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MODELLING OF COMPLETE HYDRAULIC SYSTEMS BY USING PLANE WAVE TRANSMISSION LINE THEORY

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SUMMARY

In order to design and build low noise hydraulic systems it is important to be able to examine pressure ripple (fluid borne noise) behaviour by means of a mathematical model. This work describes the application of plane wave transmission line theory to the modelling of hydraulic systems. It was shown earlier [1] that the method may be used to predict accurately the pressure ripple at any point in simple hydraulic systems consisting of a pump, pipe and load valve. This model has been extended to include systems with pipes and valves in parallel and in series. The theoretical predictions which are based on wave propagation theory are compared with experimental results obtained on complete hydraulic systems.

DÜZLEM DALGA TRANSMİSYON HAT TEORİSİ KULLANILARAK KOMPLE HİDROLİK SİSTEMLERİN MODELİNİN KURULMASI

ÖZET

Gürültü seviyesi az olan hidrolik sistemler dizayn ve inşa edebilmek için, bir matematik model yardımıyla sistemin basınç dalgalanmalarının davranışını araştırmak önemlidir. Bu çalışma, düzgün dalga transmisyon hattı teorisinin hidrolik sistemlerin modelinin kurulmasına uygulanmasını açıklamaktadır. Bu metodun, bir pompa, boru ve yükleme valfinden meydana gelen basit hidrolik sistemlerde herhangi bir noktada meydana gelen basınç dalgalanmalarının tam olarak hesaplanmasında kullanılabileceği daha önce gösterilmiştir [1] . Bu model, sistemlere borular ve yükleme valfleri paralel ve seri olarak ilave edilerek geliştirildi. Dalga yayılım teorisine dayanan teorik hesaplamalar komple hidrolik sistemlerden elde edilen deneysel sonuçlarla karşılaştırıldı.

1-INTRODUCTION

The decisions made by the system designer and installer ultimately decide whether a particular hydraulic system will be a low or high level generator of airborne noise. Although all system designs should avoid the latter condition, for certain applications, including power steering and some mobile plant and ship systems, it is imperative that the levels should be at a minimum. Therefore, it is essential that as much relevant information as possiple be made available to assist in the design and installation of low noise systems.

At both the initial design stage and during noise reduction programmes on existing systems, the designer should study the generation and transmission of pressure ripple, commonly known as fluid borne noise. These pressure ripples, which are generated primarily by the unsteady flow produced by positive displacement pumps and motors, are transmitted through the system at levels depending upon the characteristics of the pump or motor, the components and the piping configurations used in the system. Under certain conditions the pressure ripple can induce structural vibration leading to the emission of unwanted airborne noise. Both airborne noise and degradation of performance can accompany the fluctuations when they lead to cavitation conditions in suction lines or provide an excitation to system components, which in the case of valves can create dynamic instability.

It was shown in previous publications[1,2] that the generation of pressure fluctuations in hydraulic systems could be modelled using the plane wave equations of electrical transmission line theory. However, at that time the approach outlined was limited to the analysis of simple hydraulic systems consisting of a positive displacement pump, a pipe line and a load valve.

In this study techniques are described which allow the extension of the analysis to cover more realistic pipe systems with branch and valves in series.

2- BRANCH AND SERIES SYSTEMS

For the theoretical analysis outlined above to be of any value it must be extended to cover more complex system configurations. At first sight this would appear to be a daunting task owing to the wide range of possiple system layouts. When examined closely, however, even the most complex of hydraulic systems may be considered to be constructed from just two basic elements. These are pipe lines and components forming branch systems and pipe lines and components forming series systems. Hence, it is only necessary at this stage to show the analysis to be a suitable representation of wave propagation in branch and series systems. The analysis of both systems is based upon the plane wave transmission line theory [2] .

