COMPUTER AIDED DARLINGTON SYNTHESIS OF AN ALL PURPOSE IMMITTANCE FUNCTION

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Abstract: *This work is the continuation of our high precision immittance synthesis paper series introduced in IEEE TCAS-I. In the present manuscript, we modified the previously introduced high precision Bandpass LC-ladder synthesis algorithm to include the extraction of finite frequency and right half plane (RHP) transmission zeros of an impedance function as Brune/Darlington Type-C sections. Finite frequency and RHP transmission zeros are extracted employing our newly introduced modified impedance and chain parameters based algorithms one by one. After each transmission zero extraction, remaining immittance function is corrected using parametric approach. It is shown that propsed high precision synthesis algorithms can synthesize immittance functions up to 40 reactive elements with accumulated relative error in the order of* 10^{-1} *. The modified high precision synthesis package is developed in MatLab environment and it is integrated with the real frequency techniques to design matching networks over broadbands. Examples are presented to exhibit the usage of the newly proposed high precision synthesis algorithms.*

Keywords: *Darlington synthesis, Extraction of Brune/Darlington Type-C sections, Broadband Matching Networks, Equalizers, Real Frequency Techniques, Parametric Approach.*

1. Introduction

The “Real Frequency Techniques” in short, RFTs, are known the best design methods to construct lossless matching networks for Communication systems [1-5]. They work on the network functions either in complex Laplace variable $p = \sigma + j\omega$ or in Richard variable $\lambda = \Sigma + j\Omega$. In this regard, accurate synthesis of the network functions are in high demand [1-5]. In our previous publications, we introduced the high precison synthesis of immittance functions generated in *Laplace* and *Richards Domain* with transmission zeros only at DC and infinity via parametric approach [11-13]. However, some applications may demand such matching networks with transmission zeros at finite frequencies as well as at DC and infinity. Hence, realization of finite transmission zeros becomes inavitable. Therefore, in this work, previously introduced high precision lumped element synthesis techniques are extended to include transmission zeros at finite frequencies on the $j\omega$ axis and the real transmission zeros in the complex plane described by the classical Laplace variable $p = \sigma + j\omega$. These type of zeros are realized as Brune and Darlington’s Type *C-Sections* respectively using our new synthesis algorithms.

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In this paper, we presume that the driving point impedance is minimum reactance. This fact does not effect the generality of the synthesis approach since the $j\omega$ poles can be removed from the given impedance as a Foster function remaining a positive real impedance which is a minimum reactance [16].

Finite frequencies and RHP transmission zeros are realized using modified zero shifting method [14-17, 18-20]. In this paper, we introduce two different Brune/Type-C section extraction methods. The first one directly works on the given impedance function. Therefore, it is called “Impedance Based Brune/Type-C section Extraction”. The second method utilizes the chain parameters. Hence, it is called “Chain Parameters Based Brune/Type-C Section Extraction”. After each transmission zero extraction, remaining impedance is corrected employing the parametric approach to yield the desired electrical performance. In this process, left over transmission zeros are imbedded into the remaining impedance function in a similar manner to those of the algorithms introduced to synthesize LC Ladders[10-13, 16].

In the following sections, first we introduce Brune/Type-C section extraction algorithms based on impedance and chain parameters approach. Then, examples are presented. It is shown that newly proposed high precision computer aided Darlington's synthesis algorithms yield successful synthesis of positive real functions with high number of reactive elements.

2. Brune Sectıon Extractıon Usıng Impedance Based Approach

In this, section we deal with the extraction of a finite frequency transmission zero from the given minimum reactance impedance. In this regard, let

$$Z(p) = \frac{a(p)}{b(p)} = \frac{a_1p^n + a_2p^{n-1} + \cdots + a_np + a_{n+1}}{b_1p^n + b_2p^{n-1} + \cdots + b_np + b_{n+1}} \quad (1)$$

be a minimum reactance impedance with a real frequency finite transmission zero at ω_a and DC transmission zeros of order ndc to be synthesized in Darlington sense. In this case, even part of $Z\,(p)$ is given by

$$R(p) = F(p)/[b(p)b(-p)] \quad (2a)$$

Zeros of the even polynomial $F(p)$ are called the transmission zeros of $Z(p)$ including the ones at DC. For the case under consideration, $F(p)$ is given by

$$F(p) = a_0^2(p^2{+}\omega_a^2)^2(-1)^{ndc}p^{2ndc} = \mp f^2(p) \quad (2b)$$

where

$$f(p) = a_0(p^2 + \omega_a^2)p^{ndc} \quad (2c)$$

At the finite frequency transmission zeros $p = \mp j\omega_a$

$$Z(j\omega_a) = R(j\omega_a) + jX_a(\omega_a) \quad (3a)$$

where

$$R(j\omega_a) = 0 \quad (3b)$$

Referring to (3), we may extract an inductor L_a from $Z(p)$ such that

$$X_a = \omega_a L_a \quad (4)$$

In (4), the inductor L_a could be positive or negative without disturbing the positive real feature of $Z(p)$. Based on (4), we can express $Z(p)$ as follows and partially synthesize it as shown in Fig.1.

$$Z(p) = pL_a + Z_1(p) \quad (5a)$$

where

$$Z_1(p) = \frac{a_1(p)}{b_1(p)} \quad (5b)$$

$$b_1(p) \equiv b(p) \quad (5c)$$

$$a_1(p) \equiv a(p) - (pL_a)b(p) \quad (5d)$$

In (5d), $a_1(p)$ is a degree of $(n+1)$ polynomial.

