

Synthesis of Linear Antenna Arrays with Physics Based AOA, CryStAl and LA Algorithms

Lineer Anten Dizilerinin Fizik Tabanlı AOA, CryStAl ve LA Algoritmaları ile Sentezi

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| ARTICLE INFO | A B S T R A C T |
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| Article history Received : 23 May 2022 Accepted : 30 July 2022 Keywords: Linear Antenna Array Synthesis, Archimedes Optimization Algorithm, | In this article, 10, 16, and 24-Element of Linear Antenna Arrays (LAAs) synthesis are carried out with 3 different novel physics-based metaheuristic methods. These methods are called Archimedes Optimization Algorithm (AOA), Crystal Structure Algorithm (CryStAl), and Lichtenberg Algorithm (LA). While performing the LAA synthesis, Half Power Beam Width (HPBW), which is related to the directivity of the antenna, is also taken into account. The methods proposed in this study are run independently 30 times to obtain the statistical values of LAA synthesis. The minimum, maximum, median, and standard deviation values of the SLL and HPBW of the radiation patterns obtained as a result of these runs are tabulated. The performances of these three proposed novel physics-based optimization methods are given comparatively. In all simulation studies, the CryStAl method generally showed the best |
| Crystal Structure Algorithm, Lichtenberg Algorithm | performance. |
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| MAKALE BİLGİSİ | ÖZET |
| Makale Tarihleri | Bu makalede 10, 16, ve 24 elemanlı Doğrusal Anten Dizi (Linear Antenna |
| Gönderim : 23 Mayıs 2022 Kabul : 30 Temmuz 2022 | Array-LAA) sentezi literatüre yeni kazandırılmış 3 farklı fizik tabanlı metasezgisel yöntemle gerçekleştirilmiştir. Bu yöntemler Arşimet Optimizasyon Algoritması (AOA), Kristal Yapı Algoritması (CryStAl) ve Lightanharg Algoritması (LA), alarak içimlendiriligler LAA santazi |
| | gerçekleştirilirken aynı zamanda antenin yönelticiliği ile ilgili olan Yarı Demet Güç Genişliği (HPBW)'de göz önüne alınmıştır. Bu çalışmada önerilen |
| | metotlar, LAA sentezinin istatiksel değerlerini alabilmek için 30 defa bağımsız |
| Anahtar Kelimeler: | diyagramlarına ait SLL ve HPBW değerlerinin minimum, maksimum, ortalama |
| Lineer Anten Dizi Sentezi, Arşimet Optimizasyon Algoritması, Kristal Yapı Algoritması, Lichtenberg Algoritması | ve standart sapma değerleri tablolanmıştır. Önerilen 3 yeni fizik tabanlı optimizasyon metotlarının performansları karşılaştırmalı olarak verilmiştir. Yapılan tüm simülasyon çalışmalarında genellikle CryStAl metodu en iyi performansı göstermiştir. |
| | |

1. INTRODUCTION

Different problems occur in nature constantly and at the same time, nature, which is the biggest problem solver, constantly produces solutions to these problems in order to preserve its existence. There are many physical laws in nature to solve problems and keep the ecosystem functioning. Many technologies have been developed using the existing laws of physics and continue to be developed. Realizing the ability to find the solution found in the laws of physics, some researchers developed new artificial intelligence methods inspired by some physics laws to solve extremely complex problems.