3- ANALYSIS OF BRANCH SYSTEMS

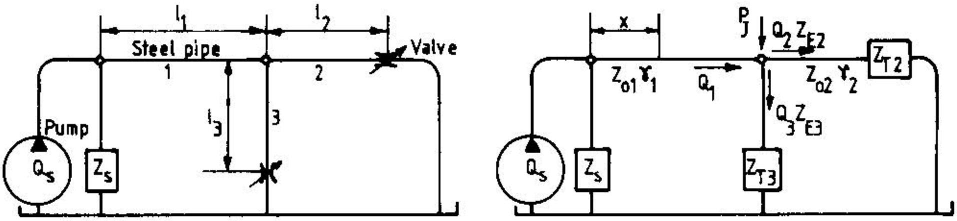
A hydraulic system with a single branch is diagrammatically shown in Fig. 1a and its corresponding impedance representation in Fig.1b. The pressure ripple at the junction is common to the three lines, i.e. it is the entry pressure for line 2 and 3 and the termination pressure for line 1. If the termination impedances and pipe line parameters for line 2 and 3 are known, then the impedance at the entry of both lines may be calculated. For example, the entry impedance,which is the ratio of entry pressure ripple to entry flow ripple, for line 2 is

$$Z_{E2} = Z_{o2} \frac{(1 + \rho_{T2} e^{-2\gamma_2 \ell_2})}{(1 - \rho_{T2} e^{-2\gamma_2 \ell_2})} \tag{1}$$

where

$$\rho_{T2} = \frac{Z_{T2} - Z_{o2}}{Z_{T2} + Z_{o2}}$$

Using the pressure at the junction and the entry impedance allows the flow ripple at the entry to lines 2 and 3 to be determined using the impedance relationships:



a) Hydraulic circuit

b) Impedance representation

Fig.1- Analysis of branch systems

$$Q_2 = \frac{P_J}{Z_{E2}} \quad \text{and} \quad Q_3 = \frac{P_J}{Z_{E3}} \tag{2}$$

For flow continuity, the algebraic sum of the flow fluctuations occurring at the junctions must be zero (hydraulic equivalent of Kirchoff's 1st law in electrical engineering). Hence,

$$Q_1 = Q_2 + Q_3 \tag{3}$$

Now, Q_1 the flow into the junction which is also the flow at the end of line 1, will be dependent upon the pressure and the impedance at the junction, such that

$$Q_1 = \frac{P_J}{Z_J} \tag{4}$$

Substituting equation (2) and (4) into (3) and re-arringing enables the junction impedance to be determined from,

$$Z_J = \frac{Z_{E2}\ Z_{E3}}{Z_{E2} + Z_{E3}} \tag{5}$$

Hence, it is possible to replace lines 2 and 3 by an effective impedance, Z_J, positioned at the junction, as Fig.2. The system has now been reduced to a simple form consisting of a pump connected to a pipe line, line 1, terminated by an inpadence, Z_j.From (2), the pressure at any position along line 1 of this system may be determined from,

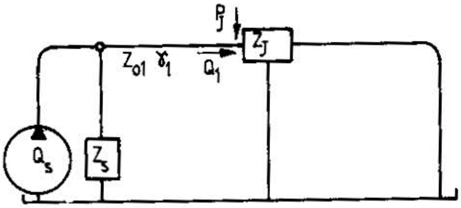


Fig.2- Equivalent circuit of fig.1

$$P_{x1} = \frac{Q_s Z_s Z_{o1}}{(Z_s + Z_{o1})} \quad \frac{(e^{-\gamma_1 x} + \rho_T e^{-\gamma_1 (2\gamma_1 - x)})}{(1 - \rho_s \rho_T e^{-2\gamma_1 \ell_1})} \tag{6}$$

where $\rho_s = \dfrac{Z_s - Z_{o1}}{Z_s + Z_{o1}}$ and $\rho_T = \dfrac{Z_J - Z_{o1}}{Z_J + Z_{o1}}$

Pressure in lines 2 and 3 may be obtained using the junction pressure, P_J, which is the pressure at $x = \ell_1$ for line 1, equation (6). e.g. for line 2,

$$P_{x2} = P_J \frac{(e^{-\gamma_2 x} + \rho_T e^{-\gamma_2 (2\ell_2 - x)})}{(1 + \rho_T e^{-2\gamma_2 \ell_2})} \tag{7}$$

