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A STUDY ON MODELING OF CONFLICT AND AGREEMENT WITH GAME THEORY

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ABSTRACT. Game theory is a mathematical approach to analyze the state of competition between players. The foundations of this theory go back about 170 years, and the main development of the subject is based on the last 55 years. In this study, the effect of game theory on political elections and political behaviors has been examined. The Nash equilibrium is investigated by creating a mathematical model of the gains and losses that two political parties obtain in the elections according to the coalition formation status of two political parties by using the Prisoners' Dilemma game model in cooperative and non cooperative games.

1. INTRODUCTION

In political science, there are several subjects in which game theory has a significant role. These include voting, political power affairs, diplomacy, negotiation and bargaining behavior, coalition formation among political groups, and political support [1].

The fact that those who participate in political life have different goals causes a rivalry and conflict between units and groups. As a result of conflicts and agreements in domestic and foreign politics, short-term or permanent political coalitions may occur [2]. Political coalitions are a common phenomenon in contemporary democratic systems [3].

Game theory is based on decision making for the most appropriate strategy selection, which can provide the best result in the face of problems. Considering the high importance of the decisions taken in politics, game theory can then be used appropriately on political issues. Through game theory, it is possible to learn how and why decision makers make mistakes in reasoning in political activities, which strategies provide the best results, why coalitions fail or persist, and what kinds of corrections can be made [2].

In this study, the gains of political parties or groups for two cases will be tried to be explained using game theory: if they enter the elections alone and if they enter the elections by forming a coalition. When the payoff matrices are examined, it will be revealed that it is necessary to decide on the formation of a political government with the best and most effective strategy.

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2. BASIC DEFINITIONS AND PRELIMINARIES

2.1. Game Theory

People have always been involved in situations where the most appropriate decisions need to be made. The decision taken may or may not affect other decision makers. The best decision may depend on one or more goals of the decision maker. The decision may relate to a static situation or a situation that evolves over time. Therefore, mathematical and algorithmic tools have been developed to model, analyze and solve such decision-making processes. Mathematical programming, multi-objective optimization, optimal control theory, and static and dynamic game theory provide the language and tools to achieve such goals.

With the development of optimization, control and game theory, it has been possible to fully achieve the analysis of many complex situations. The concepts of equilibrium and optimality are of great practical importance in policy and strategy decision-making problems, within the framework of understanding and predicting what will ultimately happen in systems across different application areas, from economics and engineering to military applications [4].

In game theory, a distinction can be made between non-cooperative games and cooperative ones. There are 2-player games and n-player games in each of these two categories.

Non-cooperative games are games in which its players are not allowed to cooperate with each other. Each player must individually decide what action to take and what strategy to follow. The best-known example of a two-player non-cooperative play is the prisoners' dilemma. In political science and other social sciences, prisoners' dilemma games have been applied to all kinds of problems.

The non-cooperative approach to political problems has some limitations. Most importantly, in most decision-making situations, actors actually cooperate to achieve certain outcomes. Decision making is often not just a conflict situation, so actors will try to find support for their views. This idea fits in with the concept of coalition, which is one of the core concepts of the cooperative game theory. Coalition is “an agreement of two or more actors who decide to cooperate in order to maximize their common return” [5].

Collaborative game theory deals with situations where at least two decision makers can increase their profits or reduce their costs by collaborating. For example, consider a situation where one person has the resources to make a particular product, someone else has the technical knowledge to make it, and a third party has the means to market it where he can sell it. While neither alone can make a profit, they can earn by working together [6].

2.2. Prisoners' Dilemma

One of the best-known examples of game theory is the prisoners' dilemma. It was revealed by Merrill Flood and Melvin Dresher between 1948-1950 [7].

The name of the Prisoners' Dilemma comes from the following scenario: Two thieves, Alice and Bob, are caught by the police and interrogated in separate cells with no communication between them. Unfortunately, the police lacks enough admissible evidence to convince the jury during a trial. Knowing this, the accused expect to be released with a sentence of less than one year. The prosecutors plan is to meet with the two prisoners and negotiate a bargain. One of the scenarios that may occur as a result of the bargaining is that both of them remain silent; the other one is that one of the detainees becomes the witness of the other friend's crime.