Basically, algorithms inspired by the laws of physics are simply called [1-4]. We can classify algorithms inspired by the laws of physics as follows. There are many different optimization methods that have emerged with different theorems of physics. These are Charged System Search inspired by Coulomb's and Newton's laws [5]; Gravitational Search Algorithm explaining the theory of gravity [6]; Henry Gas Solubility Optimization based on Henry's law [7]; Sine-Cosine Algorithm using trigonometric functions and fractals [8]; Thermal Exchange Optimization inspired by Newton's Cooling laws [9]: Lightning Search Algorithm developed based on step leader propagation mechanism using the lightning phenomenon found in nature [10]; Magnetic Optimization algorithm created by formulating magnetic force laws [11]; Electromagnetic Field Optimization based on the mathematical formula of electromagnetic physics law known as Biot-Savart [12]; They are Ion Motion Optimization methods of ions formed based on the repulsion and attraction law of electromagnets [13]. As mentioned, physics-based algorithms are quite diverse and there are many studies in the literature using physics-based algorithms [1-13]. Considering these studies, physics-based algorithms have the ability to cope with difficult problems. In order to achieve high efficiency in antennas, they must be optimized with optimization methods that overcome difficult problems. In addition, in order to increase the efficiency of the antennas, it is necessary to form antenna arrays by combining more than one antenna [14]. Antenna array synthesis is a very challenging problem in the field of computational electromagnetics. The names of the antenna arrays change according to the geometric structures of the antennas [15]. Antenna arrays in which the elements are arranged on a linear structure are called Linear Antenna Arrays (LAAs). LAAs have ease of application thanks to their flat structure. Due to this convenience, there are many studies with LAAs in the literature. Contrary to the ease of implementation, since many parameters such as position, amplitude, and phase need to be optimized, the solution of the problem is quite difficult and algorithms with the ability to cope with difficult problems are preferred for optimizing LAA [16].

In this study, LAAs with 10, 16, and 24-Element are optimized. Three novel physics-based algorithms, which are called Archimedes Optimization Algorithm (AOA), Crystal Structure Algorithm (CryStAl), and Lichtenberg Algorithm (LA) are used to determine amplitudes values of LAAs in the optimization process. To the best of our knowledge, these physics-based methods have been applied to LAA for the first time. After 30 independent runs, median, max, min, and standard deviation values of SLL, and HPBW are obtained. The results obtained using these data are presented in a comparative way.

In the rest of the study, detailed information about the problem formulation is given in Section 2. The physicsbased algorithms used are explained in Section 3. A comparative presentation of the obtained results is given in Section 4. Finally, inferences made from the conclusions are given in Section 5.

2. PROBLEM FORMULATION

The geometry of the LAA is given in Figure 1. LAA elements arranged in a linear x-axis plane are positioned equidistant from each other on both arms of the axis. LAA elements are arranged on both sides of the starting point as M pieces, a total of 2M pieces [15].



Figure 1. The geometry of LAA.

Array factor of LAA is given in the following equations:

$$AF(\varphi) = 2\sum_{m=1}^{M} I_m \cos \left[kd_m \cos(\varphi) + \varphi_m\right]$$

where, the excitation amplitude, phase, and position weight of the *m*th element in the array are represented by I_m , φ_m , and d_m respectively. The wavenumber is represented by the scanning angle *k*, which is written as $k = \frac{2\pi}{\lambda}$.

The LAA has a total of 2M elements, however, because these LAA are symmetrical, the parameter to be optimized is half of the total number of antennas, M. Cost function (CF) is also needed to integrate the antenna array's formula into metaheuristic algorithms. CF is shown below:

$$CF = WF_{SLL} \cdot F_{SLL} + WF_{HPBW} \cdot F_{HPBW}$$

(2)

(1)

where WF_{SLL} and WF_{HPBW} are the CF's weight factors. The functions F_{SLL} and F_{HPBW} are utilized to narrow HPBW and suppress SLL values, respectively.

3. ALGORITHMS

3.1. Archimedes Optimization Algorithm

AOA is discovered by Hashim et al, inspired by the buoyancy of fluids [2]. In this algorithm, firstly, an imaginary object is dropped into the water. The dropped object may be floating or submerged in water. AOA tries to find the optimum value by trying to keep the object in balance above the water. This algorithm basically has two items. The first is a fluid substance and the second is a solid body. When the solid body is released into the fluid, it is floating if it weighs no more than the weight of the fluid. In the opposite situation, it will travel into the fluid until it reaches the equilibrium state. In other words, the object has a force in the water. It is necessary to reduce this force to zero and to keep it in balance by keeping this force constant at 0. There are some necessary parameters to ensure this. AOA uses three parameters to optimize this object. These are volume, density, and acceleration. here v is the volume, p is the density, a is acceleration, and the subscripts b and o, respectively, stand for fluid and submerged object. This equation may be written as follows:

$$a_0 = \frac{p_b v_b a_b}{p_0 v_0}$$
(3)

The search strategy of AOA algorithm is explanationed in Figure 2.