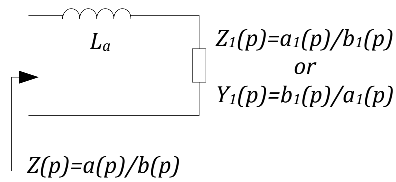

Fig. 1. Extraction of an inductor L_a from $Z(p)$

Obviously, as it is introduced above, $Z_1(p) = Z(p) - L_ap$ is zero when $p = \mp j\omega_a$. In other words,

$$Z_1(j\omega_a) = Z(j\omega_a) - j\omega_a L_a = 0 \quad (6)$$

Therefore, the numerator polynomial $a_1(p)$ must include the term $(p^2 + \omega_a^2)$ such that

$$a_1(p) = (p^2 + \omega_a^2)a_2(p) \quad (7)$$

or the admittance function $Y_1(p)$ has poles at $p = \mp j\omega_a$. That is,

$$Y_1(p) = \frac{b_1(p)}{a_1(p)} = \frac{b_1(p)}{(p^2 + \omega_a^2)a_2(p)} \quad (8)$$

By extracting the poles at $p = \mp j\omega_a$, $Y_1(p)$ can be written as

$$Y_1(p) = \frac{k_bp}{p^2 + \omega_a^2} + Y_2(p) \quad (9a)$$

where

$$Y_2(p) = \frac{b_2(p)}{a_2(p)} \quad (9b)$$

In the above formulation $a_2(p)$ and $b_2(p)$ are found as

$$a_2(p) = \frac{a_1(p)}{p^2 + \omega_a^2} \quad (10a)$$

$$b_2(p) = \left[\frac{1}{p^2 + \omega_a^2}\right][b(p) - (k_bp)a_2(p)] \quad (10b)$$

Partial synthesis of $Y_2(p)$ is depicted in Fig.2.

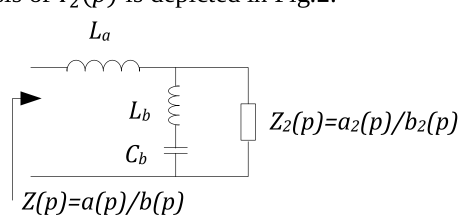

Fig. 2. Extraction of the poles at $p = \mp j\omega_a$ from the admittance function $Y_1(p)$

Finally, we set

$$Z_2(p) = \frac{a_2(p)}{b_2(p)} = L_cp + Z_3(p) \quad (11a)$$

where

$$Z_3(p) = \frac{a_3(p)}{b_3(p)} \quad (11b)$$

$$L_c = \frac{a_{21}}{b_{21}} \quad (11c)$$

$$a_3(p) = a_2(p) - (L_c p) b_3(p) \quad (11d)$$

$$b_3(p) = b_2(p) \quad (11e)$$

In (10c), a_{21} and b_{21} or denote the leading coefficients of the polynomial polynomials $a_2(p)$ and $b_2(p)$ respectively. Thus, partial synthesis of (1) is shown by Fig.3.

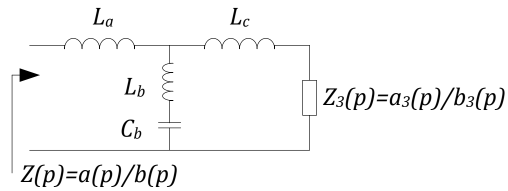

Fig. 3. Extraction of the poles at ∞ from the impedance function $Z_2(p)$

The above realization process of the finite transmission zero ω_a is called the “Brune section extraction”. In this form, one of the inductors L_a or L_c may have a negative value. However, it may be eliminated by introducing a coupled coil with mutual inductance $M > 0$ as depicted in Fig.4. At the first glance, it is straight forward to show that, the way inductors L_a, L_b and L_c are derived, must satisfy the condition [9]:

$$L_a.L_b + L_a.L_c + L_b.L_c = 0 \quad (6)$$

In this regard, the negative inductor is removed employing (12) and realization of it is depicted in Fig.4.

$$L_1 = L_a + L_b > 0 \quad (7a)$$
$$L_2 = L_b + L_c > 0 \quad (13b)$$
$$M = L_b > 0 \quad (13c)$$

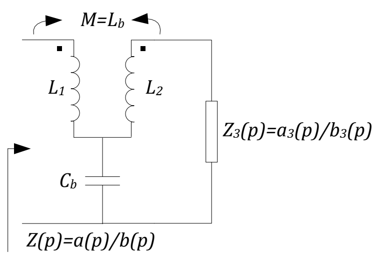

Fig. 4. Brune Section: Elimination of the negative inductor using coupled coils with a positive mutual inductance M.

In the following section, we summarize the upgraded version of our high precision synthesis algorithms introduced in [11-13] to include the extraction of finite frequency transmission zeros a Brune sections.

3. Matlab Implementation Of The New Synthesıs Algorıthm

In cascade synthesis [14-17,18-20], transmission zeros are realized as the poles of the immitance function at each step. In this regard, DC transmission zeros are realized either as series capacitors which are the poles of impedance functions at $p = 0$ or as shunt inductors which are the poles of admittance functions at $p = 0$. In a similar manner, in Brune synthesis, a finite frequency transmission zero at ω_a is realized by introducing a pole at that frequency into the admittance function in the second step of the synthesis. In this regard, synthesis algorithm can be implemented within three steps.

In step 1, at a given finite frequency transmission zero ω_a, an inductance L_a is extracted from the given impedance function $Z(p) = a(p)/b(p)$ to introduce a zero in to the remaining impedance function $Z_1(p) = a_1(p)/b_1(p)$ which is the pole of the admittance function $Y_1(p) = b_1(p)/a_1(p)$. This fact is described by equations (3-5). Thus, in $MatLab$, first we generate

$$Z(j\omega_a) = R_a + jX_a \quad (8)$$

In this step, R_a must be zero since the even part of the given impedance is zero at $p = \mp j\omega_a$ as specified by (2). However, due to numerical computational errors, R_a will not be exactly zero, rather it will be a small number. In this regard, we define an algorithmic zero such that $\varepsilon_{zero} = 10^{-m}$; $m > 0$. If $R_a \leq \varepsilon_{zero}$ then, we can go ahead with the synthesis. Otherwise, synthesis algorithm must be stoped meaning that the given impedance does not include a finite transmission zero at ω_a . In this case, if $R_a \leq \varepsilon_{zero}$, then by (4) we set

$$L_a = \frac{X_a}{\omega_a} \quad (9)$$

In (14), value of L_a may be positive or negative.