	Confess	Silent
Confess	(-5,-5)	(0,-10)
Silent	(-10,0)	(-1,-1)

TABLE 1. Prisoners' dilemma

Table 1 shows the returns of the prisoners according to their strategies. The most appropriate strategy for the accused is seen as keeping silent. However, if one of the prisoners confesses to the crime, the silent party will be sentenced to 10 years. Likewise, if both parties confess to the crime, each prisoner will be sentenced to 5 years each. These scenarios can happen because neither prisoner knows the other's strategy.

If the confession of prisoner 1 is held constant, the best strategy that prisoner 2 can employ would be to confess to the crime. Because if he confesses, he will be sentenced to 5 years, and if he does not, he will be sentenced to 10 years. With the silence of prisoner 2 held constant, prisoner 1's best choice would still be to confess. Because the 1st prisoner would prefer being released to a 1 year sentence. In this situation, confessing, regardless of what prisoner 2 does, is the best strategy for prisoner 1. The same is true for the prisoner 2. The result (confession, confession) that rational players reach while thinking about each other's behavior in the same rooms really gives the Nash balance of the game, because neither prisoner 1 nor prisoner 2 want to change their own confession strategy in the face of the other's confession strategy. However, both prefer to be sentenced to one year rather than five years each. Despite these preferences, they cannot obtain the result of cooperation (Red, Red) because they are rational and rationality is general knowledge. The word dilemma in the name of the game is derived from this [8].

If there was cooperation between the players, both players would choose the option to refuse, and they would choose the option to maximize the optimum benefit. However, it is not possible to cooperate in games played one time. Because mutual trust of the players cannot be fully established and if one player does not fulfill his obligation, the other player will not be sanctioned.

2.3. Nash Equilibrium Theorem

Each player wants to choose the strategy that will bring the highest payoff for himself. However, considering the strategies of other players, the same strategy may not always bring the best results [7]. Here, the Nash Equilibrium means that the most suitable combination of the player.

The main element in the Nash equilibrium is the existence of the equilibrium point [9]. When deciding the strategies that determine the Nash equilibrium, the value of any of the players that gives the maximum payoff against the strategy of the other players is determined. When we do the same for all players, the result will be the balance of the game [7].

The first studies on the Nash equilibrium were made by Cournot in 1838 and Bertrand in 1883 [9]. On the other hand, if a non-Nash equilibrium situation occurs, at least one player has made a mistake [10].

2.4. Repetitive Games

In static games, as in the prisoners' dilemma, players can influence each other one time. However, although the game is played in a certain period of the economic life, the effects of the decisions taken may last for the same period. The purpose of replay games is to compare short- and long-term wins. A short-term gain can be dispensed with for a strategy that yields more gains in the long run [11].

2.5. The Effect of Game Theory in Establishment of Political Power

Game theory is an effective model in both domestic and foreign policy of the countries. Formation of governments and distribution of ministries are balanced according to the voting power of the parties. Here game theory becomes an effective tool.

Again, in foreign policy, keeping the interests of a country against another or several countries at the highest level is the application of game theory.

The application of game theory on the academic studies of international relations started in 1960 with Thomas Schelling's "The Strategy of Conflict". The adaptation of the cowardly chicken game and the prisoner's dilemma game (which is still used today) to international relations started with Schelling. One of the greatest contributions of the game theory to the politics and international relations is to get rid of the idea based on states, and to use other decision-making actors for the analysis. In classical international relations, states are all considered as different sized structures with similar goals. In game theory, the characters, bureaucracies and decision-making processes of nations have gained importance [12].

Game theory in politics and international relations has arisen from the need to explain the observed issues and the questions asked. Revealing the problems in detail does not automatically create answers to the questions asked. For example, describing all the details of a negotiation does not indicate that the issue is intelligible. On the contrary, it shows how difficult a subject may be. However, this allows us to make explanations that go to the essence of the negotiation, when we treat the negotiation as a game, and the main variables and relations are handled, while other details are omitted.

