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A Nano Topology Based Assessment with Parameter Reduction in Mathematics Education

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Abstract
The aim of this study is to evaluate the effectiveness of using nano-topology to assess parameters such as readiness to learn, in-class performance, responsibility, parental awareness, behavior, interest, end-of-term achievement scores, and exam scores in mathematics education. In addition, traditional statistical methods and machine learning techniques are suggested for similar evaluations. This study aims to provide further information on the use of nano-topology, a new method for mathematics education.

Keywords: Rough set, nano topology, decision making, alternative evaluation, evaluation algorithm

INTRODUCTION

Although many definitions related to the concept of the classical set have been proposed from the past to the present, the inadequacy of these definitions emerges in the present epoch where information and data processing are inevitable. Moreover, within the framework of these definitions, it is revealed that classical set theory cannot fulfil its function in real-life problems. Carrying the concept of the set to a deeper understanding and interpretation, Zadeh (1965) introduced the fuzzy set concept. Fuzzy sets allow partial membership together with the absolute membership used in the classical sets. The rough set theory introduced by Pawlak (1982), on the other hand, has a structure that considers uncertain situations and fuzzy sets. Structures such as fuzzy and rough sets organize incomplete and uncertain information through information systems and decision tables, making the data suitable for processing. The scientific world started to work on structures that can represent and interpret ambiguous, incomplete, and indistinguishable structures upon the realization that most of the approaches designed for certain structures cannot solve some real-life problems. In the last decade, expressions defined as ambiguous or fuzzy, which have found tremendous applications in engineering, medicine, computing, space, and even social sciences, are increasingly common. In this sense, it is essential to do research on topology, generalized fuzzification, or decision making and contribute to the field (Al Shumrani, Topal, Smarandache, & Özel, 2019). Mathematical modelling of information and data is one of today's most pressing issues. Extracting relevant results and information, interpreting and creating automatic systems to interpret, especially from the information and data provided, is an unavoidable cornerstone of today's scientific understanding. Not every piece of data or information has a precise judgment or answer in real-world circumstances. One approximation type to imprecise data in this structure is the fuzzy set theory developed by Zadeh.

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Similarly, the rough set theory introduced by Pawlak is also capable of interpreting structures without strict boundaries. Structures such as fuzzy and rough sets organize incomplete, insufficient, and uncertain information, making it suitable for data analysis. Rough set theory is a successful mathematical tool that overcomes uncertainties and incomplete information. This study aims to investigate how nano-topology obtained from approximation spaces (rough sets) can help decision-making processes in systems that are much more complex and need to be evaluated together. This study has content and application that hopes that nano-topology, which has many applications in medicine (Thivagar & Antoinette, 2019; Thivagar & Priyalatha, 2017; Thivagar, Richard, & Paul, 2012; Thivagar & Richard, 2013; Thivagar & Richard, 2014; Thivagar & Richard, n.d.; Thivagar & Vijayarajan, 2016), will contribute to education and evaluation processes.

The different pedagogical approaches that have emerged so far have shown that there is no single type of education and that each education approach includes a system and methodology. For this reason, a valid and reliable understanding of assessment is needed to understand whether a pedagogical understanding is valid and reliable. Barootchi and Keshavarz (2002) suggested that alternative assessment, also known as a non-traditional assessment, is a general term for types of assessment other than conventional standardized tests. In the literature, it has been named as "alternative evaluation", "informal evaluation", "authentic evaluation", "performance evaluation", "descriptive evaluation", and "direct evaluation" (Hancock, 1994; Hamayan, 1995). In this context, considering that new measurement and evaluation approaches are emerging in education day by day, alternative assessment-evaluation; includes multiple-choice, timed, several classroom assessments, and evaluations that differ from a trial approach that characterize most standard tests, which aim to evaluate the student more fully and take their cognitive, motor, affective, ethical and other areas' development into account (Zmbicki, 2007). Alternative assessment is a type of assessment proposed as a modern approach because of the reasons such as the focus of traditional assessments on student achievement in isolation from the real world, inadequacy of assessment tools to show what students can do, assessment results that are not used effectively by teachers to identify learning difficulties and provide feedback to students, and its main aim focusing on grading student achievement rather than focusing on learning or educational experiences (Karaca, 2008). In this sense, alternative assessment involves authentic evaluations that include assessing skills by assigning students a task in the desired learning area and using assessment tools (rubrics) whose effectiveness and reliability are ensured in that task and associating the learning with real-life situations (Adanalı & Doğanay, 2007). Alderson and Banerjee (2001) claimed that alternative assessment involves procedures that are less formal than conventional tests, collected over a period rather than taken at one point at a time, and are often formative rather than summative in function, often with low risk and beneficial return effects in terms of results. Syaifuddin (2020) showed that most teachers implemented authentic assessments in mathematics teaching as designed in the learning plan. Arifin (2018)suggested that teachers use this instrument in learning linear mathematics programs to produce accessible and precise measurements. Darling-Hammond et al. (2020) drew out the implications for school and classroom practices of an emerging consensus about the science of learning and development, outlined in recent research synthesis. Mirian et al. (2020) showed that mathematics curriculum reform alone does not guarantee changes in the nature of assessment without changing mathematics teachers' conceptions regarding its purposes. Sabri et al. (2019) showed that assessment is carried out not only by the teacher but also by the student himself. Safitri et al. (2019) argued that improvement is required in the mastery of the skills competency of graduates.