4- ANALYSIS OF SERIES SYSTEMS

The hydraulic and impedance representations of the series systems are shown in Fig.3. Restrictor valves were used to terminate line 2 and to link lines 1 and 2, Fig.3. The valves are assumed to be lumped elements, with no wave propagation across the valve seat. Following the approach adopted in the branch analysis, it is convenient to represent line 2 and its valve by an entry impedance Z_{E2}. For flow continuity, the flow variations on either side of valve 1 must be the same. Hence, from Fig.3b,

$$Q_1 = Q_2 \tag{8}$$

For line 2, using the impedance relationship,

$$P_2 = Z_{E2} Q_2 \tag{9}$$

For valve 1,

$$(P_1 - P_2) = Z_{T1} Q_1 \tag{10}$$

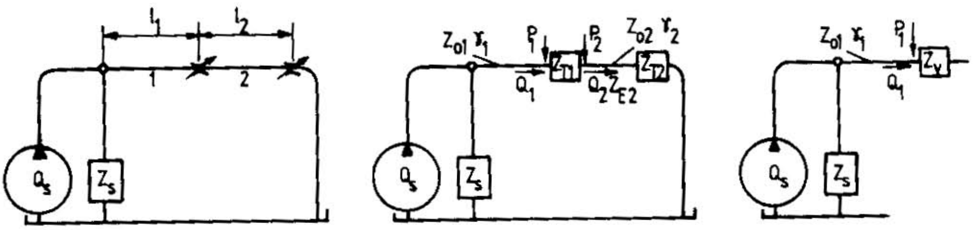
Substituting equations (8) and (9) into (10) and re-arranging gives,

$$\frac{P_1}{Q_1} = Z_{T1} + Z_{E2} \tag{11}$$

But $\frac{P_1}{Q_1} = Z_v$, the effective impedance due to valve 1 and line 2

$$Z_v = Z_{T1} + Z_{E2} \tag{12}$$

Hence, the series system may be represented as simple line 1 terminated by an impedance Z_v (Fig.3c). The pressure at any position along line 1 may be determined from equation (6) with



a) Hydraulic circuit b) Impedance representation c) Equivalent circ

Fig.3- Analysis of series systems

$$\rho_T = \frac{Z_v - Z_{01}}{Z_v + Z_{01}}$$

The pressures in line 2 may be found using the pressure P_1 at $x=\ell_1$ for line 1 and the following equation.

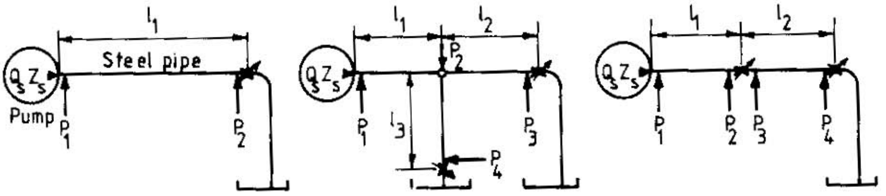
$$P_{x2} = \frac{P_1 \, Z_{E2}}{Z_v} \; \frac{(e^{-\gamma_2 x} + \rho_{T2} e^{-\gamma_2 (2\ell_2 - x)})}{(1 + \rho_{T2} e^{-2\gamma_2 \ell_2})} \tag{13}$$

5- DERIVATION OF TERMINATION AND PUMP PARAMETERS

To enable the prediction of pressure levels in the branch and series systems to be performed using the above equations, the impedance spectra of the terminations and the source impedance and source flow spectra for the pump must be known, vis.Z_T, Z_S and Q_S. Techniques were described in [1] which allowed Q_S, Z_S and Z_T to be derived with a high degree of accuracy using an analysis based upon equation (6) together with the experimental data obtained from extending pipe length tests performed on a simple system. The same approach was used to determine the system unknowns for this investigation.

6- EXPERIMENTAL INVESTIGATIONS

In order to justify the proposed theoretical analysis, a considerable number of tests were undertaken to determine experimentally the pressure ripple levels generated in a simple system and in branch and series systems. The three experimental rigs used in this investigation are shown diagrammatically in Fig.4. Each rig utilized the same axial piston pump, 14.9 mm bore rigid tubing and identical restrictor load valves. Dynamic pressures were measured using piezo-electric transducers at the positions shown. The test metod was based upon the extending pipe length approach. Instrumentation and data acquisition techniques closely followed those outlined in [3] .



a) Simple system b) Branch system c) Series system

Fig.4- Experimental rigs used during system investigation

7- DISCUSSION OF RESULTS

The derived equations were used together with the parameters found from the simple system tests, to predict the generated levels of pressure ripple in the branch and series circuits of Fig. 4a and 4b.

