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New Solutions for IOPM and (3+1)-Dimensional NLWE in Liquid with Gas Bubbles

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ABSTRACT: Generalized Kudryashov method (GKM), which is one of the solution methods of nonlinear evolution equations (NLEEs), has been used to obtain some solutions of Ivancevic option pricing model (IOPM) and (3+1)-dimensional nonlinear wave equation (NLWE) in liquid with gas bubbles. Thus, some solutions of the discussed equations have been found such as dark soliton, trigonometric and hyperbolic solutions. Two dimensional (2D) and three dimensional (3D) graphics of these solutions have been drawn with the help of Wolfram Mathematica 12.

Keywords: GKM, Ivancevic option pricing model, (3+1)-dimensional NLWE in liquid with gas bubbles, soliton solutions

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INTRODUCTION

Nonlinear evolution equations (NLEEs) are tackled in quite substantial scientific fields such as physics, biophysics, mathematical physics, optical fibers, mathematical chemistry, hydrodynamics, fluid dynamics, control theory, optics, mechanics, chemical kinematics, biogenetics and so on. With the improving world, NLEEs arise as having more hard and complex solutions. Solving these equations and finding novel methods forms a very important field of study. For this aim, several solution methods have been presented to the literature by some scientists (Kara and Ünsal, 2022; Mamun et al., 2022; Eslami and Mirzazadeh, 2014; Biswas et al., 2018; Günay et al., 2021; Duarte and da Mota, 2021; Tuluce Demiray and Bayrakci, 2021a)

Ivancevic option pricing model (IOPM) (Ivancevic, 2010; Jena et al., 2020; Ivancevic, 2011; Chen et al., 2022; González-Gaxiola et al., 2017; Chen et al., 2021) is given as:

$$iw_t = -\frac{1}{2}\sigma w_{ss} - \beta|w|^2 w, \left(i = \sqrt{-1}\right) \tag{1}$$

Where $w(s,t)$ symbolizes the option price function at time t, $|w|^2$ indicates the probability density function for the option price and denotes potential field, σ is dispersion frequency coefficient, and it is used to represent the volatility being constant or stochastic process itself, (in this paper, it is tackled as a constant). β shows the Landau coefficient symbolizing the adaptive market potential(Jena et al., 2020). IOPM defines a nonlinear wave (e.g. in Bose-Einstein condensates) described by the complex-valued wave function $w(s,t)$ of real space and time parameters. Herein, the space-like variable$s$ shows the stock (asset) price (Ivancevic, 2011). Also, Eq. (1) constitutes a connection between economy and optional pricing (Chen et al., 2022).

A lot of solutions have been procured via fractional reduced differential transform method (Jena et al., 2020), Jacobi elliptic functions (Ivancevic, 2011), rational sine-Gordon expansion method and modified exponential method (Chen et al., 2022), He's frequency amplitude formulation method (González-Gaxiola et al., 2017), and so on (Chen et al., 2021).

In the area of liquid with gas bubbles, bubble–liquid mixing equations have been improved to identify the propagation of weak nonlinear waves which are extensively observed in natural science, hydrodynamics, medical science, and engineering. Motivated by that, (3+1)-dimensional NLWE in liquid with gas bubbles is tackled as (Wang et al., 2019; Wang et al., 2020; Kumar et al.,2021; Shen et al., 2022; Liu and Zhang, 2020; Yadav and Arora, 2021; Tu et al., 2016; Liu et al., 2020),

$$\left(w_t + ww_x + \frac{1}{4}w_{xx} - w_x\right)_x + \frac{3}{4}\left(w_{yy} + w_{zz}\right) = 0 \tag{2}$$

Where $w = w(x,y,z,t)$ is a differentiable function with space coordinates x, y, z and time coordinate t (Wang et al., 2019). w is connected with the velocity of the mixture, x, y, z and t denotes the scaled spatial and temporal coordinates, sembolizing the transverse y and z perturbation on the wave propagating in the x direction (Wang et al., 2020). Eq. (2) identifies some nonlinear physical phenomena in liquid including gas bubble (Wang et al., 2019).

Many solutions have been found via generalized exponential rational function method (Kumar et al., 2021), Hirota's bilinear method (Shen at al., 2022), extended homoclinic test method (Liu and Zhang, 2020), Lie symmetry method (Yadav and Arora, 2021), Hirota bilinear method and Bäcklund transformations (Tu et al., 2016), bilinear method and KP reduction method (Liu et al., 2020) and so on our purpose in this study is to obtain some solutions of (3+1)-dimensional NLWE in liquid with gas bubbles by using GKM (Gurefe, 2020; Tuluce Demiray and Bayrakci, 2021b; Tuluce Demiray and Bayrakci, 2021c). First, the basic of GKM was presented. Afterwards, GKM was implemented to the recommended equation and some solutions were found by using the Wolfram Mathematica 12 package program.

MATERIALS AND METHODS

Let's investigate the general form of the partial differential equation with detached variables $x, y, z, \ldots, t$ as,

$$S\left(w, w_x, w_y, w_z, \ldots, w_t, \ldots, w_{xx}, w_{xy}, w_{xz}, \ldots, w_{xt}, \ldots\right) = 0 \tag{3}$$

Step 1: We regard travelling wave solution as in the following equation;

$$w(x, y, z, \ldots, t) = w(\varepsilon), \varepsilon = e_1 x + e_2 y + e_3 z + \cdots + e_p t \tag{4}$$

Using Eq. (4), Eq. (3) is transformed into an ordinary differential equation:

$$T(w, w', w'', w''', \ldots) = 0 \tag{5}$$

where superscripts indicate ordinary derivatives according ε .

Step 2. Assuming that we imagine the solutions of Eq. (5) as Eq. (6),

$$w(\varepsilon) = \frac{\sum_{k=0}^{n} a_k Z^k(\varepsilon)}{\sum_{l=0}^{m} b_l Z^l(\varepsilon)} = \frac{G[Z(\varepsilon)]}{H[Z(\varepsilon)]} \tag{6}$$

where Z is $\frac{1}{1 \pm e^{\varepsilon}}$ We must specify that Z is the solution to Eq. (7),

$$Z' = Z^2 - Z \tag{7}$$

where $Z' = \frac{dz}{d\varepsilon}$.

