction as the ratio of two exact divisible numbers. According to this definition, the set of integers and the set of fraction numbers overlap each other. The ratio of two numbers that are not exactly divisible (having a decimal notation) is not defined as a fraction. Accordingly, the expression $\frac{4}{2}$ indicates a fraction, while the expression $\frac{2}{4}$ does not.

In the literature, it is stated that the fraction has five different meanings: quotient (division), operator, ratio, measure, and part-whole (Lamon, 1999). As a result of the research, it was seen that students, teachers and prospective teachers have knowledge about all of these meanings. However, it is understood that there is confusion in the all three groups about the part-whole meaning of the fraction, about whether or not the whole must be divided equally. In the definitions and explanations made about the meaning of fraction, it is seen that there is no emphasis on the necessity of equal fragmentation of the whole, which should be emphasized mainly in the sense of the part-whole of the fraction.

Although the concepts of fraction and rational number are both shown as $\frac{a}{b}$, some academic definitions require that the values of a and b be prime between them for rational numbers (Kieren, 1976). When the categories that emerged as a result of the studies conducted with the students are examined, it is seen that the students stated the condition of being prime among them for fractions, but they did not mention such a condition for rational numbers. This situation can be interpreted as the students cannot make sense of being prime among them and therefore confuse the concepts of fractions and rational numbers.

It is seen that the common definition for the concept of fraction and rational number in students, prospective teachers and teachers is a simple verbal expression, that is, the definition that includes numerator, denominator and fraction line. This situation means that individuals make a definition by using the symbolic notation of the fraction in order to define the concepts. Thus, it can be said that in the definitions made, there is not enough information about the properties of the numbers that should be included in the numerator and denominator of the fraction or rational number, or there is a lack of awareness of the need to include this information in the definitions. Therefore, there are no differences between the concepts of fraction and rational number for individuals who make these definitions.

One of the interesting results that emerged as a result of the research is related to the signs of fraction and rational number concepts. In the studies conducted with the students, the opinion that the fraction can only take a positive value emerged, while the opinions about the rational number concept that it can only take a negative value and be positive or negative emerged. Therefore, while students associate negativity with the concept of rational number, they associate positivity with the concept of fraction. This finding is also supported by the findings regarding the fraction-rational number relationship as a result of the studies conducted with the students. In the studies conducted, some of the students stated that the fraction and the rational number are separated from each other only because of the sign difference. In this direction, two different opinions have emerged. However, the common situation in both opinions is that the fraction is associated with positivity and the rational number with negativity.

It has been revealed that teachers and prospective teachers have the opinion that there is no difference between fractions and rational numbers (that is, these expressions are two concepts with the same meaning). However, it is noticeable that no findings in this direction have been obtained in the studies conducted with the students. While it is possible for students to think of fraction and rational number concepts as the same concept under normal conditions, this situation was observed in teachers and prospective teachers. This situation can be explained by the fact that teachers and prospective teachers cannot synthesize different definitions of fraction and rational number concepts and the relationships between them, and they have a mental confusion about the differences between the two concepts.

Recommendations

In the current study, it is revealed that students, teachers and prospective teachers have various understandings of the concepts of fraction and rational number. The conceptual confusion of teachers and prospective teachers who will be teachers of the future can be shown as the reason for the different understandings that arise in students. Because the fact that the concepts are not correctly sense making in the mind of the person who teaches will naturally affect the content of the teaching they will give. Therefore, in order for students to understand the concepts correctly, it is

important that first of all, prospective teachers know the correct meanings and correct definitions of the concepts. For this reason, it is suggested that the different meanings of the concepts of fractions and rational numbers in the undergraduate education within the scope of the research, the different views in the literature on this subject and how they can be taught to students should be questioned with a critical perspective.

Fraction and rational number concepts are related to concepts such as integer, natural number, irrational number, rooted expression. However, as a result of the study, it is seen that the relationship between these concepts in individuals is not sufficiently established. For this reason, the relationship between these concepts at every learning level should be emphasized and discussed. The elementary school-middle school level fraction and integer relationship can be compared, while the high school level fraction and rational number have a rooted expression, irrational number, etc. comparisons can be made with.

