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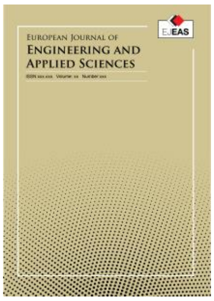

Research Article

Analysis of an Oscillation Circuit with a Linear Time-invariant Inductor and a Capacitor Modelled with Conformal Fractional Order Derivative

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Abstract: Fractional order circuit elements are being examined by researchers unremittingly. They are ever becoming more popular in the literature. The Conformable Fractional Derivative has been proposed and gained importance in the last decade. Examination of an LC tank circuit or an LC oscillator can be found in almost all undergrad physics books. There's a considerable number of studies on fractional-order capacitor circuits but, to the best of our knowledge, examination of an oscillator made of a linear time-invariant inductor and a supercapacitor modeled with Conformable Fractional Derivative has not been found in literature. In this paper, a lossless oscillator circuit containing a linear time-invariant inductor and a supercapacitor modeled with Conformable Fractional Derivative is examined for the first time in the literature. Natural response of the circuit has been found analytically. Its behavior has been illustrated with simulations for different initial conditions.

Keywords: *Circuit analysis, Conformable Fractional Derivative, Fractional Order Circuit, Natural response, Oscillation circuit, Supercapacitor.*

Lineer Zamanla Değişmeyen Endüktörlü ve Uyumlu Kesirli Dereceli Türev ile Modellenmiş Kondansatörlü Bir Salınım Devresinin Analizi

Öz: Kesirli dereceli devre elemanları araştırmacılar tarafından aralıksız olarak incelenmektedir. Literatürde giderek daha popüler hale gelmekteler. Son on yılda Uyumlu Kesirli Türev önerilmiş ve oldukça önem kazanmıştır. Bir LC tank devresinin veya bir LC osilatörünün incelenmesi, neredeyse tüm lisans fizik kitaplarında bulunabilir. Kesirli mertebeden kondansatör devreleri üzerine çok sayıda çalışma var olmasına rağmen, bildiğimiz kadarıyla, doğrusal zamanla değişmeyen bir endüktörden ve Uyumlu Kesirli Türev ile modellenmiş bir süper kondansatörden yapılmış bir osilatörün incelenmesi literatürde bulunmamaktadır. Bu makalede, literatürde ilk defa Uyumlu Kesirli Türev ile modellenmiş bir süper kondansatör ve lineer zamanla değişmeyen bir endüktör içeren kayıpsız bir osilatör devresi incelenmiştir. Devrenin doğal tepkisi analitik olarak bulunmuştur. Devrenin davranışı, farklı başlangıç koşulları için benzetimlerle gösterilmiştir.

Anahtar Kelimeler: *Devre Analizi, Doğal Yanıt, Kesirli Dereceden Devreler, Salınım devresi, Süperkondansatör, Uyumlu Kesirli Türev.*

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1. Introduction

In the late 17th century, among the classical calculus another branch of mathematics, Fractional Calculus (FD), which analyzes integrals and derivatives of non-integer order, has come to the spotlight [1]. Many mathematicians such as Liouville, Riemann and Holmgren have contributed to the progress of it [1-3]. It is an ongoing research area to seek out new fractional derivatives (FDs) [4-6]. Some applications of FDs for various systems are examined in [1-2, 4-10]. There are different definitions of FDs. Some well-known FDs are Riemann-Liouville, Caputo, Hadamard, and Weyl FDs [6]. In the Caputo approach, the derivative of a constant is zero While in Riemann-Liouville definition, the derivative of a constant is not zero [7]. All FDs satisfy the linearity property although mathematical features such as chain and product rules cannot be used. Some of the FD's drawbacks have caused another innovative approach and, in [11], the Conformable Fractional Derivative (CFD) has been suggested. This definition is satisfying the standard derivative rules. The CFD is easier to use than the Caputo and other fractional derivatives. It has analytical solutions and is easier to compute than the other FDs. Because of these properties, the CFD definition became quite popular and preferable to use in modeling fractional order (FO) systems [12]. FO derivatives have been used to model some supercapacitors. The CFD has already been used modeling electric components such as capacitors, inductors and memristors [13-16]. The capacitance of a supercapacitor is measured using a Fractional order model in [17]. In [18], due to the time dependent nature of FO models, it is shown that a FO supercapacitor model can be used to estimate its energy. The parameters of a commercial supercapacitor are determined from experimental data using a FO model [19]. Using the step voltage response of a supercapacitor, its FO model parameters are obtained [20]. Due to its time-dependency, it is hard to analyze the electrical circuits having a CFD capacitor or a supercapacitor modelled with the CFD [21]. A CFD capacitor's behavior under DC and sinusoidal waveforms are examined with incomplete gamma functions in [22]. A two-capacitor problem with a linear time-invariant (LTI) capacitor and a CFD capacitor has been inspected in [23]. A parallel resonant circuit is analyzed with Simulink since it does not have any analytical solutions [24]. Some FO modeled RC and LC electrical circuits are found in [25]. Examination of LC tank circuits is found in every physics book [26]. Such a circuit is lossless and the simplest circuit showing oscillation. A tank circuit is obtained in [27] by replacing the LTI capacitor with a memcapacitor and its analysis is given with perturbation theory. Such an LC tank circuit can also be modified replacing the LTI capacitor with a CFD capacitor and, to the best of our knowledge, it has not been examined in the literature yet. In this study, a lossless tank circuit, which consists of an LTI inductor and a CFD capacitor, has been examined analytically. An analytical solution is found. Its simulation results for different initial conditions have been given.

