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MATHEMATICAL MODELLING OF STEPPED SPILLWAYS AERATION ON THE RIVERS

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Abstract: Dissolved oxygen content (DO) is one of the most important water quality parameter in rivers and streams. Hydraulic structures may cause significant impacts on the river water dissolved oxygen concentrations. The oxygen transfer across the air-water interface at a hydraulic structure such as a weir or spillway occurs by self-aeration along the chute and by flow aeration in the hydraulic jump at the downstream end of the structure. In this study; stepped spillway was created small size in a channel. Dissolved oxygen concentrations were measured on this model with various flow rates of water by passing weir. In the light of measured values along the canal, a mathematical model has been developed to how to show a change of oxygen concentration.

Keywords: *oxygen transfer, dissolved oxygen, stepped spillway, mathematical modeling*

1. Introduction

Hydraulic structures may cause significant impacts on the river water dissolved oxygen concentrations. The oxygen transfer across the air-water interface at a hydraulic structure such as a weir or spillway occurs by self-aeration along the chute and by flow aeration in the hydraulic jump at the downstream end of the structure. Spillway type, discharge rate, flow type and channel slope affect the aeration efficiency along the channel length. Spillways with their water-air controlling mechanisms are important for their structural properties and their effects on stream ecology. Spillway types also affect the efficiency of aeration. Environmental conditions and flow rates should be taken into consideration to select the types of spillways (Berkun and Aras, 2007).

Because stepped spillways can significantly reduce the depth and size needed for a stilling basin at the toe of a dam and lead to great economic benefit, stepped spillways are increasingly attractive in engineering. But, at present, except for some formulas for the calculation, the study of stepped spillways is based only on physical models. Furthermore, most studies have focus on factors related to energy dissipation.

Thanks to the technological advances in construction of roller compacted concrete (RCC) dams over the past two decades, stepped spillways for discharging excess flood water have gained significant interest and popularity among researchers and dam engineers, both for new dams for armouring of existing embankment dams. The use of stepped spillways has enhanced the performance and economy of many RCC dams where the concrete placement in lift allows an economic and fast construction of spillway steps on the downstream dam faces (Frizell, 1992).

A stepped chute consists of an open channel with a series of drops in the invert. The flow over stepped spillways can be divided into two regimes; nappe flow and skimming flow. The nappe flow regime is defined as a succession of free falling nappes. The flowing waters bounce from one step to the next one as a series of small free falls. Nappe flows on horizontal

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steps are characterised typically by the presence of hydraulic jumps. A nappe flow without hydraulic jump might occur for relatively large discharge, before the apparition of skimming flow. Along a chute with horizontal steps, a typical nappe flow situation consists of a series of free-fall jets impinging on the next step and followed by a hydraulic jump. The flow energy is dissipated by jet break up in air, by jet impact and mixing, on the step and by the formation of a hydraulic jump on the step. Stepped channels with nappe flows can be analysed as a succession of drop structures (Fig 1).

For large discharges, the waters flow down a stepped channel as a coherent stream, skimming over the steps. In the skimming flow regime, the external edges of the steps form a pseudo-bottom over which the water skims. Beneath the pseudo-bottom, recirculating vortices develop, filling the zone between the main flow and the steps. The vortices are maintained through the transmission of shear stress from the fluid flowing past the edges of the steps. In addition small scale vorticity is generated continuously at the corner of the step bottom. Most of the flow energy is dissipated to maintain the circulation of the recirculation vortices (Aras and Berkun, 2006).

On stepped chutes with skimming flow regime, the flow is highly turbulent and the conditions for free surface aeration are satisfied. The free surface aerated flow region follows a region where the flow over the chute is smooth and glassy. Next to the boundary, however, turbulence is generated and the boundary layer grows until the outer edge of the boundary layer reaches the surface. When the outer edge of the boundary layer reaches the free surface, the turbulence initiates natural free surface aeration. The location of the start of air entrainment is called the point of inception. Downstream of inception point, a layer containing a mixture of both air and water extends gradually through the fluid. Far downstream the flow will become uniform and for a given discharge any measure of flow depth, air concentration and velocity distributions will not vary along the chute. This region is defined as the uniform equilibrium flow region (Fig 1).

