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Fuzzy linear programming model for inverse multiple criteria sorting problem: an application in a public transportation company

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Abstract

Multi-criteria decision making (MCDM) techniques are among the methods used by the decision maker in the decision phase in practice, as in the literature. MCDM problems are grouped as classifications and sorting problems according to the type of inclusion or ranking in existing classes. Inverse multi-criteria sorting problem, a new topic in recent years, is defined as choosing among existing actions in order to improve the status of alternatives. In this study, in order to find out which action plan should be chosen for providing the minimum cost fort he improvement of public transportation vehicles in case the boundaries of the classes to be assigned are uncertain, the inerse multi-criteria ranking problem is used. Because the boundaries of the classes to be assigned are fuzzy, fuzzy linear programming method will be used to solve the problem.

1. Introduction

Multi-criteria decision making (MCDM) techniques are among the methods used by the decision maker in the decision phase in practice, as in the literature. These techniques are used for solving complex problems. MCDM problems are grouped as classifications and sorting problems according to the type of inclusion or ranking in existing classes (Aires et al., 2018; Arıkan and Cıtak,2017; Parreiras et al., 2019; Hasan et al. 2019).The multi-criteria sorting problem can be described as, the assignment of alternatives under multiple criteria to the predefined classes. For example, rating the hotels according to star and classifying the students according to grading notes are a multi-criterion sorting problem (Özpeynirci et al., 2020).

Inverse multi-criteria sorting problem, a new topic in recent years, is defined as choosing among existing actions in order to improve the status of alternatives (Wolters and Mareschal,1995). In this problem, the decision makers know the initial classes of the alternatives. By performing some of the existing actions, the decision makers can improve the classes of the alternatives. The main goal is to replace the class with more alternatives at minimal cost.

In multi criteria decision making literature, a few researchers considered the effects of the actions to the alternatives according to criteria. Wolters and Mareschal (1995) studied a sorting problem and they presented three sensitivity analysis examples: The first example is the investigating the changes in the criteria values. The second and the thirds ones are respectively a criterion's object value changes effect and the minimum required change for an object to get to the best class.

Wang (2015) presented the first study about the inverse multiple criteria sorting problem. He investigated the actions and the effects of the actions on alternatives. And he proposed a model which could be applied for resorting the alternatives according to these actions.

Mousseau et al. (2018) suggested a study on the inverse multiple criteria sorting problem. They used the UTADIS, MR-sort and cumulative value function for classifying the alternatives. They studied two types of problems: firstly, they wanted to find the action plans which supply the least cost and after they assign the objects under a limited budget. Also, they propose a mathematical programming model for each case.

Ecer et al. (2020) investigated a goal programming model of an inverse multi criteria ranking problem. The goals of this study are the improvement at least cost and the number of alternatives assigned to a particular class. And they tested the model on a real-life problem.

Ecer et al. (2022) aimed an extension of inverse multiple criteria sorting problem with fuzzy parameter with a proper solution approach on area of building energy.

This study proposes a combination of inverse multiple criteria sorting problem and fuzzy linear programming. As it can be seen in the literature research, fuzzy linear programming has never been used in inverse multi-criteria sorting problem. With this aspect, this study has expanded the application of inverse multi-criteria sorting problem and made an important contribution to the literature. Also, there are a few studies about inverse multi-criteria sorting problem in the literature.

The organization of the article is as follows: We define the inverse multi-criteria sorting problem in section 2. After that in section 3; we describe the fuzzy inverse multi-criteria sorting problem and the solution of this problem by using fuzzy linear programming. The next section: we present the application of the proposed approach in real life problem. Finally, we propose the future research areas in this subject in the last section.

2. Inverse Multi-Criteria Sorting Problem (IMCSP)

Inverse multi-criteria sorting problem (IMCSP) is defined as choosing among existing actions to improve the status of alternatives (Mousseau et al., 1995). The purpose of this choosing process is to determine which actions are appropriate to achieve the desired ranking.

The stages of inverse multi-criteria sorting procedure are presented in detail as below:

- The first stage of inverse multi-criteria sorting problem is sorting the alternatives to the predefined ordered classes on several criteria.
- Decision maker can modify the alternatives to the better classes by obtaining a subset of predefined actions.
- The objective of decision maker is finding the best subset of actions with minimal cost or under a given budget.

