Celal Bayar University Journal of Science

Refraction simulation of nonlinear wave for Shallow Water-Like equation

Murat Subaşı[1], Hülya Durur[2]*

[1] Department of Mathematics, Science Faculty, Atatürk University, Erzurum, Türkiye
[2] Department of Computer Engineering, Faculty of Engineering, Ardahan University, Ardahan, Türkiye
* hulyadurur@ardahan.edu.tr
* Orcid: 0000-0002-9297-6873

Received: 19 July 2022
Accepted: 18 November 2022
DOI: 10.18466/cbayarfbe.1145651

Abstract

The generalized (3+1) dimensional Shallow Water-Like (SWL) equation, which is one of the higher dimensional evolution equations, is successfully constructed with aid of the $(1/G')$-expansion method, which is one of the analytical solution instruments in mathematics. Solitary waves are depicted by assigning specific values to the parameters in the SWL equation travelling wave solutions, which have a significant place in physical energy transport. Graphics representing the solitary wave at any given moment are displayed in 2D, 3D and contours. A simulation of the wave is created for different values of the velocity of a solitary wave, which is a physical quantity. In addition, by keeping the parameters other than the rupture event of the wave constant, the situation at which the velocity the wave reaches the breakage event is discussed.

Keywords: Exact solution, $(1/G')$-expansion method, solitary wave.

1. Introduction

When one of the independent variables in the SWL equation depends on the time parameter *t*, it is defined as the nonlinear evolution equation [1]. Observations of physical phenomena and the resulting nonlinear partial differential equations (NPDEs) corresponding to these observations have always been a subject of research since. These research topics have application areas in fluid dynamics, quantum mechanics, materials science, chemistry and similar situations [2-4]. As a result of the observations in these application areas, models of situations related to physical events are created [5-7]. For this reason, firstly, the mathematical equivalents of these models were determined. In the next process, the exact solutions of these NPDEs were investigated with the help of powerful and effective methods. Generally, the solutions obtained were associated with the soliton concept first defined by Russell [8]. From this starting point, these application areas have been continuously developed and rapid progress has been made. One of the causes for this is that the travelling wave solutions found are of various types, which enabled these studies to continue continuously. When the literature is examined, it has been observed that various methods have been developed. Some of these methods: modified sub-equation method [9], Lie transformation method [10], sine-Gordon expansion method [11], the first integral method [12], Hirota bilinear method [13] and so on [14-27].

The (3+1)-dimensional equation of generalized SWL is a nonlinear equation of evolution that has recently become very common [28].

$$u_{xxxy}+3u_{xx}u_{y}+3u_{x}u_{xy}-u_{yt}-u_{xz}=0. \quad \textbf{(1.1)}$$

Recently, there have been several articles on this equation. (G'/G)-expansion method procedure was obtained by Zayed, and travelling wave solutions were obtained with the aid of the generalized binary operator by Zhang [29-30]. With the aid of the Bernoulli sub equation process, wave solutions of Eq. (1.1) were attained by Dusunceli [31].

By using the $(1/G')$-expansion method, we aim to achieve travelling wave solutions for Eq. (1.1) in this study [32,33].

Celal Bayar University Journal of Science
Volume 19, Issue 1, 2023, p 47-52
Doi: 10.18466/cbayarfbe.1145651

The $(1/G')$ method is a method inspired by the (G'/G) method. While three different types of solutions are obtained with the (G'/G) method, the solution obtained in the $(1/G')$ method is different from these three solutions. One of the most significant reasons for choosing this method is to shed light on the shock wave phenomenon with nonlinear wave propagation in shallow waters.

