The Elasticity of Substitution and Economic Growth Rate: What Linkage?

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ABSTRACT

In this study, the effect of elasticity of substitution on output per capita, the steady-state capital-labor ratio, and the steady-state output per capita, are reconsidered respectively. The analysis is based on the constant elasticity of the substitution production function framework and the methodology used in this study is based on analytical proofs. Firstly, the squeeze theorem is employed for the elasticity of substitution and a general solution for the production function with initial and terminal conditions is derivated. The study then progressed to investigate the steady-state behavior of production function. In this vein, at steady-state the effect of elasticity of substitution on capital accumulation per labor and on output per capita, is investigated respectively. Firstly, output per capita is a decreasing function of the elasticity of substitution is demonstrated. Secondly, the result as at steady-state an increase in elasticity of substitution decreases the income per-capita and decreases the steady-state capital-intensity is demonstrated. Moreover, the finding as at steady-state an increase in elasticity of substitution strictly decreases capital share. This study challenges the studies reporting that there is a threshold of elasticity of substitution for which there is no steady-state equilibrium and for which ever-sustained growth is entailed. The findings in this study contribute to the literature by revising the influence of the elasticity of substitution on output per capita. The results of this study can explain the differences in growth paths for developed and developing countries and suggest a policy application that increasing the minimum marginal product of labor increases the growth rate of output per-capita in countries where the elasticity of substitution is higher-than-unity.

Keywords: *Economic Growth, Constant Elasticity of Substitution, Production Function, Dynamical Equilibrium, Steady-State Growth Rate.*

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INTRODUCTION

The constant elasticity of the substitution production function introduced in Solow (1956:77) exemplified a particular condition under which perpetual growth would be possible. This condition can be generalized as follows: the saving-investment obtained from the minimum marginal product of capital is greater than the growth rate of the labor force. This finding leads to an extensive analysis of the relationship between the elasticity of substitution and the growth rate of income and income per capita. In addition, the investigation of whether different growth rates among countries can be explained by the elasticity of substitution has taken a considerable place in the literature on growth theory. In growth models, technologies more flexible than Cobb-Douglas are needed for the consideration of varying factor shares, factor-biased technical change, and appropriate technology (Acemoglu, 2003, Caselli, 2005, Caselli & Coleman, 2006, Temple, 2012).

The aim of this paper is to analyze the effect of elasticity of substitution on income per capita, the growth rate of the capital-labor ratio, the growth rate of income per capita, the steady-state capital-labor ratio, and the steady-state income per capita, respectively.

The theoretical perspective examined in this study has already been investigated by various studies. De La Grandville (1989) showed that there is a threshold of elasticity of substitution for which there is no steady-state equilibrium, implying perpetually increasing capital-labor ratio and income per capita (Throughout the study, income per capita and output per capita are used interchangeably). Moreover, this threshold is increasing with the growth rate of labor and is decreasing with the saving rate. Another crucial finding reported by de La Grandville (1989) is that the growth rate of income per capita and the growth rate of capital per labor are increasing functions of the elasticity of substitution. In addition, de La Grandville (1989) analyzed the effect of elasticity of substitution conditional on steady-state and the author showed that steady-state income per capita is an increasing function of the elasticity of substitution. In a related study, the findings of Klump & de La Grandville (2000) depict that if two countries have common initial conditions, the one with the higher elasticity of substitution will always experience, other things being equal, a higher income per capita. Moreover, if any, the equilibrium values for capital per

labor and income per capita are both increasing functions of the elasticity of substitution. In addition, the results in Klump & Preissler (2000) show that a higher elasticity of substitution leads to both a higher steady-state and an increase in the probability of permanent growth. Finally, de La Grandville & Solow (2006) report that the elasticity of substitution can increase the growth rate of the economy and its effect may be greater than the contribution of the increase in the savings rate and/or technical progress. More recently, de La Grandville (2009) states that income and income per capita are increasing functions of the elasticity of substitution.

