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Book depreciation methods under intuitionistic fuzzy environment

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Abstract

In general, depreciation is the monetary value of wear and tear on capital goods inherited from previous years while creating goods and services as a result of production activities. The subject of depreciation is the addition of the loss of value that will occur in the economic values that are thought to be used in a company for more than one year. The most common depreciations methods, straight-line method, declining balance method, and unit-of-production method, are almost always handled with crisp numbers in the literature. However, the uncertainty in the parameters such as initial invest cost, salvage value, and useful life, requires the usage of fuzzy sets to represent the vagueness in these values more realistically. For this purpose, in this study, the depreciation methods are developed under intuitionistic fuzzy environment. The main contribution of this paper to the literature is that it presents the depreciation methods under intuitionistic environment with new equations for the first time. The proposed models are applied to an autonomous trolley problem to illustrate the given approach.

1. Introduction

In a broad sense, the goal of a business is to maximize the profits of the owner and/or owners in the long run. In other words, it is to increase the market value of the business. For this, businesses need to make a profit, grow and be continuous. Profit, growth, and continuity are associated with the production of goods and services of a business. The firm uses assets while performing its production function. Some of the assets used are continuous in terms of the continuation of production and are directly related to the competitive power of the enterprise. Examples of these are fixed assets with an economic life of more than one year, such as factory buildings, machinery, and equipment where production takes place. On the other hand, buildings, machinery, and equipment, which are an integral part of production, are used in production and are called tangible assets, wear out over time and lose their production power. It has made it necessary for the continuity of the business to repay the fixed asset investments, which are an important cost element for the business during its acquisition as an asset. In general, depreciation contribute to the continuity of the business by preserving the capital, which is one of the basic assumptions of accounting, by using certain methods and deducting from the profit for the period. Deducting depreciation expenses from the profit for the period may mean paying less tax for the business in the short term. However, this is the visible side of the process. On the invisible side of the process, depreciation expenses are a source of funds that do not require cash outflows.

If all of the tangible and intangible assets, which are called fixed assets, are used in the business for more than one year and lose value during this period due to physical, economic and technological reasons, these value losses make the asset worthless over time. The cost of a fixed asset that is acquired for use in the business and that will provide benefits in more than one period cannot be considered as expense in a single period. In such a case, writing the expenditure made for this asset as an expense in a single period even though it causes an increase in the operating incomes during the usage period will eliminate the possibility of comparing the said expense with the income of the relevant period. In this case, while there will be much more loss than it should be in the first period, it will appear as a profitable business since the fixed asset cost value will not be calculated in the following periods. For these reasons, the cost incurred by the enterprise for each period should be spread over the relevant periods. In accordance with the periodicity principle of accounting, the life of the enterprise, which is unlimited, should be divided into certain periods and the results of the activities of each period should be determined independently from other periods. Depreciation is the financing of the account by making periodic expense during the depletion life of the account to eliminate the negative effects of depreciation on the capital due to physical and economic reasons in the economic values of the companies. Depreciation, which is expressed as the widespread and systematic distribution of the value of an asset over its economic life, is derived from the French word 'Amortir'. Amortir means to kill slowly, to slow down, to reduce, to weaken in Latin. In the field of accounting and finance, the concept of depreciation is the reduction of the recorded amount of some items in the balance sheet by spreading over certain periods, and the elimination of the amount. Economic assets subject to depreciation in enterprises are an important element that should not be ignored is important in terms of investment from the point of view of enterprises and taxation from the point of view of the government. Depreciation is an important decision-making element for investors in that it can be written as an expense and is an incentive element. Business owners consider how much depreciation they will allocate at the establishment, operation and investment stage of the enterprises, and they see this income source as a decision-making tool. In addition, the government also affects the decisions of the enterprise by making changes in the depreciation rates in some cases. While depreciation is separated in accordance with tax principles and seen in the cost of production, it returns to the enterprise in revenue and constitutes an important auto financing source. The subject of depreciation is also of great importance in terms of determining the profits of the enterprises. In particular, it is important to make a good decision on the selection of the depreciation method in the long run in terms of the liquidity of the enterprises. Because the higher the amount of depreciation expenses that can be written according to the method to be chosen, the lower the profits of the enterprises. This, in turn, may cause the liquidity status of enterprises to be high by paying less taxes in the short term. The application of depreciation means the accumulation of funds and the recovery of the investment expenditure that is tied as capital, and the renewal of the economic asset with this income. In terms of business science, it is the allocation of profit for the replacement of a tangible asset, adding the depreciated or declining part of the tangible fixed assets to the cost of produced goods and deducting them from tax. From all these perspectives, it becomes clearer how important depreciation is for the enterprise.

