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A STUDY OF THE ACTION OF THE BEAM AND BEAMLESS (FLUSH) FLOOR SLABS OF THE MULTISTOREY BUILDINGS

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ABSTRACT

In the work is offered the method of interfacing, which essence is concluded that a given system is divided into rectangular plates – small parts and columns, and then their interfacing is made with satisfying boundary conditions by the lines of interfacing of the rectangular plates, and columns located in corner points of the rectangular elements of the floor slabs, in accordance with the columns layout and configuration of the plan. The boundary conditions by the lines of interfacing are formulated for four rectangular elements of the floor slabs.

Keywords: Beam and beamless floor slabs, The method of interfacing, Space framed system, Bending moments, Rectangular elements.

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INTRODUCTION

In the last years the beamless floor slabs have got the wide application in the construction of the apartment and public multistorey buildings. This construction is the reinforced concrete space system, in which as rigidity braces are vertical diaphragms. In comparison with the skeleton framed structure solutions of the multistorey buildings, the buildings with beamless floor slabs have got a number of architectural, technological and constructive advantages.

The object of the present work consists of development of the efficient analytical method of floor slabs solving technique adjusted for their combined action with framed system. This method is known as the method of interfacing and calculation is reduced to algebraic equations, derived from boundary conditions by the lines of interfacing of the plates, having got rigidity ribs, analogously to the equations well known as the equations of three or five moments for the continuous beams. The amount of these equations is depends on the lines of interfacing and corner points of the rectangular plates.

Many works are dedicated to the calculation of the beam and beamless floor slabs. At the same time the calculation of the lateral and longitudinal frames of the multistorey buildings is made conditionally and the action of the floor slabs is not taken into account. There are many programs for the numerical calculetion of these constructions by computer, based on the finite-element method.

THEORY

The task solution is reduced to integration of differential equations of plate's bending with observance of boundary conditions by the lines of interfacing of the rectangular parts, formed at their lines of interfacing and corner points. It is assumed that distributed load is acting on each element (fig.1).

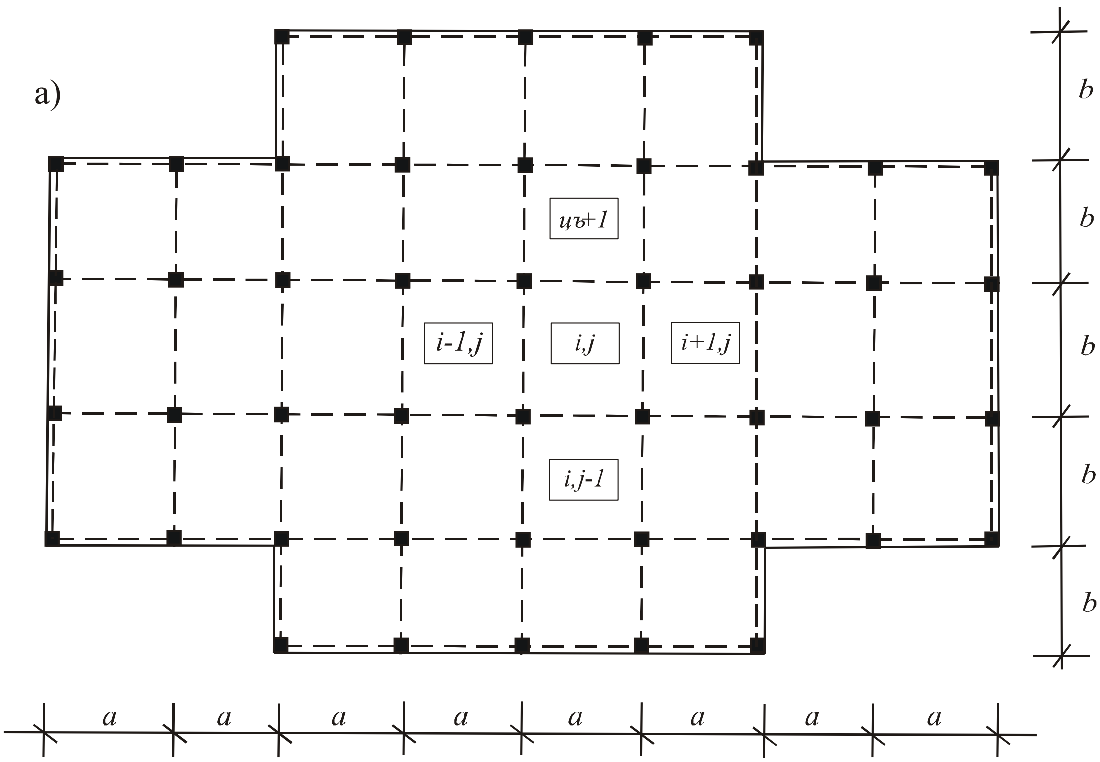

Figure1.