Any game model models the outlines of a complex issue and tries to find its solution. In this solution, it is obvious that the game has equilibrium or equilibria [13].

Coalitions, which are a form of political unity and governance, refer to the communication and cooperation of more than one political party towards the achievement of common goals. Due to the heterogeneous structure of contemporary societies, in today's democracy, parties cannot reach the necessary majority in legislative assemblies for one party government, and coalitions are needed to form the government. In this framework, the place and importance of coalition in contemporary democracies becomes better understood [3].

Political coalition building games describe the formation and disintegration of nations, as well as the formation of coalition governments, the formation of political parties and other similar events [14].

Riker is among the first political scientists to use the collaborative gaming perspective to examine political coalitions with his 1962 work of "The Theory of Political Coalitions". Many other political theorists followed suit. The coalition has found an interesting application in cabinet formations. Significant work in applying n-person cooperative game theory to coalition cabinets includes Axelrod's "Conflict of Interest", published in 1970; “A Theory of Divergent Goals with Application To Politics”, De Swaan's 1973 “Coalition Theories and Cabinet Formations”, Taylor and Laver's 1973 “Government Coalitions in Western Europe” and Dodd "Coalitions in Parliamentary Government" published in 1976 [5].

The formation of a government also corresponds to the game of forming a political coalition, in which the political parties are players. Similarly, the parties themselves can be seen as the outcome of an underlying game of political coalition-building, this time with individual legislators as the players.

Holland	--------	--------	--------	--------	--------	--------	--------	--------	--------	--------
Luxembourg	--------	--------	--------	--------	--------	--------	--------	--------	--------	--------
Germany	XX---	--------	--------	--------	--------	--------	--------	--------	--------	--------
Belgium	XXXX	--------	--------	--------	--------	--------	--------	--------	--------	--------
Finland	XXXX	X------	--------	--------	--------	--------	--------	--------	--------	--------
France	XXXX	XXX--	--------	--------	--------	--------	--------	--------	--------	--------
Italy	XXXX	XXXX	--------	--------	--------	--------	--------	--------	--------	--------
Austria	XXXX	XXXX	XXXX	XX----	--------	--------	--------	--------	--------	--------
Denmark	XXXX	XXXX	XXXX	XXXX	X------	--------	--------	--------	--------	--------
Ireland	XXXX	XXXX	XXXX	XXXX	XXXX	XX----	--------	--------	--------	--------
Portugal	XXXX	XXXX	XXXX	XXXX	XXXX	XXXX	XXXX	X------	--------	--------
Sweden	XXXX	XXXX	XXXX	XXXX	XXXX	XXXX	XXXX	XX----	--------	--------
Norway	XXXX	XXXX	XXXX	XXXX	XXXX	XXXX	XXXX	XXX--	--------	--------
Greece	XXXX	XXXX	XXXX	XXXX	XXXX	XXXX	XXXX	XXXX	XXXX	XXX--

TABLE 2. Coalition and one-party governments in Europe 1945- 1999
(X): One-Party Governments (--): Coalition Governments [3]

The term of office of the governments established in Table 2 is taken as basis. When we look at the table, it is seen that coalitions are a management style that covers almost half of the democratic government. With the establishment of coalition governments, the effect of game theory is seen on ministries and the sharing of various public authorities. There are two basic approaches put forward to explain the tendency of political parties: to join or not to join coalitions. The first of these is the "office-seeking" approach, led by Anthony Down.

This approach, which is an adaptation of the Game Theory to the political science, assumes that the main purpose of politicians is to be in power and to hold administrative units under all circumstances. Therefore, all political parties seek ways to maximize their political interests and minimize their losses. Therefore, the way to win in this system is to be in power.
Otherwise, it results that all parties entering the coalition are faced with the second best choice, and therefore they will want to share their already diminished profits with as few partners as possible. At the same time, it is accepted that these coalitions can be established between parties that are ideologically closest to each other [3].

