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Research Article

Control over Amplification in Exciton Polariton Condensate

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Abstract

Exciton polariton condensates are the most well-studied case of Bose-Einstein condensation (BEC) of quasiparticles. Together with their prominent fundamental importance, the exciton-polariton condensates have a wide spectrum of engineering applications covering interferometry and metrology, different types of SQUIDs and accelerometers, and forming a universal gate set for quantum computing via the control with external laser pulses. The efficient experimental manipulation with the polariton BEC can be realized via the bosonic final-state stimulation, matter-wave amplification, or by lasing of polaritons, but a satisfactory theoretical model for such control has not been developed yet. Here we study the polariton matter-wave amplifier based on the stimulated scattering of massive particles. The amplification of the injected quasiparticles is achieved through an elastic scattering of so-called lower polaritons (LPs). Such an amplifier has many advantages compared with a standard lasing or using a photon amplifier: it can provide a sufficient gain coefficient. To develop an efficient control algorithm for the polariton amplifier we use here the dynamical model for the LP population proposed by Ciuti, Savona, et al. in 1998. The phenomenological model for the gain coefficient is based on the experiments with cold collisions of polaritons performed by Deng, Haug, and Yamamoto in 2010 and later. We use different feedback algorithms (speed gradient vs target attractor) to track efficiently the polariton population in the amplifier. We compare the pros and cons of our alternative approaches and discuss their possible engineering applications.

Keywords: Exciton polariton condensate, Matter-wave amplifier, Cold quasiparticle collisions, Lower polaritons, Feedback control.

Eksiton Polariton Yoğunlaşmasında Amplifikasyon Üzerinde Kontrol

Öz

Eksiton polariton kondensatları, kuasiparçacıkların Bose-Einstein yoğunlaşmasının (BEC) en iyi çalışılmış halidir. Öne çıkan temel önemleriyle birlikte, eksiton-polariton kondensatları, interferometri ve metroloji, farklı SQUID türleri ve ivmeölçerleri kapsayan ve harici lazer darbeleri ile kontrol yoluyla kuantum hesaplama için evrensel bir kapı seti oluşturan geniş bir mühendislik uygulamaları yelpazesine sahiptir. Polariton BEC ile verimli deneysel manipülasyon, bozonik son durum uyarımı, madde-dalga amplifikasyonu veya polaritonların lazerlenmesi yoluyla gerçekleştirilebilir, ancak bu tür kontrol için tatmin edici bir teorik model henüz geliştirilmemiştir. Burada, büyük kütleli parçacıkların uyarılmış saçılımına dayanan polariton madde-dalga yükselticisini incelemekteyiz. Enjekte edilen kuasiparçacıkların amplifikasyonu, alt polaritonların (AP'ler) elastik saçılmasıyla sağlanır. Böyle bir yükselticinin standart bir lazerle veya bir foton yükselticisi ile karşılaştırıldığında birçok avantajı vardır: yeterli bir kazanç katsayısı sağlayabilir. Polariton amplifikatörü için verimli bir kontrol algoritması geliştirmek için burada Ciuti, Savona ve diğerleri tarafından önerilen AP popülasyonu için dinamik modeli kullanmaktayız. Kazanç katsayısı için fenomenolojik model, 2010 ve sonrasında Deng, Haug ve Yamamoto tarafından gerçekleştirilen soğuk polariton çarpışmaları deneylerine dayanmaktadır. Amplifikatördeki polariton popülasyonunu verimli bir şekilde izlemek için farklı geribesleme algoritmaları (hız gradyanı vs hedef çekici) kullanmaktayız. Alternatif yaklaşımlarımızın artılarını ve eksilerini karşılaştırır ve olası mühendislik uygulamalarını tartışmaktayız.

Anahtar Kelimeler: Eksiton polariton yoğunlaşması, Madde dalgası yükselticisi, Soğuk kuasiparçacık çarpışmaları, Alt polaritonlar, Geribesleme kontrolü.