b)

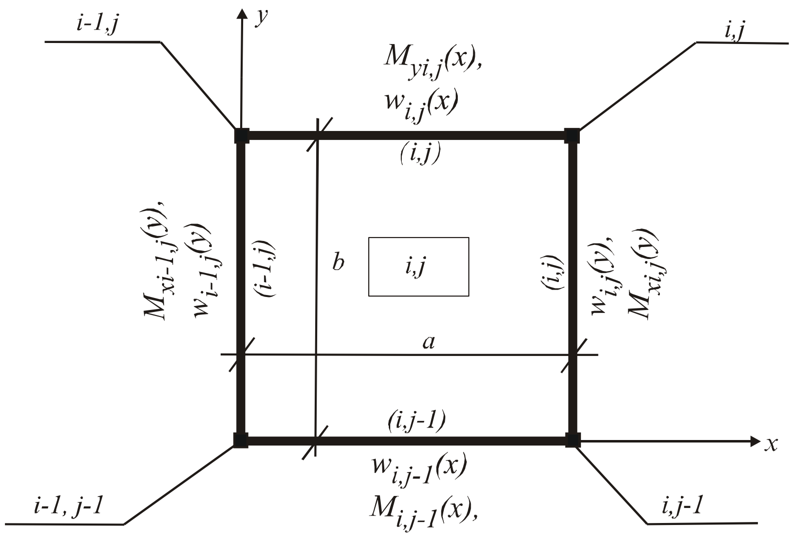

Figure1.

The differential equation of the rectangular plates bending is accepted in the form of:

$$D\Delta^2 w_{i,j} = q_{i,j}\,, \tag{1}$$

here: $D = \dfrac{Eh^3}{12(1-\nu^2)}$ – the cylindrical rigidity of the plate

E, ν – the elastic characteristics of the material of the plate , h – the thickness of the plate

– Laplace operator $\Delta = \dfrac{\partial^2}{\partial x^2} + \dfrac{\partial^2}{\partial y^2}$

$w_{i,j}$ – the function of the deflection of i, j part , $q_{i,j}$ – the active load

The boundary conditions by the lines of interfacing x=a of i,j part and x=0 of i+1, j part of the floor slabs are of the form of:

$$\begin{aligned}
&w_{i,j}(a,y) = w_{i+1,j}(0,y);\\
&\frac{\partial w_{i,j}(a,y)}{\partial x} = \frac{w_{i+1,j}(0,y)}{\partial x};\\
&M_{xi,j}(a,y) - M_{xi+1,j}(0,y) = \Delta M_x(i,j)\\
&Q^*{}_{xi,j}(a,y) - Q^*{}_{xi+1,j}(0,y) = \Delta Q_x(i,j)
\end{aligned} \tag{2}$$

Here $M_{xi,j}(a,y)$ and $Q^*_{xi,j}(a,y)$ – the bending moments and common shear forces, acting on the lines of interfacing of two rectangular parts of the floor slabs respectively, $\Delta M_x(i,j)$ and $\Delta Q_x(i,j)$ – the changes of these magnitudes because of presence of rigidity ribs of the floor slabs.