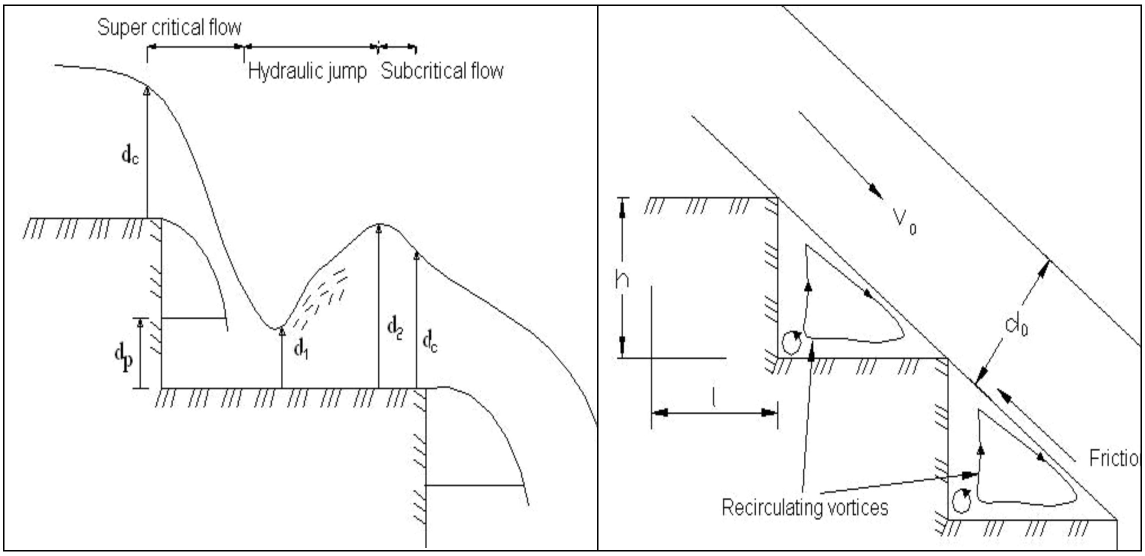

Figure 1. Nappe and skimming flow.

2. Aeration on Stepped Spillways

Air is entrained at each step by a plunging jet mechanism at the intersection of the over falling jet and the receiving waters, and at the toe of the hydraulic jump. With deep pooled steps, most of the air is entrained by the plunging jet mechanism. For flat steps with shallow waters, most of the air is entrained at the toe of the hydraulic jump. The air entrainment characteristics of hydraulic jumps were analysed by a number of researchers.

On a stepped spillway with skimming flow, the entraining region follows a region where the flow over the spillway is smooth and glassy. Next to the boundary however turbulence is generated and the boundary layer grows until the outer edge of the boundary layer reaches the surface. When the outer edge of boundary layer reaches the free surface, the turbulence can initiate natural free surface aeration. The location of the start of air entrainment is called the point of inception. Downstream of the inception point of free-surface aeration, the flow becomes rapidly aerated and free surface appears white. Far downstream the flow will become uniform and for a given discharge any measure of flow depth, air concentration and velocity distributions will not vary along the chute. This region is defined as the uniform equilibrium flow region (Fig 2).

Further presence of air within high-velocity flows may prevent or reduce the damage caused by cavitation. Cavitation erosion may occur on stepped spillways. But the risks of cavitation damage are reduced by the flow aeration. Peterka and Russell and Sheehan showed that 4 to 8 % of air concentration next to the spillway bottom may prevent cavitation damage on concrete surfaces. On stepped spillways, the high rate of energy dissipation reduces the flow momentum in comparison with a smooth chute. The reduction of flow velocity and the resulting increase of flow depth reduce also the risks of cavitation as the cavitation index increases (Chanson, 1994, Peyras et al 1992).

For the designer the flow bulking is estimated from the total quantity of air entrained while the prevention of cavitation damage requires the knowledge of the air concentration in the layers close to the spillway bottom. The reduction of the drag observed with air entrainment will reduce the energy dissipation above the spillway and hence its efficiency. It must be emphasised that the reduction of the friction factor observed with increasing mean air concentration is not completely understood. The presence of bubbles across the flow and the bubble size distribution is expected to affect the turbulence and the turbulent mixing mechanisms (Avery and Novak 1978).