Using Eq. (6), the following derivatives are obtained,

$$w'(\varepsilon) = \frac{G'Z'H - GH'Z'}{H^2} = Z'\left[\frac{G'H - GH'}{H^2}\right] = (Z^2 - Z)\left[\frac{G'H - GH'}{H^2}\right] \tag{8}$$

$$w''(\varepsilon) = \frac{Z^2 - Z}{H^2}\left[(2Z - 1)(G'H - GH') + \frac{Z^2 - Z}{H}\left[H(G''H - GH'') - 2H'G'H + 2G(H')^2\right]\right] \tag{9}$$

where $G' = \frac{dG}{dZ}$, $H' = \frac{dH}{dZ}$, $G'' = \frac{d^2G}{dZ^2}$, $H'' = \frac{d^2H}{dZ^2}$.

Step 3. The solution of the nonlinear ordinary differential equation given by Eq. (5) is sought according to the GKM as follows:

$$w(\varepsilon) = \frac{a_0 + a_1 Z + a_2 z^2 + \cdots + a_n Z^n}{b_0 + b_1 Z + b_2 z^2 + \cdots + b_m Z^m} \tag{10}$$

We use the homogeneous balance principle to find the values of m and n in Eq. (5). For this purpose, we balance between the highest order derivative and the highest order nonlinear term in Eq. (5). Thus, m and n are obtained.

Step 4**.** Then, a zero polynomial is found with respect to Z. An algebraic equation system $R(Z)$ is constituted by equating the coefficients to zero in the zero polynomial. If this found algebraic equation system is unfastened by way of Mathematica 12, $a_k(k = 0, \ldots, n), b_l(l = 0, \ldots, m), e_j(j = 0, \ldots, p)$ terms are satisfied

Application of GKM to the IOPM

To get some solutions of Eq. (1), we take into account the following equality:

$$w = e^{i(c_2 t + k_2 s)} w(\varepsilon), \varepsilon = c_1 t + k_1 s \tag{11}$$

Putting Eq. (11) into Eq. (1), we find the following equality

$$e^{i(tc_2 + sk_2)}\left(\frac{w''(\varepsilon)\sigma k_1^2}{2} + (i\sigma k_1 k_2 + ic_1)w'(\varepsilon) + w(\varepsilon)\left(\beta|w(\varepsilon)|^2 - \frac{\sigma k_2^2}{2} - c_2\right)\right) = 0 \tag{12}$$

The real and imaginary parts of Eq. (12) are obtained as follows:

$$(\sigma k_1 k_2 + c_1)\left(w'(\varepsilon)\right) = 0, \tag{13}$$

$$\frac{w''(\varepsilon)\sigma k_1^2}{2}+\beta w(\varepsilon)|w(\varepsilon)|^2-\frac{w(\varepsilon)\sigma k_2^2}{2}-w(\varepsilon)c_2=0 \tag{14}$$

If the balance principle is applied, we obtain

$$n-m+2=3n-3m\Rightarrow n=m+1 \tag{15}$$

If we choose $m=1$ and $n=2$, we get

$$w(\varepsilon)=\frac{a_0+a_1Z+a_2Z^2}{b_0+b_1Z} \tag{16}$$

$$w''(\varepsilon)=(Z^2-Z)\left[\frac{(a_1+2a_2Z)(b_0+b_1Z)-b_1(a_0+a_1Z+a_2Z^2)}{(b_0+b_1Z)^2}\right] \tag{17}$$

$$\begin{aligned}w''(\varepsilon)=&\frac{Z^2-Z}{(b_0+b_1Z)^2}(2Z-1)[(a_1+2a_2Z)(b_0+b_1Z)-b_1(a_0+a_1Z+a_2Z^2)]\\&+\frac{(Z^2-Z)^2}{(b_0+b_1Z)^3}[2a_2(b_0+b_1Z)^2-2b_1(a_1+2a_1Z)(b_0+b_1Z)\\&+2b_1^2(a_0+a_1Z\\&+a_2Z^2)]\end{aligned} \tag{18}$$

Case 1:

$$a_0=-\frac{i\sqrt{\sigma}b_0k_1}{\sqrt{\beta}},a_1=-a_2+\frac{i\sqrt{\sigma}b_0k_1}{\sqrt{\beta}},a_2=a_2,b_0=b_0,b_1=-\frac{i\sqrt{\beta}a_2}{\sqrt{\sigma}k_1},$$

$$c_1=-\sigma k_1(3k_1+k_2),c_2=-\frac{1}{2}\sigma(2k_1^2+k_2^2). \tag{19}$$

By placing Eq. (19) in Eq. (16), we get the trigonometric function solution of Eq. (1)

$$w_1(s,t)=-\frac{\sqrt{\sigma}k_1}{2\sqrt{\beta}}e^{i\left(sk_2-\frac{1}{2}\sigma(2k_1^2+k_2^2)t\right)}\left(i+tan\left[\frac{i(sk_1-\sigma k_1(3k_1+k_2)t)}{2}\right]\right) \tag{20}$$

2D and 3D graphs of real and imaginary parts of the solution (20) are demonstrated with contour simulations in Figure 1.