The conditions are formulated on the common corner points of four rectangular parts, besides ones by the lines of interfacing of two rectangular parts.

$$w_{i,j}(a,b)=w_{i+1,j}(0,b)=w_{i,j+1}(a,0)=w_{i+1,j+1}(0,0);$$

$$\frac{E_b A_{i,j}}{H}w_{i,j}=-2(1-\nu)D\left(\frac{\partial^2 w_{i,j}}{\partial x\partial y}-\frac{\partial^2 w_{i+1,j}}{\partial x\partial y}-\frac{\partial^2 w_{i,j+1}}{\partial x\partial y}+\frac{\partial^2 w_{i+1,j+1}}{\partial x\partial y}\right)+R_{i,j};$$

$$\begin{aligned}M_{xi,j}(a,b)-M_{xi+1,j}(0,b)&=\Delta\overline{M}_{xi,j};\\ M_{yi,j}(a,b)-M_{yi,j+1}(a,0)&=\Delta\overline{M}_{yi,j};\end{aligned} \quad (3)$$

$$\frac{\partial w_{i,j}}{\partial x}=\frac{\partial w_{i+1,j}}{\partial x};\quad \frac{\partial w_{i,j}}{\partial y}=\frac{\partial w_{i,j+1}}{\partial y};$$

In this conditions:

$$\Delta\overline{M}_{xi,j}=K_{xi,j}\frac{\partial w_{i,j}}{\partial x};\quad \Delta\overline{M}_{yi,j}=K_{yi,j}\frac{\partial w_{i,j}}{\partial y}$$

$E_b A_{i,j}$, $K_{xi,j}$ and $K_{yi,j}$ – the rigidity of the columns at compression and bending in two planes.

The boundary conditions by the lines of interfacing in the line of "y" are formulated analogously.

The calculation of the floor slabs of the skeleton framed multistorey buildings adjusted for action of the space framed system is reduced to formulation of the differential equation (1) with observance of the boundary conditions by the lines of interfacing (2) and (3).

The solution of the differential equation (1) is taken in the form of:

$$w_{i,j}=W_{i,j}(x,y)+\overline{W}_{i,j}(x,y)+W^{*}_{i,j}(x,y), \quad (4)$$

where $W_{i,j}(x,y)$ – the main part of the solution of the equations (1) satisfying the boundary conditions for hinged support on the whole contour of the plate and is accepted in the form of double trigonometric series [1-4]:

$$W_{i,j}(x,y)=\sum_m\sum_n B_{mn}(i,j)\sin\lambda_m x\sin\mu_n y \quad (5)$$

$\overline{W}_{i,j}(x,y)$ – the solution, which allows to satisfy the boundary conditions by the lines of interfacing, recorded in the form of single trigonometric series in combinations with the polynomial of fourth degree [1-4]:

$$\overline{W}_{i,j}(x,y)=\sum_m\Big\{\big[(1-y_b)-\nu\Phi(y)\lambda_m^2\big]f_m(i,j-1)+\big[y_b+\nu\overline{\Phi}(y)\lambda_m^2\big]\cdot f_m(i,j)+$$
$$+\frac{1}{D}\big[\Phi(y)F_m(i,j-1)-\overline{\Phi}(y)F_m(i,j)\big]\Big\}\sin\lambda_m x+\sum_n\Big\{\big[(1-x_a)-\nu\Phi(x)\mu_n^2\big]E_n(i-1,j)+ \qquad (6)$$
$$+\big[x_a+\nu\overline{\Phi}(x)\mu_n^2\big]E_n(i,j)+\frac{1}{D}\big[\Phi(x)N_n(i-1,j)-\overline{\Phi}(x)N_n(i,j)\big]\Big\}\sin\mu_n y$$

$W^*_{i,j}(x,y)$ – the solution, which allows to satisfy the boundary conditions by the lines of interfacing, recorded for the common corner points of four rectangular elements and are accepted in the form of polynomial of fourth degree [1-4]:

$$W^*_{i,j}(x,y)=(1-x_a)\big[(1-y_b)w_{i-1,j-1}+y_b w_{i-1,j}\big]+x_a\big[(1-y_b)w_{i,j-1}+y_b w_{i,j}\big]+$$
$$+\frac{1}{D}\Phi(x)\big[(1-y_b)m_x(i-1,j-1)+y_b m_x(i-1,j)\big]-$$
$$-\frac{1}{D}\overline{\Phi}(x)\big[(1-y_b)m_x(i,j-1)+y_b m_x(i,j)\big]+ \qquad (7)$$
$$+\frac{1}{D}\Phi(y)\big[(1-x_a)m_y(i-1,j-1)+x_a m_y(i,j-1)\big]-$$
$$-\frac{1}{D}\overline{\Phi}(y)\big[(1-x_a)m_y(i-1,j)+x_a m_y(i,j)\big]$$