On the other hand, the studies in the empirical literature report that cross-country regressions identify an elasticity of substitution greater than unity (Masanjala & Papageorgiou, 2004). *Among the recent studies, Mallick (2012) estimates the elasticity of substitution value for 90 countries* ranging from 0.03 to 2.18, where developing countries mostly enjoy elasticity of substitution levels higher than unity. More specifically, recent empirical studies show that the US economy historically has elasticity of substitution level lower-than-unity and continues to stay lower-than-unity. Furthermore, Oberfield & Raval (2014) estimate that over the period from 1972 to 2007 the elasticity of substitution is lower than one and has risen from 0.67 to 0.75. Cantore et al. (2017) and more recently Knoblach et al. (2020) report similar results. Similarly, a non-unitary elasticity also has implications for fiscal policy, as in the Backus et al. (2008) study of the cross-country relationship between capital-output ratios and corporate tax rates. More widely, theorists often consider the implications of variation in the elasticity of substitution, and it has been argued that the CES technology deserves much greater prominence in short-run macroeconomics. If the theoretical studies are aligned with empirical findings, one will query whether policy applications to enhance the rate of growth are sensitive to the elasticity of substitution's level with respect to unity.

This paper contributes to the related literature in various aspects. The present study examines whether the limiting values for the marginal product of input factors can be used as a policy instrument to increase the growth rate of income per capita. Second, for the elasticity of the substitution lower-than-unity policy instrument should be different from the case for higher-than-unity.

The rest of the study is organized as follows. The basis on a short theoretical review and preliminary setting in Section 2, the result as there is a threshold of elasticity of substitution for which there is no steady-state equilibrium and for which ever-sustained growth is entailed is proposed. The findings of this study are developed progressively in Sections 3,4 and 5. Particularly, Section 3 addresses whether substitution elasticity can be considered a perpetual "engine" of economic growth. The results of this section contribute to the debate from this perspective. Section 4 investigates the steady-state behavior of the production function. Section 5 proposes a policy recommendation to boost the rate of economic growth by making use of limiting marginal products. This can also be used to explain technological progress and economic development differences in countries.

PRELIMINARY ISSUES: PRODUCTION FUNCTION AND ELASTICITY OF SUBSTITUTION

The constant elasticity of the substitution production function has already been exemplified and modeled in various studies, Solow (1956), Pitchford (1960), and David & van de Klundert (1965). These studies assumed that coefficients of input factors are arbitrary free parameters. Arrow et al. (1961) contributed by proposing a mathematical derivation of CES function based on empirical observations on the relationship between wage rate and labor productivity. In their study, Arrow et al., conceptualized the free parameters as integration constants derivated from a second-order differential equation. Ozkaya (2021) showed that these integration constants are not arbitrarily free constants but are initial and terminal conditions for marginal productivity. The terms $F_K(K,0)\,\frac{\sigma-1}{\sigma}$ and $F_L(0,L)^{\frac{\sigma-1}{\sigma}}$ correspond to the free parameters that have already been introduced in previous models in Solow (1956), Pitchford (1960), Arrow et al. (1961) and David & van de Klundert (1965).

The expressions (1) and (2) are based on the studies Barelli & Abreu Pessôa (2003) and Ozkaya (2021). The elasticity of substitution is denoted by. The inputs of the production function are capital services, *K,* and labor force, *L*. Whenever σ>1, the constant elasticity of the substitution production function is as shown in (1).

$$F(K,L) = \left((F_K(K,0).K)^{\frac{\sigma-1}{\sigma}} + (F_L(0,L).L)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}$$

(1)

For σ<1, the production function is as shown in (2).

$$F(K,L) = \left((F_K(K,\infty).K)^{\frac{\sigma-1}{\sigma}} + (F_L(\infty,L).L)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}$$

(2)

For σ>1, the coefficients F_K (K,0) and F_L (0,L) denote the initial condition of the production function and denote the minimum marginal product of capital and the minimum marginal product of labor, respectively. Each is constant with respect to its input.

On the other hand, for σ<1, the coefficients F_K (K,∞) and F_L (∞,L) are terminal conditions for the production function. F_K (K,∞) and F_L (∞,L) correspond to the maximum marginal product of capital and maximum marginal product of labor, respectively.

Dividing the production function (1) and (2) by labor force quantity gives output per capita (intensive form definition) production function. In the literature, by the homogeneity assumption of the production function, it is conveniently shown as , with as *f* (*k*), with $k = \frac{K}{L}$.

By the definition of Solow (1956), the growth rate of capital per labor is written as follow: $\frac{\dot{k}}{k} = s\frac{f(k)}{k} - n$

In this setting, *f(k)* is the output per capita production function, the saving-investment ratio is denoted by *s*, and the increment of the labor force is shown by *n*.

Proposition 1.

There exists no threshold of elasticity of substitution for which there is no steady-state equilibrium and for which ever-sustained growth entails. Therefore, there exist no threshold value for saving-investment ratio, s and growth rate of labor, n as well.