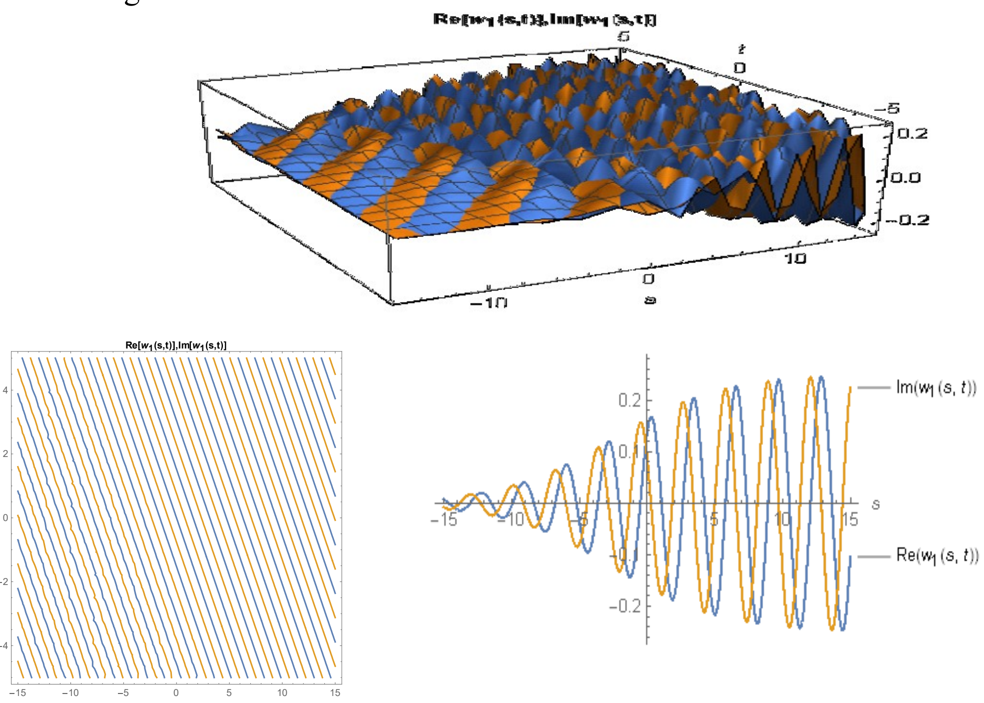

Figure 1. 3D, contour plots of solution (20) for $k_1 = 0.25, k_2 = -2, \sigma = 1, \beta = 1, -15 \leq s \leq 15,\ -5 \leq t \leq 5$ values with $-5 \leq t \leq 5$ range and 2D plot of solution for $t = 2$ with these values

Case 2:

$$a_0 = -\frac{a_2 b_0}{b_1}, a_1 = a_2\left(-1+\frac{b_0}{b_1}\right), a_2 = a_2, b_0 = b_0, b_1 = b_1, \sigma = -\frac{\beta a_2^2}{b_1^2 k_1^2},$$
$$c_1 = \frac{\beta a_2^2(3k_1+k_2)}{b_1^2 k_1}, c_2 = \frac{\beta a_2^2(2k_1^2+k_2^2)}{2b_1^2 k_1^2}. \tag{21}$$

By placing Eq. (21) in Eq. (16), we get the dark soliton solution of Eq. (1)

$$\mathrm{w}_2(s,t) = -\frac{a_2}{2b_1} e^{i\left(sk_2+\frac{\beta a_2^2(2k_1^2+k_2^2)}{2b_1^2k_1^2}\right)}\left(1+tanh\left[\frac{1}{2}\left(sk_1+\frac{\beta a_2^2(3k_1+k_2)t}{b_1^2 k_1}\right)\right]\right) \tag{22}$$

2D and 3D graphs of real and imaginary parts of the solution (22) are demonstrated with contour simulation in Figure 2.

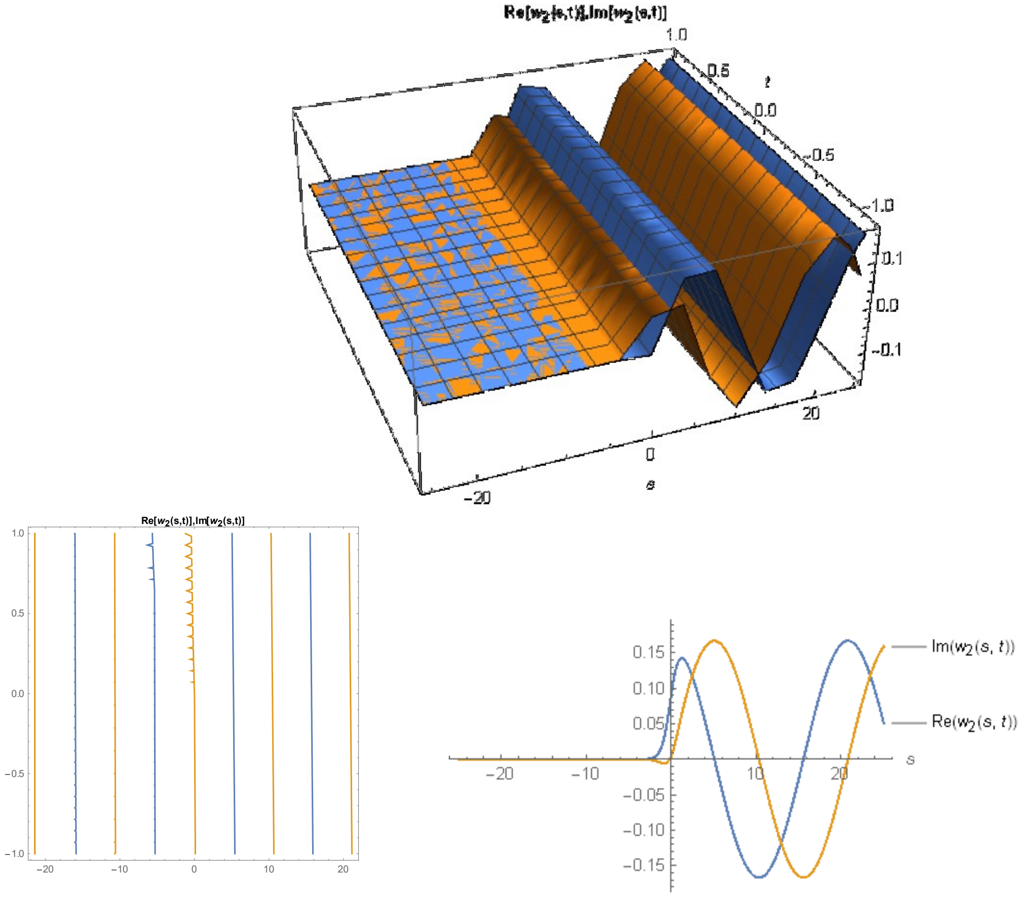

Figure 2. 3D, contour plots of solution (22) for $k_1 = 2, k_2 = 0.3, b_1 = -3, a_2 = 0.5, \beta = 2$, $-25 \leq s \leq 25, -1 \leq t \leq 1$ values with $-1 \leq t \leq 1$ range and 2D plot of solution for $t = 0.75$ with these values.