The book depreciation methods that will be handled in this study are straight-line method, declining balance, and units-of-production method. The subjective opinions of the decision maker in the depreciation methods mentioned above make them vague and ambiguous. To overcome the impreciseness in the decision makers personal evaluations, fuzzy sets theory can be affectively applied in these methods. The concept of ordinary fuzzy set, developed by Zadeh (1965) as an effective tool to handle vagueness and ambiguity. The concept of ordinary fuzzy set has been developed based on the inadequacy of classical sets expressed with binary membership function in real world problems, complex systems involving human judgments and thoughts. The membership degree, which forms the basis of fuzzy sets, proposes to express the attributes with membership degree functions. The membership degree, which takes the value of 0 or 1 in classical sets, can take all values in the range of [0,1] in fuzzy sets. In the past few decades, the ordinary fuzzy set theory proposed by Zadeh has been expanded as different approaches with different additions by researchers including intuitionistic fuzzy set (IFS) theory which was developed by Atanassov (1986).

Zadeh's ordinary fuzzy set theory consists of the degree of membership in the range of [0,1], whereas in Atanassov's IFS theory, the degree of non-membership is also defined in addition to the membership degree. In ordinary fuzzy set theory, both membership and non-membership degrees take values between 0 and 1 and the sum of membership degree and non-membership degree is limited by 1. However, in the IFS theory, the sum of these two parameters does not have to be 1. Atanassov defined a third parameter called hesitancy degree to complete this sum to 1.

The advantage of using IFS in this study is that it allows for handling the vagueness and ambiguity inherent in the decision-making process related to depreciation methods. Traditional crisp sets and ordinary fuzzy sets may not fully capture the subjective opinions and uncertainties involved in the evaluation of depreciation methods. By applying IFS theory, the study can incorporate the degree of non-membership in addition to the membership degree. This means that decision makers can express their evaluations and preferences not only in terms of belonging or not belonging to a particular set but also by considering the degree of hesitancy or non-membership. This allows for a more nuanced representation of decision makers' perspectives. The use of IFSs in this study enables the modification of depreciation methods through fuzzy reasoning, considering cash flow uncertainty. It provides investors with different depreciation alternatives and helps them make informed decisions in an environment where future outcomes are uncertain. Furthermore, the study contributes to the literature by developing original equations and considering depreciation methods under IFSs. This expands the understanding of how fuzzy sets can be applied to the field of depreciation and provides a novel perspective on decision-making in investment scenarios.

The original contribution of this study to the literature is for the first time to consider depreciation methods under IFSs and to develop original equations in this field. In the study, amortization methods are modified through IFSs to help investors make decisions in an environment of cash flow uncertainty. Options among different depreciation alternatives are provided for the future investment decisions of an autonomous trolley problem through the revision of the straight-line depreciation and declining balance depreciation methods.

The remaining sections of the paper are given as follows. Section 2 includes a literature review on crisp and fuzzy depreciation methods. Section 3 presents the preliminaries of IFSs. Section 4 presents the proposed IF-depreciation methods. Section 5 includes an illustrative application of the proposed approaches on an autonomous trolley problem. Section 6 presents the conclusions and recommendations for future study.