The paper is organized as the follows. Model of the CFD capacitor is given in the second section. Circuit The differential equation, which describes the oscillation circuit, is given and solved in the third section. The simulated waveforms, which are obtained with MATLAB, are presented in the fourth section. The paper is concluded with the last section.

2. The CFD Capacitor Constitutional Law

Before giving the constitutional law of the CFD capacitor or the supercapacitor, first the definition of the Conformable Fractional order Derivative (CFD) must be given as done in [11]. Definition 1. For a given function $f:[0,\infty)\rightarrow R$, the conformable derivative of f for order α, is defined by:

$$(T_\alpha f)(t) = \lim_{\varepsilon \to 0} \frac{f(t+\varepsilon t^{1-\alpha}) - f(t)}{\varepsilon} \quad (1)$$

For all $t > 0$ and $0 < \alpha \leq 1$ and if f is differentiable for given α and $\lim_{t\to 0^+}(T_\alpha f)(t)$ exist, it is defined as:

$$(T_\alpha f)(0) = \lim_{t\to 0^+}(T_\alpha f)(t) \quad (2)$$

If the function f is α differentiable at $t_0 > 0,\ 0 < \alpha \leq 1$, then f is continuous at t_0 and satisfies the condition:

$$(T_\alpha f)(t) = t^{1-\alpha}\frac{df(t)}{dt} \quad (3)$$

An LTI capacitor constitutional law is given as:

$$i_C(t) = C\frac{dv_C(t)}{dt} \quad (4)$$

To model a capacitor with the Conformal fractional Derivative, the following the constitutional law is obtained with replacing the ordinary derivative with the CFD:

$$i_C(t) = C_\alpha \frac{d{v^\alpha}_C(t)}{dt^\alpha} = C_\alpha t^{1-\alpha}\frac{dv_C(t)}{dt} \quad (5)$$

$i_C(t)$, $v_C(t)$, C_α and α are capacitor's current, voltage, capacitance and fractional order.

3. Analysis of the L-C_α Tank Circuit

Natural response of the well-known LC tank circuit shown in Figure 1.a shows oscillatory behavior if there is energy stored with one or both of the circuit elements. In this study, the LTI capacitor C in Figure 1.a is placed with a CFD capacitor to obtain the L-C_α tank circuit shown in Figure 1.b.

The differential equation, which describes the circuit in Figure 1.b must be derived. Starting by applying Kirchhoff's voltage and current laws,

$$v_{C_\alpha} - v_L = 0 \quad (6)$$

$$i_{C_\alpha} + i_L = 0 \quad (7)$$

where v_{C_α}, v_L, i_{C_α}, and i_L are respectively the CFD capacitor voltage, the inductor voltage, the CFD capacitor current, and the inductor current.