A comparison is made between calculated and measured pressure spectra at the four transducer positions in the branch system in Fig.5. The agreement shown is typical of the whole series of branch line tests parformed with the provision that the junction was rigidly clamped to a sufficiently large mass. Without clomping large discrepancies occurred between the results due to the effects introduced by line vibration. The differences between pressure levels with and without line vibration could, however, be accounted for in the analysis if the vibration levels were known.

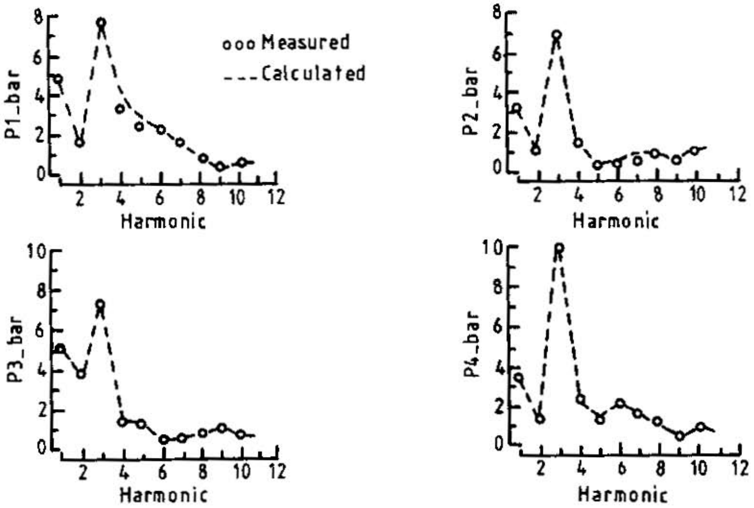


Fig.5- Pressure spectra for branch system $_1$ + $_2$= 3.6m, $_3$ = 1.15m,

mean flow Q_m= 0.7 lt/s, Q_B=0.35 lt/s, mean pressure P_m= 200 bar.

The significance of the analysis in system design is illustrated by the theoretical pressure levels shown in Fig.6. The figure shows the variation