In this study, an alternative evaluation and decision-making method is used by studying gains put forward in education. Thus, with a more holistic and formative understanding, it is aimed to present a more consistent understanding of evaluation in terms of being based on a mathematical basis. For this purpose, firstly, four different grade students studying at the 5th-grade level of Secondary School Mathematics is asked to be evaluated by four different teachers according to the subjects in the curriculum. Then, it is aimed to compare these evaluations with the exam results and other evaluation (Creswell, 2012; Fraenkel, Wallen, & Hyun, 2012.

On this axis, it is discussed whether the nano-topology method is an evaluation tool in education. For this purpose, the effect level of the nano-topology method is examined by examining whether the qualification of the success of the students evaluated with the nano-topology method in the scope of the research predicted their exam success.

The importance of this paper is that, together with the research subject's originality, the studies to be put forward are expected to have an impact and place in many artificial intelligence and decision-making systems and areas such as educational sciences and learning gains. It is aimed not only to produce a product in terms of mathematics but also to build algorithms and smart systems for decision making.

Preliminaries

Many learning approaches in the literature characterize the properties of the system by examining data sets containing all the information about a system. However, it may not always be possible to access the entire system-specific data set in real-life problems, or the data may not be complete. Rough sets are an approach developed by Pawlak(1982) for modelling imperfect and incomplete information. It is an effective mathematical tool in the reasoning and information extraction of systems used to organize data to make it suitable for analysis in the presence of incomplete, insufficient, and uncertain data.

Since the basic concepts of the rough set theory are similar to the basic concepts of topology, a relationship has been established between topology and rough sets (Thivagar & Richard, 2014). Any decomposition of a non-empty set forms the base of the topology on that set. The equivalence relation, a special kind of relation, can be passed starting from this decomposition. The equivalence relation, which forms the basis of the rough sets, has a limiting feature in studies. For example, in a set where the universal set is three cities, the neighbourhood relations on a single line should be defined. In this case, the city located in the middle of the line can switch between other cities, but the equivalence relation will not be used because there is no transition feature between the cities at both ends of the line. To overcome the limitation of this feature, in the theory of rough sets, relations containing more useful functions are suggested instead of equivalence relations during the application to daily life. Thus, the rough set theory has been generalized (Bayhan & Şen, 2018). In 1970, Levine introduced the concept of closed sets as a generalization of closed sets in topological spaces. Palaniappan & Chandrasekhara (1993) examined the concept of regular generalized closed sets in a topological domain. Maki et al. (1996) introduced the concepts of generalized pre-closed sets and pre-generalized closed sets similarly. In 1977, Gnanambal introduced the concept of generalized pre-ordered closed sets in topological spaces. In 2011, Bhattacharya introduced the representation of generalized regular closed sets in topology. Nano b-open sets in nano topological spaces and pre regular nano T1/2 spaces are studied by Parimala et al. (2016) and Parvathy and Praveena (2017).

Here, we give the basic mathematical definitions and properties to be used in this study.