Case 3:

$$a_0 = a_2, a_1 = -2a_2, a_2 = a_2, b_0 = b_0, b_1 = -2b_0, k_1 = -\frac{i\sqrt{\beta}a_2}{2\sqrt{\sigma}b_0}, \quad c_1 = \frac{3\beta a_2^2 + i\sqrt{\beta\sigma}a_2 b_0 k_2}{2b_0^2}, c_2 = \frac{\beta a_2^2}{b_0^2} - \frac{\sigma k_2^2}{2}. \tag{22}$$

By placing Eq. (22) in Eq. (16), we get the dark soliton solutions of Eq. (1)

$$\mathrm{w}_3(s,t) = \frac{a_2}{4b_0} e^{i(sk_2+c_2t)}\left(2+coth\left[\frac{1}{2}\left(\frac{-is\sqrt{\beta}a_2}{2\sqrt{\sigma}b_0}+c_1t\right)\right]+tanh\left[\frac{1}{2}\left(-\frac{is\sqrt{\beta}a_2}{2\sqrt{\sigma}b_0}+c_1t\right)\right]\right) \tag{23}$$

where $c_1 = \frac{3\beta a_2^2+i\sqrt{\beta\sigma}a_2b_0k_2}{2b_0^2}$, $c_2 = \frac{\beta a_2^2}{b_0^2} - \frac{\sigma k_2^2}{2}$

2D and 3D graphs of real and imaginary parts of the solution (23) are demonstrated with contour simulation in Figure 3.

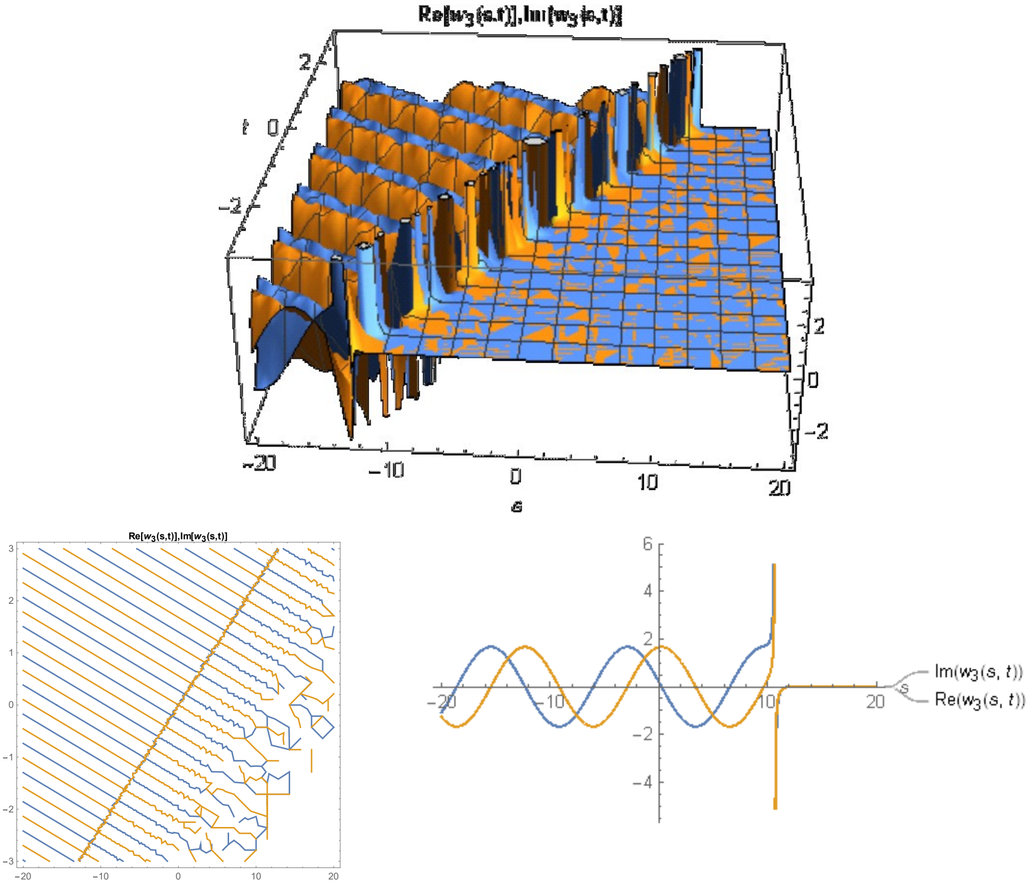

Figure 3. 3D, contour plots of solution (23) for $k_2 = 0.5, b_0 = 3, a_2 = 5, \sigma = -0.4, \beta = 2,$ $-20 \leq s \leq 20, -3 \leq t \leq 3$ values with $-3 \leq t \leq 3$ range and 2D plot of solution for $t = 2.5$ with these values.

Case 4:

$$a_0 = a_0, a_1 = -2a_0, a_2 = 2a_0, b_0 = b_0, b_1 = -2b_0, k_1 = -\frac{i\sqrt{\beta}a_0}{\sqrt{\sigma}b_0},$$

$$c_1 = -\frac{i\sqrt{\beta\sigma}a_0k_2}{b_0}, c_2 = \frac{\beta a_0^2}{b_0^2} - \frac{\sigma k_2^2}{2} \tag{24}$$

By placing Eq. (24) in Eq. (16), we get the trigonometric function solution of Eq. (1)

$$w_4(s,t) = \frac{ia_0}{b_0} e^{i\left(sk_2 + \left(\frac{\beta a_0^2}{b_0^2} - \frac{\sigma k_2^2}{2}\right)t\right)} cot\left[\frac{s\sqrt{\beta}a_0}{\sqrt{\sigma}b_0} - \frac{\sqrt{\beta\sigma}a_0k_2t}{b_0}\right] \tag{25}$$

2D and 3D graphs of real and imaginary parts of the solution (25) are demonstrated with contour simulation in Figure 4.