2. $(1/G')$-Expansion Method

This method was introduced to the literature in Yokus' Phd thesis [34]. Let us consider a NPDE as follows:

$$W\left(u,u_t,u_x,u_y,u_z,u_{xx},...\right)=0. \tag{2.1}$$

Where, let

$$u\left(x,y,z,t\right)=U\left(\xi\right)=U,\quad \xi=x+ky+mz-wt,\ w\neq 0.$$

Here k,m are numbers of wave and w is a physical quantity and the velocity parameter of the wave. We can transform (2.1) to the following ODE for $U\left(\xi\right)$:

$$S\left(U,U',U'',...\right)=0. \tag{2.2}$$

Solution of Eq. (2.2) has the form

$$U\left(\xi\right)=a_0+\sum_{i=1}^{n}a_i\left(\frac{1}{G'}\right)^i, \tag{2.3}$$

here a_i, $\left(i=0,1,2,...,n\right)$ are constants, $G(\xi)=G$ provides the following quadratic linear ordinary differential equation (ODE)

$$G''+\lambda G'+\tau=0,\quad \tau,\lambda\in R. \tag{2.4}$$

The solution format produced by the method is as follows:

$$\frac{1}{G'(\xi)}=\frac{1}{-\dfrac{\tau}{\lambda}+A\cosh\left(\xi\lambda\right)-A\sinh\left(\xi\lambda\right)},\quad A\in R. \tag{2.5}$$

Considering that the solution satisfies the equation, (2.3) substituted in Eq. (2.2). To obtain the polynomial $P\left(1/G'\right)=0$. The coefficients of all the powers of $\left(1/G'\right)$ are set to zero. Thus, a system of equations is created. By solving the system of equations, unknown constants are found.

The resulting constants are substituted in Eq. (2.3) and by applying inverse transform, a travelling wave solution of Eq. (2.1) is achieved.

3. Solutions of SWL Equation

Applying the transform $u=u\left(x,y,z,t\right)=U\left(\xi\right)$ in Eq. (1.1), we get

$$kU^4+6kU'U''+(kw-m)U''=0, \tag{3.1}$$

where w represents the velocity of the wave. After Eq. (3.1) is integrated, the following equation is attained

$$kU'''+3k\left(U'\right)^2+(kw-m)U'=0. \tag{3.2}$$

The n balancing term is a constant obtained between the highest order linear term and the highest order nonlinear term in any ODE [34]. So, balancing between highest order linear term U''' with highest nonlinear term $\left(U'\right)^2$ in Eq. (3.2), we find the balancing term $n=2$ and by considering in Eq. (2.3),

$$U\left(\xi\right)=a_0+a_1\left(\frac{1}{G'}\right)+a_2\left(\frac{1}{G'}\right)^2. \tag{3.3}$$

Let us substitute the Eq. (3.3) in the Eq. (3.2) so that the coefficients in the Eq. (3.3) can be calculated. After some mathematical operations, a polynomial equation based on $(1/G')$ is constructed. The coefficient of each term of this polynomial is equal to zero and the following system of equations is created.

$$\begin{aligned}
&\frac{1}{G'[\xi]}: && -m\lambda a_1+kw\lambda a_1+k\lambda^3a_1=0,\\
&\frac{1}{G'[\xi]^2}: && -m\tau a_1+kw\tau a_1+7k\lambda^2\tau a_1+3k\lambda^2a_1^2-2m\lambda a_2+2kw\lambda a_2+8k\lambda^3a_2=0,\\
&\frac{1}{G'[\xi]^3}: && 12k\lambda\tau^2a_1+6k\lambda\tau a_1^2-2m\tau a_2+2kw\tau a_2+38k\lambda^2\tau a_2+12k\lambda^2a_1a_2=0,\\
&\frac{1}{G'[\xi]^4}: && 6k\tau^3a_1+3k\tau^2a_1^2+54k\lambda\tau^2a_2+24k\lambda\tau a_1a_2+12k\lambda^2a_2^2=0,\\
&\frac{1}{G'[\xi]^5}: && 24k\tau^3a_2+12k\tau^2a_1a_2+24k\lambda\tau a_2^2=0,\\
&\frac{1}{G'[\xi]^6}: && 12k\tau^2a_2^2=0,
\end{aligned} \tag{3.4}$$

using a software application, obtain the k,λ,τ,w,a_1,a_2 and m constants, from Eq. (3.4).