Figure 2. The search strategy of AOA.

3.2. Crystal Structure Algorithm

The CryStAl algorithm is created, inspired by the method of adding basis to the lattice points, which is the principle underlying the formation of crystals such as pure quartz [17]. A "lattice," which displays a periodic array of points in predetermined areas but is incapable of describing the precise placements of atoms in the material, is the underlying component of a crystal. It is created by transforming the physics into an algorithm under the formation of the quartz crystal. CryStAl is created by S. Talathari et al. There are many crystal models. Here, the researchers are inspired by the model called Bravais model. The Bravais model is used to construct crystal configurations in this article since a mathematical representation of these characteristics is needed for numerical research. A periodic crystal structure is constructed in the Bravais model by considering an infinite lattice shape in which every lattice point is specified by the position of their lattice point as a vector. It is formulated as:

$$r = \sum n_i a_i$$

(4)

where n_i is the number of crystal corners, a_i is the shortest vector along with the primary crystallographic directions, and *i* is the number of crystal corners. The search strategy of the CryStAl algorithm is showed in Figure 3.



Figure 3. The search strategy of CryStAl.

3.3. Lichtenberg Algorithm

The Diffusion Limited Aggregation (DLA) theory served as the numerical foundation for the algorithm's creation. The Lichtenberg Figure (LF) is based on the model published by Witten and Sander. The LA algorithm authors are inspired by the DLA [18]. DLA has a Matrix of 0s and 1s that is formed like a map, with a particle, identified by the number one, positioned in the middle. The cluster is made up of one-valued matrix values, and vacant spaces having 0 values. Each matrix element with the value one represents a cluster particle, and the number of them in the cluster (N_c) is specified at the start of the program. The creation radius (R_c) defines the space in which the figure is built, and it is used to build a matrix with line and column numbers equal to twice R_c (diameter). Its mathematical form is as in the following equation:

$$D = \frac{\ln (N_{cluster})}{\ln (R_{cluster})}$$
(5)

The search strategy of the LA algorithm is illustrated in Figure 4.



Figure 4. The search strategy of LA.

4. NUMERICAL RESULTS

In this work, three distinct novel physic-based metaheuristic optimization approaches are utilized to construct symmetric LAAs with 10, 16, and 24-Element. The main aim of this synthesis is to maintain HPBW values stable or as narrow as possible while suppressing SLL values. AOA, CryStAl, and LA optimization techniques are utilized to identify the optimum amplitudes of the antenna array elements, and 30 independent runs are performed to assess their performance. Simulations are performed utilizing MATLAB software on a personal computer with 16 GB RAM and an i7 CPU. SLL values are given in "dB" and HPBW values are given in "o".

4.1. Case 1. LAA with 10-Element

The amplitudes for the symmetric LAA with 10-Element are found in the first example by performing the optimal array design utilizing AOA, CryStAl, and LA. Table 1 shows a statistical comparison of AOA, CryStAl, and LA methods in terms of SLL and HPBW.

| | Table 1. Comparative result of 10-Element EAA. | | | | | | | | |
|-----|--|--------------------|----------------------|--------------------|--------------------|----------------------------|-----------------------|----------------------------|----------------------------|
| nt | | SLL _{min} | SLL _{med} . | SLL _{max} | SLL _{std} | HPBW _{min} | HPBW _{med} . | HPBW _{max} | HPBW _{std} |
| eme | AOA | -26.9772 | -26.9768 | -26.9677 | 0.0019 | 12.5000 | 12.5000 | 12.5000 | 0.00000222 |
| -El | CryStAl | -26.9772 | -26.9772 | -26.9772 | 0.0000 | 12.5000 | 12.5000 | 12.5000 | 0.0000087 |
| 10 | LA | -26.9755 | -26.9610 | -26.9171 | 0.0146 | 12.4983 | 12.4998 | 12.5000 | 0.0004 |