Definition 2.1 A set-valued information system (SIS) is a quadruple $S = (U, A, V, f)$ where U is a non-empty finite set of objects, A is a finite set of condition attributes, V_a is a domain of the attribute $a \in A$, $V = \bigcup_{a\in A} V_a$ is the set of attributes values, and $f: U \times A \to P(V)$ is a mapping such that $f(x, a) \subseteq V_a$, for all $x \in U$ and $a \in A$. (Qian, Dang, Liang, & Tang, 2009)

Moreover, if a SIS contain a decision attribute, then it is called a set-valued decision information system (SDIS) and is denoted by $S = (U, A \cup \{d\}, V, D, f)$ where d is a decision attribute such that $A \cap \{d\} = \emptyset$. Here, d has two or more labels. D denotes the set of all the labels of d. For example, if d is "success", then D can consist of two labels - "0" (unsuccessful) and "1" (successful) - or of 100 labels - "0", "1", "2", ..., and "99". Besides, in an SDIS, $V = \bigcup_{a \in A} V_a$, and $f: U \times A \cup \{d\} \to P(V) \cup D$ is a mapping such that $f(x,a) \subseteq V_a$ and $f(x,d) \in D$, for all $x \in U$ and $a \in A$.

Definition 2.2 Let $S = (U, A, V, f)$ be an SIS, $x, y \in U$ and $a \in A$. Then, a is referred to as inclusion increasing preference, if $f(y,a) \subseteq f(x,a)$ means that "x is much better than y". Similarly, $a \in A$ is called to as inclusion decreasing preference, if $f(y,a) \subseteq f(x,a)$ means that "y is much better than x". If the values of some objects under a condition attribute can be ordered according to an inclusion increasing/decreasing preference, then the attribute is an inclusion criterion. In an SIS, if every condition attribute is an inclusion criterion, then the SIS is referred to as a set-valued ordered information system (SOIS). Moreover, a set-valued ordered decision information system is denoted by SODIS (Qian, Dang, Liang, & Tang, 2009)

The dominance relation in (Qian, Dang, Liang, & Tang, 2009) is presented as follows by modifying it to operate in the present paper:

Definition 2.3 Let $S = (U, A, V, f)$ be an SOIS, $A_1 \cup A_2 = A$, A_1 be the set of inclusion increasing preference attributes, and A_1 be the set of inclusion decreasing preference attributes. Then, a relation (Qian, Dang, Liang, & Tang, 2009)

$$R_A^{\geqq} := \{(x,y) \in U \times U: \forall a \in A_1, f(y,a) \subseteq f(x,a) \text{ and } \forall a \in A_2, f(x,a) \subseteq f(y,a)\}$$

is called a dominance relation on U. Here, $(x,y) \in R_A^{\geqq}$ is also denoted by $x \geq_A y$ (or commonly $y \leq_A x$) and is called "x is dominant over y" (or commonly "x is must better than y").

Here, if $A_2 = \emptyset$, then the aforesaid dominance relation can be written as follows:

$$R_A^{\geqq} := \{(x,y) \in U \times U: \forall a \in A, f(y,a) \subseteq f(x,a)\}$$

Proposition 2.4 Let $S = (U, A, V, f)$ be an SOIS and $R_A^{\geqq}$ be a dominance relation on U. Then, $R_A^{\geqq}$ is reflexive, non-symmetric, and transitive (Qian, Dang, Liang, & Tang, 2009.

Definition 2.5 Let $S = (U, A, V, f)$ be an SOIS and $R_A^{\geqq}$ be a dominance relation on U. Then, for all $x \in U$, the dominance class of x is denoted by $[x]_A^{\geqq}$ and is defined as $\{y \in U: (y,x) \in R_A^{\geqq}\}$. Moreover, $U_A^{\geqq}$ denotes the family of the dominance classes, that is, $U_A^{\geqq} := \{[x]_A^{\geqq}: x \in U\}$. Here, $U_A^{\geqq}$ is not a partition of U, but is a cover of U, that is $U = \bigcup_{x \in U} [x]_A^{\geqq}$ (Qian, Dang, Liang, & Tang, 2009).

Definition 2.6 Let $S = (U, A, V, f)$ be an SOIS, $R_A^{\geqq}$ be a dominance relation on U, and X be a subset of U. Then, the upper and the lower approximation of X are defined as $U_A^{\geqq}(X) := \{x \in U: [x]_A^{\geqq} \cap X \neq \emptyset\}$ and $L_A^{\geqq}(X) := \{x \in U: [x]_A^{\geqq} \subseteq X\}$, respectively. The boundary region of X is defined by $B_A^{\geqq}(X) := U_A^{\geqq}(X) - L_A^{\geqq}(X)$ (Qian, Dang, Liang, & Tang, 2009.