In the second step, we generate the numerator and denominator polynomials of $Z_1(p)$ as in (5) as follows.

$$a_1(p) = a(p) - pL_a b(p) \quad (15a)$$
$$b_1(p) = b(p) \quad (15b)$$

At this point, we should mention that degree of $a_1(p)$ is increased by one. Then, the numerator polynomial of $Z_2(p) = a_2(p)/b_2(p)$ is determined as in (6)

$$a_2(p) = \frac{a_1(p)}{p^2 + \omega_a^2} \quad (16)$$

Employing (9b), residue k_b is found.

$$k_b = \frac{b(j\omega_a)}{(j\omega_a)a_2(j\omega_a)} \quad (17)$$

k_b must be a real positive number. At this point, we completed the extraction of the finite transmission zero ω_a as a series resonance circuit in shunt configuration as shown in Fig.2 with element values

$$L_b = \frac{1}{k_b} > 0 \text{ and } C_b = \frac{1}{\omega_a^2 L_b} > 0 \quad (10)$$

The last computation of this step is to determine the denominator $b_2(p)$ of $Z_2(p) = a_2(p)/b_2(p)(p)$ such that

$$b_2(p) = \left[\frac{1}{p^2 + \omega_a^2}\right][b(p) - (k_b p)a_2(p)] \quad (19)$$

In $MatLab$ environment, division by $(p^2+\omega_a^2)$ is performed using the function $[q,r]=deconv(u,v)$ which performs the polynomial division operation $u(p)/v(p)$ resulting in the quotient polynomial $q(p)$ and the remainder $r(p)$. Obviously, in computing $a_2(p)$ and $b_2(p)$ using (16) and (19), remainders must be zero. However, due to accumulated numerical errors, remainders may be small numbers but not exactly zero. In this case, we can compare the norm of remainders with the algorithmic zero if they are less than ε_{zero}. If so, then we can go ahead with step 3, otherwise the algorithm is stopped indicating that extraction of the given transmission zero is not possible. At this step, degree of polynomial $a_2(p)$ is $n-1$ and degree of $b_2(p)$ is $n-2$. In this case, the impedance function $Z_2(p)=a_2(p)/b_2(p)$ must include a pole at infinity.

In step 3, the remaining pole at infinity of $Z_2(p)=a_2(p)/b_2(p)$ is removed as an inductor

$$L_c=a_2(1)/b_2(1) \tag{20}$$

It should be noted that, in $MatLab$ environment, a polynomial $P(x)=p_1x^n+p_2x^{n-1}+\cdots+p_nx+p_{n+1}$ of degree n is described by means of a vector P which includes all the coefficients $\{p_1,p_2,\dots,p_n,p_{n+1}\}$ such that

$$P=[p(1)\,p(2)\dots\,p(n)\,p(n+1)] \tag{21a}$$

Furthermore, norm of a vector P is defined as

$$norm(P)=\sqrt{p^2(1)+\,p^2(2)+\cdots+\,p^2(n+1)} \tag{21b}$$

Based on the above notation, $a_2(1)$ and $b_2(1)$ are the leading coefficients of polynomials $a_2(p)$ and $b_2(p)$ respectively. If L_a is negative, then L_c must be positive. Otherwise, L_c may take a negative value. Upon completion of this process, we end up with the remaining positive real impedance $Z_3(p)=a_3(p)/b_3(p)$. In this case,

$$a_3(p)=a_2(p)-[L_cp][b_3(p)] \tag{11a}$$

$$b_3(p)=b_2(p) \tag{22b}$$

In the above equation set, degree of $a_3(p)$ must be $n_{a3}=n-3$ or $n_{a3}=n-2$ and degree of $b_3(p)$ must be $n_{b3}=n-2$. It should be mentioned that the above Brune section extraction process is also known as zero shifting and it is programmed in $MatLab$ under the following functions:

$[Even_{Part},La] = Zeroshifting\_Step1(\,a,b,wa,eps\_zero)$,

$[\,Lb,Cb,kb,a2,b2,r_{norm}] = Zeroshifting\_Step2(wa,La,a,b,eps\_zero)$,

$[Lc,a3,b3,L1,L2,M] = Zeroshifting\_Step3(La,Lb,a2,b2)$.

Once the extraction process is completed, we plot the result of zero shifting synthesis using our general purpose $MatLab$ plot function $Plot\_Circuitv1(CT,CV)$, where CT designates the type of component to be drwan and CV is the value of that component. Based on our nomenclature $CT(i)=1$ desicribes a series inductor L_i. Components of a series resonance circuit $(pL+1/pC)$in shunt configuration is described by $CT(i)=10$ and $CT(i+1)=11$ referring to inductor L and capacitor C respectively. Terminating resistor R_T is designated by $CT(i)=9$ with the component value $CV(i)=R$. For a Brune section, we use the following $MatLab$ codes:

$$CT=[1,10,11,1,9],\ CV=[L_a,L_b,C_b,\ L_c,\ R_T]. \tag{23}$$

If n>2, then the remaining impedance $Z_3(p)=a_3(p)/b_3(p)$ is synthesized using our high precision LC ladder synthesis algorithm yielding the final circuit schematic.

It is noted that, if the driving point immittance function $F(p)=a(p)/b(p)$ is specified as an admittance, then we should flip it over to make it impedance $Z(p)=b(p)/a(p)$ to be able to apply zero shifting synthesis algorithm. In this case, $Z(p)$ may have a pole at infinity and a pole at DC. If it is so, then poles of $Z(p)$ are extracted as a Foster function as follows.

$$Z(p)=\frac{b(p)}{a(p)}=L_xp+\frac{1}{C_xp}+Z_1(p) \tag{24}$$

All the above steps are gathered under the major $MatLab$ function called

$[\,CT,CV,L1,L2,M\,]\ = SynthesisbyTranszeros(KFlag,W,ndc,a,b,eps\_zero)$.

This function synthesizes the immittance function $F(p)=a(p)/b(p)$ as described above. If the input variable $KFlag=1$ selected, $F(p)$ is an impedance, if $KFlag=0$, then $F(p)$ is an admittance. $F(p)$ may include poles at $p=0$ and/or at $p=\infty$. At the beginning of the synthesis, these poles are extracted as in (24) leaving a minimum reactance function. Then, total number of nz finite transmission zeros are extracted in a sequential manner, as they are provided by the input vector W of size nz. Thereafter, total number of ndc transmission zeros at DC are removed and finally, remaining transmission zeros at infinity are extracted using our high precision synthesis algorithms introduced in [11-12].

Remarks

- It should be emphasized that, in the above synthesis process, after each pole extraction, remaining impedance is corrected using parametric approach. Therefore, the algorithm introduced above may be called *"Direct Synthesis with Impedance Correction"* or in short "$DS-with\ ImC$".
- It is experienced that "$DS-with\ ImC$" is able to extract 10 Brune/Type-C sections with accumulated numerical error less than 10^{-1} as outlined in Example 1. However, straightforward impedance synthesis without correction fails due to over/under flows after 3 or 4 Brune section extractions.