Case 1. If

$$a_1=-2\tau,\quad a_2=0,\quad m=k\left(w+\lambda^2\right), \tag{3.5}$$

Celal Bayar University Journal of Science
Volume 19, Issue 1, 2023, p 47-52
Doi: 10.18466/cbayarfbe.1145651

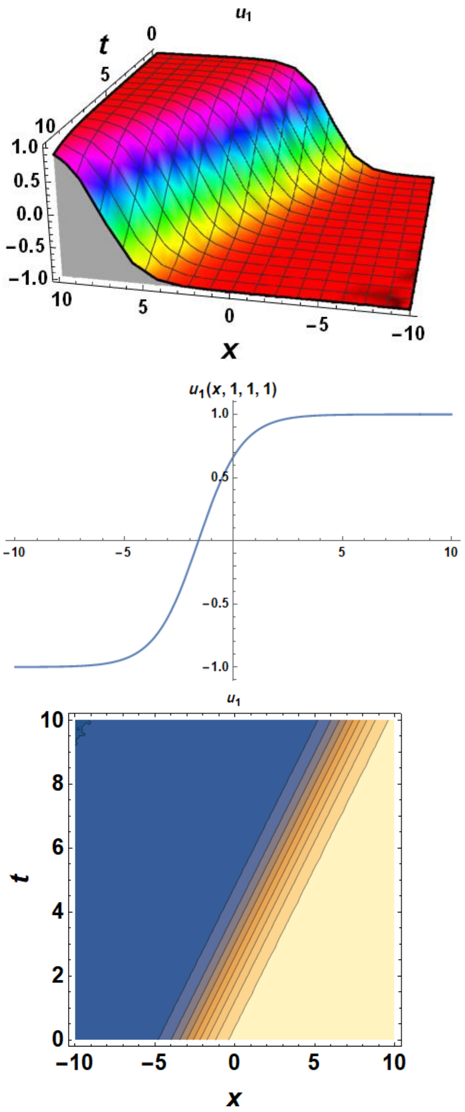

Figure1: Graphs for the Eq. (3.6) for $a_0 = 1,\ \lambda = -1,\ \tau = 0.1,\ w = 1,\ k = 0.1,\ y = 1,\ z = 1,\ A = 1.$

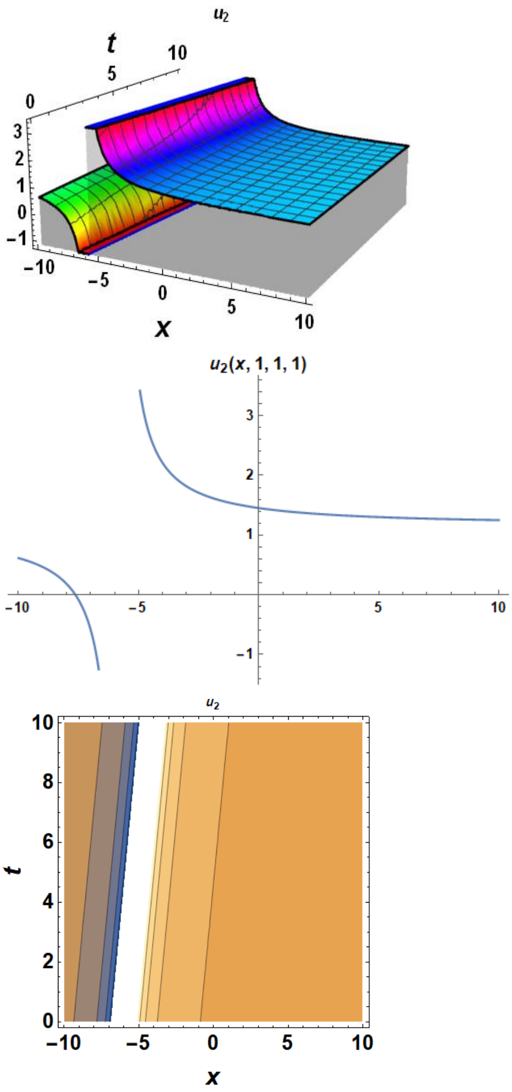

Figure2: Graphs for the Eq. (3.8) for $a_0 = 1,\ \lambda = 0.1,\ \tau = 0.1,\ w = 1,\ k = 5,\ y = 1,\ z = 1,\ A = 1,\ m = 1.$