The research carried out reveals the current understanding of fraction and rational number concepts in the Turkish sample. A similar research can be carried out with studies with a foreign sample in order to reveal the existing understandings of these concepts abroad. Thus, it can be determined how the understandings regarding of the concepts differ between the Turkish and foreign samples.

Ethic

This study was conducted in accordance with the ethical standards of the institutional and/or national research committee and with the 1964 Helsinki declaration and its later amendments.

Author Contributions

All stages of the study were organized and conducted by the authors.

Conflict of Interest

In the research, the authors declare no conflict of interest.

Funding

This study has not received funding.

References

The sources used in the study are indicated by the " * " sign.

* Aksu, Z. (2013). *The development of pre-service primary school teachers' pedagogical content knowledge in fractions* [Unpublished doctoral dissertation]. Atatürk University, Erzurum.

* Aktaş, D. G. (2019). *The concept definition of students and teachers about fraction and rational number* [Unpublished doctoral dissertation]. Gazi University, Ankara.

Argün, Z., Arıkan, A., Bulut, S., & Halıcıoğlu, S. (2014). *Temel matematik kavramların künyesi [Identity of basic mathematical concepts]*. Ankara: Gazi Kitabevi.

Bennett, A., Burton, L., Nelson, T., & Ediger, J. (2016). *Mathematics for elementary teachers: A conceptual approach* (10th. Edition). NY: The McGraw-Hill Companies.

Büyüköztürk, S., Cakmak, E. K., Akgun, O. E., Karadeniz, S., & Demirel, F. (2017). *Bilimsel araştırma yöntemleri [Scientific research methods]* (23. Baskı). Ankara: Pegem Akademi.

Carraher, D. W. (1993). Lines of thought: A ratio and operator model of rational number. *Educational Studies in Mathematics, 25*, 281-305. https://doi.org/10.1007/BF01273903

Cramer, K., & Post, T. (1995). Facilitating children's development of rational number knowledge. D. Owens, M. Reed, and G. Millsaps (Eds.), *Proceedings of the Seventeenth Annual Meeting of PME-NA* (s. 377-382). Columbus, OH: PME.

Çelik, B. (2006). *Temel matematik [Basic math]*. Ankara: Nobel Yayın.

* Çetin, H. (2020). Investigation of the definitions of pre-service elementary mathematics teachers on fraction concept. *Eurasian Journal of Teacher Education, 1*(3), 172-185. https://dergipark.org.tr/en/pub/ejte/issue/59069/795482

* Çevikbaş, M., & Argün, Z. (2017). Future mathematics teachers' knowledge of rational and irrational number concepts. *Journal of Uludağ University Faculty of Education, 30*(2), 551-581. https://doi.org/10.19171/uefad.368968

* Doğan, A. (2018). *Primary school teachers' knowledges toward fractional meanings and models used in fractions teaching* [Unpublished doctoral dissertation]. Gazi University, Ankara.

* Ercire, Y. E., Narlı, S., & Aksoy, E. (2016). Learning difficulties about the relationship between irrational number set with rational or real number sets. *Turkish Journal of Computer and Mathematics Education, 7*(2), 417-439. http://dx.doi.org/10.16949/turcomat.47225

* Erdem, E. (2016). Prospective middle school mathematics teachers'(PMTs) content knowledge about concepts 'fraction' and 'rational number'. *Journal of Education and Training Studies, 4*(6), 80-92. http://dx.doi.org/10.11114/jets.v4i6.1431

Gümüş, S. (2018). Nitel araştırmaların sistematik derlemesi: Meta-sentez [Systematic review of qualitative research: Meta-synthesis]. K. Beycioğlu, N. Özer ve Y. Kondakçı (Ed.) içinde, *Eğitim yönetiminde araştırma [Research in educational administration]* (s. 533-551). Ankara: Pegem Akademi.