Proof 1.

The expressions (A1) and (A2) in the Appendix depict the CES function formulation given by de La Grandville (1989).

The parameter $\beta^*(\sigma)\,\frac{\sigma}{\sigma-1}$ seemingly depends on . From (A3) and (A4), it is straightforward that $\beta^*(\sigma)^{\frac{\sigma}{\sigma-1}}$ is a constant with respect to both capital-labor ratio and elasticity of substitution: that is, $\frac{d\beta^*(\sigma)^{\frac{\sigma}{\sigma-1}}}{dk} = 0$ and $\frac{d\beta^*(\sigma)^{\frac{\sigma}{\sigma-1}}}{d\sigma} = 0$.

The same argument is also true for the other parameter denoted $\alpha^*(\sigma)^{\frac{\sigma}{\sigma-1}}$.

This finding stands strictly in contrast to the result of de La Grandville (1989) and de La Grandville and Solow (2009). De La Grandville (1989:479) claim that "*there is a threshold of for which there is no steady-state equilibrium. This would imply perpetually increasing capital-labor ratio and income per capita*."

De La Grandville (1989:479) claim that "*the elasticity of substitution, as a measure of the efficiency of the productive system, has to be higher when the population growth rate increases or when the savings-investment rate decreases*."

The result of this study challenges the argument of de La Grandville (1989). The reason is that the author did not derivate integration constants but propose an assumption. In the study de La Grandville (1989:479-481), it is erroneously assumed that $\beta^*(\sigma)^{\frac{\sigma}{\sigma-1}} = \frac{n}{s}$ and then σ is misidentified as a function of $\frac{n}{s}$ and β*(σ). Moreover, de La Grandville (1989) did not discriminate the case σ >1 from σ<1, in its analysis and arguments.

LABOR SHARE AND CAPITAL SHARE

The section 3 proposes the results on the relationship between elasticity of substitution, and first; labor share of income, second capital share of income, and third income per capita. For notational easiness, throughout the paper, in expression (1) let us denote $\beta = F_K(K,0)$ and $\alpha = F_L(0,L)$. Similarly, in (2), let us denote $b = F_K(K,\infty)$ and $a = F_L(\infty,L)$. Different from the parametrization in de La Grandville (1989), the parameters *β, α,b,a* used in this study are constants and do not depend on neither input factors nor elasticity of substitution. The parameter *β* and α are different from β* (σ), α* (σ), which are used in Proposition 1 of the present study.

Proposition 2.
For >1 , the effect of the elasticity of substitution on the share of capital depends on the ratio of minimum marginal products. On the other hand, for σ<1 the effect of the elasticity of substitution on the share of capital depends on the ratio of maximum marginal products.

For >1:
i.) if $\frac{F_L(0,L)}{F_K(K,0)} < k$ then $\frac{\partial \pi}{\partial \sigma} > 0$; ii.) if $\frac{F_L(0,L)}{F_K(K,0)} > k$ then $\frac{\partial \pi}{\partial \sigma} < 0$; iii.) if $F(0,L) = F(K,0)$ then $\frac{\partial \pi}{\partial \sigma} = 0$.

For < 1 :
i.) if $\frac{F_L(\infty,L)}{F_K(K,\infty)} < k$ then $\frac{\partial \pi}{\partial \sigma} > 0$; ii.) if $\frac{F_L(\infty,L)}{F_K(K,\infty)} > k$ then $\frac{\partial \pi}{\partial \sigma} < 0$; iii.) if $F(\infty,L) = F(K,\infty)$ then $\frac{\partial \pi}{\partial \sigma} = 0$.

Proof 2.
For σ>1 output per-capita is defined as $\frac{F(K,L)}{L} = f(k) = F_K(K,0)k(1+m)^{\frac{\sigma}{\sigma-1}}$ where $k = \frac{K}{L}$ and $m = \left(\frac{F_L(0,L)}{F_K(K,0)k}\right)^{\frac{\sigma-1}{\sigma}}$.