substituting values Eq. (3.5) into Eq. (3.3) and one may have an exact solution of the hyperbolic type for Eq (1.1):

$$u_1(x,y,z,t) = -\frac{2\tau}{-\frac{\tau}{\lambda} + A\cosh\left[\lambda\left(-tw + x + ky + kz\left(w+\lambda^2\right)\right)\right] - A\sinh\left[\lambda\left(-tw + x + ky + kz\left(w+\lambda^2\right)\right)\right]} + a_0, \tag{3.6}$$

Case 2. If

$$a_1 = -2\tau, \quad a_2 = 0, \quad w = \frac{m - k\lambda^2}{k}, \tag{3.7}$$

substituting Eq. (3.7) into Eq. (3.3), one may have an exact solution of the hyperbolic type for Eq. (1.1):

$$u_2(x,y,z,t) = -\frac{2\tau}{-\frac{\tau}{\lambda} + A\cosh\left[\lambda\left(x + ky + mz - \frac{t\left(m - k\lambda^2\right)}{k}\right)\right] - A\sinh\left[\lambda\left(x + ky + mz - \frac{t\left(m - k\lambda^2\right)}{k}\right)\right]} + a_0, \tag{3.8}$$

4. Results and Discussion

We have attained the travelling wave solution to the generalized SWL equation by aid of the $\left(1/G'\right)$ method in this study. While Duran and Kaya produced travelling wave solutions in hyperbolic, trigonometric and rational forms with the sub equation method in their studies [35], in the Yokus study, travelling wave solutions were produced with the modified Kudryashov method [36]. In Yokus et al. studies, complex hyperbolic type solution was obtained with modified $\left(1/G'\right)$ method [37], and in this study, hyperbolic type

Celal Bayar University Journal of Science
Volume 19, Issue 1, 2023, p 47-52
Doi: 10.18466/cbayarfbe.1145651

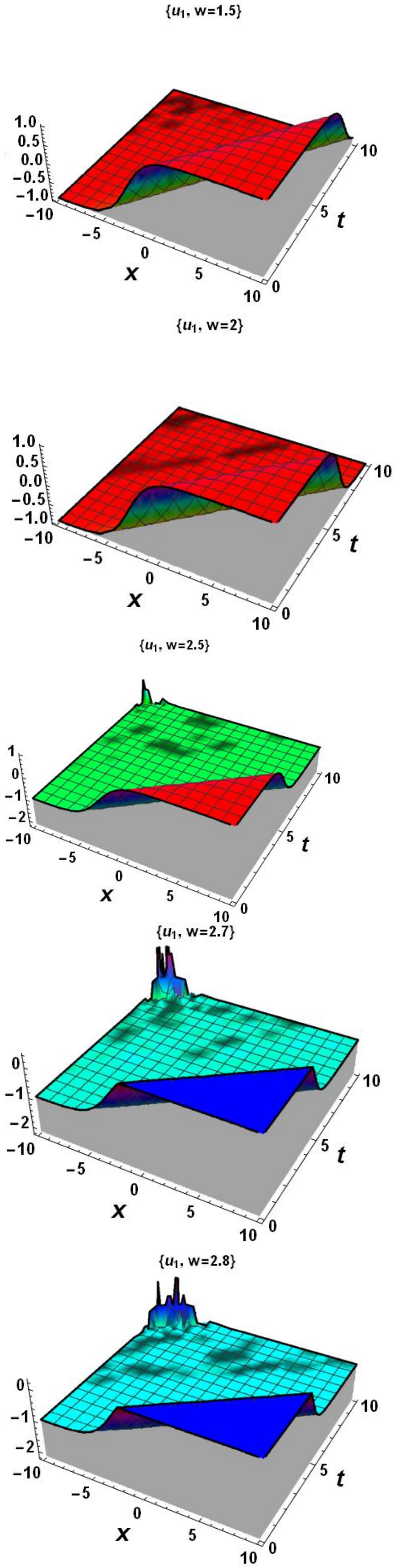

Figure3: Graphs for the Eq. (3.6) for $a_0 = 1, \lambda = -1, \tau = 0.1, k = 0.1, y = 1, z = 1, A = 1.$