2. Literature review

Although depreciation methods have been the subject of studies in the literature for many years, they also find their place in current studies published in recent years. Bawono and Handika (2023) reported that activity ratio, leverage, and depreciation method choice tend to be associated with firm profitability, based on the panel random effect regression analysis. Mendoza et al. (2022) applied the fixed percentage annual depreciation method for cost analysis in their study. Uemura (2022) analyzed the differing effects of the tax rate reduction and depreciation method reform. Li and Ding (2022) studied the process of water supply investment and labor input in the urban domestic water system with two depreciation methods: the straight-line depreciation method and the sum of years digits method. Wan and Qiu (2022) examined how asset values by industry sectors are affected by different depreciation methods. Conant and Chaille (2022) used Excel to model the effects of the straight line, declining balance, and Modified Accelerated Cost Recovery System (MACRS) depreciation methods on the annual and accumulated depreciation amounts of a company's assets. Sušková and Buchtová (2021) defined the individual types of depreciation and the methods for depreciation of tangible fixed assets in a metallurgical company. Bang and Park (2021) analyzed the effect of a decrease in the nuclear capacity factor under the nuclear phase-out policy on the depreciation cost per unit using the straight-line method and decelerated depreciation method and to provide recommendations from a sustainable management perspective. Sampaio et al. (2021) evaluated the influence of four depreciation methods on the cost of forest road transportation. Deng et al. (2020) studied the application of depreciated replacement cost in asset valuation of non-toll roads and discussed the applicability and limitations of key physical depreciation methods used worldwide. Mishin and Mishin (2020) prepared suggestions on the use of capital depreciation methods as the key tool of industrial enterprise and organization financial soundness improvement.

Although crisp depreciation methods have been frequently studied by researchers, only two fuzzy depreciation studies have been found in the literature, to the best knowledge of the authors. Akan and Kiraci (2023) developed a novel interval type-2 fuzzy depreciation approach and applied in maritime industry. Khalili et al. (2014) rewritten some classic methods for calculating depreciation in fuzzy form by using extension principle and α-cut technique. Kahraman and Kaya (2008) presented fuzzy depreciation, fuzzy tax rate, and fuzzy minimum attractive rate of return methods with numerical examples. The fact that fuzzy depreciation studies have been so little studied in the literature shows the openness of development in this area.

3. Preliminaries: intuitionistic fuzzy sets

Some basic definitions and equations on IFSs are presented in the following.

Definition 1. Let $X \neq \emptyset$ be a given set. An IFS in X is an object A given by

$$\tilde{A} = \{\langle x, \mu_{\tilde{A}}(x), v_{\tilde{A}}(x)\rangle; x \epsilon X\}, \quad (1)$$

where $\mu_{\tilde{A}}: X \rightarrow [0,1]$ and $v_{\tilde{A}}: X \rightarrow [0,1]$ satisfy the condition

$$0 \leq \mu_{\tilde{A}}(x) + v_{\tilde{A}}(x) \leq 1, \quad (2)$$

for every $x \epsilon X$. Hesitancy is equal to "1-$\left(\mu_{\tilde{A}}(x) + v_{\tilde{A}}(x)\right)$"

Definition 2. An IF number $\tilde{A}$ is defined as follows:

An IF subset of the real line

Normal, i.e., there is any $x_0 \in \mathbb{R}$ such that $\mu_{\tilde{A}}(x_0) = 1\ (so\ v_{\tilde{A}}(x_0) = 0)$

A convex set for the membership function $\mu_{\tilde{A}}(x)$, i.e.,

$$\mu_{\tilde{A}}(\lambda x_1 + (1-\lambda)x_2) \geq min(\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2)) \quad \forall x_1, x_2 \in \mathbb{R}, \lambda \in [0,1] \tag{3}$$

A concave set for the non-membership function $v_{\tilde{A}}(x), i.e.,$

$$v_{\tilde{A}}(\lambda x_1 + (1-\lambda)x_2) \leq max(v_{\tilde{A}}(x_1), v_{\tilde{A}}(x_2)) \ \forall x_1, x_2 \in \mathbb{R}, \lambda \in [0,1]. \tag{4}$$