Let $\pi = \frac{kf'}{f} = \frac{1}{1+m}$ be capital share of output per capita. The partial de rivative leads to (3).

$$\frac{\partial \pi}{\partial \sigma} = \frac{-m.\ln m}{\sigma(\sigma-1)(1+m)^2}$$
(3)

For >1 , the following result is obtained

i.) If $\ln m < 0 \leftrightarrow \frac{F_L(0,L)}{F_K(K,0)} < k$ then $\frac{\partial \pi}{\partial \sigma} > 0$, equivalently if $\frac{1-\pi}{\pi} < 1$ then $\frac{\partial \pi}{\partial \sigma} > 0$ ii.) if $\ln m > 0 \leftrightarrow \frac{F_L(0,L)}{F_K(K,0)} > k$ then $\frac{\partial \pi}{\partial \sigma} < 0$; iii.) if $F_L(0,L).L = F_K(K,0).K$ then $\frac{\partial \pi}{\partial \sigma} = 0$.

For < 1 , income per-capita is $f(k) = F_K(K,\infty)\ k(1+v)^{\frac{\sigma}{\sigma-1}}$ and v stands for $v = \left(\frac{F_L(\infty,L)}{F_K(K,\infty)k}\right)^{\frac{\sigma-1}{\sigma}}$. The following result is obtained.

i.) if $\ln v < 0 \leftrightarrow \frac{F_L(\infty,L)}{F_K(K,\infty)} > k$ then $\frac{\partial \pi}{\partial \sigma} < 0$; ii.) if $\ln v > 0 \leftrightarrow \frac{F_L(\infty,L)}{F_K(K,\infty)} < k$ then $\frac{\partial \pi}{\partial \sigma} > 0$; iii.) if $F_L(\infty,L).L = F_K(K,\infty).K$ then $\frac{\partial \pi}{\partial \sigma} = 0$.

Proposition 3.
For σ<1, an increase in the elasticity of substitution decreases the labor share of income under the condition that $\frac{F_L(\infty,L)}{F_K(K,\infty)} < k$.

Proof 3
Replacing m with v in (3) is sufficient to obtain the desired result. That is: if the ratio of the maximum marginal product of labor to the maximum marginal product of capital is greater than the capital-labor ratio, meaning that the labor share of income is lower than capital share of income, then an increase in elasticity of substitution decreases the labor share of income.

Proposition 4.
An increase in the elasticity of substitution decreases the income per capita.

Proof 4.
The logarithm of income per-capita is $\ln f(k) = \ln\beta k + \frac{\sigma}{\sigma-1}\ln(1+m)$. The derivative of both sides leads to (4). Since $\frac{m.\ln m}{1+m} < \ln(1+m)$, for all k, for all σ and $m > 0$, for all k for all σ and m > 0 equation (4) holds.

$$\frac{\partial f}{\partial \sigma} = \frac{f(k)}{(\sigma-1)^2}.\left(\frac{m.\ln m}{1+m} - \ln(1+m)\right) < 0$$
(4)

In order to investigate the effect of elasticity of substitution on the rate of growth for capital accumulation per labor, one has to differentiate both sides of the growth rate of capital per labor, k ̇/k=s f(k)/k-n
with respect to . The desired result is shown in (4).

$$\frac{\partial}{\partial\sigma}\left(\frac{\dot{k}}{k}\right)=\frac{s.\beta.(1+m)^{\frac{\sigma}{\sigma-1}}}{(\sigma-1)^2}.\,ln\left(\frac{m^{\frac{m}{1+m}}}{1+m}\right)$$
(4)

Equation (4) show that $\frac{\partial}{\partial\sigma}\left(\frac{\dot{k}}{k}\right)<0$ for all , and for both cases σ<1 and σ<1. This result clearly indicates that the rate of growth for capital per labor decreases in the elasticity of substitution.

STEADY-STATE BEHAVIOR OF PRODUCTION FUNCTION

The section 4 analyzes the effect of the elasticity of substitution on certain variables under the condition where a steady state exists.

Proposition 5.

An increase in the elasticity of substitution decreases the steady-state income per capita.
Proof 5.
In expression (4) substituting the steady-state capital-labor ratio gives the desired result.
Proposition 6.
Increasing the elasticity of substitution strictly decreases the steady-state capital share.
Proof 6.
Let π* denote the steady-state share of capital:

$$\pi^*=\left(\frac{f(k(\infty))}{\beta k(\infty)}\right)^{\frac{1-\sigma}{\sigma}}=\left(\frac{n}{s\beta}\right)^{\frac{1-\sigma}{\sigma}},\text{ where the domain is }\quad 0<\left(\frac{n}{s\beta}\right)^{\frac{1-\sigma}{\sigma}}<1.$$
$$\frac{\partial\pi^*}{\partial\sigma}=\frac{-\pi^*}{\sigma^2}ln\frac{n}{s\beta}$$
(5)

Since the steady-state requires that $\frac{n}{s}>\beta$, the steady-state capital-share absolutely decreases with the elasticity of substitution $\frac{\partial\pi^*}{\partial\sigma}<0$.
Proposition 7.
An increase in the elasticity of substitution decreases the steady-state capital-intensity. That is: $\frac{\partial k(\infty)}{\partial\sigma}<0$ for all and for all .