3. MAIN RESULTS

In this section, we consider two political parties, A and B. We aim to explain the conflict between these parties and define the situations for which we will model. Next, we will model these situations based on game theory. We will first build our model on the basis of the prisoner's dilemma in the first case and solve it. We will then expand the game for the second case, taking advantage of the prisoner's dilemma we created for the first case. In other words, we're going to make the game iterative. Finally, we will obtain the solution of the iterative game we created.

Let's assume that the political moves, interventions and similar situations of other parties or groups are ineffective in the situation that occurs between two political parties throughout the study.

Case I: The electoral base of political parties A and B is very close to each other. It is obvious that the votes will be divided when political parties A and B go to the elections alone, thinking only of their own interests. The call for a coalition or alliance of any of the parties receives positive reactions from the grassroots. The rest of the votes is distributed to other political parties in certain proportions.

Case II: The current government has been re-elected, and political parties A and B re-joined the election. In this case, let's examine what kind of attitude political parties A and B should display. Following, we model the first case with the help of game theory.

T: The situation of going to the election alone
K: State of coalition building

The row player will be the political party A, and the column player will be the political party B.
Case I:

$$G^1 = \begin{matrix} T \\ K \end{matrix} \begin{bmatrix} T & K \\ (-2,-2) & (-3,1) \\ (1,-3) & (2,2) \end{bmatrix} \quad (1)$$

The values in the payoff matrix express the percentage change in the votes of the political parties. For example, if both parties go to the elections alone and state that they are not open to the idea of coalition, this will cause a reaction from the grassroots and both parties will lose 2% of the votes.

We will find the solution of the first case with the Nash equilibrium point. To find the balance points, we will determine the largest element of the 1st component of each column element, and the largest element of the 2nd component of each row element. The marked sequential pair will be the balance point of the game.

$$G_1 = \begin{matrix} T \\ K \end{matrix} \begin{bmatrix} T & K \\ (-2,-2) & (-3,\mathbf{1}) \\ (\mathbf{1},-3) & (\mathbf{2},\mathbf{2}) \end{bmatrix} \quad (2)$$

It can be seen from the above marking that the game has a Nash equilibrium. This strategy is to form a coalition of two political parties.
Case II: Considering that there are some disagreements in the coalition in the future and that elections are held again, as it can be seen in the game tree of Figure 1. As the number of repetitions increases in the game, more strategies are obtained for the players.