In this section, the model for the situations in which we know the sorting method and the parameters used to assign the alternatives to classes is illustrated. The objective of this model is cost minimization. The cost minimization is defined as finding the best combinations of actions which is required to achieve the desired classification of the alternatives at minimum cost.

We use the notations of Özpeynirci et al. (2020) and modify if necessary. Let consider the set of alternatives $A = \{A_1, A_2, \dots. A_i\}$ evaluated on a set of criteria $K = \{K_1, K_2, \dots. K_j\}$. The set of ordered classes is denoted by $C = \{C_1, C_2, \dots. C_t\}$ where C_t is the best class of this set. In inverse sorting model; it is possible to change the class of alternative by obtaining the actions from set of actions $E = \{E_1, E_2, \dots. E_k\}$.

The parameters of the model are as shown in Table 1.

Table 1.The Parameters of The Inverse Multi-Criteria Sorting Problem Model

Parameter	Notation
c_k	The cost of action E_k
o_{ij}	The score of alternatives A_i on criterion K_j
o_{ij}'	The new score of alternatives A_i on criterion K_j after performing the selected actions.
δ_{ijk}	The impact of action E_k on the score of alternatives A_i on criterion j
w_j	The weight of criterion j;$w_j \in [0,1]$ and $\sum_{j=1}^{J} w_j = 1$
b_j^h	Upper bound of class C_t for criterion j

The decision variable of the model is;

$$x_k = \begin{cases} 1, if\ the\ action\ k\ is\ selected \\ 0, otherwise \end{cases}$$

The constraints of this model are shown as below (Mousseau et al., 1995);

$$minimize\ \sum_{k=1}^{K} c_k * x_k \tag{1}$$

$subject\ to$

$$o_{ij}' = o_{ij} + \sum_{k=1}^{K} \delta_{ijk} * X_k \qquad \forall i,j \tag{2}$$

$\sum_{j=1}^{J} w_j * o_{ij}' \geq b_j^h \qquad \forall i \qquad (3)$
$x_k \in \{0,1\} \qquad \forall k \qquad (4)$
In equation (1) shows the objective function; the objective of this model is minimizing the total cost of selected actions. In equation (2), the new scores of alternatives A_i on criterion K_j after performing the selected actions are calculated. Equation (3) ensures that all alternatives are assigned to their required classes. The decision whether to choose the actions or not, which is the decision variable of the model, is a binary integer variable and is restricted by Equation (4).

3. The Fuzzy Inverse Multi-Criteria Sorting Problem (FIMCSP)

This study proposes a combination of inverse multiple criteria sorting problem and fuzzy linear programming. In this section, we discuss the inverse multi-criteria sorting problem with the fuzzy parameter. So the model transform to the fuzzy linear model. The model is shown as below:

$$minimize \ \sum_{k=1}^{K} c_k * x_k \qquad (5)$$

$subject\ to$

$$o_{ij}' = o_{ij} + \sum_{k=1}^{K} \delta_{ijk} * X_k \qquad \forall i,j \qquad (6)$$
$$\sum_{j=1}^{J} w_j * o_{ij}' \gtrsim b_j^h \qquad \forall i \qquad (7)$$
$$x_k \in \{0,1\} \qquad \forall k \qquad (8)$$

In the Equation 7, the right-hand side parameter b_j^h is a fuzzy variable with $\left(b_j^h - p_j, b_j^h\right)$. Since it is a minimization model with a constraint containing fuzzy right-hand side value, we will adopt mathematical model proposed by H.J. Zimmermann's membership function (Zimmermann,1985). The membership function for the constraints containing fuzzy right-hand side value is written as in Equation 8:

$$\mu_{i(x)} = \begin{cases} 1 & c\sum_{j=1}^{J} w_j * o_{ij}' > b_j^h \\ 1 - \frac{D_i - \left(\sum_{j=1}^{J} w_j * o_{ij}'\right)}{p_i} & b_j^h - p_t \leq \sum_{j=1}^{J} w_j * o_{ij}' \leq b_j^h + p_t \\ 0 & \sum_{j=1}^{J} w_j * o_{ij}' \leq b_j^h \end{cases} \qquad (8)$$

