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Research Article

Smarandache Curves According to Flc-frame in Euclidean 3-space

Süleyman Şenyurt [1], **Kebire Hilal Ayvacı** [1,*], **Davut Canlı** [1]
[11] Ordu University, Faculty of Arts and Sciences, Department of Mathematics, Ordu, Türkiye
[1]senyurtsuleyman52@gmail.com, davutcanli@odu.edu.tr

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Abstract: The paper investigates some special Smarandache curves according to Flc-frame in Euclidean 3-space. The Frenet and Flc frame vectors, curvature and torsion of the new constructed curves are expressed by means of the initial curve invariants. For the sake of comparison in view, an example for Smarandache curves according to both Frenet and Flc frame is also presented at the end of paper.

Keywords: Flc-frame, natural curvatures, polynomial curves, Smarandache curves.

1. Introduction

Characterizations of curves in classical differential geometry are generally expressed with the help of Frenet framework. However, the disadvantage of this frame is that the frame cannot be settled at points where the second derivative of the curve is zero. In this case, an alternative frame is needed. Bishop defined Bishop frame which we call alternative parallel frame in 1975 [5]. This frame is formed by rotating the normal vectors at a certain angle by keeping the tangent vector in the Frenet framework constant and can be defined including the points where the second derivative of the curve is zero. Even if the Bishop frame is suitable for applications, it is not an analytical frame. Recently, Dede has introduced a new framework called Flc (Frenet like curve) frame along a given polynomial curve, and provided some insight into the geometric meaning for the nth derivative of a given curve [8]. Calculations made according to this frame are easier than the Frenet frame and Bishop frame. The most important advantage of the Flc Frame is that it has less singular points compared to the Frenet frame. Thus, by hindering the sudden rotation of the tangent vector of the curve, the deformation that may occur on the surface is prevented, and the problem of sudden ruptures and bends on the surface are removed. The Smarandache curve is defined as the regular curve with the place vector generated by the Frenet vectors of a regular curve.

In Euclidean space, the first studies for this subject were given by Ali in [2]. Turgut and Yılmaz, described the Smarandache curves in Minkowski space [15]. Later, at either Euclidean

*Correspondence: hilal.ayvaci55@gmail.com
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or Minkowski space, some features of the Smarandache curves are investigated according to the Darboux frame, Bishop frame, alternative frame, q frame and Sabban frame, [1, 3, 4, 6, 7, 9, 11–14]. In this study, we introduce special Smarandache curves according to the new Flc frame in Euclidean 3-space. The Flc apparatus of each new curve are calculated and the graphs of the curves are also presented.

2. Preliminaries

In this section, we recall some basic concepts that we refer in the context of the paper. Let $\alpha : I \subset R \to R^3$ be a regular curve in E^3. The general forms of Frenet vectors and formulas are given as

$$T(s) = \frac{\alpha'(s)}{\|\alpha'(s)\|}, \qquad N(s) = B(s) \wedge T(s), \qquad B(s) = \frac{\alpha'(s) \wedge \alpha''(s)}{\|\alpha'(s) \wedge \alpha''(s)\|}, \tag{1}$$

$$\kappa(s) = \frac{\|\alpha'(s) \wedge \alpha''(s)\|}{\|\alpha'(s)\|^3}, \qquad \tau(s) = \frac{\langle \alpha'(s) \wedge \alpha''(s), \alpha'''(s)\rangle}{\|\alpha'(s) \wedge \alpha''(s)\|^2}, \tag{2}$$

$$T'(s) = \nu\kappa(s)N(s), \qquad N'(s) = \nu\left(-\kappa(s)T(s) + \tau(s)B(s)\right), \qquad B'(s) = -\nu\tau(s)N(s), \tag{3}$$

where $\nu = \|\alpha'(s)\|$, κ is the curvature and τ is the torsion of the curve [10].

Moreover, a point $s_0 \in I$ is said to be a singular point of order 0 of the curve α, if $\alpha'(s_0)$ vanishes. Another point $s_1 \in I$ is said to be a singular point of order 1 if $\alpha''(s_1)$ vanishes. If $\alpha'(s_2) \wedge \alpha''(s_2) = 0$ that is the curvature vanishes at a point $s_2 \in I$, then s_2 is called an inflection point. Therefore as known to be the main disadvantage of the Frenet frame, it has inflection points and two type of singular points. However, recently, Dede introduced a new frame moving along a polynomial space curve of degree n and named it as Flc-frame. The vector elements of this new frame is defined as following;

$$T(s) = \frac{\alpha'(s)}{\|\alpha'(s)\|}, \qquad D_2(s) = D_1(s) \wedge T(s), \qquad D_1(s) = \frac{\alpha'(s) \wedge \alpha^{(n)}(s)}{\left\|\alpha'(s) \wedge \alpha^{(n)}(s)\right\|}, \tag{4}$$

where the prime (n) stands for the n^{th} derivative with respect to s [8]. The new vectors D_1 and D_2 are called as binormal-like and normal-like vectors, respectively. The curvatures of the Flc-frame d_1, d_2 and d_3 are defined as

$$d_1 = \frac{\langle T', D_2\rangle}{\nu}, \qquad d_2 = \frac{\langle T', D_1\rangle}{\nu}, \qquad d_3 = \frac{\left\langle {D_2}', D_1\right\rangle}{\nu}. \tag{5}$$

The relationship between the Frenet and Frenet like frame (Flc) is given by

$$\begin{bmatrix} T \\ D_2 \\ D_1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} T \\ N \\ B \end{bmatrix} \tag{6}$$

and the relations between the curvatures of two frames are

$$d_1 = \kappa cos\theta, \qquad d_2 = -\kappa sin\theta, \qquad \theta = arctan\left(-\frac{d_2}{d_1}\right), \qquad d_3 = \frac{d\theta}{\nu} + \tau, \tag{7}$$

where $\theta = \sphericalangle(N, D_2)$. Therefore, the local rate of change for the Flc-frame or namely the Frenet-like formulas can be expressed as in the following form

$$\begin{bmatrix} T' \\ {D_2}' \\ {D_1}' \end{bmatrix} = \nu \begin{bmatrix} 0 & d_1 & d_2 \\ -d_1 & 0 & d_3 \\ -d_2 & -d_3 & 0 \end{bmatrix} \begin{bmatrix} T \\ D_2 \\ D_1 \end{bmatrix}. \tag{8}$$

3. Smarandache Curves According to Flc Frame

Let us consider the curve $\beta(s) : I \subset R \to R^3$ as a regular polynomial curve in Euclidean space and denote $\{T(s),\ D_2(s),\ D_1(s)\}$ as its moving Flc frame. We define and consider the following Smarandache curves. Note that for simplicity we omit to denote the parameter s throughout the paper.