Definition 3. Suppose $\tilde{X} = (\mu_x, v_x)$ and $\tilde{Y} = (\mu_y, v_y)$ be two IFSs. Some basic mathematical operations are defined as follows (Atanassov, 1986).

$$\tilde{X} \oplus \tilde{Y} = (\mu_x + \mu_y - \mu_x\mu_y, v_x v_y) \tag{5}$$

$$\tilde{X} \otimes \tilde{Y} = (\mu_x\mu_y, v_x + v_y - v_x v_y) \tag{6}$$

$$\alpha\tilde{X} = (1-(1-\mu_x)^\alpha, v_x^\alpha) \tag{7}$$

$$\tilde{X}^\alpha = (\mu_x^\alpha, 1-(1-v_x)^\alpha) \tag{8}$$

Definition 4. Suppose $\tilde{X} = (\mu_x, v_x)$ is an intuitionistic fuzzy number (IFN). Intuitionistic Fuzzy Weighted Geometric Operator (IFWG) and Intuitionistic Fuzzy Weighted Arithmetic Operator (IFWA) with respect to, $w_i = (w_1, w_2 \ldots\ldots, w_n)$; $w_i \in [0,1]$; $\sum_{i=1}^n w_i = 1$, is defined as follows.

$$IFWG(\tilde{X}_1, \ldots\ldots, \tilde{X}_n) = \left\{\prod_{i=1}^n \mu_{x_i}^{w_i}, 1 - \prod_{i=1}^n (1 - v_{x_i})^{w_i}\right\} \tag{9}$$

$$IFWA(\tilde{X}_1, \ldots\ldots, \tilde{X}_n) = \left\{1 - \prod_{i=1}^n (1 - \mu_{x_i})^{w_i}, \prod_{i=1}^n v_{x_i}^{w_i}\right\} \tag{10}$$

Definition 5. To defuzzify IFNs, the following score function is given in Equation 11.

$$SI = Score(\tilde{X}) = \frac{\mu_{\tilde{X}}^2 + 2\mu_{\tilde{X}} - 1 + v_{\tilde{X}}}{4} \tag{11}$$

4. IF depreciation methods

In this section the IF straight line method, IF declining balance method, and IF units of production method will be presented.

4.1. IF straight line method

According to the straight-line (SL) depreciation method, an equal rate and amount of depreciation is set aside for economic assets subject to depreciation every year. This method assumes that fixed assets wear out equally every year. According to the SL depreciation method, the depreciation amount is obtained by multiplying the cost of the fixed asset with the depreciation rate or dividing the cost of the fixed asset by its useful life. In case the depreciable asset has a salvage value, the depreciation amount is calculated over the remaining value by deducting the salvage value from the cost value (Hotelling, 1925) as in Eq. (12).

$$\widetilde{D}_n = \frac{\tilde{I} - \tilde{S}}{\tilde{N}} \tag{12}$$

where

$\widetilde{D}_n$= IF depreciation during year n,

$\tilde{I}$= IF cost of the asset

$\tilde{S}$= IF salvage value

$\tilde{N}$= IF useful life.

The book value of the asset at the end of n years:

Book value in a given year = Cost basis – total depreciation charges made to date or

$$\tilde{B}_n = \tilde{I} - (\widetilde{D}_1 + \widetilde{D}_2 + \cdots + \widetilde{D}_n) \tag{13}$$

To apply Eq. (12), subtraction and division operations of IFS are needed. Eq. (14) presents the IF subtraction operation, Eq. (15) presents the IF division operation, respectively (Atanassov, 2012).

$$\tilde{X} \ominus \tilde{Y} = \begin{cases} \left(\frac{\mu_x - \mu_y}{1-\mu_y}, \frac{v_x}{v_y}\right) \text{ if } \mu_x \geq \mu_y \text{ and } v_x \leq v_y \text{ and } v_y > 0 \text{ and } v_x\pi_y \leq \pi_x v_y \\ (0,1) \; otherwise \end{cases} \tag{14}$$

$$\tilde{X} \oslash \tilde{Y} = \begin{cases} \left(\frac{\mu_x}{\mu_y}, \frac{v_x - v_y}{1-v_y}\right) \text{ if } \mu_x \leq \mu_y \text{ and } v_x \geq v_y \text{ and } \mu_y > 0 \text{ and } \mu_x\pi_y \leq \pi_x \mu_y \\ (0,1) \; otherwise \end{cases} \tag{15}$$

