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TESTING MEAN REVERSION OF STOCK PRICES IN OECD COUNTRIES: EVIDENCE FROM FOURIER THRESHOLD UNIT ROOT TEST

OECD ÜLKELERİNDE HİSSE SENEDİ FİYATLARININ ORTALAMAYA DÖNÜŞÜ: FOURİER THRESHOLD BİRİM KÖK TESTİNDEN BULGULAR

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Abstract

Researchers focus on whether stock prices have a unit root, that is, whether they contain a random walk process. If stock prices have a stationary process, that is, if they return to the mean, the effects of shocks are temporary, and it is interpreted that they will return to the trend path over time. If stock prices have transitory shocks, it allows for the prediction of future movements based on past behavior in terms of investment. This study investigates whether stock prices revert to the mean and thus have a random walk process. For this purpose, the Fourier Threshold Unit Root (FTUR) test based on the test methodology of Caner and Hansen (2001) for the period January 1990–January 2021 for 26 OECD countries is applied. The FTUR test takes into account both structural breaks and nonlinearities. The purpose of using Fourier functions to account for structural changes is that they are not affected by the number, location, or shape of breaks. Thus, the power of the test increases. According to the results of this test, stock prices in Austria, Canada, Germany, Italy, New Zealand, Spain, and the UK are linear. Therefore, Fourier Augmented Dickey-Fuller (FADF) unit root analysis was performed for these countries. The FTUR test was performed in other countries. According to the results of FTUR and FADF unit root tests, stock prices are found to contain unit roots in some countries except Italy. In some countries, stock prices have a partial unit root structure. In other words, the effects of shocks are permanent, and it is concluded that future returns cannot be predicted in these countries with the random walk process.

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Öz

Hisse senedi fiyatlarının birim köke sahip olup olmadığı yani rassal yürüyüş süreci içerip içermediği araştırmacıların odağı halindedir. Hisse senedi fiyatları durağan sürece sahip ise yani ortalamaya geri dönüyorsa şokların etkileri geçicidir ve zamanla trend yoluna döneceği yorumu yapılmaktadır. Eğer hisse senedi fiyatları geçici şoklara sahip ise yatırım açısından geçmiş davranışlara bağlı olarak gelecekteki hareketlerin tahmin edilebilmesini olanak tanımaktadır. Bu çalışma, hisse senedi fiyatlarının ortalamaya dönüp dönmediğini ve dolaysıyla rassal yürüyüş sürecine sahip olup olmadığını araştırmaktadır. Bu amaçla, 26 OECD ülkesi için Ocak 1990-Ocak 2021 dönemi için Caner ve Hansen (2001) test metodolojisine dayanan Fourier Eşik Birim Kök (FTUR) testi ile ele alınan değişkenlerin sınamaları gerçekleştirilmiştir. FTUR testi, hem yapısal kırılmaları hem de doğrusal olmayan durumları dikkate almaktadır. Fourier fonksiyonlarının yapısal değişimleri hesaba katmak için kullanılmasındaki amaç kırılmaların sayısı, yeri ve şeklinden etkilenmemesidir. Böylece, testin gücü artmaktadır. Bu test sonuçlarına göre Avusturya, Kanada, Almanya, İtalya, Yeni Zelanda, İspanya ve İngiltere'de hisse senedi fiyatlarının doğrusal olduğu görülmektedir. Bu nedenle doğrusal olduğu gözlenen bu ülkeler için Fourier Augmented Dickey Fuller (FADF) birim kök analizi yapılmıştır. Diğer ülkeler için FTUR sınaması yapılmıştır. FTUR ve FADF birim kök testi sonuçlarına göre hisse senedi fiyatlarının İtalya dışında bazı ülkelerde birim kök içerdiği bulgusuna erişilmiştir. Bazı ülkelerde hisse senedi fiyatları kısmi bir birim kök yapısına sahiptir. Diğer bir deyişle, şokların etkileri kalıcıdır ve rassal yürüyüş süreci ile bu ülkelerde gelecekteki getirilerin tahmin edilemeyeceği sonucuna varılmaktadır.

Anahtar Kelimeler: Hisse senedi fiyatları, ortalamaya dönüş, yumuşak geçişli otoregresif (STAR) model, eşik birim kök, fourier fonksiyon
JEL Sınıflandırılması: P43, E6, C4, C58

1. Introduction

Fama (1970) proposed the stock market efficiency hypothesis, in which the dynamics of stock prices are defined using a random walk process with a shift. The efficient market hypothesis assumes that future returns cannot be predicted on the basis of historical information. That is to say, a market is efficient if the prices at any given time always fully reflect all available information. Researchers and economists investigate the efficient market hypothesis, one of the fundamental principles in finance literature. For this reason, the concept of efficiency in capital markets is one of the most frequently researched topics. Today, the constantly changing market system depending on economic and financial conditions and the development of current statistical techniques have led to the investigation of market efficiency with empirical analysis (Rehman et al., 2018). Market efficiency theory, which dominates the financial field because of scarce financial resources, explains the relationship between information and stock prices in financial markets. Therefore, the behavior of stock market prices in relation to the share of scarce monetary resources is necessary (Chitenderu et al., 2014). Uncertainty is the leading obstacle to estimating stock price indices (Patel et al., 2015). Lean and Smyth (2007) stated that in the random walk theory, stock price changes maintain the same probability distribution and are independent from each other; therefore, they noted that the past movement of the stock price cannot be used to predict future movements.