In this solutions are designated:

$$\lambda_m=\frac{m\pi}{a};\quad \mu_n=\frac{n\pi}{b};\quad x_a=\frac{x}{a};\quad y_b=\frac{y}{b};$$
$$\Phi(x)=\frac{a^2}{6}\left(x_a^3-3x_a^2+2x_a\right);\quad \overline{\Phi}(x)=\frac{a^2}{6}\left(x_a^3-x_a\right);$$
$$\Phi(y)=\frac{b^2}{6}\left(y_b^3-3y_b^2+2y_b\right);\quad \overline{\Phi}(y)=\frac{b^2}{6}\left(y_b^3-y_b\right);$$

Observances of the boundary conditions by the lines of interfacing adjusted for the solution (4) have reduced to the system of the algebraic equations. The system of the algebraic equations is formulated as equations of five moments for each line of interfacing of two rectangular parts, amount of which depends on numbers of the lines of interfacing of the rectangular parts of the floor slabs.

The boundary conditions by the lines of interfacing in the common corner points of four rectangular parts is also reduced to the system of the algebraic equations, being of the form of the finite-element method equations and are formulated for each corner point of the rectangular parts of the studied floor slabs.

The method of interfacing is of special interest in such event, when given complex system is divided on smaller rectangular elements. In this case it is possible to neglect the single trigonometric series. Thereby, the unknown variables in the task's solution will only be the deflections and bending moments of the corner points of the rectangular parts. The boundary conditions by the lines of interfacing recorded for corner points are sufficient for estimation.

The developed method of calculation of beam and beamless floor slabs is general and allows to solve specifically many important tasks of the theory of ribbed plates. For this purpose of application of the developed method is given to the well- studied tasks as rectangular plates bending [1].

With reduction of the sizes of the rectangular parts, the method of interfacing goes to the known problem-solving procedure as the finite-element method.

Here is given the solution of beam and beamless floor slabs by the method of interfacing. The flat floor slabs interfaced with the longitudial and lateral framed systems are studied. For account of the action of a girder (frame beam), being included in the monolithic floor slabs, the method of interfacing is used. For this purpose given system is divided into rectangular parts by lines of axes of the longitudial and lateral frames. The girders (frame beams) of the floor slabs resist bending and torsion. These conditions allow to take into account combined action of the flat floor slabs and space framed system.

In purpose of application of the developed calculation algorithm, the square floor slabs formed from four rectangular plates interfaced with the girders of the longitudial and lateral frames of equal rigidity is studied first (fig.1 and 2).