solution was produced with the $(1/G')$ method. We especially observed that the $(1/G')$ method is applicable for the SWL equation, which is a high dimensional evolution equation. It was observed that the solution attained by this method supports the solutions obtained in the literature. Solitary wave solutions were created with the aid of particular values given to the travelling wave solutions attained in this article. The effect of changes in the velocity parameter in the solitary solution on wave behavior was analyzed. In this analysis part, other parameters are taken as constant. It has been observed that the change in velocity causes distortions at the extremes of the wave after a certain value. By aid of Mathematica package program, this situation is presented in 3D in the simulation below.

As seen in Figure3, it has been determined that the speed factor is a very effective parameter in solitary wave solutions. As seen in the simulation presented for different values of *"w"* which represents the velocity of the solitary wave, it can be observed that for $w = 2.5$, the wave begins to exhibit behaviors different from the normal behavior at the extreme points. We also observed that the wave was broken for $w = 2.7$. In future studies, taking into account the changes in the coefficients of classical wave transformation, its effect on wave behavior can be examined. When Eq. (3.6) is examined carefully, the velocity parameter w affects the distribution of the travelling wave in the direction z. The wave number in the direction z of the travelling wave cannot be observed in Figure3. Because variable z is considered as constant and $z=1$ is taken. In addition, the parameter λ, which comes from the methodology of the method and is more clearly seen in the Eq. (2.5), plays a significant role in the nonlinear distribution. It is an important parameter that affects both the wave velocity and the wave number.
The theme of discussion in this work is the physical parameter w, which represents the wave velocity. The effects of the wave velocity on the travelling wave solution and the condition that causes the wave to break are a matter of debate. Parameters other than the velocity parameter are out of the scope of this study. In the discussions about the behavior of travelling waves in the future, the k and m parameters, which affect the wave propagation in the y and z directions and represent the wave number, can be taken into account. It is also predicted that these physical discussions are valuable for experimental workers.

5. Conclusion

In this manuscript, the travelling wave solutions of the generalized SWL equation were successfully obtained with the $(1/G')$ method. It was concluded that the $(1/G')$ method for Eq. (1), which is one of the higher dimensional evolution equations, is applicable according to many methods we have given in the introduction. 2D, contour and 3D graphics of these solitary waves are presented. In Figure3, the relation between velocity parameter and $u(x,y,z,t)$ is presented in 3D, provided that the other parameters are taken as constant. The velocity values causing the breaking of the wave were investigated and determined. In future studies, many studies can be done on high dimensional nonlinear evolution equations with the help of this method.

Author's Contributions

Murat Subaşı: Assistance with the production of the manuscript, result interpretation, and analytical analysis on the structure.
Hülya Durur: Conducted literature search, drafted and authored the manuscript, performed the result analysis.

Ethics

Regarding the publication of this manuscript are no ethical issues.

References

[1]. Duran, S. 2021. An investigation of the physical dynamics of a traveling wave solution called a bright soliton. *Physica Scripta,* 96(12), 125251.

[2]. Yokus, A, Yavuz, M. 2021. Novel comparison of numerical and analytical methods for fractional Burger–Fisher equation. *Discrete & Continuous Dynamical Systems-S;* 14(7), 2591-2606.

[3]. Benetazzo, A, Barbariol, F, Pezzutto, P, Staneva, J, Behrens, A, Davison, S, Cavaleri, L. 2021. Towards a unified framework for extreme sea waves from spectral models: Rationale and applications, *Ocean Engineering;* 219: 108263.

[4]. Duran, S. 2020. Exact Solutions for Time-Fractional Ramani and Jimbo—Miwa Equations by Direct Algebraic Method, *Advanced Science, Engineering and Medicine*; 12(7): 982-988.