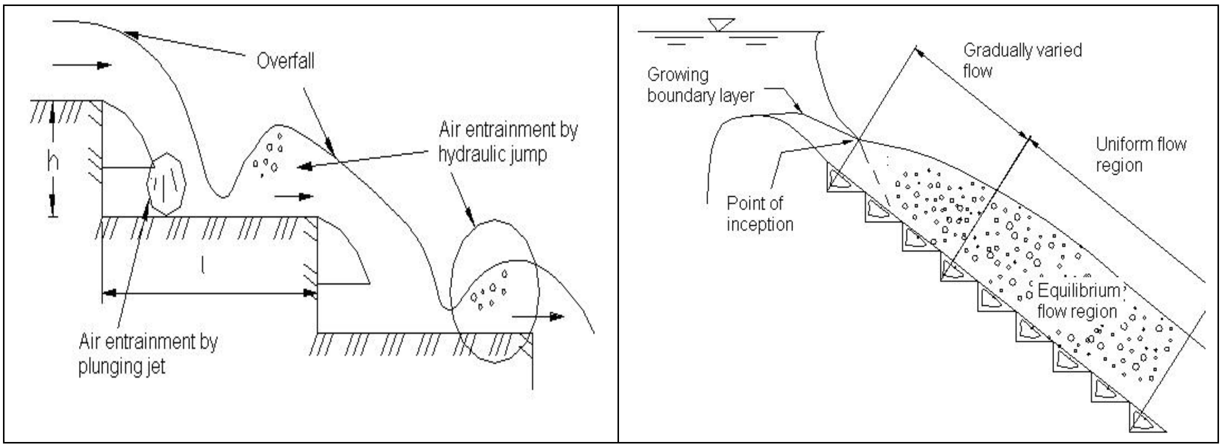

Figure 2. Aeration on stepped spillway

2.1. Dissolved oxygen content on the rivers

Aeration on weirs and photosynthetic activity may cause exceeded DO saturation levels along the streams (Bowie et al., 1985). This saturation higher status or super saturation case can cause proliferation of plants and rapid increase of algae may appear at the base of the stream (Hibbs and Gulliver, 1997). Overall gas transfer at cascades and overflow weirs can be measured by the deficit ratio (r) defined as:

$$E = \frac{C_{DS} - C_{US}}{C_S - C_{US}} = 1 - \frac{1}{r} \tag{1}$$

Where; E: Aeration efficiency, CUS: Upstream dissolved gas concentration , CDS: Dissolved gas concentration at the downstream end of the channel, CS: Saturation concentration, r: Deficit ratio (Chanson, 1994)

Upstream DO deficit greater than 2.5 mg/ℓ is normally required to obtain accuracy in the oxygen transfer efficiency measurements. Primary source of the measurement uncertainty was found to be an uncertainty in the oxygen saturation concentration determinations. In summer time the average saturation concentration about 7 mg/ℓ results an upstream DO of less than 4.5 mg/ℓ. For comparability of the results, aeration efficiency reference temperature is usually taken as 20 ºC:

$$E_{20} = 1 - (1 - E)^{1/fT} \tag{2}$$

$$f_T = 1.0 + 0.02103(T - 20) + 8.261x10^{-5}(T - 20)^2 \tag{3}$$

Where; E20 : Aeration efficiency at 20 ºC, T : Temperature, fT : Coefficient related temperature (Baylar et al., 2007)

3. Materials and Methods

This study was carried out using a laboratory model weir. Experimental set-up comprised an open channel consisting of a non-recirculated mechanism. Channel dimensions were: 400cm length x 15 cm height x 7.5 cm width. A pump causes water to flow from the main tank to the edge of the channel. Water passes through a damper and reaches the spillway. Overflowing water from the spillway flows along the channel and travels into the discharge measuring bucket system (Fig 3). Dissolved oxygen concentrations were measured at the upstream and downstream ends for various flow rates using an oxygen meter (HACH HQ10) at 10°C water temperature. Calculated deficit ratio and aeration efficiency values were converted to 20°C.