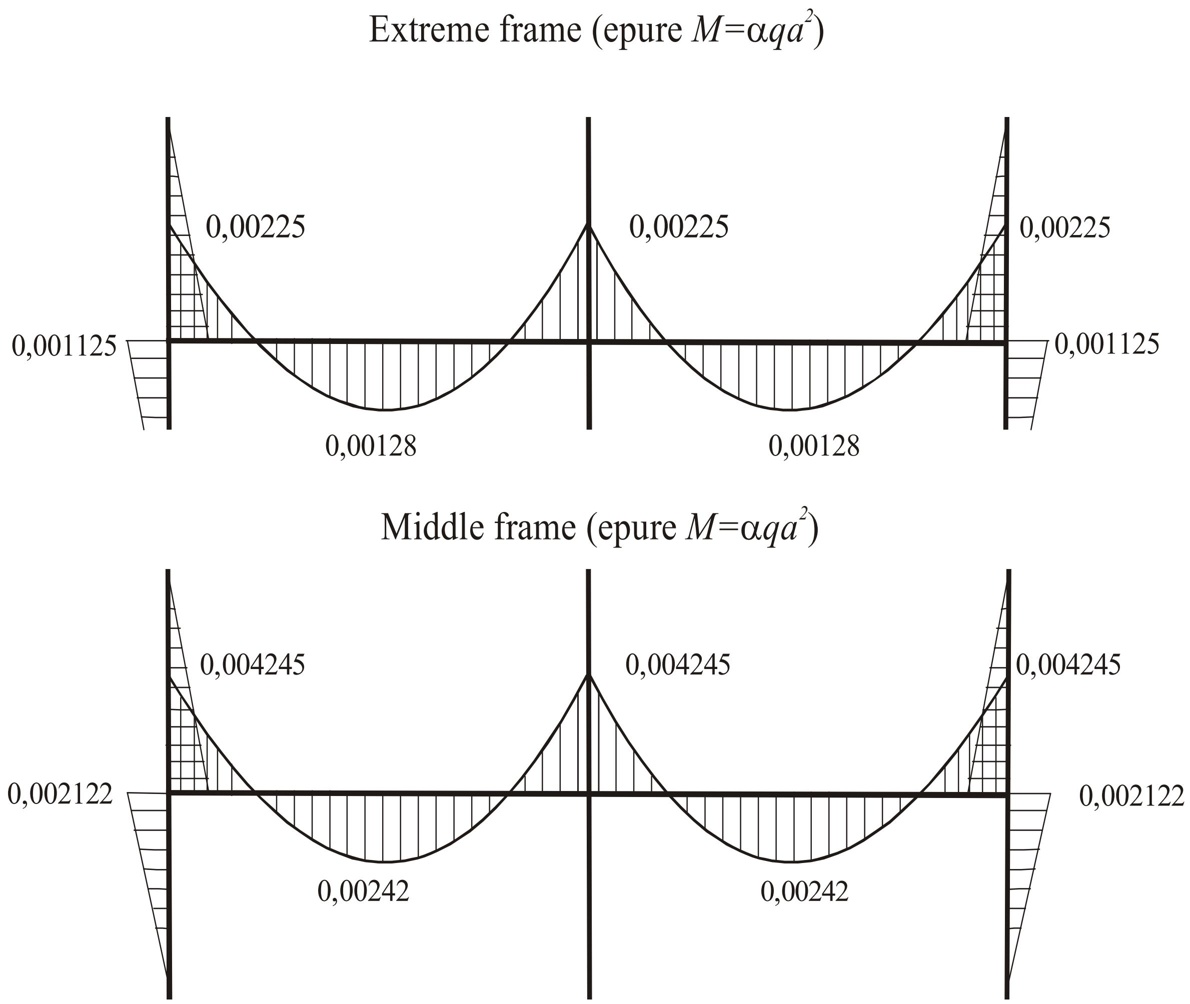

Figure 2. Epure of bending moments of beam floor slabs

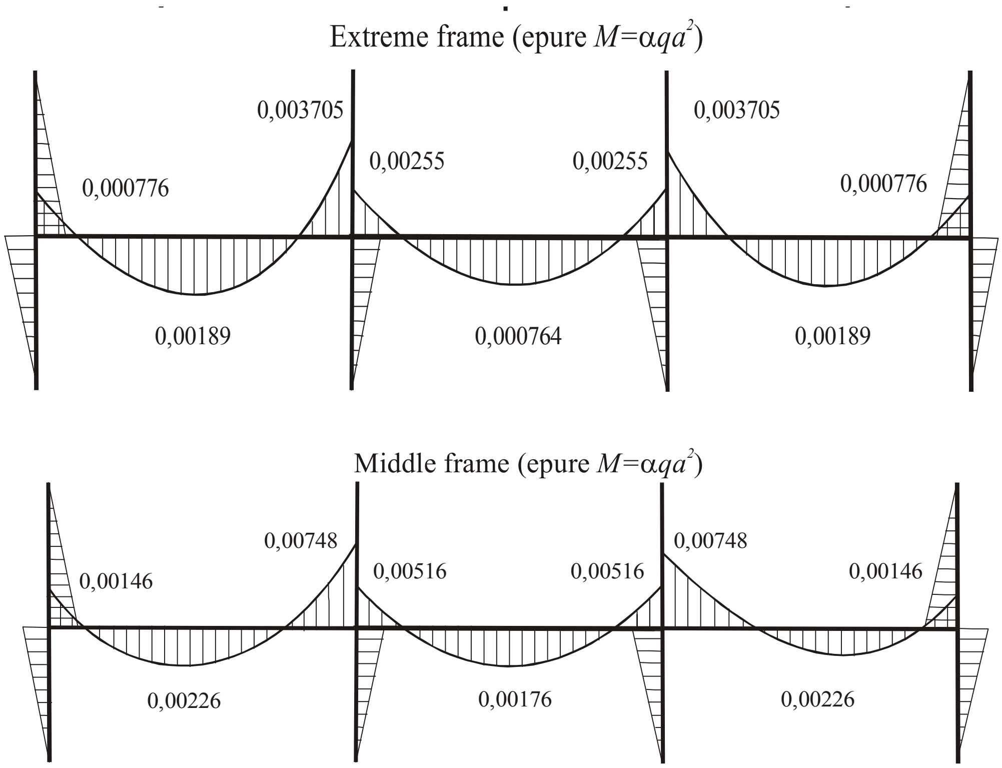