3.1. TD_2 Smarandache Curve

Definition 3.1 *The curve β_1 defined by the linear combination of two vectors T and D_2 is called the TD_2 Smarandache curve and is defined as;*

$$\beta_1(s) = \frac{1}{\sqrt{2}}(T + D_2). \tag{9}$$

We examine the Flc frame invariants of the TD_2 Smarandache curve β_1 by means of the main curve β. To do so, we first differentiate (9) with respect to s and recall the relations given at (8) to get

$$\beta_1' = \frac{\nu}{\sqrt{2}}(-d_1 T + d_1 D_2 + (d_2 + d_3) D_1).$$

By taking the norm of above and considering the equations (1), we obtain the tangent vector T_{β_1} as;

$$T_{\beta_1} = \frac{-d_1 T + d_1 D_2 + (d_2 + d_3) D_1}{\sqrt{2{d_1}^2 + (d_2 + d_3)^2}}.$$

On the other hand, by recalling (8) again, the second order derivative of (9) with respect to s is given as

$$\beta_1'' = \eta_1 T + \eta_2 D_2 + \eta_3 D_1,$$

where

$$\begin{bmatrix} \eta_1 \\ \eta_2 \\ \eta_3 \end{bmatrix} = \frac{-1}{\sqrt{2}} \begin{bmatrix} \nu^2\left({d_1}^2 + d_2 d_3 + {d_2}^2\right) + (\nu d_1)' \\ \nu^2\left({d_1}^2 + d_2 d_3 + {d_3}^2\right) - (\nu d_1)' \\ \nu^2\left(d_1 d_2 - d_1 d_3\right) - (\nu d_2)' + (\nu d_3)' \end{bmatrix}.$$

Then, the cross product of first and second order derivatives is given

$$\beta_1' \wedge \beta_1'' = \zeta_1 T + \zeta_2 D_2 + \zeta_3 D_1,$$

where

$$\begin{bmatrix} \zeta_1 \\ \zeta_2 \\ \zeta_3 \end{bmatrix} = \frac{\nu}{\sqrt{2}} \begin{bmatrix} \left(d_1\eta_3 - \eta_2\left(d_2 + d_3\right)\right) \\ \left(d_1\eta_3 + \eta_1\left(d_2 + d_3\right)\right) \\ -d_1\left(\eta_1 + \eta_2\right) \end{bmatrix}.$$

Hence, we express the principal normal and the binormal vector field of β_1 as in the following;

$$N_{\beta_1} = \frac{\nu}{\sqrt{2}} \frac{\left(\zeta_2\left(d_2 + d_3\right) - \zeta_3 d_1\right) T - \left(\zeta_1\left(d_2 + d_3\right) + \zeta_3 d_1\right) D_2 + \left(\zeta_1 d_1 + \zeta_2 d_1\right) D_1}{\left(\sqrt{2{d_1}^2 + \left(d_2 + d_3\right)^2}\right)\left(\sqrt{{\zeta_1}^2 + {\zeta_2}^2 + {\zeta_3}^2}\right)},$$

$$B_{\beta_1} = \frac{\zeta_1 T + \zeta_2 D_2 + \zeta_3 D_1}{\sqrt{{\zeta_1}^2 + {\zeta_2}^2 + {\zeta_3}^2}}.$$

The third derivative of β_1 Smarandache curve is

$$\beta_1''' = \rho_1 T + \rho_2 D_2 + \rho_3 D_1,$$

where

$$\rho_1 = \frac{1}{\sqrt{2}} \left(\begin{array}{l} \nu {d_1}^2\left(\nu^2 d_1 - 3\nu'\right) + d_1\left(\nu^3\left({d_2}^2 + {d_3}^2\right) - 3\nu^2 {d_1}' - \nu''\right) \\ -d_2\left(3\nu\nu'\left(d_2 + d_3\right) + \nu^2\left(3{d_2}' + 2{d_3}'\right)\right) - {d_2}'\nu^2 d_3 - 2\nu' {d_1}' - \nu {d_1}'' \end{array} \right),$$

$$\rho_2 = \frac{1}{\sqrt{2}} \left(\begin{array}{l} -\nu {d_1}^2\left(\nu^2 d_1 + 3\nu'\right) - d_1\left(\nu^3\left({d_3}^2 + {d_2}^2\right) + 3\nu^2 {d_1}' - \nu''\right) \\ -\nu d_2\left(\nu {d_3}' - 3\nu' d_3\right) - 3\nu'\nu {d_3}^2 - \nu^2 d_3\left(2{d_2}' + 3{d_3}'\right) + \nu {d_1}'' + 2\nu' {d_1}' \end{array} \right),$$

$$\rho_3 = \frac{1}{\sqrt{2}} \left(\begin{array}{l} -\nu^3 {d_1}^2\left(d_2 + d_3\right) - \nu d_1\left(\nu\left({d_2}' - {d_3}'\right) + 3\nu'\left(d_2 - d_3\right)\right) - \nu^3 {d_2}^2\left(d_2 + d_3\right) - \nu^3 {d_3}^3 \\ -d_2\left(\nu^2\left(\nu {d_3}^2 + 2{d_1}'\right) - \nu''\right) + \left(2\nu^2 {d_1}' + \nu''\right) d_3 + \nu\left({d_2}'' + {d_3}''\right) + 2\nu'\left({d_2}' + {d_3}'\right) \end{array} \right).$$

Hence, the Frenet curvatures κ and τ of β_1 are given as

$$\kappa_{\beta_1} = \frac{2\sqrt{2}\sqrt{{\zeta_1}^2 + {\zeta_2}^2 + {\zeta_3}^2}}{\nu^3\left(2{d_1}^2 + \left(d_2 + d_3\right)^2\right)^{\frac{3}{2}}}, \qquad \tau_{\beta_1} = \frac{\zeta_1\rho_1 + \zeta_2\rho_2 + \zeta_3\rho_3}{{\zeta_1}^2 + {\zeta_2}^2 + {\zeta_3}^2}.$$

Therefore, by using (6), the Flc apparatus of β_1 can be given by means of the Flc components of β as

$$T_{\beta_1} = \frac{-d_1 T + d_1 D_2 + (d_2 + d_3) D_1}{\mu/\sqrt{2}},$$

$$D_{2\beta_1} = \frac{1}{\mu\zeta}\begin{pmatrix} (\nu cos\theta_1 (\zeta_2 (d_2+d_3) - \zeta_3 d_1) + \mu sin\theta_1 \zeta_1) T \\ -(\nu cos\theta_1 (\zeta_1 (d_2+d_3) + \zeta_3 d_1) - \mu sin\theta_1 \zeta_2) D_2 \\ +(\nu cos\theta_1 (\zeta_1 d_1 + \zeta_2 d_1) + \mu sin\theta_1 \zeta_3) D_1 \end{pmatrix},$$

$$D_{1\beta_1} = \frac{1}{\mu\zeta}\begin{pmatrix} -(\nu sin\theta_1 (\zeta_2 (d_2+d_3) - \zeta_3 d_1) - \mu cos\theta_1 \zeta_1) T \\ +(\nu sin\theta_1 (\zeta_1 (d_2+d_3) + \zeta_3 d_1) + \mu cos\theta_1 \zeta_2) D_2 \\ -(\nu sin\theta_1 (\zeta_1 d_1 + \zeta_2 d_1) - \mu cos\theta_1 \zeta_3) D_1 \end{pmatrix},$$