Proof 7.

For $\sigma>1$ the steady-state capital intensity is given by $k(\infty)=\frac{s\alpha}{n\left(1-\left(\frac{n}{s\beta}\right)^{\frac{1-\sigma}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}}=\frac{s\alpha}{n(1-\pi^*)^{\frac{\sigma}{\sigma-1}}}$.

Let $(1-\pi^*)^{\frac{\sigma}{\sigma-1}}=w$ .Taking logarithm of both sides and differentiating with respect to σ leads to $\frac{-1}{(\sigma-1)^2}ln(1-\pi^*)+\frac{\sigma}{\sigma-1}\frac{-d\pi^*}{d\sigma}\frac{1}{(1-\pi^*)}=\frac{1}{w}.\left(\frac{\partial w}{\partial\sigma}\right)$, we substitute (4), and we obtain $\frac{1}{w}.\left(\frac{\partial w}{\partial\sigma}\right)=\frac{-1}{(\sigma-1)^2}ln(1-\pi^*)+\frac{1}{(\sigma-1)}\frac{\pi^*}{\sigma}ln\frac{n}{s\beta}\frac{1}{(1-\pi^*)}$
Since $\frac{\partial k(\infty)}{\partial\sigma}=\frac{s\alpha}{n.w}\left(-\frac{1}{w}.\left(\frac{\partial w}{\partial\sigma}\right)\right)$, equation (6) holds.

$$\frac{\partial k(\infty)}{\partial\sigma}=\frac{s\alpha}{n}\frac{(1-\pi^*)^{\frac{\sigma}{1-\sigma}}}{(\sigma-1)}\left(\frac{1}{(\sigma-1)}ln(1-\pi^*)-\frac{\pi^*}{\sigma(1-\pi^*)}ln\frac{n}{s\beta}\right)<0$$
(6)

Recall that the steady-state requires the condition $\frac{n}{s}>\beta$, for $\sigma>1$, for σ>1. Therefore, the sensitivity of steady-state capital-intensity to a change in elasticity of substitution is absolutely negative, namely $\frac{\partial k(\infty)}{\partial\sigma}<0$. To compute $\frac{\partial k(\infty)}{\partial\sigma}$ for σ<1 , one has to rearrange the parameters in (5). Since the steady-state requires the condition n/s<b it is obvious that the sensitivity of steady-state capital-intensity to a change in elasticity of substitution is negative, namely $\frac{\partial k(\infty)}{\partial\sigma}<0$. We also state that higher-than-unity elasticity of substitution does not necessarily lead to higher steady-state capital-labor ratio and/or higher per capita income.

The results of the present study are strictly in contrast to the studies in the literature, which report that the elasticity of substitution is an engine for the growth.

A POLICY RECOMMENDATION TO ENHANCE THE GROWTH RATE OF OUTPUT

This section proposes some policy implications showing how the output per capita can be perpetually increased.

Proposition 8.
For >1, increasing the minimum marginal product of labor increases the rate of growth of income per capita.

Proof 8.
The derivative of (f ̇/f) with respect to α can be defined as:

$$\frac{\partial\left(\dot{f}/f\right)}{\partial\alpha}=\left(\frac{\partial}{\partial\alpha}\left(\frac{df}{dk}\frac{k}{f}\right)\right).\frac{\dot{k}}{k}+\left(\frac{\partial}{\partial\alpha}\left(\frac{\dot{k}}{k}\right)\right)\frac{df}{dk}\frac{k}{f}\ .$$