Fama (1970) defined three types of efficiency: weak-form, semi-strong-form, and strong-form. It is assumed that all past information in weak form and all publicly available information in semi-strong form are reflected in stock prices. In strong form, stock prices reflect all publicly announced and unannounced information. Testing the mean reversion of stock prices in terms of market efficiency is at the center of research attention. The global financial crisis of 2008–2009 led to renewed interest in stock market inefficiency in emerging market economies. Stock market inefficiency may arise due to the existence of a financial crisis because investors often panic during this critical economic event, which negatively affects their ability to efficiently price stocks (Nartea et al., 2021). Therefore, whether stock prices can be characterized as random walk (unit root) or mean reversion (trend stationary) processes is constantly being investigated (Wang et al., 2015). If stock prices have a stationary process called reversion, the effects of shocks will be temporary. For this reason, they will return to the trend path over time. Stock prices with temporary shocks allow predictions of future movements based on past behavior in terms of investment. Conversely, if stock prices have a unit-rooted structure, the effects of shocks will be permanent, and they will not return to the trend path over time. Therefore, trading strategies will not be developed to obtain abnormal returns (Narayan and Prasad, 2007).

Bose (2005) mentioned the close relationship between the stock market and the real economy and stated that it is important in developing economic policies. Researchers have yet to agree on whether stock prices can be characterized as a random walk (unit root) or mean reversal (trend stationary) process. Durusu-Ciftci et al. (2019) suggested that the differences in the findings may be due to several reasons. First, they said that numerous economic events, such as economic crises, liberalization processes, or changes in economic policy, might have an impact on financial markets. Second, they stressed that it would be very restrictive to assume only a certain number of structural breaks in the financial series. They stated that it would be beneficial to use a nonlinear Fourier function-augmented Dickey–Fuller (ADF) type unit root approach that allows for an unknown number of structural breaks with unknown functional forms in time series data, as proposed by Enders and Lee (2012a). This does not require prior knowledge of numbers, dates, or the forms of multiple breaks. Many economic or financial series may contain more than one soft break at unknown dates.

According to Nartea et al. (2021), the reason for constant interest in this issue is that the main question suggested by the efficient market hypothesis is naturally an empirical question, and it is difficult to answer without returning to the data at some point. They explained that the answer to this question is that the dynamic behavior of stock prices depends on the appropriate specification. Empirical work on this topic suggests that the behavior of stock prices is due to non-linear elements attributable to institutional constraints, market friction, and transaction costs. If the process of creating real data is not linear because of reasons such as linear unit root tests and the presence of heterogeneity, transaction costs, taxes, and regulations will become unsuitable, as stated in studies by Taylor and Peel (2000) (Killian and Taylor, 2003). In this context, there are also studies that extend the literature by considering the nonlinearity in the data generation process and using individual nonlinear unit root tests. Li and Chen (2010), Tan et al. (2010), Mishra and Mishra (2011), Gozbasi et al. (2014), Lee et al. (2014), Wang et al. (2015), Mishra et al. (2015), Moghaddam and Li (2017), and Nartea et al. (2021). Apart from this, there are different tests in the literature on testing mean reversion as a way to examine market efficiency. Some studies in the literature use the variance ratio test (Fama and French (1988a); Lo and MacKinlay (1988); Poterba and Summers (1988); Kim et al. (1991); Urrutia (1995); Huber (1997); Grieb and Reyes (1999); Kawakatsu and Morey (1999); Chaudhuri and Wu (2003); Buguk and Brorsen (2003)). Some studies have used the regression coefficient approach (Fama and French, 1988b; Fama, 1990; Buguk and Brorsen, 2003). Some studies used univariate unit root tests (Liu et al. (1997), Choudhry (1997), Kawakatsu and Morey (1999); Appiah-Kusi, J., and Menyah (2003); Chaudhuri and Wu (2003); Buguk and Brorsen (2003); Worthington and Higgs (2004); Narayan and Smyth (2005); Narayan, P. K. (2006); Hamid et al. (2010); Murthy et al. (2011); Nisar and Hanif (2012); Mishra et al. (2015); Some studies have used panel unit root tests (Chaudhuri and Wu (2004); Balvers et al. (2000); Zhu (1998); Lean and Smyth (2007); Narayan and Narayan (2007); Narayan (2008); Lee et al. (2010); Ahmad et al. (2010); Yilanci (2012); Shen and Holmes (2014a); Shen and Holmes (2014b); Durusu-Çiftçi et al. (2019)). Some studies have used the opposite strategy (Richards, 1995, 1997; Balvers et al., 2000) to examine the average conversion in stock prices developed by Debondt and Thaler (1985).