where $\mu = \sqrt{4{d_1}^2 + 2(d_2+d_3)^2}$, $\zeta = \sqrt{{\zeta_1}^2 + {\zeta_2}^2 + {\zeta_3}^2}$, $\theta_1 = \sphericalangle\left(N_{\beta_1}, D_{2\beta_1}\right)$ and

$$d_{1\beta_1} = \left(\frac{2\sqrt{2}\sqrt{{\zeta_1}^2 + {\zeta_2}^2 + {\zeta_3}^2}}{\nu^3\left(2{d_1}^2 + (d_2+d_3)^2\right)^{\frac{3}{2}}}\right) cos\theta_1,$$

$$d_{2\beta_1} = -\left(\frac{2\sqrt{2}\sqrt{{\zeta_1}^2 + {\zeta_2}^2 + {\zeta_3}^2}}{\nu^3\left(2{d_1}^2 + (d_2+d_3)^2\right)^{\frac{3}{2}}}\right) sin\theta_1,$$

$$d_{3\beta_1} = -\frac{d_{2\beta_1}' d_{1\beta_1} - d_{2\beta_1} d_{1\beta_1}'}{\nu\left(d_{1\beta_1}^2 + d_{2\beta_1}^2\right)} + \frac{\zeta_1\rho_1 + \zeta_2\rho_2 + \zeta_3\rho_3}{{\zeta_1}^2 + {\zeta_2}^2 + {\zeta_3}^2}.$$

3.2. TD_1 Smarandache Curve

Definition 3.2 *The curve β_2 defined by the linear combination of two vectors T and D_1 is called the TD_1 Smarandache curve and is defined as:*

$$\beta_2(s) = \frac{1}{\sqrt{2}}(T + D_1). \tag{10}$$

We examine the Flc frame invariants of the TD_1 Smarandache curve β_2 by means of the main curve β. To do so, we first differentiate (10) with respect to s and recall the relations given at (8) to get

$$\beta_2' = \frac{\nu}{\sqrt{2}}(-d_2 T + (d_1 - d_3) \mathrm{D}_2 + d_2 \mathrm{D}_1).$$

By taking the norm of above and considering the equations (1), we obtain the tangent vector T_{β_2} as;

$$T_{\beta_2} = \frac{-d_2 T + (d_1 - d_3) \mathrm{D}_2 + d_2 \mathrm{D}_1}{\sqrt{2{d_2}^2 + (d_1 - d_3)^2}}.$$

On the other hand, by recalling (8) again, the second order derivative of (10) with respect to s is given as

$$\beta_2'' = \xi_1 T + \xi_2 D_2 + \xi_3 D_1,$$

where

$$\begin{bmatrix} \xi_1 \\ \xi_2 \\ \xi_3 \end{bmatrix} = \frac{-1}{\sqrt{2}} \begin{bmatrix} \nu^2 \left({d_2}^2 + {d_1}^2 - d_1 d_3\right) + (\nu d_2)' \\ -\nu^2 d_2 \left(d_1 + d_3\right) + \nu' \left(d_1 - d_3\right) + \nu \left({d_1}' - {d_3}'\right) \\ -\nu^2 \left({d_2}^2 - d_1 d_3 + {d_3}^2\right) + (\nu d_2)' \end{bmatrix}.$$

Next, the cross product of first and second order derivatives is given

$$\beta_2' \wedge \beta_2'' = \chi_1 T + \chi_2 D_2 + \chi_3 D_1,$$

where

$$\begin{bmatrix} \chi_1 \\ \chi_2 \\ \chi_3 \end{bmatrix} = \frac{\nu}{\sqrt{2}} \begin{bmatrix} \xi_3 \left(d_1 - d_3\right) - d_2 \xi_2 \\ d_2 \left(\xi_1 + \xi_3\right) \\ \xi_1 \left(d_1 + d_3\right) - d_2 \xi_2 \end{bmatrix}.$$

Hence, we express the principal normal and the binormal vector field of β_2 as in the following;

$$N_{\beta_2} = \frac{\nu}{\sqrt{2}} \frac{\left(\chi_2 d_2 - \chi_3 \left(d_1 - d_3\right)\right) T - d_2 \left(\chi_1 + \chi_3\right) D_2 + \left(\chi_1 \left(d_1 - d_3\right) + \chi_2 d_2\right) D_1}{\left(\sqrt{2{d_2}^2 + \left(d_1 - d_3\right)^2}\right) \left(\sqrt{{\chi_1}^2 + {\chi_2}^2 + {\chi_3}^2}\right)},$$

$$B_{\beta_2} = \frac{\chi_1 T + \chi_2 D_2 + \chi_3 D_1}{\sqrt{{\chi_1}^2 + {\chi_2}^2 + {\chi_3}^2}}.$$

The third derivative of β_2 Smarandache curve is

$$\beta_2''' = \omega_1 T + \omega_2 D_2 + \omega_3 D_1,$$

where

$$\omega_1 = \frac{1}{\sqrt{2}} \left(\begin{array}{l} \nu {d_1}^2 \left(d_2 \nu^2 - 3\nu'\right) - \nu d_1 \left(3\nu d_1 - 2\nu {d_3}' - 3\nu' d_3\right) + \nu^3 {d_2}^3 - 3\nu {d_2}^2 \nu' \\ + \left(\nu^2 \left(\nu {d_3}^2 - 3{d_2}'\right) - \nu''\right) d_2 + {d_1}' \nu^2 d_3 - 2\nu' {d_2}' - \nu {d_2}'' \end{array} \right),$$

$$\omega_2 = \frac{1}{\sqrt{2}} \left(\begin{array}{l} -\nu^3 {d_1}^2 \left(d_1 - d_3\right) - d_1 \left(\nu^3 \left({d_2}^2 + {d_3}^2\right) + 2{d_2}' \nu^2 + 3\nu d_2 \nu' - \nu''\right) + \nu^3 {d_2}^2 d_3 \\ -\nu d_2 \left(\nu \left({d_1}' + {d_3}'\right) + 3\nu' d_3\right) + \nu^3 {d_3}^3 - d_3 \left(2\nu^2 {d_2}' + \nu''\right) + \nu \left({d_1}'' - {d_3}''\right) + 2\nu' \left({d_1}' - {d_3}'\right) \end{array} \right),$$

$$\omega_3 = \frac{1}{\sqrt{2}} \left(\begin{array}{l} -\nu^3 d_2 {d_1}^2 + \nu d_1 \left(\nu {d_3}' + 3\nu' d_3\right) - \nu {d_2}^2 \left(\nu^2 d_2 + 3\nu'\right) - d_2 \left(\nu^2 \left(\nu {d_3}^2 + 3{d_2}'\right) - \nu''\right) \\ -3\nu' \nu {d_3}^2 + d_3 \left(2\nu^2 {d_1}' - 3\nu^2 {d_3}'\right) + \nu {d_2}'' + 2\nu' {d_2}' \end{array} \right).$$