In the next section, we propose an alternative method to synthesize Brune sections using the impedance based chain parameters in a similar maner to that of described by [5].

4. Synthesıs Vıa Chaın Matrıx Method

Referring to Fig. 5a and (1), a lossless two-port terminated in a unit resistance can be described by means of its driving point impedance based chain paramter matrix $T(p)$ such that

$$T = \frac{1}{f_T}\begin{pmatrix} A_T & B_T \\ C_T & D_T \end{pmatrix} \tag{125}$$

where

$A_T(p) = \{either\ even\ or\ odd\ part\ of\ a(p)\}$,
$B_T(p) = \{either\ odd\ or\ even\ part\ of\ a(p)\}$,
$C_T(p) = \{either\ odd\ or\ even\ part\ of\ b(p)\}$, $D_T(p) = \{either\ even\ or\ odd\ part\ of\ b(p)\}$

and the polynomial $f_T(p)$ inclueds all the finite transmission zeros of $Z(p)$ and may be expressed as in (2) such that

$$f_T(p) = a_0 p^{ndc} \prod_{i=1}^{nz} (p^2 + \omega_i^2) \tag{26}$$

In (26), ω_i is a finite transmission zero of $Z(p)$ which is realized as a Brune section. Referring to Fig.5b, chain matrix of Brune section with a transmission zero $\omega_i = \omega_a$ is given as

$$T_1 = \frac{1}{f_1}\begin{pmatrix} A_1 & B_1 \\ C_1 & D_1 \end{pmatrix} \tag{13a}$$

where

$$A_1(p) = RC_b(L_a + L_b)p^2 + R \tag{27b}$$
$$B_1(p) = (L_a + L_c)p \tag{27c}$$
$$C_1(p) = RC_b p \tag{27d}$$
$$D_1(p) = C_b(L_b + L_c)p^2 + 1 \tag{27e}$$
$$f_1(p) = a_{01}(p^2 + \omega_a^2) = L_b C_b (p^2 + \omega_a^2) \tag{27f}$$
$$a_{01} = L_b C_b \tag{27g}$$

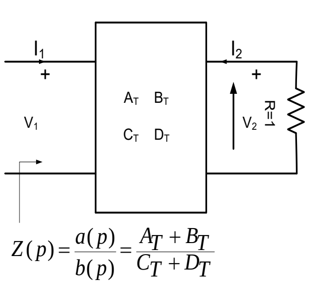

Fig.5a. Chain parameters representation of a lossless two-port

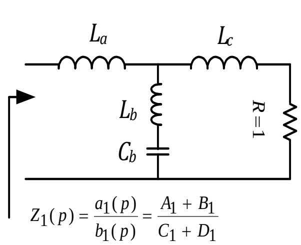

Fig.5b. Chain parameters representation of a Brune/Type-C Section

Referring to (27a), Fig.5a and Fig.6, synthesis of $Z(p)$ may be initiated by extracting the first finite transmission zero ω_1 as a Brune section as described by the chain matrix T_1 of (27).

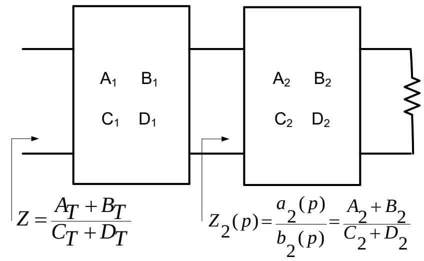

Fig.6. Extraction of a Type-C Section from $T = T_1 T_2$

In this case, $T = T_1 T_2$ or

$$T = \frac{1}{f_T}\begin{bmatrix} A_T & B_T \\ C_T & D_T \end{bmatrix} = \left\{\frac{1}{f_1}\begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix}\right\}\left\{\frac{1}{f_2}\begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix}\right\} \tag{14}$$

where T_2 is the chain matrix of the remaining lossless two-port after the extraction of T_1 from the given chain matrix T, i.e.

$$T_2 = \frac{1}{f_2}\begin{pmatrix} A_2 & B_2 \\ C_2 & D_2 \end{pmatrix} = T_1^{-1} T \tag{29a}$$

where

$$A_2 = (A_T D_1 - C_T B_1)/f_1^2, \tag{29b}$$
$$B_2 = (B_T D_1 - D_T B_1)/f_1^2, \tag{29c}$$
$$C_2 = (C_T A_1 - A_T C_1)/f_1^2, \tag{29d}$$
$$D_2 = (D_T A_1 - B_T C_1)/f_1^2. \tag{29e}$$

Eventually, the driving point input impedance $Z_2(p) = a_2(p)/b_2(p)$ is given by

$$Z_2 = \frac{A_2 + B_2}{C_2 + D_2} = \frac{a_2(p)}{b_2(p)} \tag{30a}$$

where

$$a_2(p) = [a(p)D_1(p) - b(p)B_1]/f_1^2, \tag{30b}$$
$$b_2(p) = [b(p)A_1(p) - a(p)C_1]/f_1^2. \tag{30c}$$

Thus, the alternative method of zero shifting may be implemented as described in the following algorithm.

Algorithm*: *Impedance Synthesis via Chain Matrix

Inputs: Polynomials $a(p)$ and $b(p)$ of degree n of an impedance $Z(p) = a(p)/b(p)$ with nz finite transmission zeros of $W = \{\omega_1, \omega_2, \ldots, \omega_{nz}\}$ and ndc transmission zeros at DC. It should be noted that $n \geq ndc + 2nz$.

Computational steps:

Step 1: Compute the chain parameters of T_1 as in (27).
Step 2: Compute the chain parameters of T_2 as in (29).
Step 3: Compute the driving point impedance $Z_2(p) = a_2(p)/b_2(p)$ employing (30).

Step 4a: Determine the component values of the extracted Brune section as in (14), (18), (19), (21) and (12). Store all the component codes and values.

Step 4b: Correct the impedance $Z_2(p) = a_2(p)/b_2(p)$ by inserting the remaining transmission zeros in its even part employing the parametric approach.

Step 5: Repeat the Steps 1-4 nz times until all the finite transmission zeros are extracted. Then, store the remaining impedance as $Z_2(p) = Z_r(p) = a_r(p)/b_r(p)$ which will only have transmission zeros at DC and infinity.

Step 6: Finally, synthesize $Z_r(p) = a_r(p)/b_r(p)$ using the high precision synthesis method introduced in [13].