There are many studies in the literature on efficient markets. These studies present very different results for different samples. These different results show that there is no consensus on efficient markets in the literature. The efficient market hypothesis will continue to be tested with the development of new and powerful testing methodologies. Because this study tests the efficient market hypothesis for OECD countries, some studies on OECD countries will be mentioned. However, there are not many studies investigating efficient markets for OECD countries. The first study is known as the work of Cheung and Lai (1995). Of the 18 countries included in this study, 16 are OECD countries. Although not all OECD countries have been examined comprehensively, since the majority of them are among the OECD countries, this study is included in the literature as the first study examining the returns of stock markets for OECD countries. In this study, a modified rescaled range test and fractional differencing test methods were used between January 1970 and August 1992. The results of this study generally show that stock returns do not have long-term memory. Anagnostidis et al. (2016) also partially examined OECD countries. In the study involving 12-euro countries, daily stock prices between August 24, 2004 and September 15 were examined using the Hurst Exponent method. The results of this study show that the 2008 crisis had a negative impact on stock prices, resulting in stock prices reverting to the mean. In the study by Shen and Holmes (2014), the mean reversion trend of stock prices was investigated for 16 OECD countries, covering the years 1970–2011. For this purpose, linear (SURADF) and non-linear panel unit root (SURKSS) tests based on seemingly unrelated regression (SUR) models were used. The results of this study show that most OECD countries have a mean-reverting trend. Lee et al. (2014) tested the efficient market hypothesis of stock prices for country groups consisting of seven different panels. These country groups consist of panels covering low, middle, and high-income countries as well as OECD, G6, Asian, and European countries. For this purpose, a nonlinear panel unit root test was used. The results for the OECD countries show that stock prices do not have a mean-reverting trend in any country except Poland. In his study, Adekoya (2021) tested the efficient market hypothesis using monthly data that differed by country for 26 OECD countries. For this purpose, the fractional frequency nonlinear unit root test was used. The results of the study showed that in linear models, the market was efficient in 8 countries and inefficient in the other 18 countries. Additionally, nonlinearity exists in only nine countries. For non-linear countries, it was concluded that only Greece has an efficient market.

This study investigated the mean reversion of stock prices in 26 OECD countries. In this context, an answer was sought with a test that considers nonlinear and structural breaks and whether the shocks to stock prices are temporary or permanent. Depending on whether the series contains a unit root or not is important in terms of predicting future movements by examining price movements in the past. This study is expected to contribute to the literature in three aspects. First, the use of Fourier functions to consider structural breaks. Second, determining whether the effect of structural breaks is permanent using fractional frequencies. Finally, both linearity and nonlinearity are considered. In this respect, the second section includes the dataset and economical method used within the scope of the study, and the findings obtained because of the Fourier Threshold Unit Root (FTUR) test are given in the third section of the study. Finally, in the conclusion and evaluation section, the subject examined within the scope of the study and the empirical findings obtained because of the analysis are evaluated.

2. Dataset and Econometric Methodology

In this study, the aim was to investigate whether stock prices returned to the average for 26 OECD countries with monthly data covering the period from January 1990 to January 2021. The Fourier Threshold Unit Root (FTUR) test based on Ylanc et al. (2020) and the tests of Caner and Hansen (2001) and Christopoulos and León-Ledesma (2010) was used to determine if the stock prices for the selected nation group are stationary or have a random wandering process. In this test, a model with a Fourier function is first predicted, and residuals are estimated to eliminate the effects of structural breaks. Next, nonlinear models with residuals are predicted to test the stationarity of the series. Yılancı et al. (2020) chose to use a threshold-type unit root test instead of a smooth transition autoregressive (STAR) model-type unit root test in the last stage of their proposed test. Furthermore, the Fourier functions included in the model contribute to the validity and dependability of the findings gained by capturing smooth transitions as opposed to abrupt breaks in the series.

2.1. Fourier Threshold Unit Root (FTUR) Test

Perron (1989) suggested that structural breaks should be considered in stationarity tests. Zivot and Andrews (1992), Lee and Strazicich (2003), and Lumsdaine and Papell (2004) created tests in which the shape and timing of structural breaches are known and the effect of the breaks is detected abruptly. These tests provide accurate results for situations where the times and numbers of breaks are known and their effects cause sudden shocks. Recently, it has been considered that the effects of this structural change may be softer and more gradual than sudden. In other words, nonlinear unit root tests were introduced in the literature for cases where the break structures were not sharp. These tests are Fourier function approximations that do not require predetermined break numbers or structures. In this approach, the movements of unknown functions are captured by using trigonometric terms. In this context, unit root tests based on frequency component selection were developed using the Fourier function approach, where it is not necessary to determine the break numbers and structure in advance. Studies have revealed that unit root tests developed using the Fourier approach can detect unknown behaviors even if the breaks do not occur in their own period. Yılancı et al. (2020) considered the two-step methodology of Christopoulos and León-Ledesma (2010) to allow structural breaks while testing the stationarity of a series. They expressed the data generation process as follows:

$$y_t = a_0 + a_1 \sin\left(\frac{2\pi kt}{T}\right) + a_2 \cos\left(\frac{2\pi kt}{T}\right) + u_t \tag{1}$$

Here, k is the frequency number of the Fourier function, t is the time term, and T is the sample size. The true value of k is usually unknown, and to find the appropriate number of frequencies, the value of k that gives the least residual square sum is chosen as the appropriate number of frequencies. The existence of unknown breaks in the data generation process of y_t is investigated by testing the null hypothesis $(H_0: a_1 = a_2 = 0)$ against the alternative hypothesis $(H_1: a_1 = a_2 \neq 0)$. F statistics can be used to test this basic hypothesis. This test for constrained (temporary) structural breaks performs particularly well compared with other tests when the breaks are temporary and tend to be in opposite directions. In the second stage of the test, the OLS residuals of equation (1) are obtained and are as follows:

$$\hat{u}_t = y_t - (\hat{a}_0 + \hat{a}_1 \sin(2\pi\hat{k}t/T) + \hat{a}_2 \cos(2\pi\hat{k}t/T)) \tag{2}$$