The membership function for the objective function is written as in Equation 9:

$$\mu_{c(x)} = \begin{cases} 1 & c(x) \leq z_0 \\ 1 - \frac{c(x) - z_0}{z_1 - z_0} & z_0 \leq c(x) \leq z_1 \\ 0 & c(x) \geq z_1 \end{cases} \qquad (9)$$

We will write the fuzzy inverse multi-criteria sorting mathematical model shown as below:
$maksimize\ \alpha$

$$\alpha \leq 1 - \frac{c(x) - z_0}{z_1 - z_0} \qquad \forall k$$

$$\alpha \leq 1 - \frac{D_i - \left(\sum_{j=1}^{J} w_j * o_{ij}'\right)}{p_i} \qquad \forall I$$

$$o_{ij}' = o_{ij} + \sum_{k=1}^{K} \delta_{ijk} * X_k \qquad \forall i,j$$

$$x_k \in \{0,1\} \qquad \forall k$$

With the solution of the new model transformed to fuzzy model, the actions to be carried out will be determined.

4. The Application of The Transportation Vehicle Improvement

In this study, a problem for the improvement of trams used for public transportation in a company in Turkey has been addressed. Company managers were asked to determine actions on trams and improve them in order to meet the increasing demand.

Eight trams (A_1, A_2, A_3, ... ,A_8) used by the company for public transportation has been used. And they are classified into the predefined classes (A, B, C, D and E) according to 5 criterions. The criteria stated by the decision makers as important in the classification of the trams and the weights of these criterions are as shown in Table 2.

Table 2. Criteria Used in Evaluation

Criterion	Description	Weights
Passenger Capacity (K_1)	Shows the seated passenger carrying capacity of the vehicle.	0,23
Comfort (K_2)	It shows the numerical value in the range of 0-1 given by the decision makers for all the factors such as ergonomics of the seats, ventilation of the vehicle, ease of use of the handles. It is calculated as passenger capacity(sitting)/passenger capacity(standing).	0,06
Air Conditioner (K_3)	It shows the power of air conditioner.	0,18
Door Number (K_4)	It shows the total number of doors that passengers can get on and off in the vehicle.	0,25
Engine Number (K_5)	It shows the total number of engines used in the trams.	0,28

Performance values of the trams at the beginning of the application are as shown in Table 3.

Table 3. Current Scores of Trams

	Passenger Capacity	**Comfort Rate**	**Air Conditioner**	**Door Number**	**Engine Number**
Trams	**K_1**	**K_2**	**K_3**	**K_4**	**K_5**
A_1	24	22	21	23	19
A_2	33	25	30	28	29
A_3	26	29	19	25	18
A_4	38	36	18	22	17
A_5	23	11	11	12	12
A_6	32	23	20	25	20
A_7	22	29	26	27	15
A_8	25	26	21	17	17

It is thought that the efficiency of the trams (ET) decreases as the initial performance values given in the Table 3 increase. While "0" means the best performance for each criterion; "40" represents the weakest performance.

The efficiency values of trams (ET) are calculated as in Equation (10):

$$ET_i = \sum_{j=1}^{5} o_{ij} * w_j \, ; \qquad \forall I \tag{10}$$

In this problem, the trams are classified into 5 classes called A, B, C, D and E where the best class is A. Fuzzy boundaries belonging to these classes are as given in Table 4.

Table 4. Fuzzy Boundaries Belonging to Tram Classes

Classes	**Fuzzy Boundaries of Classes** $\left(b_{j-1}^h + p_j, b_j^h\right)$
A	$0 + 1 < ET \leq 8$
B	$8 + 3 < ET \leq 13$
C	$13 + 2 < ET \leq 18$
D	$18 + 3 < ET \leq 24$
E	$24 + 3 < ET \leq 30$

The current efficiency classes of the trams calculated according to the intervals given in Table 4 are given in Table 5.

Table 5. Initial Efficiency Classes of Vehicles

	A1	A2	A3	A4	A5	A6	A7	A8
ET	21,69	29,61	22,43	24,4	14,29	24,19	22,43	20,1
Classes	D	E	D	E	C	E	D	D

The improvement actions can be listed as:

1. Adding air-conditioner
2. Increasing the number of effective doors
3. Increasing engine power
4. Updating the power of engine power on the air conditioner
5. Improving the air conditioner in front of the doors

With the assumption that the improvement actions will increase the tram efficiency, the effects of the actions on the tram performance are expressed with negative values. Some actions can affect more than one criterion at the same time. The effects of the actions on the trams and the cost of each action are shown in Table 6.