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1. Introduction

Bose-Einstein Condensate (BEC) plays an important role in many engineering applications due to its unique properties: it is a state of matter which brings quantum effects to a macroscopic scale. Usually, speaking about BEC we keep in mind a condensation of real quantum particles. Nevertheless, one can consider condensates of quasiparticles. Exciton polariton condensates are the most well-studied case of such BECs (Kasprzak, et al., 2006; Deveaud-Plédran, 2012; Byrnes, et al., 2014).

The first theoretical prediction of polariton BEC has been done as early as in 1996 in (Imamoğlu, et al., 1996), and later it was confirmed experimentally in (Sun, et al., 2017). Exciton polaritons (often called just polaritons for short) appear in the optical cavities, where the excitons (i.e. other quasiparticles made from electron-hole pairs) can couple to photons (Allen, 2018). The details of the environment in which the condensation takes place are also of great importance for forming polariton BEC (Richard, et al., 2010).

Additional very interesting and prominent effects can happen in the system with topological defects, topological corner modes, and low dimensional structures (Flayac, 2012; Xu, et al., 2022).

1.1. Applications of Polariton BEC

Together with their fundamental importance, the exciton polariton condensates have a wide spectrum of engineering applications covering interferometry, metrology, different types of SQUIDs and accelerometers (Moxley, et al., 2021).

Exciton polariton condensates possess many attractive features for quantum computing and quantum communication: they may operate under room temperature, they demonstrate high dynamical speed, they are easy to probe, and finally, they are flexible for different fabrication techniques (Ghosh, Liew, 2020; Moxley, et al., 2021, Kavokin, et al., 2022).

The polariton condensate may help to form a universal gate set for quantum computing via the control with external laser pulses (Ghosh, Liew, 2020; Stroev, 2021). The polariton BEC can work also as a processor in which quantum superpositions are produced, while the optical field is used for distributing the generated quantum coherence (Lüders, et al., 2021).

1.2. Applications of Polariton BEC

The efficient experimental manipulation with the polariton BEC can be realized via the bosonic final-state stimulation, matter-wave amplification, or by lasing of polaritons (Deng, et al., 2010), but a satisfactory theoretical model for such control has not been developed yet.

In the lasing method, the exciton polariton condensate can be separated from the incoherent high-energy excitonic reservoir located at the pumping laser position. There are many efficient experimental techniques to do it, for instance, optical trapping (Pieczarka, et al., 2022).

1.3. Our Model

Here we study the polariton matter-wave amplifier based on the stimulated scattering of massive particles. The amplification of the injected quasiparticles is achieved through an elastic scattering of so-called lower polaritons (LPs). Such an amplifier has many advantages compared with a standard lasing or using a photon amplifier: it can provide a sufficient gain coefficient.

To develop an efficient control algorithm for the polariton amplifier we use here the dynamical model for the LP population proposed in (Ciuti, et al., 1998; Ciuti, et al. 2003). The phenomenological model for the gain coefficient is based on the experiments with cold collisions of polaritons performed in (Deng, et al., 2010) and later. We use different feedback algorithms (speed gradient vs target attractor) to track efficiently the polariton population in the amplifier. We compare the pros and cons of our alternative approaches and discuss their possible engineering applications.

2. Model

Mixing of cavity photon with exciton can be described as appearance of new type of quasiparticles: polaritons. leads to Describe in detail. Due to a reversible spontaneous emission, the polariton spectrum splitting into an upper and lower branches, which for short are called ‘upper polaritons’ (UPs) and ‘lower polaritons’ (LPs) (Vladimirova, et al., 2010; Pinsker, et al., 2017). When the cavity-photon resonance is detuned to higher than that of the quantum well exciton (so called ‘blue detuning’), LPs have a longer lifetime, and a shorter cooling time, facilitating thermalization (Deng, et al., 2010). For the process of the polariton condensation, we will focus here on the lower polariton spectrum.