Christopoulos and León-Ledesma (2010) applied the unit root test to the OLS residuals obtained in equation (2) and proposed three different models for the unit root test in linear and nonlinear form, as follows:

$$\Delta u_t = \alpha_1 u_{t-1} + \sum_{j=1}^{p} \beta_j \Delta u_{t-j} + e_t \tag{3}$$

$$\Delta u_t = \rho u_{t-1}(1 - \exp(-\theta \Delta u_{t-i}^2)) + \sum_{j=1}^{p} \alpha_j \Delta u_{t-j} + e_t, \;\; i = 1,2, \ldots L \tag{4}$$

$$\Delta u_t = \lambda_1 u_{t-1}^3 + \sum_{j=1}^{p} \beta_j \Delta u_{t-j} + e_t \tag{5}$$

They named equations (3), (4), and (5) the Fourier ADF, Fourier KSS, and Fourier KJ unit root tests, respectively. However, Yılancı et al. (2020) applied Caner and Hansen's (2001) test procedure in the second stage instead of using these unit root tests. They named this test the Fourier Threshold Unit Root (FTUR) test. The following model is used to implement the FTUR test:

$$\Delta\hat{u}_t = \theta_1' \hat{u}_{t-1} 1_{\{Z_{t-1} < \lambda\}} + \theta_2' \hat{u}_{t-1} 1_{\{Z_{t-1} \geq \lambda\}} + \varepsilon_t \tag{6}$$

Where $\hat{u}_t$ is the residual from Equation (2) and r_t represents the deterministic component vector containing a constant and probable trend term $\hat{u}_{t-1} = (\hat{u}_{t-1} r_t' \Delta\hat{u}_{t-1}, \ldots, \hat{u}_{t-k})$. Z_{t-1} is the threshold variable defined as $\hat{u}_t - \hat{u}_{t-m}$ for $m \geq 1$. λ represents the unknown threshold parameter and has a value between λ_1 and λ_2. The values of λ_1 and λ_2 are selected as $P(Z \leq \lambda_1) = \pi_1 >$ and $P(Z \leq \lambda_2) = \pi_2 < 1$. It is $\pi_1 = 1 - \pi_2$. Yilancı et al. (2020), following the suggestion of Andrews (1998), set them to $\pi_1 = 0.15$ and $\pi_2 = 0.85$. The components of the parameter vectors are defined as $\theta_1 = \begin{pmatrix} \rho_1 \\ \beta_1 \\ \alpha_1 \end{pmatrix}$ and $\theta_2 = \begin{pmatrix} \rho_2 \\ \beta_2 \\ \alpha_2 \end{pmatrix}$. Here (ρ_1, ρ_2) represents the slope coefficients, (β_1, β_2) the slope coefficients of the deterministic components, and (α_1, α_2) the slope coefficients of the lagged differences in the dependent variable. For each value of the threshold parameter, equation (6) is estimated using the least squares method and is expressed as follows:

$$\Delta\hat{u}_t = \hat{\theta}_1(\lambda)'\hat{u}_{t-1}1_{\{Z_{t-1}<\lambda\}} + \hat{\theta}_2(\lambda)'\hat{u}_{t-1}1_{\{Z_{t-1}\geq\lambda\}} + \hat{\varepsilon}_t(\lambda) \tag{7}$$

The least squares estimator of the λ parameter is reached by minimizing the residual variance of $\sigma^2(\lambda)$ as follows:

$$\hat{\lambda} = \underset{\lambda\in\Lambda}{\operatorname{argmin}}\hat{\sigma}^2(\lambda) \tag{8}$$

After the model is estimated, the basic hypothesis established to test the linearity and threshold effect is established as $H_0: \theta_1 = \theta_2$. To test linearity, Caner and Hansen (2001) proposed the following test statistic:

$$W_t(\lambda) = \mathrm{T}\left(\frac{\hat{\sigma}_0^2}{\hat{\sigma}^2(\hat{\lambda})-1}\right) \tag{9}$$

Here, $\hat{\sigma}_0^2$ is the residual variance under the null hypothesis, and $\hat{\sigma}^2$ shows the variance of the threshold model. In addition, Yılancı et al. (2020) proposed using the Fourier ADF unit root test process for linearity. The prediction of the unit root process is based on the threshold autoregressive (TAR) model, and the null and alternative hypothesis is established as follows:

$H_0: \rho_1 = \rho_2 = 0$

$H_1: \rho_1 < 0, \qquad \rho_2 < 0$

where the null hypothesis indicates the unit root structure in both regimes. The one-way Wald test statistics for the alternative hypothesis are as follows:

$$R_T = t_1^2 1_{(\hat{\rho}_1<0)} + t_2^2 1_{(\hat{\rho}_1<0)} \tag{10}$$

Here, t_1 and t_2 are the *t* statistics of the coefficients $\hat{\rho}_1$ and $\hat{\rho}_2$ obtained from the OLS estimation. Caner and Hansen (2001) determined the critical values using the bootstrap method for situations in which the threshold values are known and unknown.

3. Empirical Findings

In this study, 26 OECD countries were considered to investigate the reversionary mean of stock prices. The descriptive statistics for these countries are presented in Table 1. As can be seen from the descriptive statistics, all countries except Spain and Finland have a non-normal distribution. Denmark is mesokurtic, whereas Greece, Japan, and New Zealand are leptokurtic. All the remaining countries are platykurtic. The country with the highest mean is Greece, while Mexico has the lowest mean. In addition, according to descriptive statistics, the country with the highest standard deviation is Greece. This shows that stock prices in Greece during the analysis had great volatility. We also present the time paths for the series with Fourier approximations in Appendix I.