Using the constitutive laws of the elements:

$$v_L(t) = L \cdot \frac{di_L(t)}{dt} \quad (8)$$

$$i_{C_\alpha}(t) = C_\alpha t^{1-\alpha} \frac{dv_C(t)}{dt} \quad (9)$$

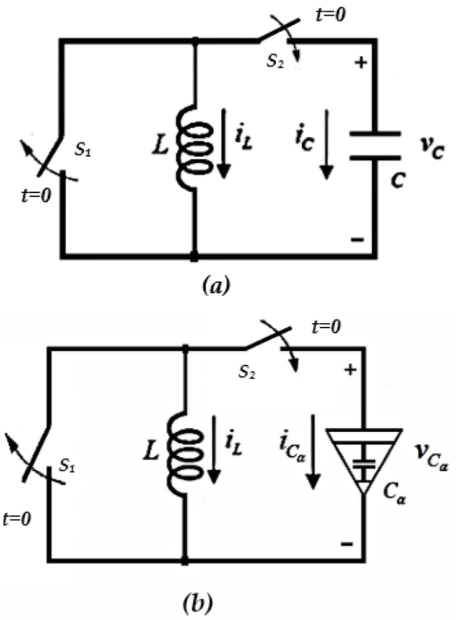


Figure 1. *LC Tank Circuit modeled with a) an LTI capacitor and b) a CFD capacitor*

By combining the Eqs. 8 and 9, the differential equation, which describes the circuit, is obtained as

$$v_C(t) - v_L(t) = 0 \quad (10)$$

$$v_C - L\frac{di_L(t)}{dt} = 0 \quad (11)$$

$$i_L(t) = -i_C(t) = -Ct^{1-\alpha}\frac{dv_C(t)}{dt} \quad (12)$$

$$v_C(t) + L\frac{d}{dt}\left(Ct^{1-\alpha}\frac{dv_C(t)}{dt}\right) = 0 \quad (13)$$

$$v_C(t) + LC\left((1-\alpha)t^{-\alpha}\frac{dv_C(t)}{dt} + t^{1-\alpha}\frac{d^2v_C(t)}{dt^2}\right) = 0 \quad (14)$$

$$LCt^{1-\alpha}\frac{d^2v_c(t)}{dt^2} + LC(1-\alpha)t^{-\alpha}\frac{dv_C(t)}{dt} + v_C(t) = 0 \quad (15)$$

Such a differential equation is named as Sturm-Liouville [28]. Wolfram| Alpha, an online symbolic calculator, is used for its solution. The differential equation solution is given as

$$v_C(t) = (\alpha+1)^{\frac{-\alpha}{\alpha+1}}c_1(CL)^{\frac{-\alpha}{2(\alpha+1)}}t^{\frac{\alpha}{2}}\Gamma\left(\frac{1}{1+\alpha}\right)J_{-\frac{\alpha}{\alpha+1}}\left(\frac{2t^{\frac{\alpha+1}{2}}}{(\alpha+1)\sqrt{LC}}\right) + (\alpha+1)^{\frac{-\alpha}{\alpha+1}}c_2(CL)^{\frac{-\alpha}{2(\alpha+1)}}t^{\frac{\alpha}{2}}\Gamma\left(\frac{\alpha}{1+\alpha}+1\right)J_{\frac{\alpha}{\alpha+1}}\left(\frac{2t^{\frac{\alpha+1}{2}}}{(\alpha+1)\sqrt{LC}}\right) \quad (16)$$

Here Jv(x) is the Bessel function of the first kind of order v. In this solution, the Bessel function order is given as non-integer and it is singular at x=0. More about the Bessel function can be found in [29]. The series expansion of the Bessel function around x = 0 can be expressed as

$$J_v(x) = \sum_{m=0}^{\infty}\frac{(-1)^m}{m!\,\Gamma(m+v+1)}\left(\frac{x}{2}\right)^{2m+v} \quad (17)$$

where $\Gamma(x)$ is the Complete Gamma function. It is an important function used to calculate factorials [29].

The Complete gamma function is given by the integral definition as

$$\Gamma(x) = \int_0^{\infty} y^{x-1}e^{-y}dy \quad (18)$$

Let's define the following parameters:

$$A = (\alpha+1)^{-\frac{\alpha}{\alpha+1}}(CL)^{-\frac{\alpha}{2(\alpha+1)}}\Gamma\left(\frac{1}{1+\alpha}\right) \quad (19)$$

$$B = (\alpha+1)^{-\frac{\alpha}{\alpha+1}}(CL)^{-\frac{\alpha}{2(\alpha+1)}}\Gamma\left(\frac{\alpha}{1+\alpha}+1\right) \quad (20)$$