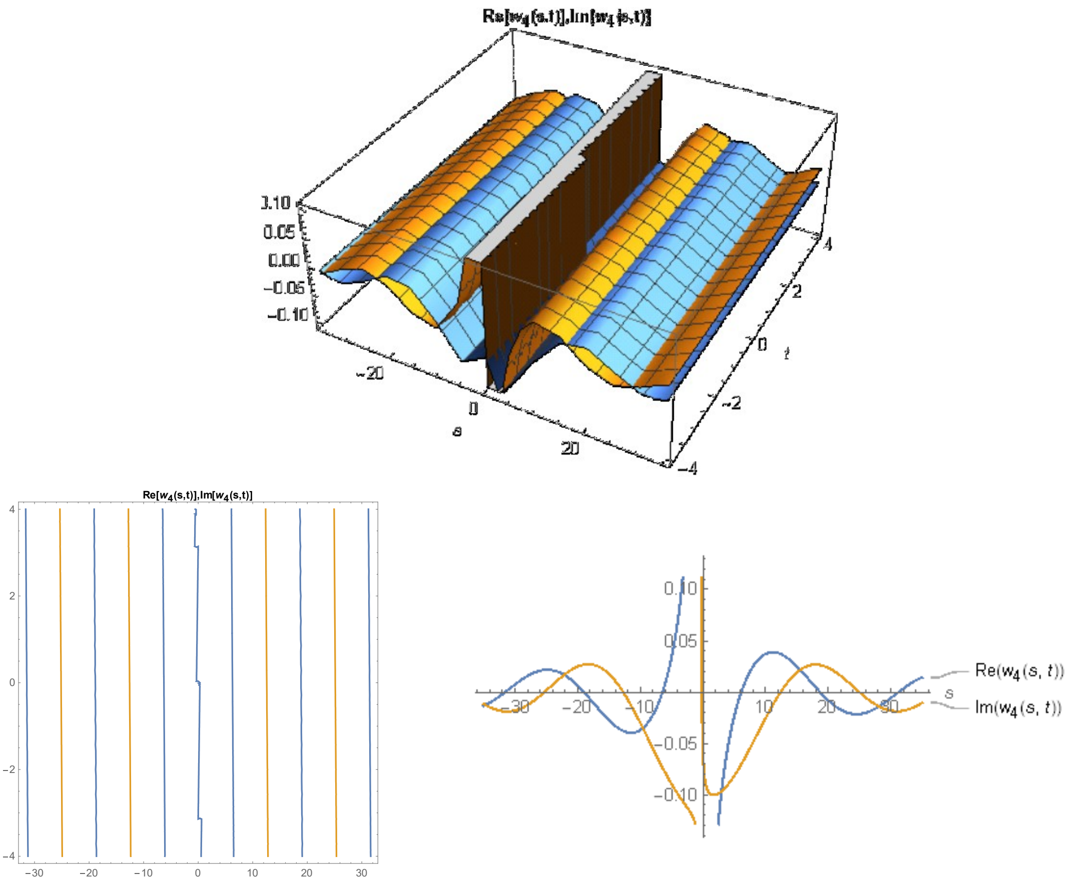

Figure 4. 3D, contour plots of solution (25) for $k_2 = 0.25, b_0 = -0.7, a_0 = 0.01, \sigma = -0.4, \beta = 2,$ $-35 \le s \le 35, -4 \le t \le 4$ values with $-4 \le t \le 4$ range and 2D plot of solution for $t = 3$ with these values

Case 5:

$$a_0 = 0, a_1 = 0, a_2 = -\frac{i\sqrt{\sigma}b_1k_1}{\sqrt{\beta}}, b_0 = -\frac{b_1}{2}, c_1 = \sigma k_1(6k_1 - k_2), c_2 = -\frac{1}{2}\sigma(8k_1^2 + k_2^2) \tag{26}$$

By placing Eq. (26) in Eq. (16), we get the dark soliton solution of Eq. (1)

$$w_5(s,t) = \frac{i\sqrt{\sigma}k_1}{\sqrt{\beta}} e^{i\left(sk_2 - \frac{1}{2}\sigma(8k_1^2 + k_2^2)t\right)}(-1 + coth[sk_1 + \sigma k_1(6k_1 - k_2)t]) \tag{27}$$

2D and 3D graphs of real and imaginary parts of the solution (27) are demonstrated with contour simulation in Figure 5.