The above algorithm is programed under a $MatLab$ function called "$Zeroshifting\_viaChainMatrixImpedanceCorrection$".

5. Real And Complex Transmission Zeros

A driving point input impedance may as well include real and complex transmission zeros in the complex frequency domain. A real transmission zero $\sigma_a > 0$ appears as the suqare of a second order even polynomial $(p^2 - \sigma_a^2)^2$; and the complex conjugate mirror image paired transmission zeros appear as the square of a fourth order even polynomial $(p^4 + dp^2 + e)^2$ in the numerator polynomial $F(p)$ of the even part impedance function $R(p)$. Thus,

$$F(p) = a_0^2(p^2 - \sigma_a^2)^2(p^4 + dp^2 + e)^2\hat{F}(p) \quad (31a)$$

$$\hat{F}(p) = (-1)^{ndc}p^{2ndc}\prod_{i=1}^{nz}(p^2 + \omega_i^2)^2 \quad (31b)$$

A positive - real transmission zero σ_a can be extracted as a Darlington Type-C section like a transmission zero located at a finite frequency ω_a. Thus, one can employ the Brune section extraction algorithm replacing ω_a^2 by $-\sigma_a^2$. In this case, even part $R(p)$ of $Z(p)$ will be zero at $p = \mp\sigma_a$ such that

$$Z(p) = R(p) + odd(p) \quad (32a)$$

with

$$R(\sigma_a) = 0 \quad (32b)$$

At this point, we presume that $odd(p)$ consist of a single positve inductor L_a such that $odd(p) = pL_a$ or

$$L_a = \frac{a(\sigma_a)b(-\sigma_a) - a(-\sigma_a)b(\sigma_a)}{2\sigma_a b(\sigma_a)b(-\sigma_a)} \quad (33)$$

After extracting pL_a from $Z(p)$, the remaining impedance, $Z_1(p) = Z(p) - pL_a$ must have a zero at $p = \mp\sigma_a$ or the admittace $Y_1(p)$ has a pole at $p = \mp\sigma_a$. Then,

$$Y_1(p) = \frac{b_1(p)}{a_1(p)} = \frac{k_b p}{p^2 - \sigma_a^2} + Y_2(p) \quad (34a)$$

where $a_1(p)$, $b(p)$ are as in (5c,d) and

$$a_2(p) = \frac{a_1(p)}{p^2 - \sigma_a^2} \quad (34b)$$

$$k_b = \frac{b(\sigma_a)}{\sigma_a a_2(\sigma_a)} > 0 \quad (34c)$$

Partial synthesis of the impedance $Z_2(p) = 1/Y_2(p) = a_2(p)/b_2(p)$ can be obtained as a series LC circuit with

$$L_b = -\frac{1}{k_b} < 0\,,\ \ C_b = \frac{k_b}{\sigma_a^2} > 0 \quad (35)$$

The rest of the elements of Darlington Type-C section is given as in (21-22). As we did before, the negative inductor L_b is removed using a perfectly coupled coil as specified by (12) with a negative coupling coefficient $M = -1/k_b = L_b$. The negative coupling coefficient M is realized by reversely winding $L_1 = L_a + L_b > 0$ and $L_2 = L_c + L_b > 0$ of Fig.4.

Complex transmission zeros in quadruplet symmetry are realized as a Darlington Type-D sections. However, usage of Type-D section does not have much practical importance. Therefore, its synthesis and realization details are skipped in this paper. Interested readers are referred to [18-20].

6. Impedance Correction Via Paremetric Approach

In this section, we presume that a minimum reactance impedance $Z_k(p) = a_k(p)/b_k(p)$ of degree of nk is obtained with a random numerical error as the result of Brune/Type-C section extractions and/or any pole extractions at DC and infinity. However, its remaining transmission zeros are precisily known in advance and they are specified under a $MatLab$ vector $W = [\omega_1\ \omega_2 \ldots \omega_{nz}]$ and as p^{2ndc}. Due to the random numerical errors, the computed even part of $Z_k(p)$ which is given by

$$R(p) = \frac{a_k(p)b_k(-p) + a_k(-p)b_k(p)}{2b_k(p)b_k(-p)} \quad (36a)$$

$$R(p) = \frac{A_1p^{2m} + A_2p^{2(nk-1)} + \cdots + A_mp^2 + A_{m+1}}{b_k(p)b_k(-p)} \quad (36b)$$

with $m \geq nk$, may not precisely yield the pre-defined transmission zeros at finite frequencies $\{\omega_1\ \omega_2 \ldots \omega_{nz}\}$ and/or at DC of order $2ndc$. In this case, keeping the same denominator $b(p)$, we can generate a corrected even part function such that

$$R_{ck}(p) = \frac{a_0^2(-1)^{ndc}p^{2ndc}\prod_{i=1}^{nz}(p^2 + \omega_i^2)^2}{b_k(p)b_k(-p)} \quad (36c)$$

where a_0 is determined from the leading coefficient of the even polynomial $F(p) = a_k(p)b_k(-p) + a_k(-p)b_k(p)$ such that a_0=$\sqrt{|A_1|}$. Hence, using (31), even part of $Z_k(p)$ is re-generated precisely to include exact transmission zeros. In this case, corrected minimum reactance impedance $Z_{kc}(p)$ may be determined by means of parametric approach [16]. In this method firstly, roots $p_{ik}$of $b_k(p) = b_{1k}p^{nk} + \cdots + b_{nk}p + 1$ are computed. Then, the corrected impedance is expressed as

$$Z_{kc} = Z_{0k} + \sum_{i=1}^{nk} \frac{K_i}{p - p_{ik}} = \frac{a_{kc}(p)}{b_{kc}(p)} \tag{157a}$$

where the residues K_i are given by

$$K_i = (-1)^{nk} \frac{(-1)^{ndc} a_0^2}{p_{ik} B_{1k} \prod_{\substack{j=1 \\ j\neq i}}^{n} p_{ik}^2 - p_{jk}^2} \tag{37b}$$

$$B_{1k} = (-1)^{nk} b_{1k}^2 \tag{37c}$$

$$Z_{ok} = \frac{a_{1k}}{b_{1k}} \tag{37d}$$

Thus, the given impedance $Z_k(p)$ is corrected to warrant the desired network topologywith prescribed transmission zeros.