The derivative of growth rate of capital per labor ratio with respect to α equals $\frac{\partial(\dot{k}/k)}{\partial\alpha} = \frac{s\beta}{\alpha} m(1+m)^{\frac{1}{\sigma-1}}$. And, the derivative of the capital share of income with respect to α equals $\frac{\partial}{\partial\alpha}\left(\frac{df}{dk}\frac{k}{f}\right) = \frac{-(\sigma-1)}{\sigma.\alpha}\frac{m}{(1+m)^2}$. Substituting $\frac{\partial(\dot{k}/k)}{\partial\alpha}$ and $\frac{\partial}{\partial\alpha}\left(\frac{df}{dk}\frac{k}{f}\right)$ gives the derivative of $\left(\frac{\dot{f}}{f}\right)$ with respect to α. That is:

$$\frac{\partial(\dot{f}/f)}{\partial\alpha} = \frac{s\beta}{\alpha}\frac{m}{(1+m)}\left(\frac{(1+m)^{\frac{1}{\sigma-1}}}{\sigma}\right) + \frac{(\sigma-1)}{\sigma.\alpha}\frac{m.n}{(1+m)^2}$$

. Rearranging above formulation leads to the following result: $\frac{\partial(\dot{f}/f)}{\partial\alpha} = \frac{m}{\alpha.\sigma(1+m)^2}\left(\frac{sf}{k} + (\sigma-1)n\right) > 0$ where $\frac{sf}{k} + (\sigma-1)n > 0$ for all k.

We already know that: $\frac{sf}{k} + (\sigma-1)n > 0$ holds for all k. Therefore, an increase in minimum marginal product of labor strictly increases the growth rate of income per capita. In order to perpetually increase the rate of growth of output, the policy makers should enhance the minimum marginal product of labor with respect to that of capital.

CONCLUSION AND DISCUSSION

This study contributes to the literature on the properties of constant elasticity of substitution production function (Solow, 1956, Pitchford, 1960, Arrow et al., 1961, David & van de Klundert, 1965, Barro & Sala-i-Martin, 1995). More specifically, this study revises the aforementioned findings in the literature (de La Grandville, 1989, Klump & Preissler, 2000), and corrects some inconsistencies and misinterpretations. The results of the present study shed light on the misinterpretation that the elasticity of substitution is a "magic tool" for perpetual economic growth. Instead, the finding as an increase in the minimum marginal product of labor enhances the rate of growth for output per capita is shown. This result appears as an appropriate policy tool to be set in the first place. On the other hand, the result as output per capita is a decreasing function of the elasticity of substitution is demonstrated. Moreover, the result as at steady-state an increase in elasticity of substitution decreases the output per capita and decreases the capital per labor is demonstrated. Furthermore, it is demonstrated at steady-state that an increase in elasticity of substitution strictly decreases capital share. This study challenges the studies reporting that there is a threshold of elasticity of substitution for which there is no steady-state equilibrium and for which ever-sustained growth is entailed. We believe that the extensive approach proposed in this study will be a source for different points of view. The future studies which will be based on existing literature on modern growth theory should consider the findings of the present study. Future studies may suggest that the economic implications of limiting the behavior of factor productivities are deeper than previously considered. This gives more room to design alternative growth-enhancing policies across the world.

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APPENDIX

For our purposes; first of all, we have to focus on the formulation proposed in de La Grandville (1989), which introduced a variant of the CES production function. That is given in (A1).

$$f(k) = A(\sigma)\left(\left(1 - c(\sigma)\right)k^{\frac{\sigma-1}{\sigma}} + c(\sigma)\right)^{\frac{\sigma}{\sigma-1}} \quad \text{(A1)}$$

De La Grandville (1989) defines the CES function parameters as:

$$A(\sigma)^{\frac{\sigma-1}{\sigma}}\left(1 - c(\sigma)\right) = \beta(\sigma) \text{ and } A(\sigma)^{\frac{\sigma-1}{\sigma}}c(\sigma) = \alpha(\sigma) \quad \text{(A2)}$$

Comparing the parameters in (A2) with the CES production function given in this study by (1), for σ>1, we obtain the relations given in (A3):

$$\beta^*(\sigma) = F_K(K,0)^{\frac{\sigma-1}{\sigma}} \text{ and } \alpha^*(\sigma) = F_L(0,L)^{\frac{\sigma-1}{\sigma}} \quad \text{(A3)}$$

On the other hand, assume the other case, σ<1. Comparing the parameters of the production function (A1) with the CES production function given in the present study by (2), leads the following identities depicted in expression (A4):

$$\beta^*(\sigma) = F_K(K,\infty)^{\frac{\sigma-1}{\sigma}} \text{ and } \alpha^*(\sigma) = F_L(\infty,L)^{\frac{\sigma-1}{\sigma}} \quad \text{(A4)}$$