[5]. Raissi, M, Karniadakis, G, E. 2018. Hidden physics models: Machine learning of nonlinear partial differential equations. *Journal of Computational Physics*; 357: 125-141.

[6]. Duran, S. 2020. Solitary Wave Solutions of the Coupled Konno-Oono Equation by using the Functional Variable Method and the Two Variables (G'/G, 1/G)-Expansion Method. *Adıyaman Üniversitesi Fen Bilimleri Dergisi;* 10(2): 585-594.

[7]. Yokuş, A. 2018. Comparison of Caputo and conformable derivatives for time-fractional Korteweg–de Vries equation via the finite difference method. *International Journal of Modern Physics B;* 32(29): 1850365.

[8]. Russell, J. S. 1845. Report on Waves.

[9]. Duran, S, Yokuş, A, Durur, H, Kaya, D. 2021. Refraction simulation of internal solitary waves for the fractional Benjamin–Ono equation in fluid dynamics. *Modern Physics Letters B;* 35(26): 2150363.

[10]. Saleh, R, Mabrouk, S. M, Wazwaz, A, M. 2021. Lie symmetry analysis of a stochastic gene evolution in double-chain deoxyribonucleic acid system. *Waves in Random and Complex Media*; 1-15.

[11]. Nestor, S, Houwe, A, Rezazadeh, H, Betchewe, G, Bekir, A, Doka, S, Y. 2022. Chirped W-shape bright, dark and other solitons solutions of a conformable fractional nonlinear Schrödinger's equation in nonlinear optics. *Indian Journal of Physics;* 96,243-255.

[12]. Feng, Z. 2002. The first-integral method to study the Burgers–Korteweg–de Vries equation. *Journal of Physics A: Mathematical and General;* 35(2): 343.

[13]. Alhami, R., & Alquran, M. 2022. Extracted different types of optical lumps and breathers to the new generalized stochastic potential-KdV equation via using the Cole-Hopf transformation and Hirota bilinear method. *Optical and Quantum Electronics,* 54(9), 1-12.

[14]. Silambarasan, R, Kilicman, A. 2023. Solitons of nonlinear dispersive wave steered from Navier-Bernoulli hypothesis and Love's hypothesis in the cylindrical elastic rod with compressible Murnaghan's materials. Mathematics and Computers in Simulation; 203, 699-720.

[15]. Yokuş, A. 2021. Construction of different types of traveling wave solutions of the relativistic wave equation associated with the Schrödinger equation. *Mathematical Modelling and Numerical Simulation with Applications,* 1(1), 24-31.

[16]. Kumar, S, Kumar, A, Wazwaz, A, M. 2020. New exact solitary wave solutions of the strain wave equation in microstructured solids via the generalized exponential rational function method. *The European Physical Journal Plus;* 135: 870.

[17]. Park, C, Khater, M, M, Abdel-Aty, A, H, Attia, R, A, Rezazadeh, H, Zidan, A, M, Mohamed, A, B. 2020. Dynamical analysis of the nonlinear complex fractional emerging telecommunication model with higher–order dispersive cubic–quintic. *Alexandria Engineering Journal;* 59(3): 1425-1433.

[18]. Duran, S. 2021. Travelling wave solutions and simulation of the Lonngren wave equation for tunnel diode. *Optical and Quantum Electronics*; 53(8): 58.

[19]. Yokuş, A, Durur, H, Abro, K, A, Kaya, D. 2020. Role of Gilson–Pickering equation for the different types of soliton solutions: a nonlinear analysis. *The European Physical Journal Plus;* 135: 657.

[20]. Li, Z, Manafian, J, Ibrahimov, N, Hajar, A, Nisar, K. S, Jamshed, W. 2021. Variety interaction between k-lump and k-kink solutions for the generalized Burgers equation with variable coefficients by bilinear analysis. *Results in Physics*; 28: 104490.