Then, Eq. 16 turns into

$$v_C(t) = c_1 A\, J_{-\frac{\alpha}{\alpha+1}}\left(\frac{2t^{\frac{\alpha+1}{2}}}{(\alpha+1)\sqrt{LC}}\right) + c_2\, B\, J_{\frac{\alpha}{\alpha+1}}\left(\frac{2t^{\frac{\alpha+1}{2}}}{(\alpha+1)\sqrt{LC}}\right) \quad (21)$$

c_1 and c_2 are the integral constants of the solution of the differential equation, which must be found. The given initial conditions of the energy storing elements are used for defining the constants. In the expression found for the CFD capacitor voltage in the circuit, the Bessel functions are used as series for simplicity and the initial conditions $v_C(0)$ and $i_L(0)$ are used at t=0 instant. To determine c_1, at $t=0$, the initial condition $v_C(0)$ is substituted in Eq.16:

$$v_C(0) = c_1 A\cdot 0\cdot\sum_{m=0}^{\infty}\frac{(-1)^m}{m!\,\Gamma\left(m-\frac{\alpha}{\alpha+1}+1\right)}\cdot\left(\frac{0}{(\alpha+1)\sqrt{LC}}\right)^{2m-\frac{\alpha}{\alpha+1}} + c_2 B\cdot 0\cdot\sum_{m=0}^{\infty}\frac{(-1)^m}{m!\,\Gamma\left(m+\frac{\alpha}{\alpha+1}+1\right)}\cdot\left(\frac{0}{(\alpha+1)\sqrt{LC}}\right)^{2m+\frac{\alpha}{\alpha+1}} \quad (22)$$

The following expression is undetermined since it turns into "zero times infinity"

$$v_C(0) = \begin{cases} 0 form = 1,2,3,4\ldots \\ c_1 A \dfrac{1}{\Gamma\left(\dfrac{1}{\alpha+1}\right)} (0 \cdot \infty)(LC)^{\frac{\alpha}{2(\alpha+1)}}(\alpha+1)^{\frac{\alpha}{\alpha+1}} + 0 form = 0 \end{cases} \tag{23}$$

By taking the limit of the term that causes singularity at t=0 for m=0,

$$\lim_{t\to 0}(t^{\frac{\alpha}{2}} \cdot t^{-\frac{\alpha}{2}}) = \lim_{t\to 0} t^0 = 1 \tag{24}$$

Therefore, using the limit at t=0,

$$v_C(0) = c_1(\alpha+1)^{-\frac{\alpha}{\alpha+1}}(LC)^{-\frac{\alpha}{2(\alpha+1)}}\Gamma\left(\frac{1}{\alpha+1}\right)\frac{1}{\Gamma\left(\frac{1}{\alpha+1}\right)}(LC)^{\frac{\alpha}{2(\alpha+1)}}(\alpha+1)^{\frac{\alpha}{\alpha+1}}$$
$$\Rightarrow c_1 = v_C(0) \tag{25}$$

To determine c_2, at $t = 0$,

$$i_C(0) = i_L(0) \tag{26}$$

By differentiating $v_C(t)$,

$$\frac{dv_C}{dt} = \frac{d}{dt}\left[c_1 A \cdot t^{\frac{\alpha}{2}} \cdot \sum_{m=0}^{\infty} \frac{(-1)^m}{m!\,\Gamma\left(1 - \frac{\alpha}{\alpha+1} + 1\right)}\left(\frac{t^{\frac{\alpha+1}{2}}}{(\alpha+1)\sqrt{LC}}\right)^{2m-\frac{\alpha}{\alpha+1}} + c_2 B \cdot t^{\frac{\alpha}{2}} \cdot \sum_{m=0}^{\infty} \frac{(-1)^m}{m!\,\Gamma\left(1 + \frac{\alpha}{\alpha+1} + 1\right)}\left(\frac{t^{\frac{\alpha+1}{2}}}{(\alpha+1)\sqrt{LC}}\right)^{2m+\frac{\alpha}{\alpha+1}}\right] \tag{27}$$