7. Assesment Of The Synthesıs Error

We can device several methods to assess the accumulated numerical errors in the course of immittance synthesis. For example, as described in Section VI, we can generate a positive real impedance function $Z(p) = a(p)/b(p)$ with transmission zeros located in the right half plane as well as on the finite frequencies using parametric approach. Then, synthesis is carried out, which in turn yields the lossless Darlington two-port in resistive termination. In this regard, impedance based accumulated numerical error may be derived by generating the actual driving point impedance $Z_T(p) = a_T(p)/b_T(p)$ from the element values of the synthesized network. Thus, relative error may be described by

$$Error_R = norm(a - a_T)/C_{max} + norm(b - b_T)/C_{max} \tag{38a}$$

where the vector pairs $\{a, a_T\}$ and $\{b, b_T\}$ include coefficients of the numerator and denominator polynomials of $Z(p)$ and $Z_T(p)$ respectively and $C_{max} = max(a, a_T, b, b_T)$. Similarly, we can define an absolute error as

$$Error_A = norm(a - a_T) + norm(b - b_T) \tag{38b}$$

8. Examples

In this section two examples are presented to exhibit the utilization of the impedance synthesis algorithms introduced in this paper. In the first one, we test the numerical robustness of the newly proposed synthesis algorithms on the randomly generated minimum reactance function. In the second example, the proposed impedance synthesis algorithm is integrated with the real frequency technique to design a lossless matching network for an actual monopole antenna.

Example 1

In this example, general form of a driving point input impedance $Z(p) = a(p)/b(p)$ is synthesized in Darlington sense employing the algorithms developed in this paper. For this purpose, $Z(p)$ is generated from its even part $R(p^2)$ employing the parametric approach of Section VI as a minimum reactance. $R(p^2)$ is constructed with $nr = 3$ positive real - right half plane zeros, $nz = 3$ finite frequency zeros and $ndc = 3$ zeros at DC. Thus, as in (36), its numerator polynomial $F(p^2)$ is given by

$$F(p) = a_0^2(-1)^{ndc} p^{2ndc} \prod_{i=1}^{nr} \left(p^2 - \sigma_i^2\right)^2 \prod_{j=1}^{nz} \left(p^2 + \omega_i^2\right)^2 \tag{16}$$

Furthermore, its denominator polynomial $B(p^2) = b(p)b(-p)$ is expressed by means of an auxiliary polynomial $c(\omega)$ such that

$$B(p^2) = b(p)b(-p) = \frac{1}{2}[c^2(\omega) + c^2(-\omega)]|_{\omega^2=-p^2} > 0 \tag{40}$$

where $c(\omega)$ is selected as an 18th degree polynomial (i.e. $n = 18$.) with real arbitrary coefficients c_i such that

$$c(\omega) = \left[\sum_{i=1}^{n=18} c_i \omega^i\right] + 1 \tag{41}$$

Then, $Z(p)$ is generated as described in Section VI, using our $MatLab$ function :

$[a, b] =$
$ZeroShiftingMinimum\_FunctionRHP(ndc, W, S, a0, c)$.

In this function, input vectors $W^T = [\omega_1\ \omega_2\ ...\ \omega_{nz}]$ includes all the selected finite frequency transmission zeros, $S^T = [\sigma_1\ \sigma_2\ ...\ \sigma_{nr}]$ contains all the selected positive real-right half plane zeros of $R(p^2)$ and $c^T = [c_1\ c_2\ ...\ c_n]$ includes the real coefficients of the auxiliary polynomial $c(\omega)$. Input variable a_0 is the leading coefficient of (39) and ndc is the total number of transmission zeros located at DC. In this regard, degree n is given by $n = 2(nr + nz) + ndc + n_L$, where n_L designates the total number of transmission zeros at infinity. For the example under consideration $n_L = 3$.

Once the input variables are set, our Matlab function, $ZeroShiftingMinimum\_FunctionRHP$ generates $Z(p) = a(p)/b(p)$ as $MatLab$ vectors a and b such that

$$a(p)=\sum_{i=1}^{n+1=19} a_i p^i \text{ and } b(p)=\sum_{i=1}^{n+1=19} b_i p^i \quad (42)$$

All the input variables are listed in Table 1a and the resulting coefficents of $a(p)$ and $b(p)$ are listed in Table 1b.

Table 1a. Generation of $Z(p)=a(p)/b(p)$ with transmission zeros located in RHP [S], on the finite frequencies [W], at DC (n_{DC}) and infinity(n_L)

$\varepsilon_{zero} = 10^{-8}$	a_0 =500	$ndc=3$	$n_L=3$	$KFlag=1$
$c(\omega)=c_1\omega^n+c_2\omega^{n-1}+\cdots+c_n\omega+1$				
$nr=3;[S]$	$nz=3;[W]$	$c_1 \dots c_6$	$c_7 \dots c_{12}$	$c_{13} \dots c_{18}$
$\boldsymbol{\sigma_1}=0.1$ $\boldsymbol{\sigma_2}=0.3$ $\boldsymbol{\sigma_3}=1.7$	$\omega_1=0.2$ $\omega_2=1.5$ $\omega_3=2.0$	1.6294 1.8116 0.2540 1.8268 1.2647 0.1951	0.5570 1.0938 1.9150 1.9298 0.3152 1.9412	1.9143 0.9708 1.6006 0.2838 0.8435 1.8315

Table 1b. $Z(p)=a(p)/b(p)$ such that $n=18$, $ndc=3$, $n_L=3$, $nr=3$ with $S^T=[0.1\ 0.3\ 1.7]$, and $nz=3$ with $W^T=[0.2\ 1.5\ 2]$

$a(p)$	$b(p)$
0	$1.629400000000002e+00$
$7.640386374191374e+04$	$2.021199578409565e+01$
$9.477565802382027e+05$	$1.240994058599176e+02$
$5.748126748609327e+06$	$4.997625503065354e+02$
$2.255362397132513e+07$	$1.476782670622500e+03$
$6.393411837380380e+07$	$3.396224286336301e+03$
$1.384849904981637e+08$	$6.293526108606870e+03$
$2.364596946610408e+08$	$9.601210825123586e+03$
$3.238068035051984e+08$	$1.221637655172800e+04$
$3.583006791852569e+08$	$1.305433366323806e+04$
$3.204420015909864e+08$	$1.173859256942452e+04$
$2.304831538762949e+08$	$8.858425592653462e+03$
$1.319330848881640e+08$	$5.567059212761518e+03$
$5.898551922123835e+07$	$2.873327778659241e+03$
$1.993957497958470e+07$	$1.190802369500236e+03$
$4.811178659928725e+06$	$3.821715429382096e+02$
$7.402976617981475e+05$	$8.935564490704259e+01$
$5.461218267980077e+04$	$1.355554064078911e+01$
0	$1.000000000000000e+00$

$Z(p)$ is synthesized employing the proposed algorithms; $CTRHP, CVRHP, LL1, LL2, MM, aR, bR\,] = SynthesisbyRHPTranszeros(KFlag, W, S, ndc, a, b, eps\_zero)$ and $[\,CT, CV, L1, L2, M\,] = SynthesisbyTranszeros(KFlag, W, ndc, aR, bR, eps\_zero)$.