Table 1. Comparative result of 10-Element LAA

CryStAl achieved the best SLL based on the values given in Table 1. When HPBW values are examined, there is no noticeable difference between algorithms. When the CPU time values are examined according to the standard deviation, the most stable algorithm is LA with 0.778 seconds. The amplitude values of the best SLL value results are given in Table 2.

| Table 2. Amplitude values of LAA with 10-Element. | | | | | | |
|---|---------|------------------------------------|--|--|--|--|
| | AOA | 0.2169 0.1932 0.1518 0.1028 0.0719 | | | | |
| 10-Element | CryStAl | 0.6342 0.5649 0.4440 0.3007 0.2102 | | | | |
| | LA | 0.8340 0.7430 0.5840 0.3955 0.2765 | | | | |

Each algorithm spent CPU time while obtaining the median value. The algorithm that consumes the least CPU time is AOA with 13.073 seconds. The radiation patterns created by these values using the amplitude values given in Table 2 are shown in Figure 5.



Figure 5. The radiation pattern for LAA with 10-Element.

There is no difference in the radiation pattern of the sequence of 10-Element LAA in Figure 5 since all algorithms reach almost the same SLL value.

The algorithm with the fastest approach curve has been the LA algorithm. It is clearly seen in Figure 6 that AOA does not have a good convergence curve of 10-Element LAA.

4.2. Case 2. LAA with 16-Element

Comparative results of 16-Element LAAs are tabulated in Table 3. The values given in Table 3 are presented with various data such as standard deviation, minimum and maximum values of SLL an HPBW.

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Figure 6. Convergence curve of LAA with 10-Element.

Table 3. Comparative result of LAA with 16-Element.

| nt | | SLLmin | SLLmed. | SLL _{max} | SLLstd | HPBW _{min} | HPBW _{med} . | HPBW _{max} | HPBW _{std} |
|------|--------|----------|----------|---------------------------|--------|----------------------------|-----------------------|----------------------------|----------------------------|
| Ă A | OA | -40.2374 | -36.7346 | -28.1659 | 3.1819 | 8.7221 | 8.9562 | 9.0000 | 0.0852 |
| E Cr | ryStAl | -40.2423 | -40.2209 | -40.2027 | 0.0094 | 8.9985 | 8.9994 | 8.9999 | 0.0004 |
| ۲ LA | 4 | -40.2083 | -39.2288 | -37.5938 | 0.7255 | 8.9962 | 8.9989 | 8.9999 | 0.0011 |

When the values shown in Table 3 are compared, the algorithm with the best SLL value is the CryStAl algorithm. According to the standard deviation, the least scattering algorithm is the CryStAl algorithm. HPBW values are very close to each other. The most stable algorithm in terms of CPU time has been the LA with 0.121 seconds. Table 4 showed the amplitude value of the best SLL achieved.

| | Table 4. Amplitude values of LAA with 16-Element. | | | | | | | |
|----------------|---|--|--|--|--|--|--|--|
| 16- Element | AOA | $0.2223\ 0.2078\ 0.1812\ 0.1466\ 0.1090\ 0.0732\ 0.0432\ 0.0248$ | | | | | | |
| | CryStAl | 0.6527 0.6102 0.5321 0.4305 0.3200 0.2151 0.1267 0.0727 | | | | | | |
| | LA | 0.5826 0.5446 0.4751 0.3843 0.2856 0.1921 0.1132 0.0649 | | | | | | |
| | | | | | | | | |

For the median of the 16-Element LAA, the fastest algorithm to arrive at the result is AOA with 16.796 seconds. Table 4 shows the amplitude values of LAA with 16-Element obtained by algorithms. The 2-Dimension radiation pattern obtained using these amplitudes is shown in Figure 7.



Figure 7. The radiation pattern for LAA with 16-Element.

The convergence curve of the algorithms when obtaining the 16-Element LAA is given in Figure 8.



Figure 8. The convergence curve of LAA with 16-Element.

Examining the convergence curve of 16-Element LAA, AOA has a very bad convergence curve. The algorithm with the best convergence curve of 16-Element LAA is CryStAl.