[21]. Zhang, Z, Li, B, Chen, J, Guo, Q. 2021. Construction of higher-order smooth positons and breather positons via Hirota's bilinear method. *Nonlinear Dynamics;* 105(3): 2611-2618.

[22]. Wang, K, J, Liu, J, H. 2022. On abundant wave structures of the unsteady korteweg-de vries equation arising in shallow water. *Journal of Ocean Engineering and Science.*

[23]. Seadawy, A, R, Rehman, S, U, Younis, M, Rizvi, S, T, R, Althobaiti, S, Makhlouf, M, M. 2021. Modulation instability analysis and longitudinal wave propagation in an elastic cylindrical rod modelled with Pochhammer-Chree equation. *Physica Scripta;* 96(4): 045202.

[24]. Alotaibi, M, F, Omri, M, Khalil, E, M, Abdel-Khalek, S, Bouslimi, J, Khater, M, M. 2022. Abundant solitary and semi-analytical wave solutions of nonlinear shallow water wave regime model. *Journal of Ocean Engineering and Science.*

[25]. Yilmaz, E, U, Khodad, F, S, Ozkan, Y, S, Abazari, R, Abouelregal, A, E, Shaayesteh, M, T, ... Ahmad, H. 2022. Manakov model of coupled NLS equation and its optical soliton solutions. *Journal of Ocean Engineering and Science.*

[26]. Isah, M. A., & Yokuş, A. (2022). The investigation of several soliton solutions to the complex Ginzburg-Landau model with Kerr law nonlinearity. *Mathematical Modelling and Numerical Simulation with Applications*, 2(3), 147-163.

[27]. Özkan, Y, S, Eslami, M, Rezazadeh, H. 2021. Pure cubic optical solitons with improved $$ tan (\varphi/2) $$ tan (φ/2)-expansion method. *Optical and Quantum Electronics;* 53(10): 1-13.

[28]. Baskonus, H, M, Eskitascioglu, E, I. 2020. Complex wave surfaces to the extended shallow water wave model with (2+1)-dimensional. *Computational Methods for Differential Equations;* 8(3): 585-596.

[29]. Zayed, E, M, E. 2010. Traveling wave solutions for higher dimensional nonlinear evolution equations using the G'/G-expansion method, *Journal of Applied Mathematics & Informatics*; 28(1_2): 383-395.

[30]. Zhang, Y, Dong, H, Zhang, X, Yang, H. 2017. Rational solutions and lump solutions to the generalized (3+1)-dimensional shallow water-like equation, *Computers & Mathematics with Application;* 73(2): 246-252.

[31]. Dusunceli, F. 2019. Exact Solutions for Generalized (3+1)-Dimensional Shallow Water-Like (SWL) Equation, *In Conference Proceedings of Science and Technology;* 2(1): 55-57.

[32]. Durur, H, Yokuş, A. 2020. Analytical solutions of Kolmogorov–Petrovskii–Piskunov equation. *Balıkesir Üniversitesi Fen Bilimleri Enstitüsü Dergisi;* 22(2): 628-636.

[33]. Yokus, A, Durur, H, Ahmad, H. 2020. Hyperbolic type solutions for the couple Boiti-Leon-Pempinelli system. *Facta Universitatis, Series: Mathematics and Informatics;* 35(2): 523-531.

[34]. Yokus, A. 2011. Solutions of some nonlinear partial differential equations and comparison of their solutions, Ph.Diss., Fırat University.

[35]. Duran, S., & Kaya D. (2021). Applications of the Sub Equation Method for the High Dimensional Nonlinear Evolution Equation. Erzincan University Journal of Science and Technology, 14(3), 898-906.

[36]. Yokuş, A. (2021). Solitary Wave Solutions of the Generalized (3+ 1)-Dimensional Shallow Water-Like Equation by Using Modified Kudryashov Method. Adıyaman University Journal of Science, 11(1), 157-165.

[37]. Yokuş, A., Duran, S., & Durur, H. (2022). Analysis of wave structures for the coupled Higgs equation modelling in the nuclear structure of an atom. *The European Physical Journal Plus*, 137(9), 1-17.