Using Eq. 9, the CFD capacitor current can be expressed as

$$(t) = C_\alpha t^{1-\alpha}\frac{dv_C}{dt} = C_\alpha t^{1-\alpha}\frac{d}{dt}\left[c_1 A \cdot t^{\frac{\alpha}{2}} \cdot \sum_{m=0}^{\infty} \frac{(-1)^m}{m!\,\Gamma\left(m - \frac{\alpha}{\alpha+1} + 1\right)}\left(\frac{t^{\frac{\alpha+1}{2}}}{(\alpha+1)\sqrt{LC}}\right)^{2m-\frac{\alpha}{\alpha+1}} + c_2 B \cdot t^{\frac{\alpha}{2}} \cdot \sum_{m=0}^{\infty} \frac{(-1)^m}{m!\,\Gamma\left(m + \frac{\alpha}{\alpha+1} + 1\right)}\left(\frac{t^{\frac{\alpha+1}{2}}}{(\alpha+1)\sqrt{LC}}\right)^{2m+\frac{\alpha}{\alpha+1}}\right]$$

$$\Rightarrow i_C(t) = C_\alpha t^{(1-\alpha)}\left[c_1 A \sum_{m=0}^{\infty} \frac{(-1)^m}{m!\,\Gamma\left(m - \frac{\alpha}{\alpha+1} + 1\right)}\left(Y \cdot m(\alpha+1) \cdot t^{(m(\alpha+1)-1)}\right) + c_2 B \sum_{m=0}^{\infty} \frac{(-1)^m}{m!\,\Gamma\left(m + \frac{\alpha}{\alpha+1} + 1\right)}(Z \cdot ((m(\alpha+1)+\alpha) \cdot t^{(m(\alpha+1)+\alpha-1)})\right] \tag{28}$$

$$\Rightarrow i_C(t) = C_\alpha\left[c_1 A \sum_{m=0}^{\infty} \frac{(-1)^m}{m!\,\Gamma\left(m - \frac{\alpha}{\alpha+1} + 1\right)}\left(Y \cdot m(\alpha+1) \cdot t^{(m(\alpha+1)-\alpha)}\right) + c_2 B \sum_{m=0}^{\infty} \frac{(-1)^m}{m!\,\Gamma\left(m + \frac{\alpha}{\alpha+1} + 1\right)}(Z \cdot (m(\alpha+1)+\alpha) \cdot t^{(m(\alpha+1))})\right]$$

Where

$$Y = \left(\frac{1}{(\alpha+1)\sqrt{LC}}\right)^{\left(2m-\frac{\alpha}{\alpha+1}\right)} \qquad (29)$$

$$Z = \left(\frac{1}{(\alpha+1)\sqrt{LC}}\right)^{\left(2m+\frac{\alpha}{\alpha+1}\right)} \qquad (30)$$

Substituting t=0 in equation (28),

$$\begin{aligned} i_C(t) &= C_\alpha \Bigg[c_1 A \sum_{m=0}^{\infty} \frac{(-1)^m}{m!\,\Gamma\left(m-\frac{\alpha}{\alpha+1}+1\right)} \left(Y \cdot m(\alpha+1) \cdot 0^{(m(\alpha+1)-\alpha)}\right) \\ &+ c_2 B \sum_{m=0}^{\infty} \frac{(-1)^m}{m!\,\Gamma\left(m+\frac{\alpha}{\alpha+1}+1\right)} \left(Z \cdot (m(\alpha+1)+\alpha) \cdot 0^{(m(\alpha+1))}\right) \Bigg] \end{aligned} \qquad (31)$$

The following expression is undetermined since it turns into "Zero to the power of zero":

$$i_C(t) = \begin{cases} 0 & for\ m = 1,2,3 \\ 0 + c_2 B \dfrac{1}{\Gamma\left(\frac{\alpha}{\alpha+1}+1\right)} \left(\dfrac{1}{(\alpha+1)\sqrt{LC}}\right)^{\frac{\alpha}{\alpha+1}} \cdot \alpha \cdot 0^0 & for\ m = 0 \end{cases} \qquad (32)$$

In order to get rid of the indeterminant form, taking the limit as time variable approaches to zero of $t^{(m(\alpha+1))}$as follows

$$\lim_{t\to 0}(t^0) = 1 \qquad (33)$$

Therefore, using the limit at t=0, the second coefficient $\boldsymbol{c_2}$ is found as

$$\begin{aligned} i_C(0) &= c_2 C_\alpha (a+1)^{-\frac{\alpha}{\alpha+1}} (LC)^{-\frac{\alpha}{2(\alpha+1)}} \Gamma\left(\frac{\alpha}{\alpha+1} + 1\right) \frac{1}{\Gamma\left(\frac{\alpha}{\alpha+1}+1\right)} \alpha \left(\frac{1}{(\alpha+1)\sqrt{LC}}\right)^{\frac{\alpha}{\alpha+1}} \end{aligned} \qquad (34)$$

$$\Rightarrow c_2 = \frac{i_C(0)}{\alpha C_\alpha (\alpha+1)^{-\frac{2\alpha}{\alpha+1}} (CL)^{-\frac{\alpha}{(\alpha+1)}}} \qquad (35)$$