4.2. IF declining balance method

The declining-balance (DB) method of calculating depreciation allocates a fixed fraction of the beginning book balance each year (Kranz and Worrell, 2001). The fraction, α, is obtained as follows:

$$\tilde{\alpha} = \left(\frac{1}{\tilde{N}}\right)(multiplier) \tag{16}$$

The most used multipliers in the 1.5 and 2.0 (called double-declining balance). The book value at the end of n years is calculated as in Eq. (17).

$$\tilde{B}_n = \tilde{I}(1-\tilde{\alpha})^{\tilde{n}} \tag{17}$$

4.3. IF units of production method

It is based on the logic of calculating the depreciation expense of the fixed asset to be used in production over a ratio based on the production output of that asset (Liapis and Kantianis, 2015). For each fixed asset, its service life, which is measured in units, is determined according to the service it receives. For example, measures such as mileage for a vehicle, the number of prints for a printing machine, the number of units to be produced for a production machine can be used to determine the service life. The depreciation shares to be allocated for each service unit is determined by dividing the net initial cost value, which is found by subtracting the estimated salvage value from the initial cost of the fixed asset, by the service life unit specified above. Thus, at the end of each year (and based on the months that make up the year), the number of service units performed by the fixed asset is multiplied by the depreciation share per service unit to find the annual depreciation share required. When the total number of service units by years reaches the number of service life units, depreciation is no longer calculated. Eq. (18) shows the depreciation in any year with IF units of production method

$$\tilde{D}_{\tilde{n}} = \frac{Servixe\ units\ consumed\ during\ year\ n}{Total\ service\ units}\left(\tilde{I} - \tilde{S}\right) \tag{18}$$

5. Application on autonomous trolley investment problem

The proposed IF depreciation methods will be applied on an illustrative example in this section. Suppose that a firm has bought an autonomous trolley for their warehouse where the corresponding IF data is presented in Table 1.

Table 1. Parameters and IF values of the autonomous trolley

Parameters	IF Value
$\tilde{I}$	(\$10,000; 0.95, 0.05)
$\tilde{S}$	(\$2,000; 0.90, 0.10)
$\tilde{N}$	(5; 0.90, 0.10)

Economic life here is the expected period of time during the asset remains useful to for its owner. The economic life of an asset can be different than its actual physical life. As it can be understood from its definition, the usage period of an asset cannot be known with certainty at the beginning. This account is accompanied by many variables. For example, reasons such as technological innovations, workplace accidents, usage errors, not operating in a suitable temperature environment or operating more than originally planned may cause a machine's life to be much shorter than originally anticipated. For this reason, this parameter, which is actually imprecise and contains uncertainty, should be expressed with fuzzy numbers instead of crisp numbers. The initial investment cost may be similarly affected by the exchange rate difference, price rise, inflation etc., and the amounts initially taken into account for the calculation may soon become obsolete. Since the salvage value is also a forecast value for a future date, assuming it is precise can lead to erroneous results. Therefore, in this study, these parameters are also considered under intuitionistic fuzziness.

The SL depreciation rate is $\frac{1}{(5;\,0.90,0.10)}$. Therefore, the annual depreciation charge, which is the standard yearly rate at which depreciation is charged to a fixed asset, becomes as follows by using Eq. (12).

$$D_n = \frac{((10{,}000;\,0.95{,}0.05)-(2{,}000;\,0.90{,}0.10))}{(5;\,0.90{,}0.10)} = \frac{(8{,}000;\,0.50{,}0.50)}{(5;\,0.90{,}0.10)} = (1{,}600;\ 0.56, 0.44)$$

Then, the autonomous trolley has the book values during its useful life as in Table 2, where B_{n-1} is the book value of before the depreciation charge for year n.