Hence, the Frenet invariants κ and τ of β_2 can be expressed as

$$\kappa_{\beta_2} = \frac{2\sqrt{2}\sqrt{{\chi_1}^2 + {\chi_2}^2 + {\chi_3}^2}}{\nu^3 \left(2{d_2}^2 + \left(d_1 - d_3\right)^2\right)^{\frac{3}{2}}}, \qquad \tau_{\beta_2} = \frac{\chi_1 \omega_1 + \chi_2 \omega_2 + \chi_3 \omega_3}{{\chi_1}^2 + {\chi_2}^2 + {\chi_3}^2}.$$

By using again (6), the Flc apparatus of β_2 can be given by means of the Flc components of β as

$$T_{\beta_2} = \frac{\sqrt{2}}{\vartheta}\Big(- d_1 T + d_1 D_2 + (d_2 + d_3)\, D_1\Big),$$

$$D_{2\beta_2} = \frac{1}{\vartheta\chi}\left(\begin{array}{c} (\nu cos\theta_2\,(\chi_2 d_2 - \chi_3\,(d_1 - d_3)) + \vartheta sin\theta_2 \chi_1)\, T \\ -\,(\nu cos\theta_2\,(d_2\,(\chi_1 + \chi_3)) - \vartheta sin\theta_2 \chi_2)\, D_2 \\ +\,(\nu cos\theta_2\,(\chi_1\,(d_1 - d_3) + \chi_2 d_2) + \vartheta sin\theta_2 \chi_3)\, D_1 \end{array} \right),$$

$$D_{1\beta_2} = \frac{1}{\vartheta\chi}\left(\begin{array}{c} -\,(\nu sin\theta_2\,(\chi_2 d_2 - \chi_3\,(d_1 - d_3)) - \vartheta cos\theta_2 \chi_1)\, T \\ +\,(\nu sin\theta_2\,(d_2\,(\chi_1 + \chi_3)) + \vartheta cos\theta_2 \chi_2)\, D_2 \\ -\,(\nu sin\theta_2\,(d_2\,(\chi_1 + \chi_3)) - \vartheta cos\theta_2 \chi_3)\, D_1 \end{array} \right),$$

where $\vartheta = \sqrt{4{d_2}^2 + 2\,(d_1 - d_3)^2}$, $\chi = \sqrt{{\chi_1}^2 + {\chi_2}^2 + {\chi_3}^2}$, $\theta_2 = \sphericalangle\left(N_{\beta_2}, D_{2\beta_2}\right)$ and

$$d_{1\beta_2} = \left(\frac{2\sqrt{2}\sqrt{{\chi_1}^2 + {\chi_2}^2 + {\chi_3}^2}}{\nu^3 \left(2{d_2}^2 + (d_1 - d_3)^2\right)^{\frac{3}{2}}} \right) cos\theta_2,$$

$$d_{2\beta_2} = -\left(\frac{2\sqrt{2}\sqrt{{\chi_1}^2 + {\chi_2}^2 + {\chi_3}^2}}{\nu^3 \left(2{d_2}^2 + (d_1 - d_3)^2\right)^{\frac{3}{2}}} \right) sin\theta_2,$$

$$d_{3\beta_2} = -\frac{d'_{2\beta_2} d_{1\beta_2} - d_{2\beta_2} d'_{1\beta_2}}{\nu\left(d_{1\beta_2}^2 + d_{2\beta_2}^2\right)} + \frac{\chi_1\omega_1 + \chi_2\omega_2 + \chi_3\omega_3}{{\chi_1}^2 + {\chi_2}^2 + {\chi_3}^2}.$$

3.3. $D_2 D_1$ Smarandache Curve

Definition 3.3 *The curve β_3 defined by the linear combination of two vectors D_2 and D_1 of Flc frame is called the $D_2 D_1$ Smarandache curve and is defined as;*

$$\beta_3(s) = \frac{1}{\sqrt{2}}\,(D_2 + D_1)\,. \tag{11}$$

We examine the Flc frame invariants of the $D_2 D_1$ Smarandache curve β_3 by means of the main curve β. By differentiating (11) with respect to s, first and referring the relations given at (8) to get

$$\beta_3' = \frac{\nu}{\sqrt{2}}\,(-(d_1 + d_2)T - d_3 D_2 + d_3 D_1)\,.$$

By taking the norm of above and considering the equations (1), we obtain the tangent vector T_{β_2} as;

$$T_{\beta_3} = \frac{-(d_1 + d_2)T + d_3 \mathrm{D}_2 + d_3 \mathrm{D}_1}{\sqrt{2{d_3}^2 + (d_1 + d_2)^2}}.$$

On the other hand, by recalling (8) again, the second order derivative of (11) with respect to s is given as

$$\beta_3'' = \phi_1 T + \phi_2 D_2 + \phi_3 D_1,$$

where

$$\begin{bmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} \nu^2 d_3 (d_1 - d_2) - (\nu d_1)' - (\nu d_2)' \\ -\nu^2 \left({d_1}^2 + d_2 + {d_3}^2\right) - (\nu d_3)' \\ -\nu^2 \left(d_1 d_2 + {d_2}^2 + {d_3}^2\right) + (\nu d_3)' \end{bmatrix}.$$

Next, the cross product of first and second order derivatives is given

$$\beta_3' \wedge \beta_3'' = \upsilon_1 T + \upsilon_2 D_2 + \upsilon_3 D_1,$$

where

$$\begin{bmatrix} \upsilon_1 \\ \upsilon_2 \\ \upsilon_3 \end{bmatrix} = \frac{\nu}{\sqrt{2}} \begin{bmatrix} -d_3 (\phi_2 + \phi_3) \\ \phi_3 (d_1 + d_2) + \phi_1 d_3 \\ -\phi_2 (d_1 + d_2) + \phi_1 d_3 \end{bmatrix}.$$

Hence, we express the principal normal and the binormal vector field of β_3 as in the following;

$$N_{\beta_3} = \frac{\nu}{\sqrt{2}} \frac{(d_3 (\upsilon_2 + \upsilon_3)) T - (\upsilon_1 d_3 + \upsilon_3 (d_1 + d_2)) D_2 - (\upsilon_1 d_3 - \upsilon_2 (d_1 + d_2)) D_1}{\left(\sqrt{2{d_3}^2 + (d_1 + d_2)^2}\right)\left(\sqrt{{\upsilon_1}^2 + {\upsilon_2}^2 + {\upsilon_3}^2}\right)},$$

$$B_{\beta_3} = \frac{\upsilon_1 T + \upsilon_2 D_2 + \upsilon_3 D_1}{\sqrt{{\upsilon_1}^2 + {\upsilon_2}^2 + {\upsilon_3}^2}}.$$