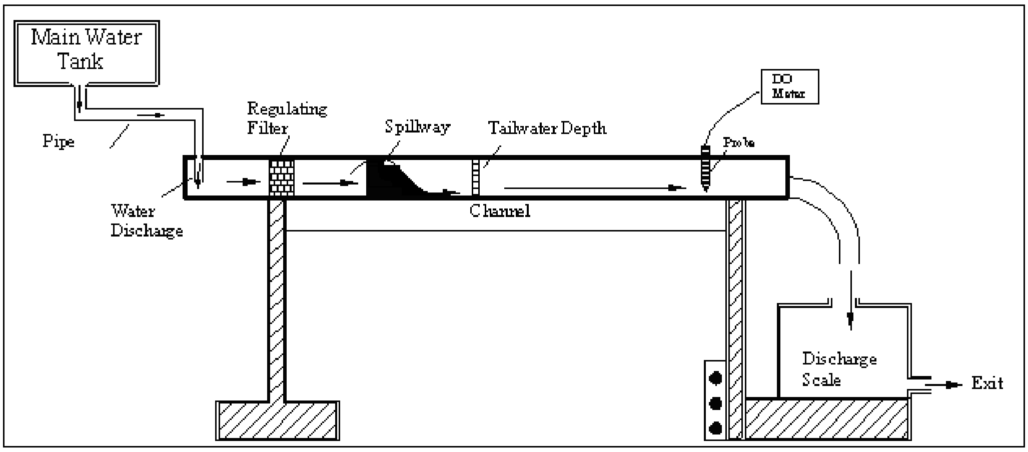

Figure 3. Experimental setup

Dissolved oxygen concentration measured at upstream of the spillway (A) and four different point (B, C, D, E) of downstream of the spillway (Fig 3). The discharge of the spillway was 0.10, 0.15, 0.20, 0.32, 0.40 lt/sn respectively and channel slope 0°, 0.5°, 1.0°, 1.5° respectively. Dissolved oxygen concentration has measured by oxygen meter. Two distinct flow regimes occur on stepped spillways, so-called nappe and skimming flow. Whereas in nappe flow the steps act as a series of overfalls with the water plunging from one step to another, the water flows as a coherent stream over the pseubottom formed by the outer step edges in skimming flow, without air pockets under the jets. Nappe flow is found for low discharges (0.10, 0.15, 0.20 lt/sn) and skimming flow is found high discharges (0.32 and 0.40 lt/sn) (Aras, 2009).

4. Result and Discussion

Regression studies were made with 5 different methods (linear, quadratic, cubic, 4th degree and exponential) to analyze the dissolved oxygen variations at four points along the channel. Under Matlab program, created equations and curves can be used in accordance with the intended targets. Discharge variations at each point were investigated. Equations were developed to obtain the dissolved oxygen concentrations for different flow rates in the channel (Aras, 2009).

Regression analysis about the dissolved oxygen concentrations and discharge changes at four points on the channel were made and analysis of each of the following equations were obtained (x : Discharge, t : Points, f : Dissolved oxygen concentration):

Linear approach:

$$f=\begin{cases}4.233x+13.65, & C_B \quad 0\le t\le 1\\ 4.045x+13.46, & C_C \quad 1\le t\le 2\\ 3.779x+13.3, & C_D \quad 2\le t\le 3\\ 4.398x+12.97, & C_E \quad 3\le t\le 4\end{cases} \tag{4}$$

Quadratic approach:

$$f=\begin{cases}6.169x^2+1.127x+13.96, & C_B \quad 0\le t\le 1\\ 11.81x^2-1.899x+14.06, & C_C \quad 1\le t\le 2\\ 16.08x^2-4.317x+14.12, & C_D \quad 2\le t\le 3\\ 10.93x^2-1.107x+13.53, & C_E \quad 3\le t\le 4\end{cases} \tag{5}$$

Cubic approach:

$$f=\begin{cases}187.2x^3-132.8x^2+32.02x+11.95, & C_B \quad 0\le t\le 1\\ 170.8x^3-115x^2+26.29x+12.22, & C_C \quad 1\le t\le 2\\ 206.9x^3-137.4x^2+29.82x+11.9, & C_D \quad 2\le t\le 3\\ 259.8x^3-181.8x^2+41.76x+10.74, & C_E \quad 3\le t\le 4\end{cases} \tag{6}$$

4th degree approach:

$$f=\begin{cases}948.9x^4-696.6x^3+153.7x^2-5.596x+13.29, & C_B \quad 0\le t\le 1\\ 416.3x^4-155.2x^3-46.6x^2+25.77x+11.3, & C_C \quad 1\le t\le 2\\ 1564x^4-1336x^3+379.7x^2-37.35x+14.31, & C_D \quad 2\le t\le 3\\ 2029x^4-1816x^3+551.5x^2-62.51x+15.39, & C_E \quad 3\le t\le 4\end{cases} \tag{7}$$

Exponential approach

$$f=\begin{cases}13.2709e^{0.4335x}, & C_B \quad 0\le t\le 1\\ 13.4918e^{0.2777x}, & C_C \quad 1\le t\le 2\\ 13.3328e^{0.2626x}, & C_D \quad 2\le t\le 3\\ 13.0142e^{0.3101x}, & C_E \quad 3\le t\le 4\end{cases}$$

(8)