Table 2. Book values of the autonomous trolley with SL method

$\tilde{n}$	$\tilde{B}_{n-1}$	$\tilde{D}_n$	$\tilde{B}_n$
(1; 0.90, 0.10)	($10,000; 0.95, 0.05)	($1,600; 0.56, 0.44)	($8,400; 0.89, 0.11)
(2; 0.90, 0.10)	($8,400; 0.89, 0.11)	($1,600; 0.56, 0.44)	($6,800; 0.75, 0.25)
(3; 0.90, 0.10)	($6,800; 0.75, 0.25)	($1,600; 0.56, 0.44)	($5,200; 0.43, 0.57)
(4; 0.90, 0.10)	($5,200; 0.43, 0.57)	($1,600; 0.56, 0.44)	($3,600; 0.00, 1.00)
(5; 0.90, 0.10)	($3,600; 0.00, 1.00)	($1,600; 0.56, 0.44)	($2,000; 0.00, 1.00)

In the following, the same autonomous trolley problem will be handled with IF-DB method where the salvage value is expected to be $\tilde{S}$ = ($778; 0.24, 0.87) this time. Here, double declining balance (DDB) depreciation will be used to calculate the annual depreciation and the book values.

The book value at the beginning of the first year is $10,000, and the DDB is $\frac{2}{(5;\,0.90,0.10)}$. Then Table 3 is obtained for the DDB in terms of the book value of time.

Table 3. Book values of the autonomous trolley with DDB method

$\tilde{n}$	$\tilde{B}_{n-1}$	$\tilde{D}_n$	$\tilde{B}_n$
(1; 0.90, 0.10)	($10,000; 0.95, 0.05)	($4,000; 0.70, 0.30)	($6,000; 0.83, 0.17)
(2; 0.90, 0.10)	($6,000; 0.83, 0.17)	($2,400; 0.51, 0.49)	($3,600; 0.66, 0.34)
(3; 0.90, 0.10)	($3,600; 0.66, 0.34)	($1,440; 0.35, 0.65)	($2,160; 0.48, 0.52)
(4; 0.90, 0.10)	($2,160; 0.48, 0.52)	($864; 0.23, 0.77)	($1,296; 0.32, 0.68)
(5; 0.90, 0.10)	($1,296; 0.32, 0.68)	($518; 0.14, 0.86)	($778; 0.21, 0.79)

The depreciation deduction for the first year which shown bold in Table 3 is calculated by using Eq. (7) as follows.

$$(\$4{,}000;\ 0.70, 0.30) = \left(\frac{\$10{,}000\times 2}{5};\, 1-(1-0.70)^{2/5}, 0.30^{2/5}\right)$$

The estimated salvage value ($778; 0.24, 0.87) and the final book value ($778; 0.21, 0.79) are defuzzified by using Eq. (11). With these parameter values, the final crisp book value is found to be equal to the estimated crisp salvage value of $130 which is generally never the situation in real cases. When $\tilde{B}_{\tilde{n}} \neq \tilde{S}$, some adjustments needed to be done in the depreciation methods. In this case, there can be two options: either $\tilde{B}_{\tilde{n}} > \tilde{S}$ or $\tilde{B}_{\tilde{n}} < \tilde{S}$. If $\tilde{B}_{\tilde{n}} > \tilde{S}$, the book value (BV) can be equal to the salvage value by switching form DB to SL depreciation. Then, Eq. (19) is used to calculate the SL depreciation in any year n.

$$\tilde{D}_n = \frac{BV\ at\ the\ beginning\ of\ year\ n-\tilde{S}}{Remaining\ useful\ life\ at\ the\ beginning\ of\ year\ n} \tag{19}$$

Now, suppose that all the parameters given are the same except the $\tilde{S}$. Here, the $\tilde{S}$ will be considered as ($550; 0.35, 0.65). Then, it is needed to determine the optimal time to switch form DB to SL depreciation. Table 4 presents the DDB depreciation of each year as before.