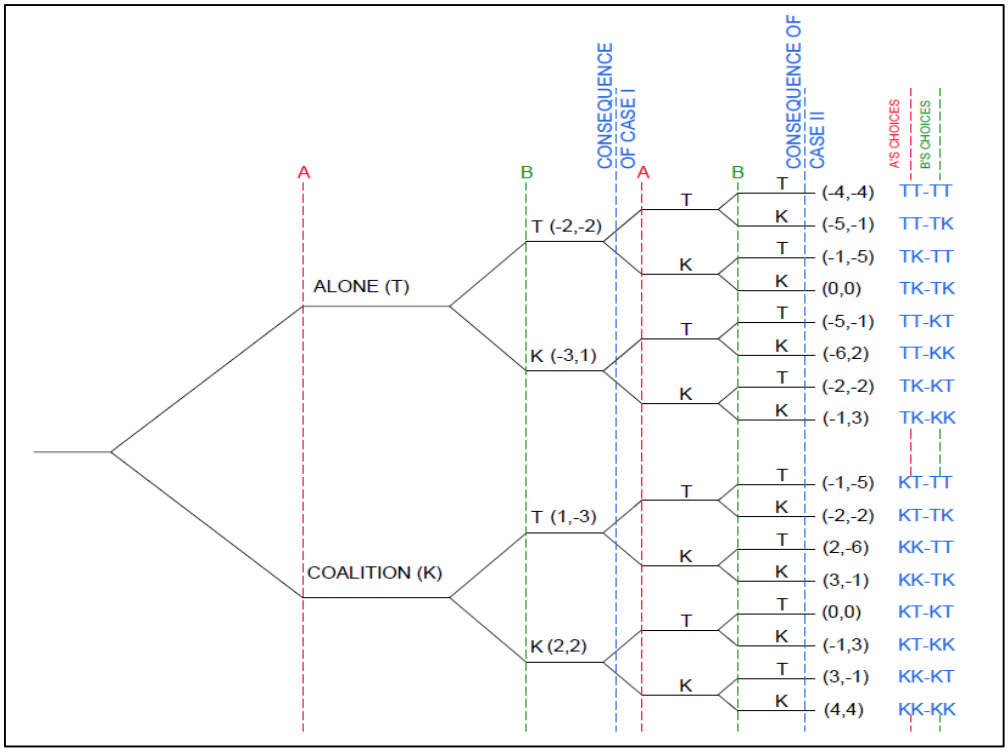

FIGURE 1. Demonstrating Case II with game tree

	TT	TK	KT	KK
TT	(-4,-4)	(-5,-1)	(-5,-1)	(-6,2)
TK	(-1,-5)	(0,0)	(-2,-2)	(-1,3)
KT	(-1,-5)	(-2,-2)	(0,0)	(-1,3)
KK	(2,-6)	(3,-1)	(3,-1)	(4,4)

TABLE 3. Strategies in the game tree

In Table 3, the expressions TT, TK, KT, KK in the row are the strategies chosen by party A in the 1st and 2nd elections. Similarly, the expressions TT, TK, KT, KK in the columns are the strategies of party B. For example, the expression TTKT (-5,1) in the 1st row, 3rd column element of the matrix means that party A wants to enter alone in both elections, while party B is open to coalition in the 1st election and wishes to enter alone in the 2nd election. The first two moves of the TTKT expression are for party A, and the other two moves are for party B.

$$G_2 = \begin{bmatrix} (-4,-4) & (-5,-1) & (-5,-1) & (-6,2) \\ (-1,-5) & (0,0) & (-2,-2) & (-1,3) \\ (-1,-5) & (-2,-2) & (0,0) & (-1,3) \\ (2,-6) & (3,-1) & (3,-1) & (4,4) \end{bmatrix} \quad (3)$$

Again, looking at the payoff matrix, if both parties decide to enter the elections on their own, and state that they are not open to the idea of coalition, they will see a reaction from the grassroots

again. Both parties will lose 4% of the vote. When we look at the 2nd row and 1st column element, when parties A and B went to the election alone in the first election and declared that party A was open to the idea of coalition in the second election, there was a 1% gain in votes compared to the first election.

Then, there is the Nash equilibrium point. To find the balance points, let's determine the largest element of the 1st component of each column element and the largest element of the 2nd component of each row element. The marked ordered pairs will be the balance points of the game.

$$G_2 = \begin{bmatrix} (-4,-4) & (-5,-1) & (-5,-1) & (-6,\mathbf{2}) \\ (-1,-5) & (0,0) & (-2,-2) & (-1,\mathbf{3}) \\ (-1,-5) & (-2,-2) & (0,0) & (-1,\mathbf{3}) \\ (\mathbf{2},-6) & (\mathbf{3},-1) & (\mathbf{3},-1) & (\mathbf{4},\mathbf{4}) \end{bmatrix} \tag{4}$$

As a result, we analyzed the participation status of the two political parties in two stages with the Nash equilibrium. Increasing the stages will create more strategies. In both stages, forming a coalition of two parties is seen as the most optimal solution.

4. CONCLUSION

The problem of forming a coalition between two political parties has been handled with the help of the game theory. The moves that the political parties should take to keep their own interests at the highest level are examined with the Nash equilibrium. The gains and losses that they can achieve in case of cooperation and going to the elections alone have been modeled. In this cooperative game, the gains of the political parties according to the decisions they will take in the first election are shown in the yield matrix. It is seen that the balance point, that is, the most appropriate move for the two political parties, is to form a coalition. Afterwards, it has been researched which path would offer the most gains for the political parties if the government held elections again. In this repetitive cooperative game, it is seen that the political parties forming a coalition in both elections is the most lucrative strategy. The equilibrium concept of game theory can contribute to our understanding of political decision-making by predicting predictions that determine optimal strategies. Within the framework of these results, it is clear that cooperative games can be used in future international and political studies.

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