2.1. Lower Polariton Dynamics Model

To describe the LP dynamics, let’s follow the model proposed in (Ciuti, et al., 1998), see also (Ciuti, et al. 2003) for its further development. The population N_{LP} Ciuti of the lower polaritons depends on the exciton polariton population in the reservoir N_{EP} as:

$$\frac{dN_{\mathrm{LP}}}{dt} = I_{\mathrm{LP}} - \frac{N_{\mathrm{LP}}}{\tau_{\mathrm{LP}}} + a_{\mathrm{LP}} N_{\mathrm{EP}} (1 + N_{\mathrm{LP}}) + \\ + b_{\mathrm{LP}} N_{\mathrm{EP}}^2 (1 + N_{\mathrm{LP}}) \,. \tag{1}$$

Here I_{LP} stands for the external pumping rate of the lower polaritons by the optical field, τ_{LP} is the bottleneck LP lifetime. Two last terms represent the scattering processes: scattering between acoustic phonons and LPs with $a_{\mathrm{LP}} = 47\ \mathrm{s}^{-1}$, and LP-LP scattering with $b_{\mathrm{LP}} = 0.45\ \mathrm{s}^{-1}$.

The reasonable maximum number of the exciton polaritons in the reservoir can be evaluated like 10^3 (Deveaud-Plédran, 2012).

2.2. Gain Phenomenological Model

To develop the gain phenomenological model based on the experimental data, let’s consider the plot in Fig.1. One can see that in the high gain regime is consists of two parts: the first fast growth, and then the exponential decay (it looks like a linear part in the logarithmic scale) up to the time about 200 ps.

To model Fig.1, let’s approximate it with the function te^{-t} as:

$$\ln(G-1) = \ln(700 \cdot t) - 700 \cdot t + 31 \,. \tag{2}$$

In (2) we rescaled the time to ps.

The gain coefficient G increases with the reservoir population N_{EP} as (Deng, et al., 2010):

$$G - 1 = G_0 \exp\{c N_{EP}^2\} \ . \quad (3)$$

Making logarithm of (3) and comparing with (2), we fit the constants: $\ln(G_0) = 31$, and $c = 30/(10^3)^2 = 3\cdot 10^{-5}$; for the second constant we matched the peak $G - 1$ in Fig. 1 which is around $10^{1.5}$, i.e. around 30, to the maximum number of N_{EP} which is around 1000.

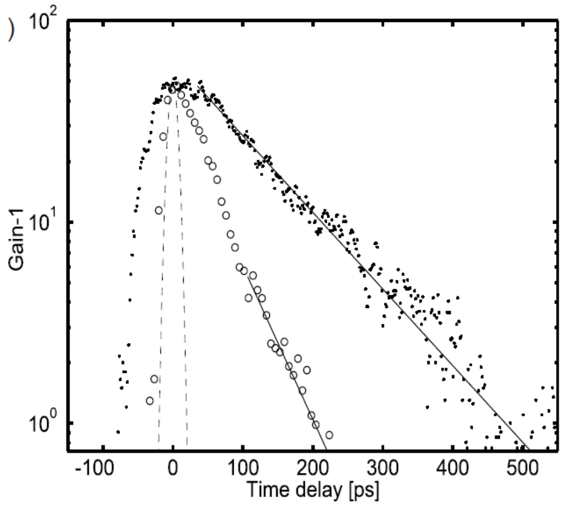

Fig. 1 Open circles: The gain function $G - 1$ vs. the time delay between the pump and probe optical pulses. Solid dots: The intensity of bottleneck LP emission at a low pumping intensity. Source: Fig.7c in (Deng, et al., 2010).

Thus the function $N_{EP}(t)$ can be restored by the gain model (2)-(3).

2.3. Time Scale Hierarchy in the Model

Now, before we develop the control algorithm, let's sum up the time hierarchy in the model. The shortest scale corresponds to the bottleneck LP lifetime τ_{LP}. It can be evaluated experimentally as 10-20 ps (Tassone, et al., 1997), although it can be increased up to 200-300 ps with the special experimental setup (Snoke, 2015; Sun, et al., 2017).

Two other time scales come from the phonon-LP scattering: $1/a_{LP} = 0.02$ s, and from the LP-LP scattering: $1/b_{LP} = 2$ s.

Thus, the control process in the model takes place in the time diapason from 20 ps (it should be greater than the LP lifetime) till 200 ps (it should be faster than the gain function is decreasing, see Fig.1).