Definition 2.7 Let $S = (U, A, V, f)$ be an SOIS and B be a subset of A. Then, B is said to be a criterion reduction of S if $R_A^{\geqq} = R_B^{\geqq}$ and $R_M^{\geqq} \neq R_A^{\geqq}$, for all $M \subseteq B$. That is, a criterion reduction B is a minimal attribute subset of A satisfying $R_A^{\geqq} = R_B^{\geqq}$. (Qian, Dang, Liang, & Tang, 2009)

Definition 2.8 Let $S = (U, A, V, f)$ be an SOIS and $R_A^{\geqq}$ be a dominance relation on U. Then, the core of A is defined as $CORE(A) := \{a \in A : R_A^{\geqq} \neq R_{A-\{a\}}^{\geqq}\}$. (Qian, Dang, Liang, & Tang, 2009)

Definition 2.9 Let $S = (U, A, V, f)$ be an SOIS, X be a subset of U, and $R_A^{\geqq}$ be a dominance relation on U. Then, $\tau_A^{\geqq}(X) := \{U, \emptyset, U_A^{\geqq}(X), L_A^{\geqq}(X), B_A^{\geqq}(X)\}$ is a topology on U concerning X and is referred to as nano-topology corresponding to the dominance relation. Here, $\beta_A^{\geqq}(X) := \{U, \emptyset, L_A^{\geqq}(X), B_A^{\geqq}(X)\}$ is basis of $\tau_A^{\geqq}(X)$. (Thivagar & Richard, 2014; Thivagar & Richard, n.d.)

Definition 2.10 Let $S = (U, A, V, f)$ be an SOIS and $\beta_A^{\geqq}$ be the basis of $\tau_A^{\geqq}$. Then, $CORE\ (A) = \{a \in A : \beta_A^{\geqq} \neq \beta_{A-\{a\}}^{\geqq}\}$. (Thivagar & Richard, 2014)

The $CORE(A)$ algorithm provided in (Thivagar & Richard, 2014) is presented as follows by modifying it to operate in the present paper:

Algorithm 2.11

Input: An SODIS $S = (U, A \cup \{d\}, V, D, f)$,

A dominance relation $R_A^{\geq}$ over U,

$X = \{x \in U : f(x,d)$ satisfies the consired label of the decision attribute$\}$

Output: The set of all the indispensable attributes $CORE(A)$

The Reduced SODIS $S_{R_A^{\geq}} = (U, CORE(A) \cup \{d\}, V, D, f)$

Step 1 Represent S as an information table whose columns labelled by attributes and rows by objects.

Step 2 Obtain the lower approximation $L_A^{\geq}(X)$ and the boundary region $B_A^{\geq}(X)$ of X concerning $R_A^{\geq}$.

Step 3 Generate the basis $\beta_A^{\geq}(X)$ of the nano-topology $\tau_A^{\geq}(X)$.

Step 4 Obtain $L_{A-\{a\}}^{\geq}(X)$ and $B_{A-\{a\}}^{\geq}(X)$ of X concerning $R_{A-\{a\}}^{\geq}$, for all $a \in A$.

Step 5 Generate the basis $\beta_{A-\{a\}}^{\geq}(X)$ of the nano-topology $\tau_{A-\{a\}}^{\geq}(X)$, for all $a \in A$.

Step 6 Obtain $CORE(A) = \{a \in A : \beta_A^{\geq} \neq \beta_{A-\{a\}}^{\geq}\}$

METHOD

To obtain the data, 80 students in 5th-grade from 4 different secondary school classes in Bitlis province were determined as the population. At the end of the semester, five students were selected from each class with the classification of very good level, good level, intermediate level, passed level and failed level in terms of extreme case sampling, which is among the purposeful sampling techniques since extreme Case Sampling focuses on participants with unique or special characteristics in which the student's level of the success is taken as unique characteristics. As a result, a sub-sample of 20 students was created from this sample.