Table 4. DDB depreciation of each year

$\tilde{n}$	$\tilde{D}_n$	$\tilde{B}_n$
(1; 0.90, 0.10)	($4,000; 0.70, 0.30)	($6,000; 0.83, 0.17)
(2; 0.90, 0.10)	($2,400; 0.51, 0.49)	($3,600; 0.66, 0.34)
(3; 0.90, 0.10)	($1,440; 0.35, 0.65)	($2,160; 0.48, 0.52)
(4; 0.90, 0.10)	($864; 0.23, 0.77)	($1,296; 0.32, 0.68)
(5; 0.90, 0.10)	($518; 0.14, 0.86)	($778; 0.21, 0.79)

By using Eq. (19) the SL depreciation of each year is calculated as in Table 5. Then, defuzzified SL and DDB depreciation values are compared for each year, and it is switched from DDB to SL when the DB depreciation is less than or equal to SL. The transition from DB to SL depreciation can occur at any point within the n-year period, with the aim of determining the most advantageous year for the switch. The rule governing this transition is as follows: If the depreciation amount under DB in any given year is lower than (or equal to) the corresponding amount under SL, it is recommended to switch to the SL method and adhere to it for the entirety of the depreciable lifespan of the project.

Table 5. Comparison of SL and DDB depreciation

If Switch to SL At the Beginning of Year	IF SL Depreciation	Defuzzified SL Depreciation	IF DDB Depreciation	Defuzzified DDB Depreciation	Switching Decision
(2; 0.90, 0.10)	(($6,000; 0.83, 0.17) - ($550; 0.35, 0.65)) / (4; 0.90, 0.10)	$1,079	($2,400; 0.51, 0.49)	$1,081	Do not switch
(3; 0.90, 0.10)	(($3,600; 0.66, 0.34) - ($550; 0.35, 0.65)) / (3; 0.90, 0.10)	$475	($1,440; 0.35, 0.65)	$423	Switch to SL

The bold value ($1,079) in Table 5 is calculated by using Eqs. (14-15) and Eq. (11), respectively as follows.

$$\frac{(\$6{,}000;\ 0.83, 0.17) - (\$550;\ 0.35, 0.65)}{(4;\ 0.90, 0.10)} = \frac{(\$5{,}450;\ 0.75, 0.25)}{(4;\ 0.90, 0.10)} = (1362.50;\ 0.83, 0.17) = \$1{,}079$$

Since it is found that the optimal time to switch from DB to SL is (3; 0.90, 0.10), the depreciation result is calculated as in Table 6.

Table 6. Depreciation results

Year	DDB with Switch to SL	End of Year BV
(1; 0.90, 0.10)	($4,000; 0.70, 0.30)	($6,000; 0.83, 0.17) = $4,798
(2; 0.90, 0.10)	($2,400; 0.51, 0.49)	($3,600; 0.66, 0.34) = $2,174
(3; 0.90, 0.10)	($1,017; 0.53, 0.47)	($2,583; 0.28, 0.72) = $586
(4; 0.90, 0.10)	($1,017; 0.53, 0.47)	($1,143; 0.00, 1.00) = $0
(5; 0.90, 0.10)	($1,017; 0.53, 0.47)	($279; 0.00, 1.00) = $0
	Total: ($9,450; 0.98, 0.02)	

Although, end of year BV is finalized as $0, the total IF DDB depreciation value is not exactly equal to the difference between IF initial cost and IF estimated salvage value. It is because that with the existing IF subtraction and division operations, the difference and the quotient of two IFNs cannot be defined in some cases and therefore have to be assigned to some specific values which means the developed operations are incomplete (Du, 2021).

This is why many different IF subtraction and division operations are proposed in the literature but there is still no consensus on which one should be preferred certainly.

