Erc.Üniv.Fen Bil.Derg., 5, 1-2 (1989), 871-874

ON RELATION BETWEEN CONVERGENCE AND ABSOLUTE SUMMABILITY $|N, p_n|$ OF INFINITE SERIES

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SUMMARY

In this paper, we gave a relation between summability method $|\bar{N}, p_n|$ of that series with a given series Σa_n.

SONSUZ SERİLERİN YAKINSAKLIK VE MUTLAK TOPLANABİLİRLİĞİ ARASINDAKİ İLİŞKİ ÜZERİNE

ÖZET

Bu çalışmada verilen bir Σa_n serisi ile 0 serisinin $|\bar{N}, p_n|$ toplabilme metodu arasındaki ilişkiyi verdik.

I. INTRODUCTION

Let (p_n) be a sequence such that $p_n > 0$, $P_n = \sum_{v=0}^{n} p_v \to \infty$ as $n \to \infty$. The transformation $(\bar{N}, p_n)$ maps a sequence (S_n) into the sequence (U_n) by means of the equation :

$$U_n = P_n^{-1} \sum_{v=0}^{n} p_v S_v \qquad (1)$$

where the sequence (S_n) is the n.th partial sum of a given series Σa_n [1].

A matrix method is said to be regular if it is preserving the limit for convergent sequences [2].

The necessary and sufficient conditions for the regularity of (1) is $P_n \to \infty$ as $n \to \infty$ [2].

A series Σa_n with its partial sum S_n is said to be summable $(\bar{N}, p_n)$ to sum S if

$$U_n = P_n^{-1} \sum_{v=o}^{n} p_v S_v \to S \quad \text{as } n \to \infty \qquad [1].$$

A series Σa_n with its partial sum S_n is said to be absolutely summable $(\bar{N}, p_n)$ or summable $|\bar{N}, p_n|$ if

$$\sum_{n=o}^{\infty} |U_n - U_{n+1}| < \infty \ [1].$$

Given two summability methods A and B, we write $(A) \subseteq (B)$ if each series summable A is summable B [1].

2. WE SHALL NOW PROVE THE FOLLOWING THEOREMS

Theorem 1.Let (a_n) be a sequence of positive numbers. Then the series Σa_n is convergent if and only if the series Σa_n is the summable $|\bar{N}, p_n|$.

Proof : (i) necessity : suppose that the series Σa_n is convergent and let S_n be the n.th partial sum of given series Σa_n.

Then we must show that

$$\sum_{n=1}^{\infty} |U_n - U_{n-1}| < \infty$$

By applying Abel's transformation to the sums in the right side of equality:

$$U_n - U_{n-1} = P_n^{-1} \sum_{v=o}^{n} p_v S_v - P_{n-1}^{-1} \sum_{v=o}^{n-1} p_v S_v$$

we have

$$U_n - U_{n-1} = p_n(P_n\ P_{n-1})^{-1} \sum_{v=1}^{n} P_{v-1}a_v \tag{2}$$

Let

$$\sum_{n=1}^{m} |U_n - U_{n-1}| = \sum_{n=1}^{m} p_n(P_n\ P_{n-1})^{-1} \sum_{v=1}^{n} P_{v-1}a_v = W_m \tag{3}$$

To show that $\sum_{n=1}^{\infty}|U_n - U_{n-1}| < \infty$, it is sufficient to show that the sequence (W_m) is convergent. Let us prove this now. Since

$$p_m(P_mP_{m-1})^{-1} \sum_{v=1}^{m} P_{v-1}a_v > 0$$

for all $m \in \mathbb{N}$,

$$W_m - W_{m-1} = \sum_{n=1}^{m} p_n(P_nP_{n-1})^{-1} \sum_{v=1}^{n} P_{v-1}a_v - \sum_{n=1}^{m-1} p_n(P_nP_{n-1})^{-1} \sum_{v=1}^{n} P_{v-1}a_v$$

$$= p_m(P_mP_{m-1})^{-1} \sum_{v=1}^{m} P_{v-1}a_v$$

is positive for all $m \in \mathbb{N}$. That is, the sequence (W_m) is monotonically Moreover

$$W_m = \sum_{n=1}^{m} p_n(P_nP_{n-1})^{-1} \sum_{v=1}^{n} P_{v-1}a_v$$

$$= \sum_{n=1}^{m} P_{v-1}a_v \sum_{n=v}^{n} (P_{n-1}^{-1} - P_n^{-1})$$

$$= \sum_{v=1}^{m} P_{v-1}a_v(P_{v-1}^{-1} - P_m^{-1})$$

$$= \sum_{v=1}^{m} a_v$$

for all $m \in \mathbb{N}$. Hence the sequence (W_m) is convergent.

(ii) sufficiency : suppose that the series Σa_n is summable $|\bar{N}, p_n|$.

It means that $\sum_{n=1}^{\infty} |U_n - U_{n-1}| < \infty$. By (3), considering the fact that $P_M \to \infty$ as $M \to \infty$, we have

$$\sum_{n=1}^{\infty} p_n (P_n P_{n-1})^{-1} \sum_{v=1}^{n} P_{v-1} a_v$$

$$= \sum_{v=1}^{\infty} P_{v-1} a_v \sum_{n=v}^{\infty} p_n (P_n P_{n-1})^{-1}$$

$$= \sum_{v=1}^{\infty} P_{v-1} a_v \lim_{M \to \infty} \sum_{n=v}^{M} (P_{n-1}^{-1} - P_n^{-1})$$

$$= \sum_{v=1}^{\infty} P_{v-1} a_v \lim_{M \to \infty} (P_{v-1}^{-1} - P_M^{-1})$$

$$= \sum_{v=1}^{\infty} P_{v-1} a_v P_{v-1}^{-1} = \sum_{v=1}^{\infty} a_v. \text{ Thus,}$$

$$\sum_{n=1}^{\infty} |U_n - U_{n-1}| = \sum_{v=1}^{\infty} a_v$$

is obtained which means that the series Σa_n is convergent.

This completes the proof of the theorem.

Now, we want to give another theorem for the series with arbitrary terms:

Theorem 2. Suppose that the serie Σa_n is convergent. Then the series Σa_n is summable $|\bar{N}, p_n|$.

Proof: The proof of this theorem can be made similarly as in theorem 1

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