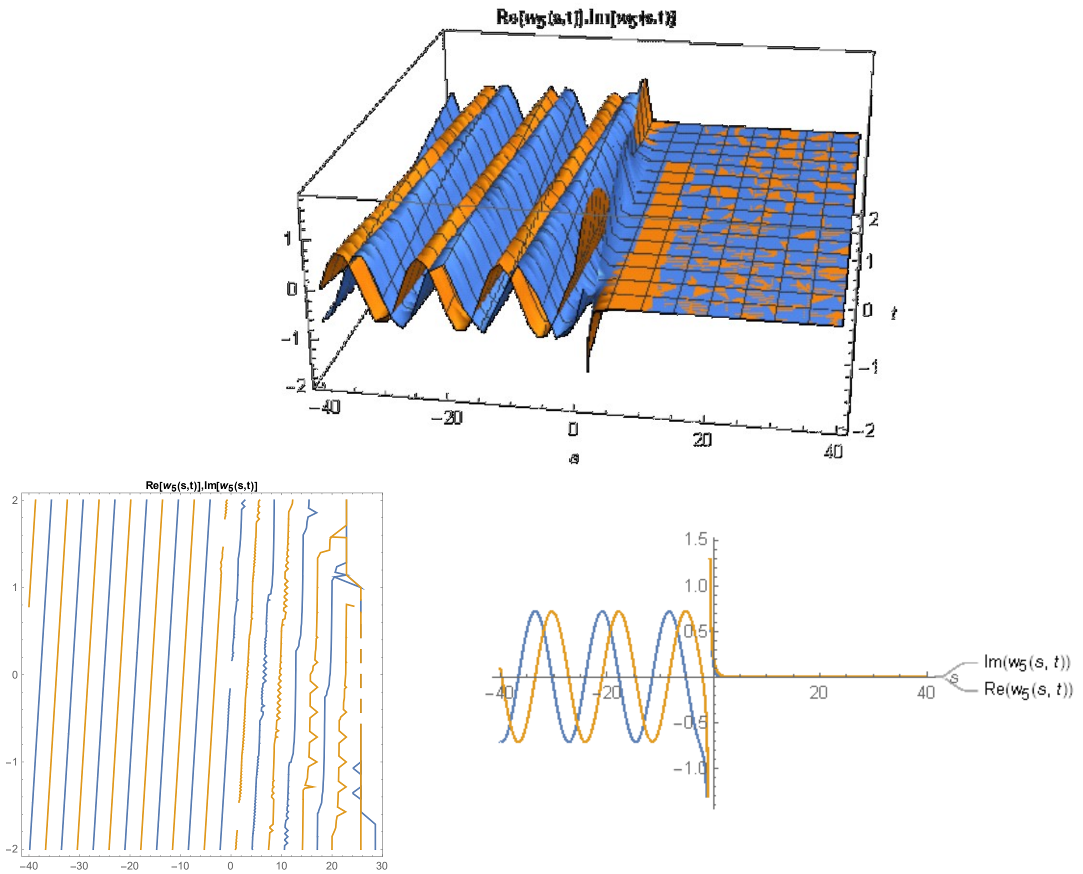

Figure 5. 3D, contour plots of solution (27) for $k_1 = 0.8, k_2 = 0.5, \sigma = 0.2, \beta = 1,$ $-40 \le s \le 40, -2 \le t \le 2$ values with $-2 \le t \le 2$ range and 2D plot of solution for , $t = 1$ with these values

Case 6:

$$a_1 = -a_0 - \frac{i\sqrt{\sigma}b_1k_1}{\sqrt{\beta}}, a_2 = \frac{i\sqrt{\sigma}b_1k_1}{\sqrt{\beta}}, b_0 = \frac{i\sqrt{\beta}a_0}{\sqrt{\sigma}k_1}, \quad c_1 = -\sigma k_1(3k_1 + k_2), c_2 = -\frac{1}{2}\sigma(2k_1^2 + k_2^2) \tag{28}$$

By placing Eq. (28) in Eq. (16), we get the dark soliton solution of Eq. (1)

$$w_6(s,t) = -\frac{i\sqrt{\sigma}k_1}{2\sqrt{\beta}} e^{i\left(sk_2 - \frac{1}{2}\sigma\left(2k_1^2 + k_2^2\right)t\right)} \left(1 + tanh\left[\frac{1}{2}(sk_1 - \sigma k_1(3k_1 + k_2)t)\right]\right) \tag{29}$$

2D and 3D graphs of real and imaginary parts of the solution (29) are demonstrated with contour simulation in Figure 6.

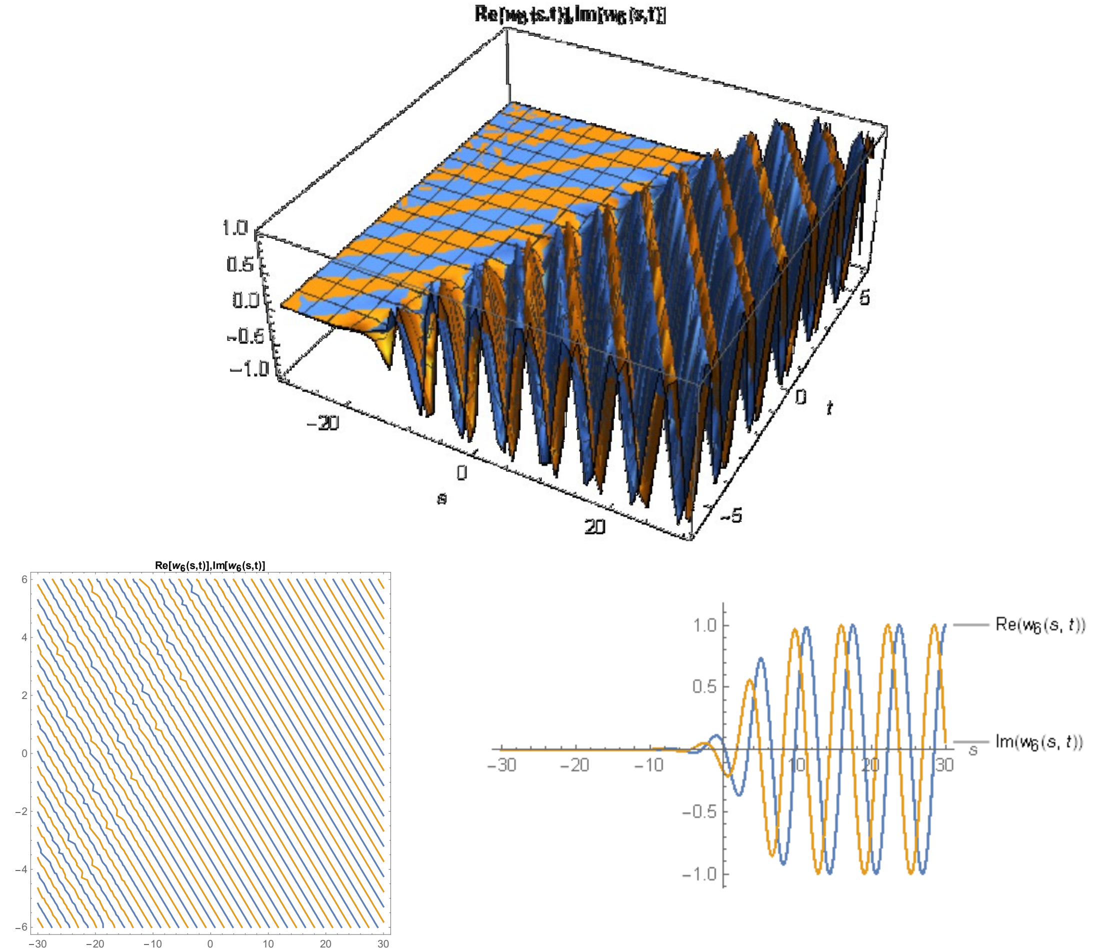

Figure 6. 3D, contour plots of solution (29) for $k_1 = 0.5, k_2 = -1, \sigma = 4, \beta = -1, -30 \le s \le 30,$ $-6 \le t \le 6$ values with $-6 \le t \le 6$ range and 2D plot of solution for $t = 1.5$ with these values

Practice of GKM to the (3+1)-dimensional NLWE in liquid with gas bubbles

To get some soliton solutions of Eq. (2), we take into account the following equality:

$$w(x, y, z, t) = w(\varepsilon), \ \ \varepsilon = x + y + z - ct, \tag{30}$$

where x, y, z are space coordinates and t temporal coordinate.