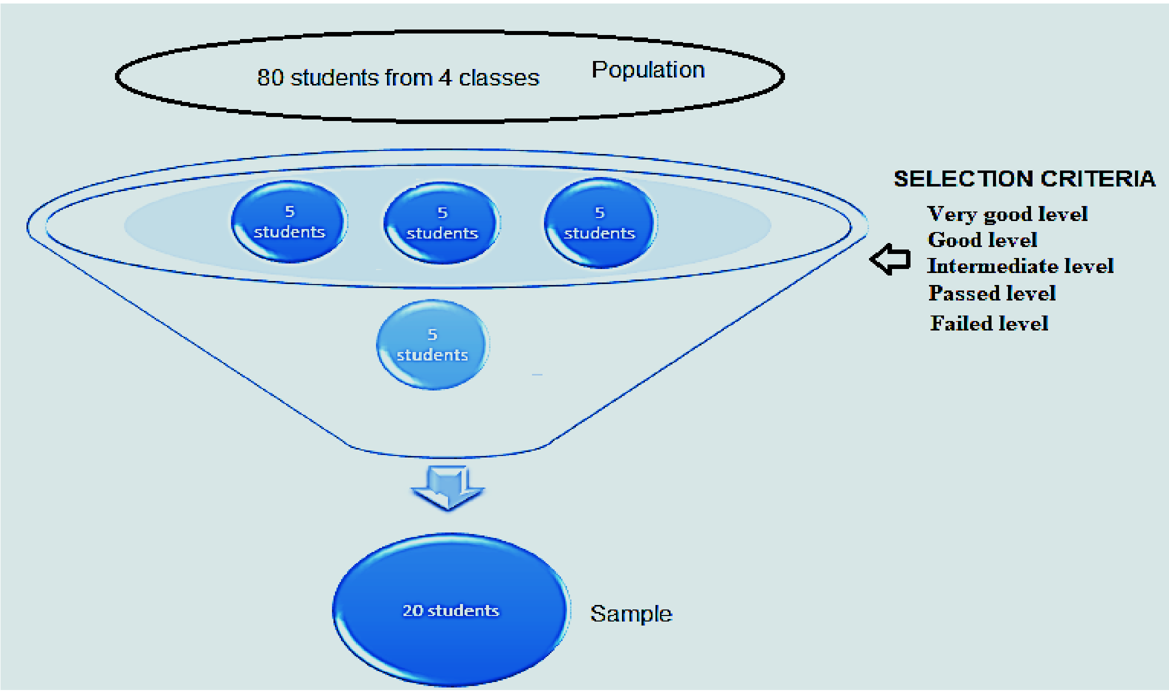

Fig. 1 The population and the sample of the study

As in every course, many features need to be acquired beforehand or afterwards to learn the mathematics lesson and reach the desired level of success. In other words, specific symptoms must be seen for mathematical success to occur. In addition to these features, for the measurement of success to be healthy, the situation of the students should be examined in a certain period. This review period has been accepted as a period in our opinion at the secondary school level. In addition, the exams are given to students during one semester, and the average of the exam applied at the end of the semester is also considered as a feature for student success. The appendix, which includes the final exam data, can be found in the last section of the paper.

Table 1 shows the characteristics determined for mathematics achievement, how much the students have these qualities are (sufficient/moderate/insufficient), the average results of their exams, and general scores (for success, 1 and failure, 0) assigned by their teachers.

These characteristics of the students specified in Table 1 and their degree of having these characteristics were examined by the course teachers as a result of the holistic examinations in one semester. For example, the awareness of responsibility was rated due to the general impression of the student's responsibilities during and outside the lesson. The in-class performance evaluation grades to be given to the student are not included in the end-of-term success score for objectivity.

Just as in the case of the data obtained for the health status of a person and the disease status are evaluated by experts or doctors in this field, the success level of the student can be evaluated according to the characteristics of the student by teachers who are the experts in this profession. As a result of the characteristics of the students and the grades they got from the exams, success and failure situations were created by the teachers of the course.

Table 1 Considered Information System (S: Sufficient, IS: Insufficient, M: Moderate, Succesfull:1, Unsuccesfull:0)

U /A	Learning readiness	In-class performance	Responsibility	Parental awareness	Behaviour	Interest	Success points at the end of the term	Exam points	Success
s_1	S	S	S	S	M	S	92,5	86	1
s_2	S	S	S	S	M	S	74	63	0
s_3	S	S	S	S	S	S	99	100	1
s_4	M	M	S	IS	S	S	69	46	0
s_5	S	S	S	S	S	S	89,5	85	1
s_6	M	M	S	M	IS	M	77,5	58	1
s_7	S	S	S	S	S	S	77,5	55	0
s_8	IS	IS	IS	M	IS	IS	39,5	38	0
s_9	IS	S	S	IS	S	S	44,5	38	0
s_{10}	S	S	S	S	S	S	100	100	1