In the second case, where $B_n < S$, the depreciating should stop when it gets down $B_n = S$. Here, suppose that $\tilde{S}$ = ($2,000; 0.40, 0.60) = $680 this time. If the full deduction ($1,440; 0.35, 0.65) = $423 was taken, $B_{(3;\,0.90,0.10)}$ would have been less than $\tilde{S} =$ ($2,000; 0.40, 0.60). Thus, $D_{(3;\,0.90,0.10)}$ is adjusted to ($1,600; 0.43, 0.57) by making $B_{(3;\,0.90,0.10)}$ equal to ($2,000; 0.40, 0.60). Then, the final adjusted depreciation results are presented in Table 7.

Table 7. Adjusted depreciation results

End of Year	D_n	B_n
(1; 0.90, 0.10)	($4,000; 0.70, 0.30)	($6,000; 0.83, 0.17)
(2; 0.90, 0.10)	($2,400; 0.51, 0.49)	($3,600; 0.66, 0.34)
(3; 0.90, 0.10)	($1,600; 0.43, 0.57)	($2,000; 0.40, 0.60)
(4; 0.90, 0.10)	($0; 0.00, 1.00)	($2,000; 0.40, 0.60)
(5; 0.90, 0.10)	($0; 0.00, 1.00)	($2,000; 0.40, 0.60)
	Total: ($8,000; 0.92, 0.08)	

In the following the same autonomous trolley problem will be handed with unit of production depreciation method. Suppose that the trolley's estimated net cost is ($10,000; 0.95, 0.05) and salvage value is ($2,000; 0.90, 0.10) same as before. The trolley is expected to give service for 150,000 miles. Then, the allowed depreciation amount for the trolley usage of 30,000 miles is computed by using Eq. (18).

$$\frac{30{,}000}{250{,}000}\big((\$10{,}000;\ 0.95, 0.05) - (\$2{,}000;\ 0.90, 0.10)\big)$$

$$= \frac{30{,}000\ miles}{250{,}000\ miles}(\$8{,}000;\ 0.50, 0.50)$$

= ($960; 0.08, 0.92) = $59

6. Conclusion

Businesses need new investments in order to grow, increase their sustainability and competitiveness. In economic terms, investment refers to the net additions made to capital goods in a certain period. The company pays attention to many evaluation criteria such as payback period, internal productivity rate, net present value while making investment decision.

As a result of these evaluations, the depreciation amount at the end of the period is also effective in the decision of the enterprise. In the calculation of the depreciation amount, the selected depreciation method is effective. While determining the method, besides the legal legislation, the fund-raising function of depreciation should not be overlooked in terms of business to the extent permitted by the legislation. The same business and the same transactions may cause the values in the financial statement to change with different methods chosen and may also affect the tax amount that the business has to pay, as well as the financing need and cost.

The most used depreciation methods, straight-line method, declining balance method, and unit-of-production method, are analyzed under IF environment in this study to represent the uncertainty in the parameters in a more realistic way. All numbers were left fuzzy until the last possible stage and no defuzzification was made. In this way, fuzzy data is preserved and taken into account until the last stage. Initial invest cost, salvage value, and useful life parameters are evaluated as IF values.

This study makes a significant contribution to the literature by incorporating the use of IFS) in the analysis of depreciation methods, providing a more comprehensive and nuanced approach to decision-making. By considering both membership and non-membership degrees, the study addresses the inherent vagueness and ambiguity in evaluating depreciation methods. It proposes the modification of traditional depreciation methods, such as the straight-line method and declining balance method, through fuzzy reasoning based on IFSs, taking into account cash flow uncertainty and offering investors different depreciation alternatives for informed decision-making. The development of original equations and a framework for considering depreciation under IFSs expands the understanding of how fuzzy sets can be applied in the context of depreciation.

For the further research, it is recommended to perform a comparison analysis with other extensions of fuzzy sets such as Pythagorean fuzzy sets, spherical fuzzy sets, or picture fuzzy sets. Also, the tax depreciation method like MACRS depreciation can be analyzed under fuzzy environment to include the uncertainties in the parameters and gain better and more realistic results.

Conflicts of Interest

The author declared that there is no conflict of interest.

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