The capacitor current for t=0 instance is related to Eq. **7** with the inductor current at that instance. Therefore,

$$i_C(0) = -i_L(0) \qquad (36)$$

Finally, the equation of the CFD capacitor voltage can be rearranged as:

$$\begin{aligned} v_C(t) &= v_C(0) \cdot A \cdot t^{\frac{\alpha}{2}} \cdot J_{\frac{-\alpha}{\alpha+1}}\left(\frac{2t^{\frac{\alpha+1}{2}}}{(\alpha+1)\sqrt{LC}}\right) \\ &- \left(\frac{i_L(0)}{\alpha C_\alpha (\alpha+1)^{\frac{-2\alpha}{\alpha+1}} (LC)^{\frac{-\alpha}{\alpha+1}}}\right) \cdot B \cdot t^{\frac{\alpha}{2}} \\ &\cdot J_{\frac{\alpha}{\alpha+1}}\left(\frac{2t^{\frac{\alpha+1}{2}}}{(\alpha+1)\sqrt{LC}}\right) \end{aligned} \qquad (37)$$

4. Circuit waveforms

In this section, using Eq. 37, the plots of the inductor current and the CFD voltage of the tank circuit are given in Figures 2-4. All simulations are made for the parameters given in Table 1 and three different initial conditions. A short investigation of the waveforms it is concluded that the amplitude of the super-capacitors voltage is decreasing as the time progresses although the amplitude of its current is increasing with respect to time. This is caused by the time-dependent term $t^{1-\alpha}$. It can be thought that the equivalent capacitor capacitance increases by time and this results in falling down of the circuit's characteristic impedance, and an increase in the circulation current. For low values of α , the effect of the CFD capacitor is more dominant and this results in higher CFD capacitor current amplitudes. Another interesting result is that when α becomes 1 (integer order), the $\boldsymbol{L\text{-}C_\alpha}$ tank circuit is oscillating with the same amplitude as the well-known LC circuit with the LTI components.

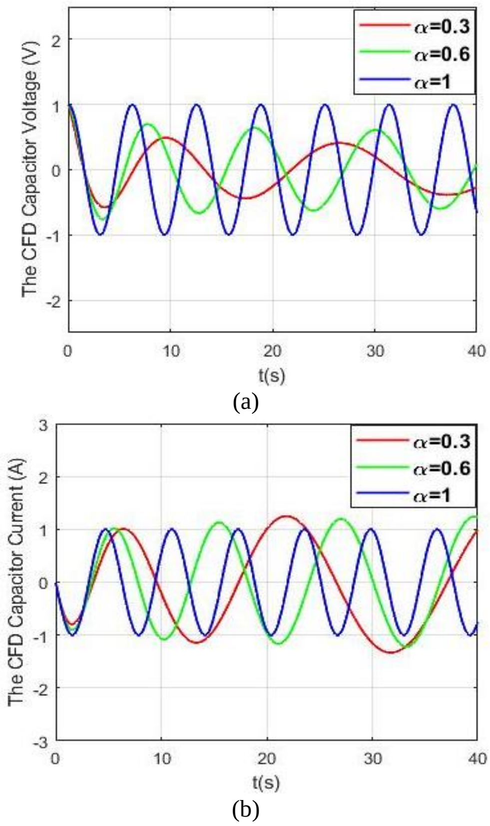
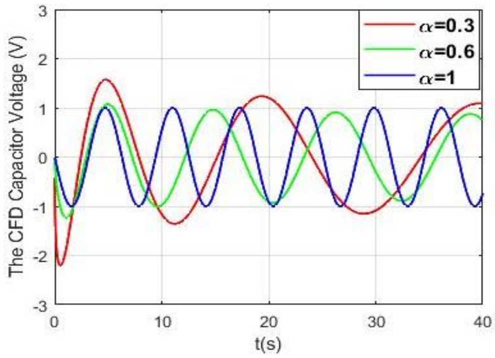
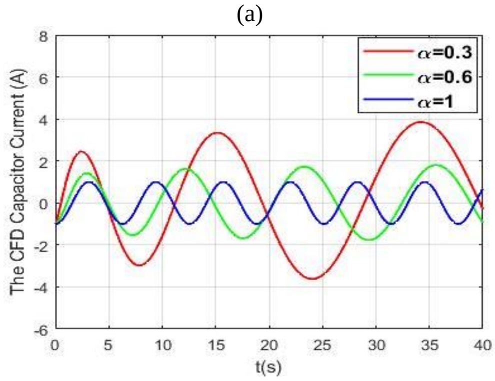