As a result of the regression analysis, curves and equations of dissolved oxygen concentration changes under different discharge values at the downstream of spillway can be given as follows, (X: Distance, Y:Dissolved oxygen concentration):

Equations for $Q_1=0.10$ l/sn:

$$F(X)=Y=\begin{cases}-0.1947X+14.07\\ -0.05332X^2-0.0347X+14.01\\ -0.02665X^3+0.0666X^2-0.1599X+14.02\\ -0.006055X^4+0.009677X^3+0.1236X+14.02\\ 13.5983e^{-0.0145X}\end{cases} \tag{9}$$

Equations for $Q_2 = 0.15$ l/sn:

$$F(X) = Y = \begin{cases} -0.2533X + 14.35 \\ 0.001X^2 - 0.2833X + 14.36 \\ -0.01113X^3 + 0.0601X^2 - 0.3357X + 14.37 \\ -0.005464X^4 + 0.02165X^3 + 0.3029X + 14 \\ 14.3555e^{-0.0181X} \end{cases}$$

(10)

Equations for $Q_3 = 0.20$ l/sn:

$$F(X) = Y = \begin{cases} -0.1687X + 14.52 \\ 0.04502X^2 - 0.3037X + 14.57 \\ 0.0102X^3 - 0.00005X^2 - 0.2567X + 14.57 \\ 0.000004545X^4 + 0.009989X^3 + 0.2567X + 14.57 \\ 14.525e^{-0.0118X} \end{cases} \quad (11)$$

Equations for $Q_4 = 0.32$ l/sn :

$$F(X) = Y = \begin{cases} -0.2693X + 14.71 \\ 0.03X^2 - 0.3593X + 14.74 \\ 0.03553X^3 - 0.1299X^2 - 0.1923X + 14.73 \\ 0.01181X^4 - 0.03532X^3 + 0.2632X + 14.73 \\ 14.7093e^{-O.O188X} \end{cases} \quad (12)$$

Equations for $Q_5 = 0.40$ l/sn :

$$F(X) = Y = \begin{cases} -0.172X + 15.47 \\ 0.02997X^2 - 0.2619X + 15.5 \\ 0.01555X^3 - 0.04X^2 - 0.1888X + 15.5 \\ 0.003636X^4 - 0.006268X^3 + 0.2107X + 15.5 \\ 15.4752e^{-0.0113X} \end{cases} \quad (13)$$

5. Concslusion

The results of our experimental study indicate that the aeration efficiency of nappe flows is primarily a function of the discharge and channel length. The oxygen transfer increases with increasing chute length. Also the oxygen transfer increases with the discharge for a constant ratio Hdam/dc. In skimming flow situations, aeration efficiency is nearly zero as long as no free-surface aeration occurs. When free-surface aeration takes place on the stepped channel, the oxygen transfer increases sharply with the dam height. The largest aeration efficiencies are obtained with small discharges. The analysis shows also that the channel slope and number of steps have little effect on the aeration efficiency of skimming flows.

According to the method of least squares, the magnitude of regression errors, the estimated standard error, the determination and correlation coefficient must be calculated to determine the significance of the regression analysis model (Table 1-2).

Table 1. Error analysis for discharge-dissolved oxygen change

METHODS	Size of Regression Error	Standard Deviation	Estimated Standard Error	Coefficient of Determination	Correlation Coefficient
Linear	0.1162	0.5503	0.1968	0.9041	0.9508
Quadratic	0.1023	0.5503	0.2262	0.9156	0.9568
Cubic	0.0001245	0.5503	0.0112	0.9999	0.9999
4th Degree	0.4796	-	-	-	-
Exponential	0.4262	-	-	-	-

Table 2. Error analysis for dissolved oxygen-distance change

METHODS	Size of Regression Error	Standard Deviation	Estimated Standard Error	Coefficient of Determination	Correlation Coefficient
Linear	0.0965	0.5657	0.2197	0.8994	0.9484
Quadratic	0.0852	0.5657	0.2918	0.9113	0.9546
Cubic	2.1955	-	-	-	-
4th Degree	11.1720	-	-	-	-
Exponential	0.0963	0.5657	0.2194	0.8990	0.9480

Quadratic approach showed very good results for dissolved oxygen-distance correlation. Supersaturated dissolved oxygen concentrations occurred during the laboratory model studies. Aeration efficiency is over 100%. Therefore, obtained graphs and equations can be used to predict the dissolved oxygen concentrations for the supersaturasyon cases.

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