Figure 2. Epure of bending moments of beam floor slabs. (Continue)

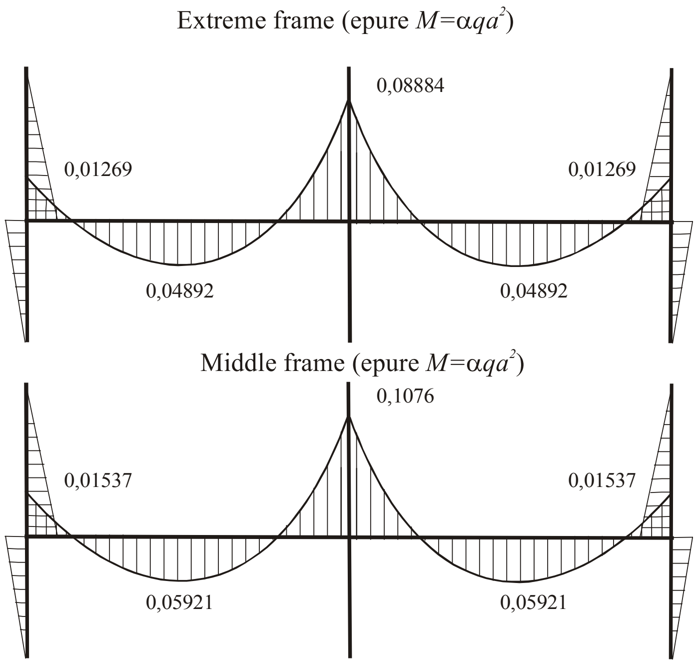

Figure 3. Epure of bending moments of beamless (flush) floor slabs.