Figure 2*. a) v_C and b) i_C vs. time (t) for $v_C(0)$ = 1 V and $i_L(0)$ = 0 A.*

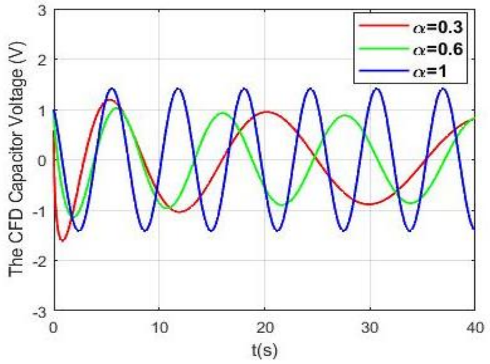


(a)

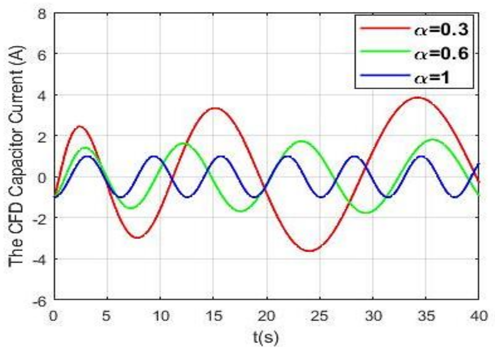


(b)

Figure 3. *a)* v_C *and b)* i_C *vs. time (t) for* $v_C(0) = 0$ *V and* $i_L(0) = 1$ *A.*



(a)



(b)

Figure 4. *a)* v_C *and b)* i_C *vs. time (t) for* $v_C(0) = 1$ *V and* $i_L(0) = 1$ *A.*

Table 1 The circuit parameters

Parameter	Value
C_α	$1\ \mathrm{F}/s^{1-\alpha}$
L	1 H

If the circuit with a more realistic super capacitor model were to be analyzed, a series resistor had to be added to it, and, then, the current would go down to zero due to the power dissipation within the resistor. An inductor with a core gets saturated if its current gets sufficiently high enough. During saturation, the solution given in the last section and the waveforms given in this section cannot be used to analyze such a circuit.

5. Conclusions

The combination of every circuit element with others must be examined analytically. Analysis of physical and engineering systems modelled with FDs have become a hot research area with promising applications. The CFD is much simpler than other FDs. Some supercapacitors can be modeled with CFD. The CFD capacitor's combination with an inductor has been examined in this study. Its analytical solution has been found using the Bessel functions of the first kind of order and the Complete Gamma function. Using simulation, the behavior of $\boldsymbol{L}$-$\boldsymbol{C_\alpha}$ tank circuit for different initial conditions is examined. The solution given here is important from the circuit theory point of view. It may help to tank circuit and undamped oscillator designers. Also, it may find usage in other areas if there is an electrical and mechanical analogy existing.

The results given here presents a lossless $\boldsymbol{L}$-$\boldsymbol{C_\alpha}$ tank circuit. In such a case, the amplitude of the CFD capacitor and the LTI inductor currents keeps increasing. The damping and analysis of an ***R-L-C*** circuit is of great importance and can be found in all circuit theory books. As a future work, we suggest that, by adding a series resistor to the circuits, the analytical solution and behaviour of an $\boldsymbol{R}$-$\boldsymbol{L}$-$\boldsymbol{C_\alpha}$ circuit can be examined.

Author Contribution

Formal analysis – Mendi Arapi (MA)-Reşat Mutlu (RM); Investigation – RM; Experimental Performance - none; Data Collection – MA; Processing – MA; Literature review – RM; Writing – MA, RM; Review and editing – MA, RM;

Declaration of Competing Interest

The authors declared no conflicts of interest with respect to the research, authorship, and/or publication of this article.

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