3. Control Algorithm

The external pumping rate I_{LP}, the first term in (1), plays a role of control parameter in the model (1)-(3). We propose here two alternative feedback algorithms to track the LP population, or, in other words, to reproduce the goal function $N_{LP}^{[g]}(t)$ chosen arbitrary. We remind here, that the control is called stabilization for the constant target parameter, and tracking for time-dependent one.

These two approaches are based on different control 'philosophy', and we will compare their pros and cons.

3.1. Speed Gradient Control Algorithm

Let's define non-negative goal function for the LP number tracking:

$$\psi_{SG}(t) = \frac{1}{2}\left(N_{LP}(t) - N_{LP}^{[g]}(t)\right)^2 \ . \quad (4)$$

Speed Gradient (SG) algorithm minimizes the function (3) by driving the system along the gradient of its derivative in the space of control parameter (Fradkov, 2007). Particularly, in our case we define the control signal I_{LP} with the 1D speed gradient, i.e. via the partial derivative:

$$I_{LP}^{[SG]} = -\frac{1}{T_{SG}} \frac{\partial}{\partial I_{LP}} \left(\frac{d\psi_{SG}}{dt} \right) . \quad (5)$$

Here the positive constant T_{SG} stands for the SG control time scale. To compute the inner time derivative in (5), we use (4) with the substitution of (1) for dN_{LP}/dt, and after evaluation of the partial derivative we get:

$$I_{LP}^{[SG]} = -\frac{1}{T_{SG}} \left(N_{LP} - N_{LP}^{[g]} \right) . \quad (6)$$

This control signal should be substituted to RHS(1) to drive the system towards the control goal.

3.2. Target Attractor Control Algorithm

An alternative Target Attractor (TA) approach of feedback has been developed in (Kolesnikov, 2012). The idea is to form in the phase space of (1) an artificial attractor locking the dynamics of the system in the neighborhood of control goal.

Let's consider the goal function as:

$$\psi_{TA}(t) = N_{LP}(t) - N_{LP}^{[g]}(t) \quad (6)$$

(compare it with (4)), and then demand the exponencial convergence toward the attractor in the form:

$$\frac{d\psi_{TA}(t)}{dt} = -\frac{1}{T_{TA}} \psi_{TA}(t) \ , \quad (7)$$

the positive constant T_{TA} represents here the typical scale of TA control. Eq.(7) provides the solution:

$$\psi_{TA}(t) = \psi_{TA}(0) e^{-t/T_{TA}} \ . \quad (8)$$

Substituting (6) and (8) into (1), we can express the TA control signal explicitely:

$$I_{LP}^{[TA]} = -\frac{1}{T_{TA}} \left(N_{LP} - N_{LP}^{[g]} \right) + \frac{dN_{LP}^{[g]}}{dt} + \frac{N_{LP}}{\tau_{LP}} - a_{LP} N_{EP} (1 + N_{LP}) - b_{LP} N_{EP}^2 (1 + N_{LP}) \ . \quad (9)$$

To compare with the SG algorithm (6), the TA signal is more complex.

We emphasize here that both algorithms do not have a memory: in (6) and (9) we use only the instantaneous values for $N_{LP}(t)$ and $N_{EP}(t)$, the target LP population $N_{LP}^{[g]}(t)$ is an arbitrary smooth differentiable positively defined function.

4. Achievability of the Control Goal

Now let's evaluate the achievability of the control goal for the equation (1). To do that, let's analyze first the order of the corresponding terms in RHS(1). We will use here the inverse time scale in ps^{-1}.

As we mentioned in the discussion on the time hierarchy, the control time constants $1/T_{SG}$ and $1/T_{TA}$ for both algorithms should be of the order 10^{-2}-10^{-1} ps^{-1}. The dissipation term with the polariton lifetime has a similar order 10^{-1} ps^{-1}.

On the contrary, the orders of both scattering terms are much less: if we take the N_{EP} to be of the order 10^3, then the term with a_{LP} gets the order 10^{-8} ps^{-1}, while the term with b_{LP} becomes around 10^{-7} ps^{-1}. The contribution of both terms to the achievability of the control goal is much less, and we will focus on the role of the control and dissipation terms.