Table 6. Effects of Actions on Trams Efficiency

Vehicle	E1			E2		E3			E4			E5		
	K1	K2	K3	K2	K4	K1	K4	K5	K2	K3	K5	K1	K3	K4
A1	-4	-10	-14	-20	-10	-2	-5	-13	-18	-13	-12	-8	-19	-15
A2	-11	-10	-18	-3	-7	-15	-3	-8	-6	-17	-7	-8	-7	-10
A3	-16	-18	-2	-11	-18	-6	-8	-10	-12	-16	-18	-14	-17	-14
A4	-1	-9	-20	-17	-5	-2	-1	-1	-6	-10	-7	-17	-17	-2
A5	-11	-17	-2	-12	-13	-3	-5	-3	-19	-9	-13	-9	-11	-12
A6	-15	-14	-4	-9	-15	-2	-3	-13	-4	-12	-19	-4	-17	-5
A7	-5	-10	-9	-20	-16	-2	-14	-18	-17	-17	-19	-7	-15	-16
A8	-1	-19	-1	-16	-17	-9	-19	-14	-4	-5	-14	-18	-7	-19
Cost	20			30		40			80			10		

The results obtained when the right-side values of the model constraints are solved as if they are at the lower and upper bound are as in the Table 7.

Table 7. The Results of Models

The Right-Side Values of The Model Constraints	In the Lower Bound (z_0)	In the Upper Bound (z_1)
Objective function	10	30
Decision Variables		
x_1	0	1
x_2	0	0
x_3	0	0
x_4	0	0
x_5	1	1

According to these data; the membership function of the fuzzy model's objective function can be written as in Eq. (9):

$$\mu_{c(x)} = \begin{cases} 1 & c(x) \leq 10 \\ 1 - \frac{c(x)-10}{20} & 10 \leq c(x) \leq 30 \\ 0 & c(x) \geq 30 \end{cases}$$

The new fuzzy model created according to this objective function can be written as follows:

$maksimize\ \alpha$

$$\alpha \leq 1 - \frac{c(x)-10}{20} \qquad \forall k$$

$\alpha \leq 1 - \frac{D_i - \left(\sum_{j=1}^{J} w_j * o'_{ij}\right)}{p_i}$ $\forall I$

$o_{ij}' = o_{ij} + \sum_{k=1}^{K} \delta_{ijk} * X_k$ $\forall i, j$

$x_k \in \{0,1\}$ $\forall k$

The obtained fuzzy model has been solved by using Excel Solver and the optimum values of the problem are obtained as in Table 8.

Table 8. The Solution of the Fuzzy Model

Objective function	1
Decision Variables	
x_1	0
x_2	0
x_3	0
x_4	0
x_5	1

According to the results of fuzzy mathematical model, it was decided to choose the fifth action. And this action will allow improving the passenger capacity (K_1), air conditioner (K_3) and number of doors (K_4) criteria of the trams.

5. Conclusions

Inverse multi-criteria sorting method is used to determine the actions that will improve the status of the alternatives in the problems with ordered classes. Therefore, it can be used in many areas.

In this study, the inverse multi-criteria ranking method was used for the selection of alternatives that can improve the determined criteria in the trams belonging to the urban public transportation company. In the problem, the lower boundaries of the classes to which the trams are assigned are considered to be fuzzy and the inverse multi-criteria sorting method has been extended to solve a fuzzy mathematical model. There are five possible actions in the study which is carried out by considering five criteria in company with eight alternative trams. Then one of these possible actions are selected to improve the trams situations according to criteria.

The model proposed in this study can be developed by using different classification approaches or by determining different purposes. In addition, in the study only the lower boundaries of the classes were accepted as fuzzy variables. The model can be expanded if other parameters are also fuzzy variable.
Data Availability Statement: My manuscript has no associated data.

Contribution of researchers

Authors have equal contribution in all the sections.

Conflict of interest

The authors declared that there is no conflict of interest.

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