Putting Eq. (30) into Eq. (2), we find the following

$$\left(-cw' + ww' + \frac{1}{4}w''' - w'\right)_x + \frac{3}{4}(w'' + w'') = 0 \tag{31}$$

If the integration constant of Eq. (31) is taken as zero and by integrating Eq. (31) with respect to ε, we get,

$$(2 - 4c)w + 2w^2 + w'' = 0 \tag{32}$$

If the balance principle is applied between w^2 and w'' in Eq. (32), we obtain

$$n - m + 2 = 2n - 2m \Rightarrow n = m + 2 \tag{33}$$

If we choose $m = 1$ and$n = 3$, we get,

$$w(\varepsilon) = \frac{a_0 + a_1 Z + a_2 Z^2 + a_3 Z^3}{b_0 + b_1 Z}, \tag{34}$$

$$w'(\varepsilon) = (Z^2 - Z)\left[\frac{(a_1 + 2a_2 Z + 3a_3 Z^2)(b_0 + b_1 Z) - b_1(a_0 + a_1 Z + a_2 Z^2 + a_3 Z^3)}{(b_0 + b_1 Z)^2}\right] \tag{35}$$

$$w''(\varepsilon) = \frac{Z^2-Z}{(b_0+b_1Z)^2}(2Z-1)[(a_1+2a_2Z+3a_3Z^2)(b_0+b_1Z)-b_1(a_0+a_1Z+a_2Z^2+a_3Z^3)] + \frac{(Z^2-Z)^2}{(b_0+b_1Z)^3}[(b_0+b_1Z)^2(2a_2+6a_3Z)-2b_1(b_0+b_1Z)(a_1+2a_2Z+3a_3Z^2)+2b_1^2(a_0+a_1Z+a_2Z^2a_3Z^3)] \tag{36}$$

Case 1:

$$a_0 = 0, a_1 = -\frac{b_1}{2}, a_2 = 3b_1, a_3 = -3b_1, b_0 = 0, c = \frac{1}{4} \tag{37}$$

By placing Eq. (37) in Eq. (34), we get the dark soliton solution of Eq. (2)

$$w_1(x,y,z,t) = \frac{1}{4}\left(1-3tanh^2\left[\frac{1}{2}\left(-\frac{t}{4}+x+y+z\right)\right]\right) \tag{38}$$

2D and 3D graphs of the solution (38) is demonstrated with contour simulations in Figure 7.

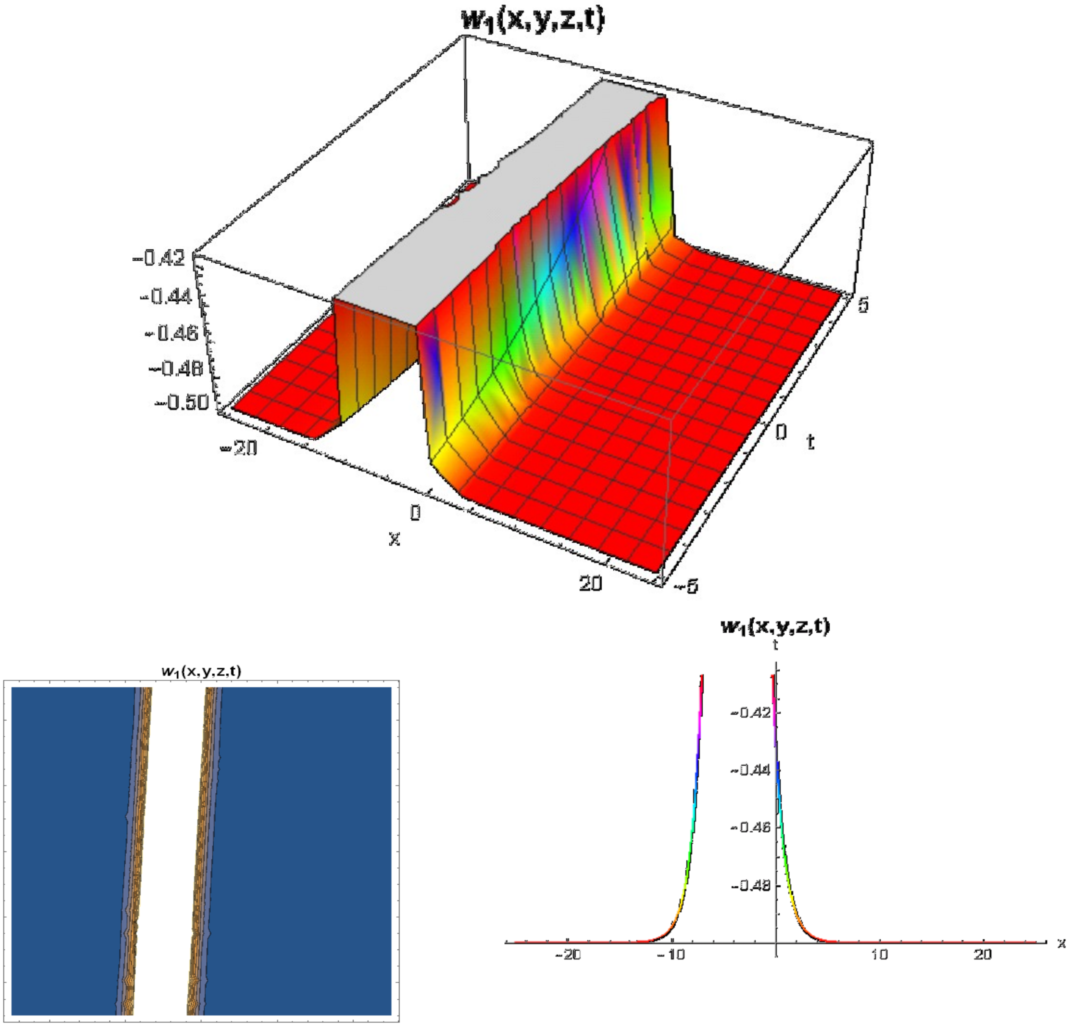

Figure 7. 3D, contour plots of solution (38) for $y = 0.2, z = 4, -5 \le t \le 5, -25 \le x \le 25$, values with $-5 \le t \le 5$ range and 2D plot of solution for $t = 2$ with these values