* Günaydın, R. (2018). *Investigation in the APOS theoretical framework of the process of creating fraction concept in the 5th grade students'* [Unpublished master's thesis]. Eskişehir Osmangazi University, Eskişehir.

* Gürel, Z. Ç., & Okur, M. (2016). 6th and 7th grade secondary school students' misconceptions about fractions. *Erzincan University Journal of Education Faculty, 18*(2), 922-952. https://doi.org/10.17556/jef.30116

Jones, J. C. (2012). *Visualizing elementary and middle school mathematics methods*. USA: John Wiley & Sons.

Kieren, T. (1976). On the mathematical, cognitive and instructional foundations of rational numbers. R. Lesh (Ed.), *Number and measurement: Papers from a research workshop* (s. 101-144). Athens, Georgia: The Georgia Center for the Study or Learning and Teaching Mathematics.

Lamon, S. J. (1999).*Teaching fractions and ratios for understanding*. New Jersey: Lawrence Erlbaum Associates.

Lamon, S. J. (2007). Rational numbers and proportional reasoning: Toward a theoretical framework. F. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (s. 629-668). Charlotte, NC: Information Age Publishing.

* Macit, E. (2020). *An investigation of the relationship between 6th grade students' images, misconceptions and academic achievement about fraction* [Unpublished doctoral dissertation]. Inonu University, Malatya.

* Macit, E., & Nacar, S. (2019). Concept images for rational number and fraction of the students at the elementary mathematics education department. *Inonu University Journal of the Graduate School of Education, 6*(11), 50-62. https://doi.org/10.29129/inujgse.547277

Milli Eğitim Bakanlığı [MEB] (2018). *Matematik dersi öğretim programı (İlkokul ve ortaokul 1, 2, 3, 4, 5, 6, 7, 8. Sınıflar) [Mathematics curriculum (primary and secondary school grades 1, 2, 3, 4, 5, 6, 7 and 8)]*. Ankara: Talim Terbiye Kurulu Başkanlığı.

Musser, G. L., Peterson, B. E., & Burger, W. F. (2014). *Mathematics for elementary teachers: A contemporary approach* (10th edition). USA: John Wiley & Sons.

Niven, I. (1961). *Numbers: Rational and irrational*. Washington D.C.: Mathematical Association of America.

Patton, M. Q. (2014). *Qualitative research & evaluation methods: Integrating theory and practice*. London: Sage publications.

Pluye, P., Gagnon, M. P., Griffiths, F., & Johnson-Lafleur, J. (2009). A scoring system for appraising mixed methods research, and concomitantly appraising qualitative, quantitative and mixed methods primary studies in mixed studies reviews. *International Journal of Nursing Studies, 46*(4), 529-546. https://doi.org/10.1016/j.ijnurstu.2009.01.009

* Şen, C. (2021). Assessment of a middle-school mathematics teacher's knowledge for teaching the 5th-grade subject of fractions. *Turkish Journal of Computer and Mathematics Education (TURCOMAT), 12*(1), 96-138. https://doi.org/10.16949/turkbilmat.742136

* Temel, H., & Eroğlu, A. O. (2014). A study on 8th grade students' explanations of concepts of number. *Kastamonu Education Journal, 22*(3), 1263-1278. https://dergipark.org.tr/en/pub/kefdergi/issue/22601/241491

Thomas, J., & Harden, A. (2008). Methods for the thematic synthesis of qualitative research in systematic reviews. *BMC Medical Research Methodology, 8*(1), 1-10. https://doi.org/10.1186/1471-2288-8-45

* Yetim, S., & Alkan, R. (2010). Common mistakes and misconceptions of 7th grade students about the rational numbers and placement of the rational numbers on the number line. *Manas Journal of Natural Sciences, 2*(11), 87–109.

Yıldırım, A., & Şimşek, H. (2018). *Sosyal bilimlerde nitel araştırma yöntemleri [Qualitative Research Methods in Social Sciences]* (11. Baskı). Ankara: Seçkin Yayıncılık.