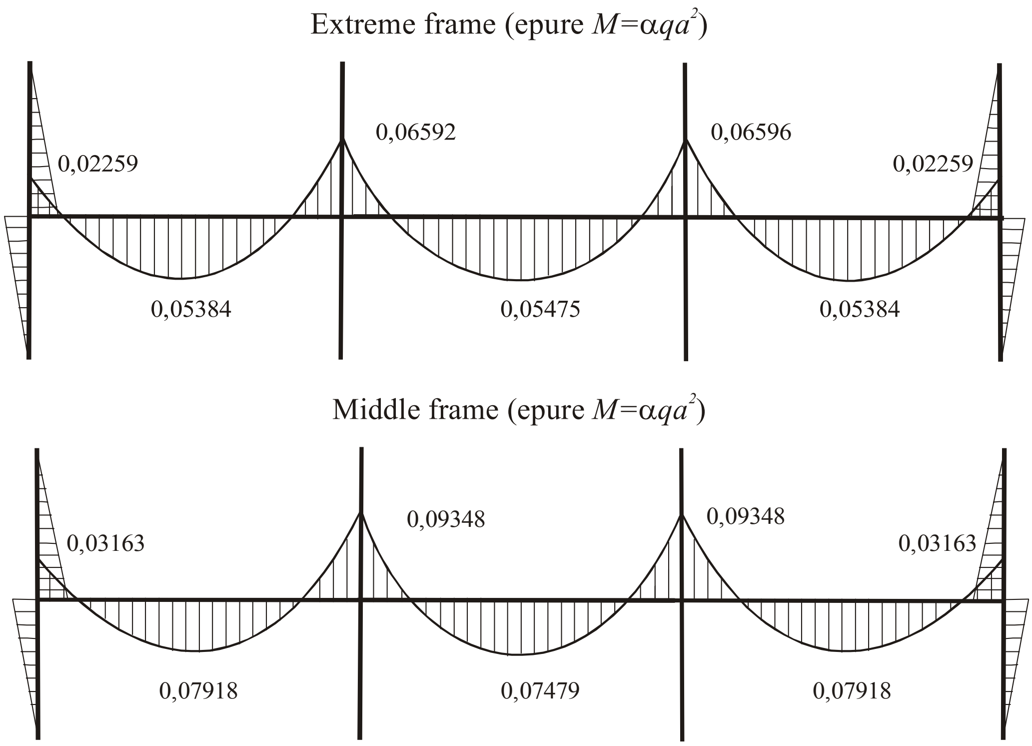

Figure 3. Epure of bending moments of beamless (flush) floor slabs. (Continue)

The second example is studied. The flat floor slabs is supported by lateral and longitudial girders on its contour, and by two longitudial and lateral frames in the middle, as a result the floor slabs are divided on nine equal parts. The rigidity of the girders is equal (δ_c=4,86; δ_b=16,57; K_x=0,85).

For the solution for beamless floor slabs, the given system is divided into rectangular parts by the lines of axes of the columns net. The boundary conditions by the lines of interfacing of the rectangular parts are formed on analogy with the beam floor slabs.

The specific examples are studied:
The floor slabs in the form of plate is connected with a column in corner points.
2. The double-spanned beamless floor slabs is interfacing with columns. The number of pattern rectangular parts, lines and corner points are shown on fig.1 and 3.
3. The triple-spanned beamless floor slabs of the elastic interfacing with columns of the buildings. In this case the floor slabs are free from fastening on the contour except the points of the interfacing with the columns (fig.1 and 3).
4. The double-spanned floor slabs in longitudial and single-spanned floor slabs in lateral direction, supported by the columns of the multistorey skeleton buildings.

CONCLUSIONS

On the grounds of study of the solutions for flat floor slabs, interfacing with the frames of the multistorey buildings one may draw conclusions:

The method of interfacing for calculation the ribbed and flush floor slabs, interfacing with the elements of multistorey skeleton buildings is offered and it is a latest technique of structural mechanics.
The studied multiple examples of calculation of the single, double, and triple space framed systems displays its efficiency.

Many examples with bending moment diagrams construction are presented and the analysis of the influence of the girders rigidity on the mode of deformation of flat floor slabs is given. In the beamless floor slabs the rigidity of the space systems in vertical plane in bending is considerably lower, than in ribbed floor slabs. Therefore it is necessary to take into account this circumstance in the practical calculations.

REFERENCE

1. Ashkan Khoda Bandeh Lou. Design of ribbed plates, supported by the columns. "Works Scientific –research institute of construction and Architecture". Baku, 2002. p.106-112. ISSN: 0208-3663.
2. Ashkan Khoda Bandeh Lou. Design of vertical loading of skeleton frame buildings with beamless floor slabs "Technical Sciences" №2, Baku, 2003. p. 63-68. ISSN: 1683-7622.
3. Seyfullayev Kh.Q. To Solution of equations of theory of plane coverings of the variable thickness and curvature under arbitrary boundary conditions. Applied Mechanics, edition XVI, №10, 1980.
4. Timoshenko S.P., Gudder D. "Theory of elasticity" Moscow, Science, 1975.

NOTATION

a,b – Length and width of rectangular in plan of plates;
h – the thickness of plate;
$w_{i,j}$ – deflection function;
D – is a cylindrical rigidity of the plate;
- Laplace operator;
q – the active load;
v - Puasson coefficient;
m_x, m_y – bending moments of main points of rectangular plates;
K_x – column rigidity in bending and compression;
B_{mn} – coefficients of double trigonometrical rows;
f_m – rib deflection in direction x;
E_n – rib deflection in direction y;
F_m – bending moments in section contours slabs in direction – *x*;
N_n – bending moments in section contours slabs in direction – *y*;
δ_c – relative rib rigidity in torsion;
δ_b – relative rib rigidity in bending;
M_x, M_y *and* $M_{x,y}$ – bending and torsion moments.
$\lambda = \frac{b}{a}$ - lengthening parameter;
$x_a = \frac{x}{a};\ y_b = \frac{y}{b}$ - sizeless co-ordinates.