Table 1: Descriptive Statistics

Countries	Mean	Median	Max.	Min	Std. Dev.	Skewness	Kurtosis	Jarque-Bera
Australia	71.02	71.69	127.13	23.30	28.77	0.02	1.74	24.69*
Austria	86.17	82.67	196.87	35.51	39.13	0.73	2.68	34.77*
Belgium	64.22	63.63	112.84	23.43	25.32	-0.01	1.87	19.99*
Canada	68.44	67.95	124.22	21.49	29.94	-0.03	1.67	27.50*
Denmark	49.19	35.87	165.46	9.27	36.36	1.01	3.01	63.19*
Finland	74.87	77.52	201.35	6.78	40.69	0.13	2.80	1.75
France	73.00	74.80	124.39	26.70	27.42	-0.11	1.84	21.71*
Germany	65.53	64.60	117.97	25.24	25.59	0.19	1.91	20.76*
Greece	244.79	159.57	815.84	66.40	177.22	1.22	3.50	96.01*
Ireland	71.82	75.93	156.82	18.15	33.31	0.14	2.16	12.21*
Italy	92.81	92.33	174.61	30.03	34.34	0.35	2.47	11.97*
Japan	87.72	88.83	177.84	47.05	22.28	0.29	3.59	10.50*
South Korea	65.32	53.96	153.09	15.52	31.90	0.29	1.65	33.62*
Mexican	47.31	31.90	116.68	1.01	40.22	0.29	1.40	44.89*
Netherlands	79.04	81.28	135.32	23.29	30.11	-0.26	2.04	18.54*
New Zealand	81.18	68.82	189.85	34.97	30.61	1.33	4.47	144.15*
Norway	56.40	41.40	161.00	6.73	43.47	0.76	2.44	40.70*
Portugal	86.86	92.97	173.88	21.72	35.97	-0.27	2.28	12.75*
Spain	75.07	80.69	157.07	17.91	32.60	-0.04	2.64	2.10
Sweden	56.79	52.78	152.88	7.86	34.19	0.54	2.45	22.54*
Switzerland	69.85	73.11	127.13	15.41	29.01	-0.29	2.14	16.47*
Turkey	47.62	32.29	189.15	0.033	46.62	0.62	2.14	35.34*
England	78.84	84.85	116.69	31.55	22.98	-0.45	2.12	24.65*
USA	66.81	64.25	138.83	16.64	30.75	0.16	2.13	13.25*
Chile	62.77	49.78	153.45	4.26	42.85	0.40	1.73	35.11*
Israel	51.38	49.61	105.94	4.48	31.30	0.027	1.45	37.34*

Note: ** denote the significance level at 1%.*

Before proceeding with the unit root analysis, the first step is to consider the significance of Fourier terms and the appropriate frequency selection for the analysis of stock prices. For this purpose, optimal frequency values, which are the first step of Christopoulos and León-Ledesma's (2011) two-step method, were determined. The optimal frequency value was determined in the model where the sum of the squares residual is the minimum. Christopoulos and León-Ledesma (2011) considered fractional frequencies in the range of $k = (0{,}5,\ 1,\ \ldots,\ 2{,}5,\ 3)$ for optimal frequency values. The main purpose of using fractional frequencies is to determine whether the effect of structural breaks is permanent. The fractional value of the optimal frequency indicates that the effect of structural breakage is permanent. Table 2 presents the F statistics and optimal frequency values of the analysis regarding the significance of Fourier terms. According to these results, the optimal frequency value was fractional in all countries except Greece and the Netherlands. Therefore, the impact of ruptures is permanent in all countries except Greece and the Netherlands. In addition, Fourier terms are meaningful for all countries. The meaning of the Fourier terms requires that this analysis be continued with unit root tests based on the Fourier approach. Otherwise, there will be a specification error.

Table 2: Significance of Fourier Terms

Countries	k	Min. SSR	F	Countries	k	Min. SSR	F
Australia	0.5	38299.77	1302.621*	Mexican	0.5	29057.80	3645.58*
Austria	0.5	276177.29	196.467*	Netherlands	2	201317.16	124.9457*
Belgium	0.5	76278.93	393.2698*	New Zealand	0.5	114620.67	377.5147*
Canada	0.5	28491.13	1979.683*	Norway	0.5	58663.53	2031.939*
Denmark	0.5	55251.10	1461.975*	Portugal	0.5	163466.73	359.6848*
Finland	0.5	311094.06	181.228*	Spain	0.5	109796.96	481.0276*
France	0.5	106703.47	300.0104*	Sweden	0.5	67811.11	1001.06*
Germany	0.5	80273.92	376.337*	Switzerland	0.5	87318.73	478.1086*
Greece	1	4875221.58	258.378*	Turkey	0.5	32165.07	4465.182*
Ireland	0.5	246892.32	124.2367*	England	0.5	64195.46	381.321*
Italy	1.5	209174.14	203.0235*	USA	0.5	52082.37	1064.862*
Japan	0.5	105014.29	140.4143*	Chile	0.5	54423.42	2136.752*
South Korea	0.5	44272.13	1396.841*	Israel	0.5	21423.25	2962.595*

Note: ** denote the significance level at 1%. Critical values at 1% for constant and trend are 6.730 and 6.873, respectively.*

After determining the significance of the Fourier terms and obtaining the appropriate frequency values, a linearity test is performed on the residuals obtained from equation (2) to determine whether the stock prices of the countries are linear. The linearity test results are shown in Table 3. According to the linearity test results, stock prices in Austria, Canada, Germany, Italy, New Zealand, Spain, and England are linear. As a result, these nations were subjected to the FADF test devised by Christopoulos and León-Ledesma (2011). According to the FADF test results, stock prices in Austria, Canada, Germany, New Zealand, Spain, and England are unit-rooted, and therefore, these markets are weakly efficient. For Italy, it is concluded that stock prices are stationary, they are not weak-form efficient, and there is mean reversion.