During the execution of the above statements, firstly $nr=3$ positive real right half plane zeros $\sigma_1=0.1$, $\sigma_2=0.3$ and $\sigma_3=1.7$ are extracted from $Z(p)=a(p)/b(p)$ as Type-C sections using "$SynthesisbyRHPTranszeros$". In this function, the remaning impedance $Z_R(p)=a_R(p)/b_R(p)$ is returned as $Matlab$ vectors aR and bR. Then, $Z_R(p)=a_R(p)/b_R(p)$ is synthesized employing the $MatLab$ function $SynthesisbyTranszeros$. In this regards, finite frequency transmission zeros $\omega_1=0.2$, $\omega_2=1.5$ and $\omega_3=2.0$ are extracted as Brune sections. Then, $ndc=3$ transmission zeros at DC, and finally, $nL=3$ transmission zeros at infinity are removed. Hence, we end up with the synthesis as depicted in Fig. 7. Element values of the final circuit is listed in Table 1c.

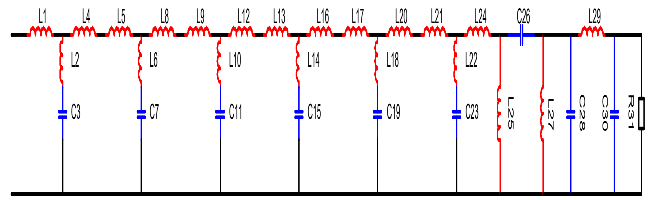

Fig.7. Synthesis of Z(p) as specified by Table 1a and b

Table 1c. Element values of the synthesized network

Type-C Sections		Brune Sections		Highpass Sections
σ_1	L1=53.9342 kH L2=-668.814 mH C3=149.518 mF L4=677.212 mH	ω_1	L13=883.859 µH L14=168.818 µH C15=148.089 kF L16=-141.744 µH	L25=23.7736 µH C26=521.685 kF L27=107.029 µH
σ_1	L5=663.21 mH L6=-86.7987mH C7=128.01 F L8=99.8692 mH	ω_2	L17=-1.04075µH L18=15.9144 µH C19=27.9272 kF L20=1.11358 µH	Lowpass Sections
σ_3	L9=481.815 µH L10=-387.053 µH C11=893.989 F L12=1.96795 mH	ω_3	L21=3.92545 µH L22=34.2182 µH C23=7.30606 kF L24=-3.52147 µH	C28=49.7319 kF L29=1.9386 µH C30=10.0286 kF R31=8.03857µ**Ω**

error accumulated in the course of synthesis		
$error\_a$	$error\_b$	$Cmax$
1.64×10^{-9}	2.54×10^{-14}	3.58×10^{8}

At the end of synthesis, accumulated numerical error is assessed by re-generating the driving point input impedance $Z_T(p)=a_T(p)/b_T(p)$ from the synthesized circuit. Eventually, the accumulated numerical error is computed using (39). It is found that $error_a=1.64\times10^{-9}$ and $error_b=2.54\times10^{-14}$ or total relative error is found as $Error_R=error_a+error_b=7.45\times10^{-9}$.

All the above computations are gathered under a $Matlab$ program called $Zeroshifting\_ExampleGeneralFormZ.m$ and it is provided as an open source code for the interested readers in [17].

Remarks:

- It should be mentioned that size of the accumulated numerical error occurs in the course of synthesis, depends on the total number and the value of the real and finite transmission zeros as well as the values of the selected coefficients of the auxiliary polynomial $c(\omega)$.
- In order to asses the robustness of the newly developed high precision zero shifting extraction methods, we run several tests. In this regard, finite transmission zeros are selected in ascending order and total number of $n=2\times nz$

coefficients of the auxiliarly polynomial $c(\omega)$ are initialized using the random number generator of $MatLab$. For example, when we run the program with $nz = 9$ finite transmission zeros as given in the vector $W =$ [0.1 0.3 1 1.2 1.4 1.6 1.8 2 2.1], at the end of the synthesis, we end up with 36 reactive elements and various size of relative erros ranging between 10^{-10} and 10^{-4}.

- It is experienced that, we can employ *"impedance based- zero shifting"* algoithm with impedance correction, to extract 13 or beyond Brune sections whereas the *"chain matrix based-finite transmission zero extraction with impedance correction"* fails after 10 Type-C and/or Brune section extractions. On the other hand, It was not possibe to utilize impedance or chain matrix based extraction algorithms without impedance correction beyond $nr = nz = 3\ or\ 4$ Type-C/Brune section extractions. Furthermore, if we increase the number of finite transmission zeros up to $n = 2 \times nz = 20$, then the accumulated relative error increases up to 10^{-1}. In this case, synthesis includes total number of 40 reactive elements. The above results can be reproduced by using our $MatLab$ program called $Zeroshifting\_Example1b.m$ which is provided as an open source code in our web-page [17].

Example 2

In this example, we design a wideband monopole antenna matching network over $40\,MHz$ to $85\,MHz$ employing the real frequency technique via parametric approach [16]. Measured Antenna impedance data is given in Table 2a. In the design, we use the real part form of the driving point impedance as expressed by (2). In this regard, we insert two transmission zero at the normalized angular frequencies $\omega_1 = 0.2$ and $\omega_2 = 1.0$ (meaning that $f_1 = 20MHz$ and $f_2 = 100\ MHz$).