4.3. Case 3. LAA with 24-Element

Compared to the results obtained, there is no noticeable difference in the 16-Element LAA, which is an easier problem than the 24-Element LAA. Comparative results for LAA with 24-Element are tabulated in Table 5.

| | | | Table 5. Co | mparative r | result of 2 | 24-Element l | LAA. | | |
|----------|---------|----------|-------------|---------------------------|--------------------|----------------------------|----------------------|----------------------------|----------------------------|
| nt | | SLLmin | SLLmed. | SLL _{max} | SLL _{std} | HPBW _{min} | HPBW _{med.} | HPBW _{max} | HPBW _{std} |
| 24-Eleme | AOA | -38.9896 | -33.2765 | -25.7744 | 3.2153 | 5.5013 | 5.8552 | 5.9999 | 0.1779 |
| | CryStAl | -40.8412 | -40.0569 | -36.6468 | 1.0083 | 5.9719 | 5.9956 | 6.0000 | 0.0070 |
| | LA | -40.2575 | -38.9898 | -37.2272 | 0.8543 | 5.9857 | 5.9973 | 6.0000 | 0.0038 |

According to Table 5, the algorithm that reaches the best values is CryStAl. In addition, the best algorithm in terms of the standard deviation of SLL is the LA algorithm. The HPBW values for all algorithms had almost the same values. When the CPU time values are examined, the algorithm with the best standard deviation value is the AOA with 0.110 seconds. The Amplitude values obtained by the algorithms while reaching the best values are given in Table 6.

| | | Table 6. Amplitude values of 24-Element LAA. |
|----------------|---------|---|
| | AOA | 0.0815 0.0787 0.0741 0.0678 0.0593 0.0499 0.0417 0.0325 0.0229 0.0181 0.0117 0.0092 |
| 24- Element | CryStAl | 0.7921 0.7728 0.7214 0.6651 0.5793 0.5014 0.4060 0.3203 0.2411 0.1652 0.1145 0.0839 |
| | LA | 0.7990 0.7731 0.7401 0.6649 0.5888 0.5023 0.4049 0.3358 0.2385 0.1698 0.1138 0.0829 |

By processing these values, a radiation pattern is obtained for the 24-Element LAA. When the median values are compared according to the CPU time, the AOA with 24.120 seconds has the best value. The radiation pattern of the algorithms obtained LAA with 24-Element is shown in Figure 9.

As can be clearly seen from Figure 9, the algorithm that obtained the best radiation pattern is the CryStAl algorithm. The convergence curves of the algorithms are plotted in Figure 10.

When the convergence curve of 24-Element LAA is examined, the fastest converging algorithm is LA. The algorithm that achieves the best results is CryStAl. The algorithm with the worst convergence curve of 24-Element LAA and the worst result is AOA.



Figure 9. The radiation pattern for LAA with 24-Element.



Figure 10. Convergence curve of LAA with 24-Element.

5. CONCLUSIONS

In this study, the AOA, CryStAl, and LA are applied to optimize amplitude values of the 10, 16, and 24-Element LAAs. The main purpose of this study is to test the performance of novel physics-based algorithms in synthesizing LAA. These three proposed new physics-based metaheuristic optimization methods have been applied to LAA synthesis for the first time according to the literature study we have done. In all simulations, HPBW is kept constant, and SLL is tried to be reduced to the minimum value. HPBW values during this process are determined as 12.5°, 9°, and 6° for 10, 16, and 24-Element LAA, respectively. The algorithm that achieves the best SLL median value in all LAA simulations is CryStAl. When examined in terms of the standard deviation of SLL, the method that reached the best value for the 10 and 16-Element LAA is CryStAl. In the 24-Element LAA synthesis, the LA method reached its value with the least scattering. When all cases are examined in terms of CPU time, even if the algorithm that reaches the fastest result is AOA, it is insufficient in terms of obtaining a good result. Novel physics-based optimization methods can be used in future work to solve different antenna arrays and other computational electromagnetic problems.

Author Contributions

Each author contributed an equal amount to the study.

Conflict of Interest

There is no conflict of interest between the article authors.

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