Moreover, the third derivative of β_3 Smarandache curve is

$$\beta_3''' = \epsilon_1 T + \epsilon_2 D_2 + \epsilon_3 D_1,$$

where

$$\epsilon_1 = \frac{1}{\sqrt{2}} \left(\begin{array}{l} {d_1}^2 \nu^3 (d_1 + d_2) + d_1 \left(\nu^3 \left({d_2}^2 + {d_3}^2\right) + 3\nu d_3 \nu' + 2\nu^2 {d_3}' - \nu''\right) + \nu^2 d_3 \left({d_1}' - {d_2}'\right) \\ +{d_2}^3 \nu^3 + d_2 \left(\nu d_3 \left(\nu^2 d_3 - 3\nu'\right) - 2\nu^2 {d_3}' - \nu''\right) - 2\nu' \left({d_1}' + {d_2}'\right) - \nu \left({d_1}'' + {d_2}''\right) \end{array} \right),$$

$$\epsilon_2 = \frac{1}{\sqrt{2}} \left(\begin{array}{l} \nu {d_1}^2 \left(\nu^2 d_3 - 3\nu'\right) - \nu d_1 \left(3\nu' d_2 + \nu \left(3{d_1}' + 2{d_2}'\right)\right) + {d_2}^2 \nu^3 d_3 \\ -\nu \left({d_1}' d_2 \nu - \nu^2 {d_3}^3 + 3{d_3}^2 \nu' + {d_3}''\right) - d_3 \left(-3\nu^2 {d_3}' + \nu''\right) - 2\nu' {d_3}' \end{array} \right),$$

$$\epsilon_3 = \frac{1}{\sqrt{2}} \left(\begin{array}{l} -{d_1}^2 \nu^3 d_3 - \nu d_1 \left(3\nu' d_2 + {d_2}' \nu\right) - \nu {d_2}^2 \left(\nu^2 d_3 + 3\nu'\right) \\ -\nu^2 d_2 \left(2{d_1}' + 3{d_2}'\right) - \nu {d_3}^2 \left(\nu^2 d_3 + 3\nu'\right) - d_3 \left(3\nu^2 {d_3}' - \nu''\right) + 2\nu' {d_3}' + \nu {d_3}'' \end{array} \right).$$

Therefore, the Frenet curvatures κ and τ of β_3 can be expressed as

$$\kappa_{\beta_3} = \frac{2\sqrt{2}\sqrt{{\upsilon_1}^2 + {\upsilon_2}^2 + {\upsilon_3}^2}}{\nu^3 \left(2{d_3}^2 + (d_1 + d_2)^2\right)^{\frac{3}{2}}}, \qquad \tau_{\beta_3} = \frac{\upsilon_1 \epsilon_1 + \upsilon_2 \epsilon_2 + \upsilon_3 \epsilon_3}{{\upsilon_1}^2 + {\upsilon_2}^2 + {\upsilon_3}^2}.$$

By using again (6), the Flc apparatus of β_3 can be given by means of the Flc components of β as

$$T_{\beta_3} = \frac{\sqrt{2}}{\delta}\Big(-(d_1+d_2)T + d_3 D_2 + d_3 D_1\Big),$$

$$D_{2\beta_3} = \frac{1}{\delta\upsilon}\left(\begin{array}{r} (\nu cos\theta_3\left(d_3\left(\upsilon_2+\upsilon_3\right)\right)+\delta sin\theta_3\upsilon_1)\,T \\ -\left(\nu cos\theta_3\left(\upsilon_1 d_3+\upsilon_3(d_1+d_2)\right)-\delta sin\theta_3\upsilon_2\right)D_2 \\ +\left(\nu cos\theta_3\left(\upsilon_1 d_3-\upsilon_2(d_1+d_2)\right)+\delta sin\theta_3\upsilon_3\right)D_1 \end{array}\right),$$

$$D_{1\beta_3} = \frac{1}{\delta\upsilon}\left(\begin{array}{r} -\left(\nu sin\theta_3\left(d_3\left(\upsilon_2+\upsilon_3\right)\right)-\delta cos\theta_3\upsilon_1\right)T \\ +\left(\nu sin\theta_3\left(\upsilon_1 d_3+\upsilon_3(d_1+d_2)\right)+\delta cos\theta_3\upsilon_2\right)D_2 \\ +\left(\nu sin\theta_3\left(\upsilon_1 d_3-\upsilon_2(d_1+d_2)\right)+\delta cos\theta_3\upsilon_3\right)D_1 \end{array}\right),$$

where $\delta=\sqrt{4{d_3}^2+2\left(d_1+d_2\right)^2}$, $\upsilon=\sqrt{{\upsilon_1}^2+{\upsilon_2}^2+{\upsilon_3}^2}$, $\theta_3=\sphericalangle\left(N_{\beta_3},D_{2\beta_3}\right)$ and

$$d_{1\beta_3} = \left(\frac{2\sqrt{2}\sqrt{{\upsilon_1}^2+{\upsilon_2}^2+{\upsilon_3}^2}}{\nu^3\Big(2{d_3}^2+(d_1+d_2)^2\Big)^{\frac{3}{2}}}\right)cos\theta_3,$$

$$d_{2\beta_3} = -\left(\frac{2\sqrt{2}\sqrt{{\upsilon_1}^2+{\upsilon_2}^2+{\upsilon_3}^2}}{\nu^3\Big(2{d_3}^2+(d_1+d_2)^2\Big)^{\frac{3}{2}}}\right)sin\theta_3,$$

$$d_{3\beta_3} = -\frac{d_{2}{}'_{\beta_3} d_{1\beta_3} - d_{2\beta_3} d_{1}{}'_{\beta_3}}{\nu\left(d_{1\beta_3}^2+d_{2\beta_3}^2\right)} + \frac{\upsilon_1\epsilon_1+\upsilon_2\epsilon_2+\upsilon_3\epsilon_3}{{\upsilon_1}^2+{\upsilon_2}^2+{\upsilon_3}^2}.$$

3.4. TD_2D_1 Smarandache Curve

Definition 3.4 *The curve β_4 defined by the linear combination of the vectors T, D_2 and D_1 of Flc frame is called the TD_2D_1 Smarandache curve and is defined as;*

$$\beta_4(s) = \frac{1}{\sqrt{3}}\left(T+D_2+D_1\right). \tag{12}$$

We examine the Flc frame invariants of the D_2D_1 Smarandache curve β_4 by means of the main curve β. By differentiating (12) with respect to s, first and referring the relations given at (8) to get

$$\beta_4' = \frac{\nu}{\sqrt{3}}\left(-(d_1+d_2)T+(d_1-d_3)D_2+(d_2+d_3)D_1\right).$$

By taking the norm of above and considering the equations (1), we obtain the tangent vector T_{β_2} as;

$$T_{\beta_3} = \frac{-(d_1+d_2)T+(d_1-d_3)D_2+(d_2+d_3)D_1}{\sqrt{(d_1+d_2)^2+(d_1-d_3)^2+(d_2+d_3)^2}}.$$

On the other hand, by recalling (8) again, the second order derivative of (12) with respect to s is given as

$$\beta_4'' = \gamma_1 T + \gamma_2 D_2 + \gamma_3 D_1,$$

where

$$\begin{bmatrix} \gamma_1 \\ \gamma_2 \\ \gamma_3 \end{bmatrix} = \frac{1}{\sqrt{3}} \begin{bmatrix} -\nu^2\left({d_1}^2 - d_1 d_3 + {d_2}^2 + d_2 d_3\right) - \nu'\left(d_1 + d_2\right) - \nu\left({d_1}' + {d_2}'\right) \\ -\nu^2\left({d_1}^2 + d_1 d_2 + d_2 d_3 + {d_3}^2\right) + \nu\left({d_1}' - {d_3}'\right) + \nu'\left(d_1 - d_3\right) \\ -\nu^2\left({d_2}^2 + d_1 d_2 - d_1 d_3 + {d_3}^2\right) + \nu\left({d_2}' + {d_3}'\right) + \nu'\left(d_2 + d_3\right) \end{bmatrix}.$$