Table 3: Results of Fourier Threshold Unit Root Tests

Countries	$\hat{\lambda}$	m	W_T	R_T	t_1	t_2	FADF
Australia	11	11	57.2 (0.055)***	0.00 (0.990)	0.0198 (0.810)	-0.028 (0.834)	
Austria	-2.98	1	43.7 (0.123)				-3.284
Belgium	-4.24	4	64.4 (0.009)*	6.76 (0.263)	-0.461 (0.899)	2.56 (0.097)***	
Canada	-1.31	1	42.4 (0.195)				-3.420
Denmark	1.80	1	63.6 (0.043)**	0.141 (0.985)	0.374 (0.718)	0.029 (0.817)	
Finland	4.63	1	100.00 (0.000)*	15.6 (0.019)**	3.20 (0.321)	2.31 (0.131)	
France	15.6	11	46.7 (0.068)***	5.07 (0.407)	1.40 (0.382)	1.77 (0.285)	
Germany	-7.54	6	37.7 (0.230)				-2.664
Greece	34.8	3	65.6 (0.001)*	21.5 (0.003)*	4.17 (0.002)*	-2.02 (0.992)	

Ireland	-6.60	6	59.8 (0.007)*	19.2 (0.009)*	4.00 (0.009)*	1.80 (0.277)	
Italy	14.1	6	39.1 (0.123)				-3.312***
Japan	-6.36	2	40.8 (0.051)***	9.29 (0.108)	1.78 (0.255)	2.47 (0.102)	
South Korea	13.8	12	53.9 (0.004)*	5.81 (0.332)	2.12 (0.179)	1.15 (0.487)	
Mexican	-0.275	1	73.0 (0.000)*	3.60 (0.545)	1.17 (0.464)	1.49 (0.354)	
Netherlands	-2.08	1	55.9 (0.012)**	12.1 (0.053)***	3.07 (0.038)**	1.63 (0.317)	
New Zealand	14.3	12	43.5 (0.176)				-0.9883
Norway	9.78	8	88.1 (0.004)*	25.2 (0.007)*	-0.836 (0.940)	4.95 (0.002)*	
Portugal	15.8	7	63.0 (0.001)*	0.680 (0.936)	0.565 (0.663)	-0.601 (0.911)	
Spain	-1.95	1	38.7 (0.129)				-2.9179
Sweden	9.69	8	72.3 (0.019)**	0.882 (0.912)	-0.736 (0.934)	0.583 (0.675)	
Switzerland	-3.84	3	50.5 (0.030)**	3.04 (0.637)	-0.544 (0.900)	1.71 (0.299)	
Turkey	1.23	1	47.2 (0.048)**	15.8 (0.015)**	2.68 (0.070)***	-2.94 (0.999)	
England	9.49	12	36.1 (0.273)				-2.6817
USA	-1.53	1	67.0 (0.028)**	0.541 (0.948)	0.682 (0.628)	-0.277 (0.878)	
Chile	-3.76	4	82.2 (0.001)	8.91 (0.153)	-1.54 (0.980)	2.56 (0.101)	
Israel	11.3	12	70.3 (0.009)*	10.8 (0.107)	-0.333 (0.881)	3.27 (0.033)**	

Note: **, **, and *** denote significance at 1%, 5%, and 10%, respectively. The critical values at the 10% significance level for the FADF unit root test for values of k=0.5 and k=1.5 are – 3.64 and – 3.12, respectively. The number of simulations was 10000.*

The underlying reasons for the nonlinearity of financial data are related to market dynamics, participant behavior, and exogenous factors. Because these factors may cause financial series to contain nonlinear components, it is expected to obtain more accurate results with nonlinear analysis methods. The results of the FTUR test, which is a nonlinear test, are presented in Table 3. According to these results, Austria, Denmark, France, Japan, South Korea, Mexico, Portugal, Sweden, Switzerland, the United States, and Chile have unit roots both together and in separate regimes. According to these results, the stock markets of these countries have weak form efficiency and exhibit the same behavior in both regimes. While stock prices in Finland are stationary for the unrestricted model, they contain a unit root in both regimes. In this case, it can be stated that Finland is stationary in the case where the regimes are evaluated together, i.e., mean reversion is in question. However, when the regimes are evaluated separately, they have a unit root, and the stock market has weak form efficiency. Belgium, Norway, and Israel have unit roots in the first regime and are stationary in the second regime. Greece, the Netherlands, Ireland, and Turkey were stationary in the first regime and had unit roots in the second regime. Based on these results, the stock markets of Belgium, Norway, Israel, Greece, the Netherlands, Ireland, and Turkey have partial weak-form efficiency because they have a unit root in one of the two regimes.