Table 2a. Measured Impedance Data for Monopole Antenna over 20 MHz to 100 MHz

Frequency (MHz)	Real Part $RLA(f)$ ($\boldsymbol{\Omega}$)	ImaginaryPart $XLA\ (f)$ ($\boldsymbol{\Omega}$)
20	0.6000	-6.0000
30	0.8000	-2.2000
40	0.8000	0.0000
45	1.0000	1.4000
50	2.0000	2.8000
55	3.4000	4.6000
60	7.0000	7.6000
65	15.0000	8.8000
70	22.4000	-5.4000
75	11.0000	-13.0000
80	5.0000	-10.8000
90	1.6000	-6.8000
100	1.0000	-4.4000

These transmission zeros helps to concentrate the power delivered to the antenna within the desired band of operation. On the other hand, we wish terminate the lossless equalizer in $R_0 = 50$ ohm which corresponds to a normalized unit termination. In this case we fixed the coefficient of the numerator as $a_0 = 1/(\omega_1^2\omega_2^2) = 25$.

Eventually, using the RFT-parametric method, transducer power gain of the matched antenna is optimized yielding the normalized driving point input impedance $Z_B(p) = a(p)/b(p)$ of the matching network as listed in Table 2b. During the optimization of the transducer power gain, impedances are normalized with respect to $R_0 = 50\boldsymbol{\Omega}$. For the frequency normalization, we use $f_0 = 100\ MHz$.

Table 2b. Coefficients of the driving point input impedance $Z_B(p) = a(p)/b(p)$ of the matching network for Example 2

$a(p)$	$b(p)$
0	1.0000
0.1285	36.7818
4.7280	46.9881
5.9732	62.7998
5.6160	44.0998
3.7764	26.1939
0.8576	5.0048
0.2657	0.2657

Synthesis of the normalized impedance $Z(p) = a(p)/b(p)$ is depicted in Fig. 8a. In this figure, normalized element values are denormalized by replacing inductors L_i by $L_{iA} = L_i \times R_0/\omega_0$ and capacitors C_i by $C_{iA} = C_i/(R_0\omega_0)$. The resulting element values are as in Table 2c.

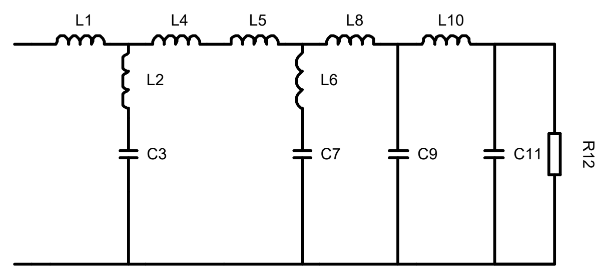

Fig. 8a. Synthesis of the matching network for the monopole antenna over 40-85 MHz.

Table 2c. Element Values of the Matching Network

$First\ Brune\ Section$			
$L1(mH)$	$L2(H)$	$L4(H)$	$C3(F)$
-936.584	1.47569	2.5637	16.9412
$Second\ Brune\ Section$			
$L5(H)$	$L6(H)$	$L8(H)$	$C7(mF)$
-1.52892	4.49579	2.31683	222.43
$Lowpass\ Section\ with\ resistive\ termination\ R$			
$C9(F)$	$L10(mH)$	$C11(mF)$	$R12\ (\Omega)$
1.64324	812.306	27.1874	1

Furthermore, inductive tees (L1,L2,L4) and (L5,L6,L8) can be replaced by coupled coils which eliminates the negative inductors as reviewed by (12) of Section III. Thus, we obtained the matching network with actual elements as depicted in Fig. 8b with element values shown in Table 2d. Corresponding transducer power gain versus normalized angular frequency is shown in Fig. 9. Close examination of Fig.9 reveals that at $\omega = 0.2$ and $\omega = 1.0$ transducer power gain goes down to minus infinity as forced by means of finite transmission zeros located at $\omega_1 = 0.2$ and $\omega_2 = 1$.

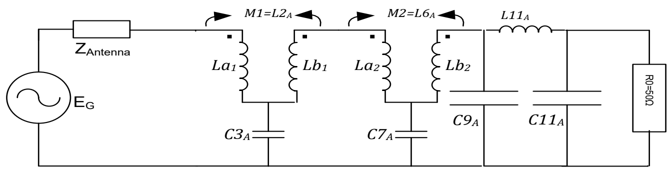

Fig. 8b. Antenna matching network with coupled coils

Table 2d. Element Values of Antenna Matching Network with Coupled Coils

$La1(nH)$	$Lb1(nH)$	$M1(nH)$	$C3A(pF)$
42.90	321.44	117.43	539.2
$La2(nH)$	$Lb2(nH)$	$M2(nH)$	$C7A(pF)$
236.1	542.13	357.76	7.08
$C9A(pF)$	$L10A(nH)$	$C11A(pF)$	$R12\ (\Omega)$
52.306	64.64	0.865	50

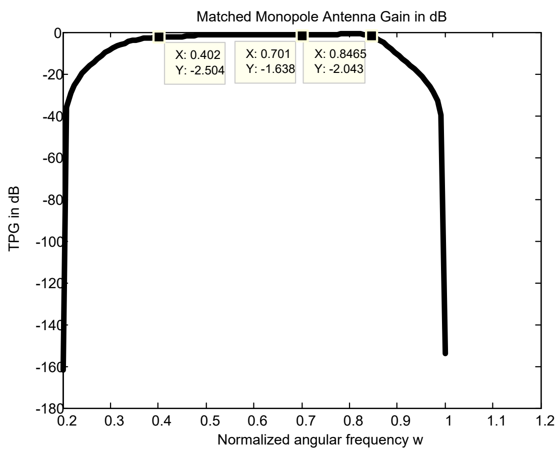

Fig. 9. Large view of TPG over $\omega = 0.2$ and $\omega = 1.0$.

9. Conclusion

As the continuation of our previous works, in this paper, "High precision Bandpass LC Ladder Synthesis" algorithm is modified to include extraction of real and finite frequency transmission zeros as Darlington Type-C and Brune sections respectively. The modified algorithm also utilizes the parametric approach at each transmission zero extraction to correct the remaining impedance. It is verified that the proposed algorithm can safely be utilized to synthesize immittance functions up to 40 elements yielding the accumulated numerical error about 10^{-1}. The new high precision synthesis algorithm is integrated with the real frequency direct computational technique to construct matching networks with optimum transducer power gain and circuit topology. The new synthesis algorithm is provided as open source codes to all the users.

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