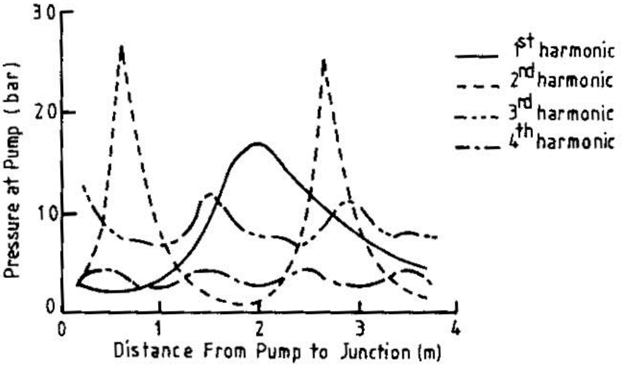


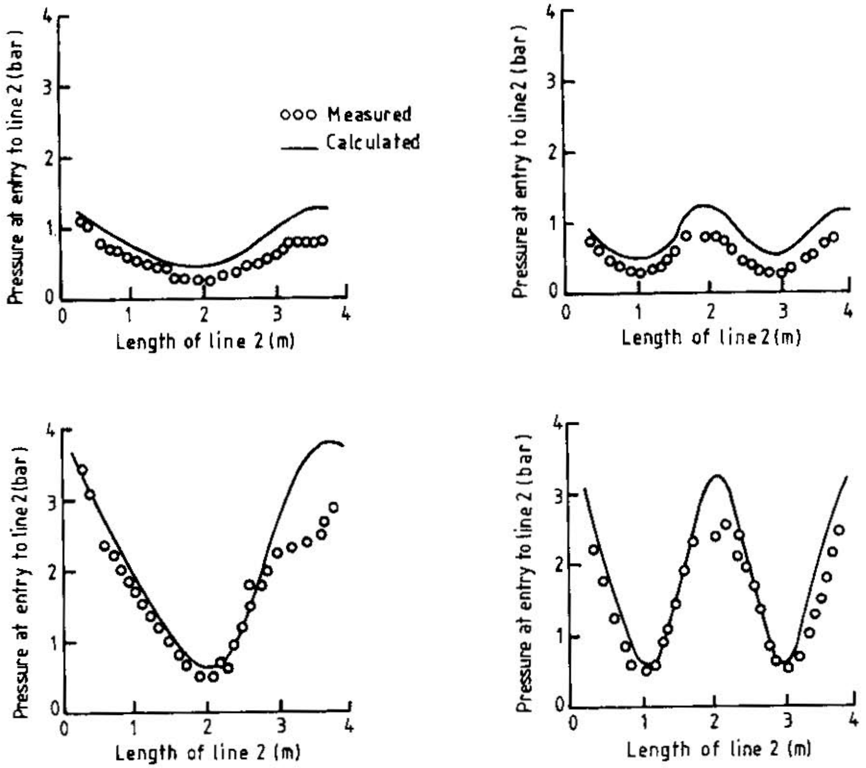
Fig.6- Variation in pressure ripple with junction position $\overset{-}{x}$ single branch system.

$_1 + _2$ = 3.97m, $_3$=1.15m, Q_m= 0.7 lt/s, Q_B= 0.35 lt/s, P_m=200 bar.

in the first four harmonics of the pressure ripple generated at the pump outlet resulting from changes in the position of the junction. Clearly, if high pressure ripples are to be avoided at the pump, the junction should not be positioned at about 0.6 m, 2. m or 2.6 m from the pump. A junction position of either 1.2 m or close to 4. m is likely to be best solution. However, when assessing the best configuration for a system it may also be necessary to take into consideration the pressures occurring at other positions within the system.

A similar approach was used to verify the analysis for the series systems. In Fig.7 a comparison is made between the calculated and measured pressures occurring at the transducer positioned at the entry of line 2 (transducer P_3 of Fig.3). Although the experimental results show rather more scatter, particularly around resonant conditions, than that obtained from simple and branch system tests, the correlation is generally good. From these results, it would appear that the propagation of pressure waves in the system is unaffected by the high local velocities, turbulence and possible cavitation conditions likely to be generated by the first valve.

An interesting effect occured during the tests when the first valve was fully open and the majority of the system pressure was generated at the second valve. The small pressure drop of approximatelly 6 bar at the fully open valve was found to be sufficient to reduce considerably the resonant pressure levels in the system compared with those obtained from the simple system tests. For example, at piston frequency the pressure was reduced from 23 bar (RMS) to under 15 bar for a mean system pressure of 200 bar. The lengths at wich resonance occurred and the pressure levels at anti-resonant conditions remained unaltered.



a) Ist Harmonic b) 2nd Harmonic

Fig.7- Pressure ripple variation in a series system. Line 1=1.2m, mean pressure at line 1= 200 bar, mean flow = 0.7 lt/s.

8- SYSTEM DESIGN

Although in this investigation the analysis of pressure ripple has been limited to constituent parts which make up hydraulic systems, the theoretical approach adopted is ideally suited to the analysis of a complete system. It is a relatively simple task, using the concept of an entry impedance, to link branch an series systems and to introduce into a system additional generators of pressure ripple such as extra pumps or hydraulic motors.

Techniques have been developed which allow the pressure ripple levels generated in systems to be assessed using digital computer metods. The configuration of a particular circuit together with the system components may be entered using an interactive program. In addition to predicting pressure ripple levels for circuits with differing layouts and components, the analysis can be used in assessing the performance of silencers when introduced in the system.

9- CONCLUSIONS

Step have been taken to develop analytical techniques, based upon the plane wave transmission line theory, which will allow the potential levels of pressure ripple generated in hydraulic systems to be determined.

The analysis has been successfully applied to circuits having pipelines and components arranged as branch systems and series systems. The analysis may be easily extended and linked to bueld-up circuits having multiple branches, combination of branch series systems, etc. If the necessary system components parameters are known, then the pressure levels at positions in a wide range of circuit configurations may be determined.

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