According to FADF and FTUR test results, Italy does not have weak form efficiency and has a return-to-average process. Stock markets in Belgium, Finland, Greece, Ireland, the Netherlands, Norway, Turkey, and Israel were found to have partially weak form efficiency and partially return to average. It was determined that Australia, Austria, Canada, Denmark, France, Germany, Japan, South Korea, Mexico, New Zealand, Portugal, Spain, Sweden, Switzerland, England, the United States, and Chile stock markets have unit root processes, and these markets have weak form efficiency.

4. Conclusion and Discussion

Market efficiency means that prices respond quickly and accurately to relevant information. An efficient market is characterized by a random walk process, which indicates that the returns of a stock market cannot be predicted from previous price changes. During the random walk process, shocks have a permanent effect. The persistence of shocks causes stock prices to reach a new equilibrium, which means that future returns cannot be predicted according to historical movements of stock prices. Unit root analysis helps establish whether the shocks are lasting or not. According to Shively (2003), several studies show that financial series are not linear. For these reasons, it is essential to use tests that consider nonlinearity in the analysis of stock prices, which is a financial series.

This study examined whether the stock markets of 26 OECD countries from January 1991 to January 2021 had weak form efficiency. For this purpose, the FADF test developed by Christopoulos and León-Ledesma (2011), considering structural breaks, and the FTUR test of Yılancı et al. (2020), which developed the nonlinear unit root test of Caner and Hansen (2001) with Fourier functions, were used. First, whether the Fourier functions are meaningful for structural breaks was investigated. Fourier functions were found to be significant for structural breaks in all units. Fractional frequencies are used for Fourier functions. It was determined that the optimal frequency values are fractional for all countries except Greece and the Netherlands. According to this result, the structural breaks in the stock prices of Greece and Holland were permanent; for other countries, the effect was temporary. According to the linearity test results, Austria, Canada, Germany, Italy, New Zealand, Spain, and England have a non-linear process; in other countries, stock prices are linear. Because of the linear and nonlinear unit root tests, the stock markets of some countries, except Italy, are weakly efficient, and some are partially weakly efficient. The findings of this study are similar to those of Lee et al. (2014), who used panel data analysis among the studies reviewed in the literature. The results are also similar to those of Appiah-Kusi, J., and Menyah (2003), Worthington and Higgs (2004), Hamid et al. (2010), Nisar and Hanif (2012), Shen and Holmes (2014a), and Shen and Holmes (2014b), which use time series methods and make evaluations for countries or country groups.

This study examines the stock markets of 26 OECD countries using monthly data from January 1990 to January 2021. Future research could also consider equity markets from different geographical regions or emerging markets to provide a broader perspective. They can also obtain new results using data with different frequencies, compare different analysis methods or models, and evaluate the efficiency of stock markets under the influence of macroeconomic variables. They can also examine market efficiency in a broader context by investigating the efficiency of stock prices within different sectors or industries.

Author Contribution

CONTRIBUTION RATE	EXPLANATION	CONTRIBUTORS
Idea or Concept	Creating the research idea or hypothesis	Gökhan KONAT Hüseyin İŞLEK
Literature Review	Scanning the necessary literature for the study	Gökhan KONAT
Research Design	Designing the method, scale and pattern of the study	Hüseyin İŞLEK
Data Collection and Processing	Collecting, organizing and reporting data	Gökhan KONAT
Discussion and Comment	Taking responsibility in evaluating and concluding the findings	Hüseyin İŞLEK

Conflict of Interest

There is no conflict of interest reported by the authors.