The cross product of first and second order derivatives is given

$$\beta_4' \wedge \beta_4'' = \psi_1 T + \psi_2 D_2 + \psi_3 D_1,$$

where

$$\begin{bmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \end{bmatrix} = \frac{\nu}{\sqrt{3}} \begin{bmatrix} \gamma_3\left(d_1 - d_3\right) - \gamma_2\left(d_2 + d_3\right) \\ \gamma_3\left(d_1 + d_2\right) + \gamma_1\left(d_2 + d_3\right) \\ \gamma_1\left(d_3 - d_1\right) - \gamma_2\left(d_1 + d_2\right) \end{bmatrix}.$$

Hence, we express the principal normal and the binormal vector field of β_4 as in the following;

$$N_{\beta_4} = \frac{\nu}{\sqrt{3}} \frac{\begin{array}{c}\left(\psi_2\left(d_2 + d_3\right) - \psi_3\left(d_1 - d_3\right)\right) T - \left(\psi_1\left(d_2 + d_3\right) + \psi_3\left(d_1 + d_2\right)\right) D_2 \\ + \left(\psi_1\left(d_1 - d_3\right) + \psi_2\left(d_1 + d_2\right)\right) D_1\end{array}}{\left(\sqrt{(d_1 + d_2)^2 + (d_1 - d_3)^2 + (d_2 + d_3)^2}\right)\left(\sqrt{{\psi_1}^2 + {\psi_2}^2 + {\psi_3}^2}\right)},$$

$$B_{\beta_4} = \frac{\psi_1 T + \psi_2 D_2 + \psi_3 D_1}{\sqrt{{\psi_1}^2 + {\psi_2}^2 + {\psi_3}^2}}.$$

Moreover, the third derivative of β_4 Smarandache curve is

$$\beta_4''' = \iota_1 T + \iota_2 D_2 + \iota_3 D_1,$$

where

$$\iota_1 = \frac{1}{\sqrt{3}} \left(\begin{array}{l} {d_1}^3 \nu^3 + \nu {d_1}^2\left(\nu^2 d_2 - 3\nu'\right) + d_1\left(\nu^3\left({d_2}^2 + {d_3}^2\right) + 3\nu'\nu d_3 - \nu'' - \nu^2\left(3{d_1}' - 2{d_3}'\right)\right) \\ \nu {d_2}^2\left(\nu^2 d_2 - 3\nu'\right) + d_2\left(\nu d_3\left(\nu^2 d_3 - 3\nu'\right) - \nu'' - \nu^2\left(3{d_2}' + 2{d_3}'\right)\right) \\ \nu^2 d_3\left({d_1}' - {d_2}'\right) - 2\nu'\left({d_1}' + {d_2}'\right) - \nu\left({d_1}'' + {d_2}''\right) \end{array} \right),$$

$$\iota_2 = \frac{1}{\sqrt{3}} \left(\begin{array}{l} -{d_1}^3 \nu^3 + \nu {d_1}^2\left(\nu^2 d_3 - 3\nu'\right) - d_1\left(\nu^3\left({d_2}^2 + {d_3}^2\right) + 3 d_2 \nu'\nu - \nu'' + \nu^2\left(3{d_1}' + 2{d_2}'\right)\right) \\ +\nu^3 {d_2}^2 d_3 - \nu d_2\left(3\nu' d_3 + \nu\left({d_1}' + {d_3}'\right)\right) + \nu {d_3}^2\left(\nu^2 d_3 - 3\nu'\right) \\ -d_3\left(\nu'' + \nu^2\left(3{d_3}' + 2{d_2}'\right)\right) + 2\nu'\left({d_1}' - {d_3}'\right) + \nu\left({d_1}'' - {d_3}''\right) \end{array} \right),$$

$$\iota_3 = \frac{1}{\sqrt{3}} \left(\begin{array}{l} -\nu^3 {d_1}^2\left(d_3 + d_2\right) - d_1\left(3\nu\nu'\left(d_2 + d_3\right) - \nu^2\left({d_3}' - {d_2}'\right)\right) - \nu^3 {d_2}^3 \\ +d_2\left(\nu'' - \nu^3 {d_3}^2 - \nu^2\left(2{d_1}' + 3{d_2}'\right)\right) - \left(\nu {d_2}^2 + \nu {d_3}^2\right)\left(\nu^2 d_3 + 3\nu'\right) \\ +\left(\nu'' - \nu^2\left(3{d_3}' - 2{d_1}'\right)\right) d_3 + 2\nu'\left({d_3}' + {d_2}'\right) + \left({d_2}'' + {d_3}''\right)\nu \end{array} \right).$$

Therefore, the Frenet curvatures κ and τ of β_4 can be expressed as

$$\kappa_{\beta_4} = \frac{3\sqrt{3}\sqrt{{\psi_1}^2+{\psi_2}^2+{\psi_3}^2}}{\nu^3((d_1+d_2)^2+(d_1-d_3)^2+(d_2+d_3)^2)^{\frac{3}{2}}}, \qquad \tau_{\beta_4} = \frac{\psi_1\iota_1+\psi_2\iota_2+\psi_3\iota_3}{{\psi_1}^2+{\psi_2}^2+{\psi_3}^2}.$$

By using again (6), the Flc apparatus of β_4 can be given by means of the Flc components of β as

$$T_{\beta_4} = \frac{\sqrt{3}}{\Delta}\Big(-(d_1+d_2)T+d_3D_2+d_3D_1\Big),$$

$$D_{2\beta_4} = \frac{1}{\Delta\psi}\left(\begin{array}{c} (\nu cos\theta_4\left(\psi_2\left(d_2+d_3\right)-\psi_3\left(d_1-d_3\right)\right)+\Delta sin\theta_4\psi_1)\,T \\ -\left(\nu cos\theta_4\left(\psi_1\left(d_2+d_3\right)+\psi_3\left(d_1+d_2\right)\right)-\Delta sin\theta_4\psi_2\right)D_2 \\ +\left(\nu cos\theta_4\left(\psi_1\left(d_1-d_3\right)+\psi_2\left(d_1+d_2\right)\right)+\Delta sin\theta_4\psi_3\right)D_1 \end{array}\right),$$

$$D_{1\beta_4} = \frac{1}{\Delta\psi}\left(\begin{array}{c} -\left(\nu sin\theta_4\left(\psi_2\left(d_2+d_3\right)-\psi_3\left(d_1-d_3\right)\right)-\Delta cos\theta_4\psi_1\right)T \\ +\left(\nu sin\theta_4\left(\psi_1\left(d_2+d_3\right)+\psi_3\left(d_1+d_2\right)\right)+\Delta cos\theta_4\psi_2\right)D_2 \\ -\left(\nu sin\theta_4\left(\psi_1\left(d_1-d_3\right)+\psi_2\left(d_1+d_2\right)\right)-\Delta cos\theta_4\psi_3\right)D_1 \end{array}\right),$$