Case 2:

$$a_0 = 0, a_1 = 3b_0, a_2 = 3(-b_0+b_1), a_3 = -3b_1, c = \frac{3}{4} \tag{39}$$

By placing Eq. (39) in Eq. (34), we get the hyperbolic function solution of Eq. (2)

$$w_2(x,y,z,t) = -\frac{3}{2+2cosh\left[\frac{3t}{4}-x-y-z\right]} \tag{40}$$

2D and 3D graphs of the solution (40) is demonstrated in with contour simulations in Figure 8.

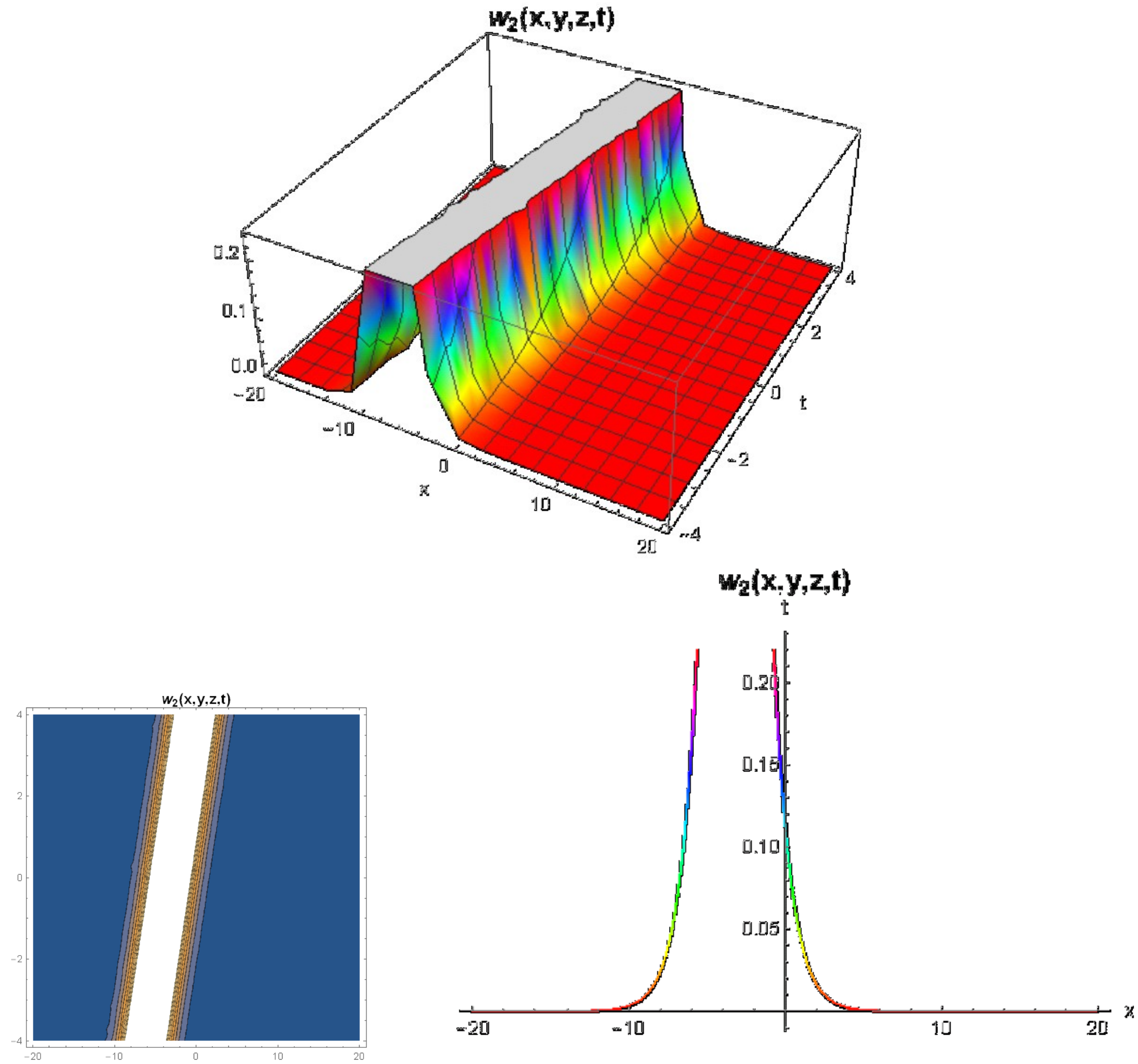

Figure 8. 3D, contour plots of solution (40) for $y = 3, z = 0.2, -20 \leq x \leq 20, -4 \leq t \leq 4$values with $-4 \leq t \leq 4$ range and 2D plot of solution for $t = 0{,}02$ with these values

CONCLUSION

In this study, GKM was applied to acquire solutions of IOPM and (3+1)-dimensional NLWE in liquid with gas bubbles. Thus, solutions of these equations were procured such as dark soliton, trigonometric and hyperbolic solutions. In addition, for some certain values, 3D and 2D graphical representations of these solutions were given with contour simulations with the help of Wolfram Mathematica 12. As far as we know, GKM has not been applied to IOPM and (3+1)-dimensional NLWE in liquid with gas bubbles before. In the light of the results, we deduce that GKM is an effective method in understanding various nonlinear phenomena. In future studies, GKM can be used in research of other NLEEs.

Conflict of Interest

The article authors declare that there is no conflict of interest between them.

Author's Contributions

The authors declare that they have contributed equally to the article.

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