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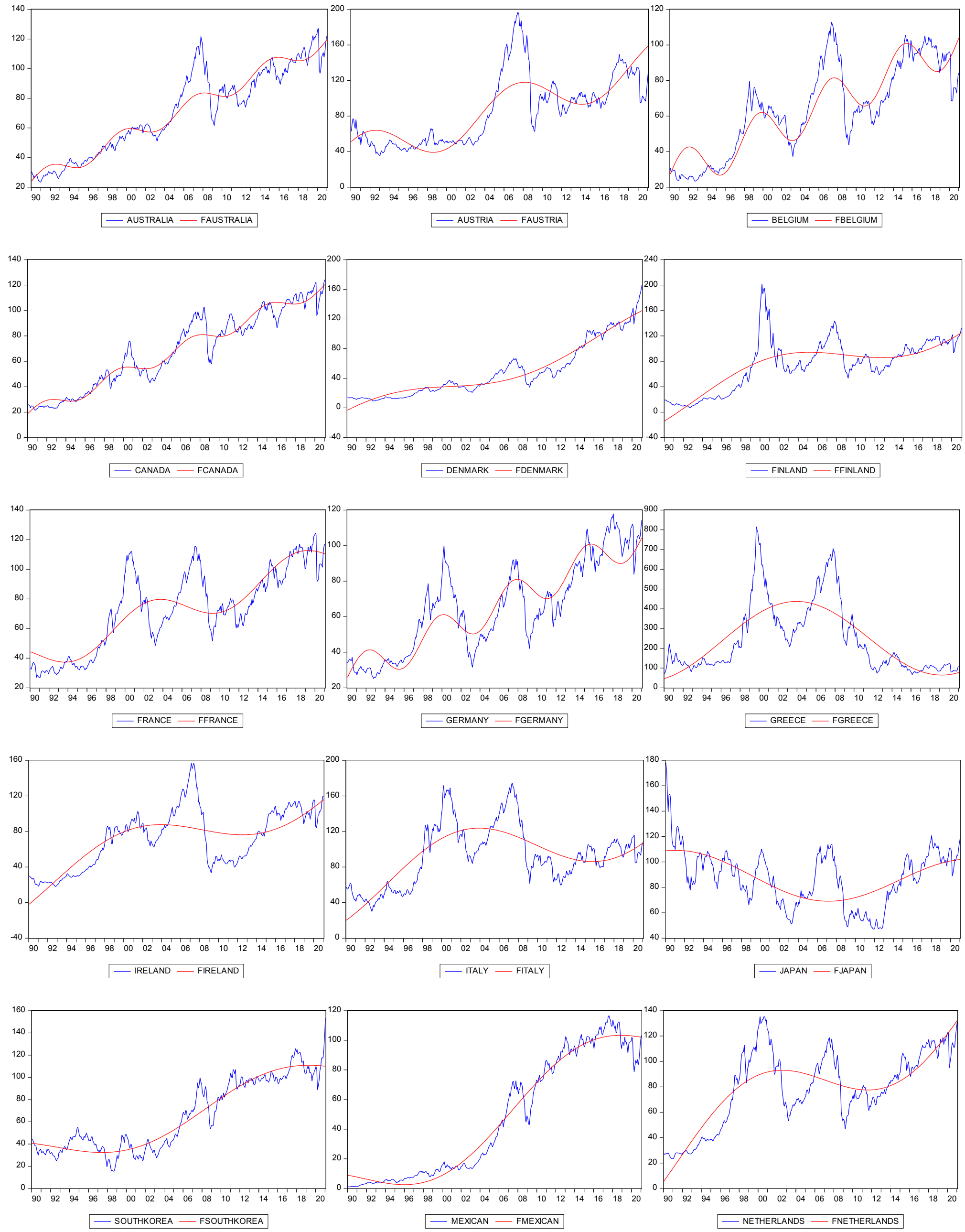
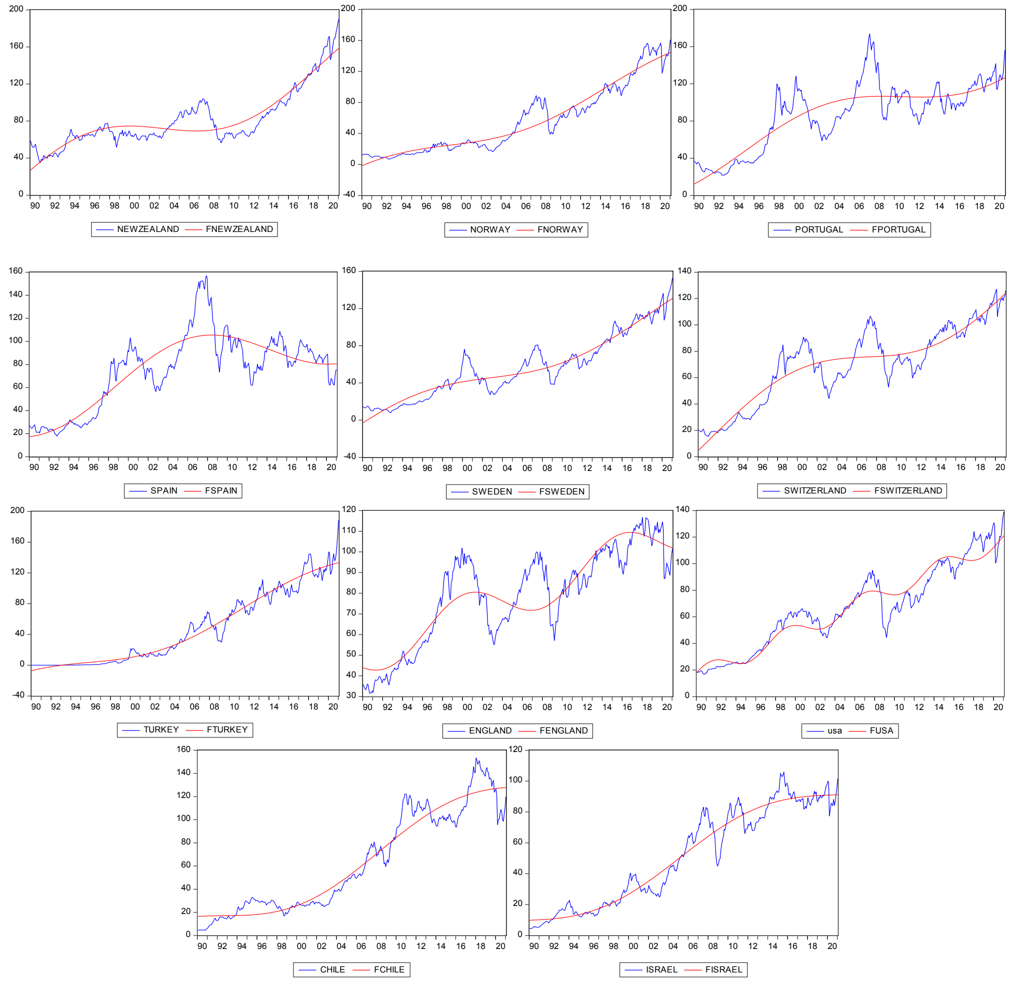
Appendix I

Relative output and the Fourier functions





Resume

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