where $\Delta = \sqrt{3\left((d_1+d_2)^2+(d_1-d_3)^2+(d_2+d_3)^2\right)}$, $\psi = \sqrt{{\psi_1}^2+{\psi_2}^2+{\psi_3}^2}$, $\theta_4 = \sphericalangle\left(N_{\beta_4}, D_{2\beta_4}\right)$ and

$$d_{1\beta_4} = \left(\frac{3\sqrt{3}\sqrt{{\psi_1}^2+{\psi_2}^2+{\psi_3}^2}}{\nu^3((d_1+d_2)^2+(d_1-d_3)^2+(d_2+d_3)^2)^{\frac{3}{2}}}\right)cos\theta_4,$$

$$d_{2\beta_4} = -\left(\frac{3\sqrt{3}\sqrt{{\psi_1}^2+{\psi_2}^2+{\psi_3}^2}}{\nu^3((d_1+d_2)^2+(d_1-d_3)^2+(d_2+d_3)^2)^{\frac{3}{2}}}\right)sin\theta_4,$$

$$d_{3\beta_4} = -\frac{{d_2}'_{\beta_4}d_{1\beta_4}-d_{2\beta_4}{d_1}'_{\beta_4}}{\nu\left(d_{1\beta_4}^2+d_{2\beta_4}^2\right)}+\frac{\psi_1\iota_1+\psi_2\iota_2+\psi_3\iota_3}{{\psi_1}^2+{\psi_2}^2+{\psi_3}^2}.$$

Example 3.5 *Let us consider $\alpha=\alpha(t)$ be a 4^{th} order polynomial curve parametrized by*

$$\alpha(t) = \left(t^2, \frac{2t^3}{3}, \frac{t^4}{4}\right).$$

The corresponding Frenet apparatus of this curve are given as

$$T(t) = \left(\tfrac{2t}{|t|(t^2+2)}, \tfrac{2|t|}{t^2+2}, \tfrac{|t|t}{t^2+2}\right), \quad N(t) = \left(-\tfrac{2|t|}{t^2+2}, -\tfrac{|t|t-2sign(t)}{t^2+2}, \tfrac{2|t|}{t^2+2}\right),$$

$$B(t) = \left(\tfrac{t^2}{t^2+2}, -\tfrac{2t}{t^2+2}, \tfrac{2t(|t|t+sign(t))}{|t|(t^2+2)}\right), \quad \kappa = \tfrac{2}{|t|(t^2+2)^2}, \quad \tau = \tfrac{2}{t(t^2+2)^2}.$$

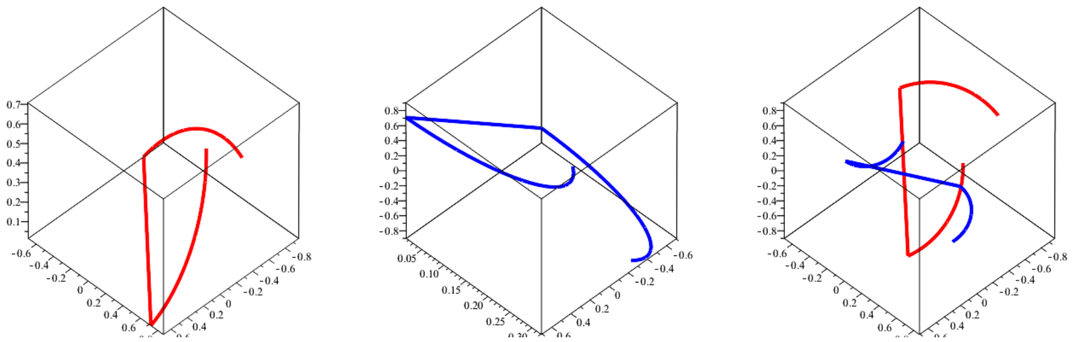
On the other hand, as $\|\alpha'\| = |t|\,(t^2+2)$*, the corresponding FLC apparatus of* α *are*

$$
\begin{aligned}
T(t) &= \left(\tfrac{2t}{|t|(t^2+2)}, \tfrac{2|t|}{t^2+2}, \tfrac{|t|t}{t^2+2}\right),\\
D_2(t) &= \left(-\tfrac{t|t|}{(t^2+2)\sqrt{t^2+1}}, -\tfrac{|t|^3}{(t^2+2)\sqrt{t^2+1}}, \tfrac{2(t|t|+sign(t))}{(t^2+2)\sqrt{t^2+1}}\right),\\
D_1(t) &= \left(\tfrac{t}{\sqrt{t^2+1}}, -\tfrac{1}{\sqrt{t^2+1}}, 0\right),\ \ d_1 = \tfrac{t}{\sqrt{t^2+1}},\ \ d_2 = -\tfrac{sign(t)}{\sqrt{t^2+1}},\ \ d_3 = \tfrac{t|t|}{2(t^2+1)}.
\end{aligned}
$$

To compare the two frames namely the Frenet frame and the Frenet like frame, let us denote α_1 *and* β_1 *as the* TN*-Smarandache curve and* TD_2*-Smarandache curve, and define these as*

$$
\alpha_1(t) = \frac{T(t)+N(t)}{\sqrt{2}}, \qquad \beta_1(t) = \frac{T(t)+D_2(t)}{\sqrt{2}}.
$$

The graph of these curves are given in Figure 1.



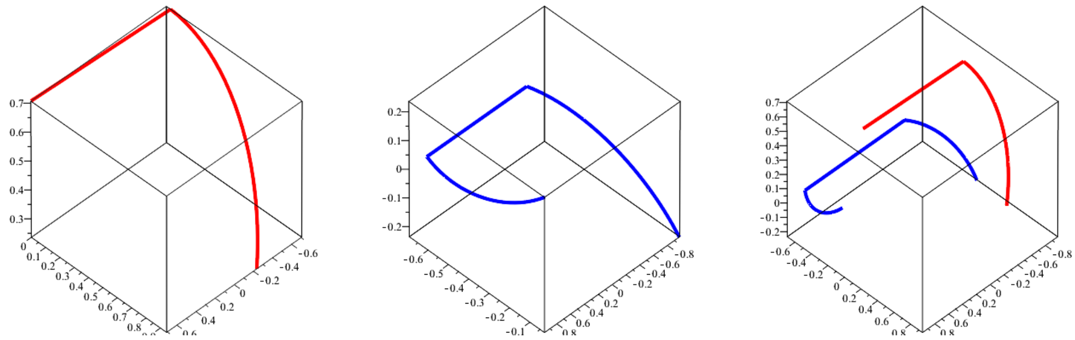
(a) α_1 alone (b) β_1 alone (c) α_1 and β_1 together

Figure 1: $TN-$ vs TD_2- Smarandache curves for $t \in (-1,1)$

Next let us denote this time α_2 and β_2 as the TB and TD_1-Smarandache curves, respectively, and define these as

$$
\alpha_2(t) = \frac{T(t)+B(t)}{\sqrt{2}}, \qquad \beta_2(t) = \frac{T(t)+D_1(t)}{\sqrt{2}}.
$$

The corresponding pictures for these curves are provided in Figure 2.