For the speed gradient feedback we can evaluate the achievability through the simplified dynamical equation:

$$\frac{dN_{LP}(t)}{dt} = \frac{N_{LP}^{[g]}(t)}{T_{SG}} - \left(\frac{1}{T_{SG}} + \frac{1}{\tau_{LP}}\right) \cdot N_{LP}(t) \quad , \tag{10}$$

with the solution:

$$N_{LP}(t) = e^{-\gamma t} N_{LP}(0) + \frac{e^{-\gamma t}}{T_{SG}} \int_0^t e^{\gamma \cdot \xi} N_{LP}^{[g]}(\xi) d\xi \; ; \qquad \gamma = \frac{1}{T_{SG}} + \frac{1}{\tau_{LP}} \, . \tag{11}$$

Particularly, for the linear growth:

$$N_{LP}^{[g]} = t \tag{12}$$

(200 LP quasiparticles per 200 ps) the solution is given by:

$$N_{LP}(t) = \frac{1}{\gamma T_{SG}} \left(t - \frac{1}{\gamma}\right) + e^{-\gamma t} N_{LP}(0) \, . \tag{13}$$

The value $1/\gamma^2 T_{SG}$ represents the systematic error of the speed gradient algorithm in goal achievability.

For the target attractor approach, the achievability of the control goal (for the case of differentiable smooth functions) is already guaranteed by (8). The only thing that we need to check is the constrain for the time derivative of the target LP population in the RHS(9) for the control signal. It is safe to demand it not to be greater than the control tracking term with T_{TA} and the dissipation term with τ_{LP}:

$$\left|\frac{dN_{LP}^{[g]}(t)}{dt}\right| << \frac{N_{LP}(t)}{\max\{T_{TA}, \tau_{LP}\}} \, . \tag{14}$$

Under such condition, the control signal does not become too large

5. Discussion

Both methods, SG and TA, demonstrate the achievability of the control over the LP population and, thus, can be applied for the improvement of amplifier characteristics in the polariton BEC.

As one can observe from the results for SG, this approach is easy to formulate, and it does not demand sufficient computational sources. Additionally, speed gradient control works only if the goal is not achieved, and it becomes switched off as soon as the system dynamics enters the target neighbourhood. From another hand, SG works virtually as a decay term ('friction') at the level of the population N_{LP} tracking, and by that, it cannot avoid the energy dissipation in the control process. As a result, in its standard form, the speed gradient cannot guarantee the precise achievement of the control goal. It can be modified slightly, which makes it more similar to the alternative TA method.

Target attractor feedback, from another side, is very rigid and locks the system in the neighbourhood of the control goal exponentially fast. Nevertheless, is has two serious handicaps. First, as one can easily see from (9), it is more sophisticated from the point of computation. Second, the support of the artificially created attractor in the system demands the continuous pumping of the energy via the control field, even if the control goal is achieved: the substitution of $N_{LP} = N_{LP}^{[g]}$ to (9) does not make the signal $I_{LP}^{[TA]}$ to be zero. The instantaneous power of the TA control can be evaluated as $I_{LP}^{[TA]}\bar{\varepsilon}_{LP}$, where $\bar{\varepsilon}_{LP}$ is the average energy for LP, which can be taken closed to 1.59 eV, i.e. about $2.5 \cdot 10^{-19}$ J (Deveaud-Plédran, 2012).

To sum up, we can say that the choice between SG vs. TA is the choice between computational simplicity and energy-consuming vs. computational accuracy and fast convergence. Thus, the practical task may define the best algorithm.

6. Conclusions

The algorithm proposed here is virtually the first example of feedback control applied to the condensate of quasiparticles. Recently we used it successfully for other quantum systems (Borisenok, 2018; Borisenok, 2020; Borisenok, 2021; Borisenok, 2022); and in both versions. SG and TA, it demonstrated the robustness, flexibility and stability under the relatively small perturbation of the dynamical systems. Both forms of our algorithm are not sensitive very much to the initial conditions, which is also important for the quantum application.

Our approach opens the new gate to many practical applications of specific quantum phases of quasiparticles (like Bose-Einstein condensate) to quantum engineering, quantum metrology, quantum computations, and quantum communications.

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