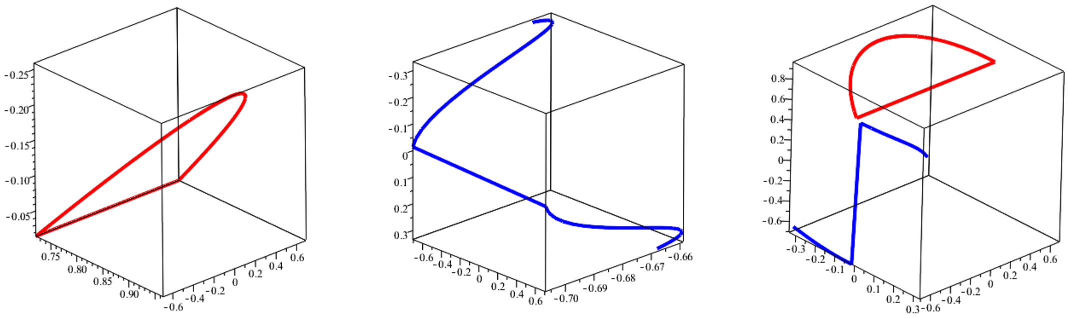
(a) α_2 alone (b) β_2 alone (c) α_2 and β_2 together

Figure 2: $TB-$ vs TD_1- Smarandache curves for $t \in (-1, 1)$

If we define α_3 and β_3 as the $NB-$ and D_2D_1- Smarandache curves, respectively, then we have

$$\alpha_3(t) = \frac{N(t) + B(t)}{\sqrt{2}}, \qquad \beta_3(t) = \frac{D_2(t) + D_1(t)}{\sqrt{2}},$$

where these curves are presented in Figure 3.



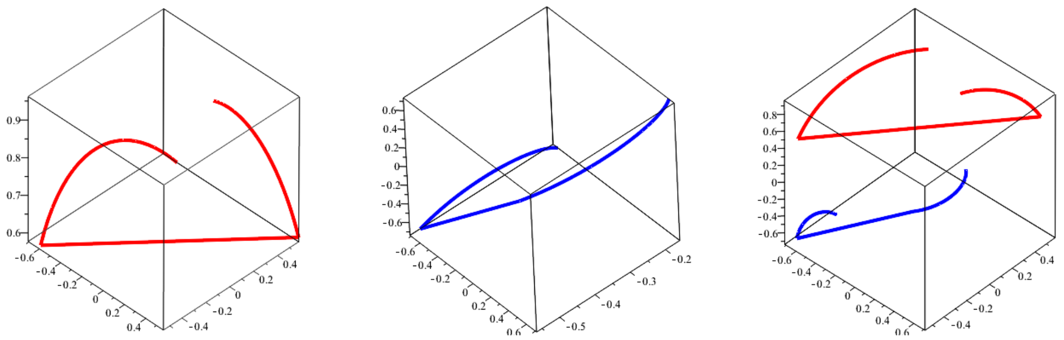
(a) α_3 alone (b) β_3 alone (c) α_3 and β_3 together

Figure 3: $NB-$ vs D_2D_1- Smarandache curves for $t \in (-1, 1)$

Finally, if we take α_4 and β_4 as the $TNB-$ and TD_2D_1- Smarandache curves, respectively, then we write

$$\alpha_4(t) = \frac{T(t) + N(t) + B(t)}{\sqrt{3}}, \qquad \beta_4(t) = \frac{T(t) + D_2(t) + D_1(t)}{\sqrt{3}}.$$

The Figure 4 shows these curves.



(a) α_4 alone (b) β_4 alone (c) α_4 and β_4 together

Figure 4: $TNB-$ vs TD_2D_1- Smarandache curves for $t \in (-1, 1)$

Declaration of Ethical Standards

The authors declare that the materials and methods used in their study do not require ethical committee and/or legal special permission.

Authors Contributions

Author [Süleyman Şenyurt]: Thought and designed the research/problem, contributed to research method (%40).

Author [Kebire Hilal Ayvacı]: Evaluation of data, wrote the manuscript, Contributed to completing the research and solving the problem (%30).

Author [Davut Canlı]: Contributed to completing the research and solving the problem, visualization (%30).

Conflicts of Interest

The authors declare no conflict of interest.

References

[1] Abdel-Aziz H.S., Saad M.K., *Computation of Smarandache curves according to Darboux frame in Minkowski 3-space*, Journal of the Egyptian Mathematical Society, 25(4), 382-390, 2017.

[2] Ali A.T., *Special Smarandache curves in the Euclidean space*, International Journal of Mathematical Combinatorics, 2, 30-36, 2010.

[3] Alıç Ş., Yılmaz B., *Smarandache curves according to alternative frame in* E^3, Journal of Universal Mathematics, 4(2), 140-156, 2021.

[4] Bektaş O., Yüce S., *Special Smarandache curves according to Darboux frame in* E^3, Romanian Journal of Mathematics and Computer Science, 3, 48-59, 2013.

[5] Bishop R.L., *There is more than one way to frame a curve*, The American Mathematical Monthly, 82(3), 246-251, 1975.

[6] Çalşkan A., Şenyurt S., *Smarandache curves in terms of Sabban frame of spherical indicatrix curves*, General Mathematics Notes, 31(2), 1-15, 2015.

[7] Çetin M., Tuncer Y., Karacan M.K., *Smarandache curves according to Bishop frame in Euclidean 3-space*, General Mathematics Notes, 20, 50-66, 2014.

[8] Dede M., *A new representation of tubular surfaces*, Houston Journal of Mathematics, 45(3), 707-720, 2018.

[9] Ekici C., Göksel M.B., Dede M., *Smarandache curves according to q-frame in Minkowski 3-Space*, Conference Proceedings of Science and Technology, 2(2), 110-118, 2019.

[10] Oneill, B., Semi Riemann Geometry, Academic Press, 1983.

[11] Şenyurt S., Eren K., *Some Smarandache curves constructed from a spacelike Salkowski curve with timelike principal normal*, Punjab University Journal of Mathematics, 53(9), 679-690, 2021.

[12] Şenyurt S., Eren K., *Smarandache curves of spacelike anti-Salkowski curve with a spacelike principal normal according to Frenet frame*, Gümüşhane Üniversitesi Fen Bilimleri Dergisi, 10(1), 251-260, 2020.

[13] Şenyurt S., Eren K. *Smarandache curves of spacelike anti-Salkowski curve with a timelike principal normal according to Frenet frame*, Erzincan University Journal of Science and Technology, 13(2), 404-416, 2020.

[14] Taşköprü K., Tosun M., *Smarandache curves on* S^2, Boletim da Sociedade Paranaense de Matematica 3rd Serie, 32(1), 51-59, 2014.

[15] Turgut M., Yılmaz S., *Smarandache curves in Minkowski spacetime*, International Journal of Mathematical Combinatorics, 3, 51-55, 2008.