$\mathbf{S_{11}}$	S	S	S	S	S	S	90	76	1
$\mathbf{S_{12}}$	IS	IS	M	S	IS	M	66	60	0
$\mathbf{S_{13}}$	S	S	M	S	M	S	75,5	65	1
$\mathbf{S_{14}}$	S	M	S	S	S	M	77,5	55	1
$\mathbf{S_{15}}$	IS	IS	IS	IS	IS	IS	7,5	21	0
$\mathbf{S_{16}}$	IS	M	IS	M	S	S	53	45	0
$\mathbf{S_{17}}$	S	S	IS	M	M	S	83,5	90	1
$\mathbf{S_{18}}$	S	S	S	M	S	S	65	65	0
$\mathbf{S_{19}}$	S	S	S	S	S	S	93	95	1
$\mathbf{S_{20}}$	M	M	S	IS	M	M	63,5	70	0

In Table 1, all the features that will affect the student's success were examined. The success and failure situations were determined by experts, namely the teachers of the course. For example, for S_{16}, the student was deemed unsuccessful because the readiness level was insufficient, the in-class performance was moderate, the responsibility level was insufficient, the parental awareness was moderate, the behaviour was sufficient, the interest in the course was sufficient, and the grade average was low.

Table 3 in the appendix shows where the grades in the exam points column, which we put forward as a feature, come from. In addition, it is to demonstrate that there is no visible difference between the data obtained from the table and the exam we made. The subjects in this exam will be considered as a criterion since they include most of the achievements of the 5th grade 1st semester mathematics subjects. Children's psychological perspectives on the lesson were added as criteria to consider in their academic dominance as well as their behaviours.

For the formation of dominance classes and the emergence of nano topology, students with appropriate criteria should be selected in Table 2. Students with the same characteristics can be removed and added to the table. In other words, if S_{11} is included instead of S_7, the same topology can be obtained again.

Since nano topology is formed thanks to the student space selected according to the criteria, $CORE(A)$ is made on the criteria of these students. First, the topology must be created.

DATA ANALYSIS

This section applies Algorithm 2.11 to determine the main characteristics affecting the 5th-grade students' mathematics achievements and their success levels. To this end, 8 students are randomly selected from Table 1. The purpose here is to determine a random sub-sample. Random sample selection is to increase the scope validity of our method and to make the method feasible.

Let $U = \{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8\}$ represent eight students randomly selected from Table 1, d: Success, $D = \{0 \text{ (unsuccessful)}, 1 \text{ (successful)}\}$, $X = \{x \in U : f(x,d) = 1\} = \{x_2, x_4, x_7, x_8\}$ be the set of their successful ones, and $A = \{a_1, a_2, a_3\}$ be the condition-attribute set – non-student-derived characteristics, the characteristics of the student before and during the education, and the characteristics that show their performances – such that

$a_1 = \{\text{Parental Awareness(PA)}\}$

$a_2 = \{\text{Learning Readiness (LR), Behaviour(B), Interest(I)}\}$

$a_3 = \{\text{In} - \text{Class Performance(P), Responsibility(R), Grade(G)}\}$

Remark 4.1 Grade (G) represents here**,** for a mathematics course, a course is the average of two exams (Success points at the end of the term and Exam points in Table 1) taken during the semester. In-class performance grade is not included in this average in terms of objectivity.

Step 1 The table representation of the SODIS is as follows:

Table 2 Considered SODIS $S = (U, A \cup \{d\}, V, D, f)$

U /A	a_1	a_2	a_3	d
$x_1 = S_2$	{PA}	{LR, I}	{R}	0
$x_2 = S_{15}$	{PA}	{LR, B}	{R, G}	1
$x_3 = S_{18}$	{PA}	{LR, B}	{P}	0
$x_4 = S_1$	{PA}	{LR, I}	{P, R, G}	1
$x_5 = S_7$	{PA}	{LR, B, I}	{P, R}	0
$x_6 = S_9$	∅	{B, I}	{P, R}	0
$x_7 = S_{13}$	{PA}	{LR, I}	{P, G}	1
$x_8 = S_{17}$	∅	{LR, B}	{P, G}	1

Step 2 According to Table 2, the dominance classes of $x_i \in U$ and $U_A^{\gtrsim}$ are as follows:

$[x_1]_A^{\gtrsim} = \{x_1, x_4, x_5\}$

$[x_2]_A^{\gtrsim} = \{x_2\}$

$[x_3]_A^{\gtrsim} = \{x_3, x_5\}$

$[x_4]_A^{\gtrsim} = \{x_4\}$

$[x_5]_A^{\gtrsim} = \{x_5\}$

$[x_6]_A^{\gtrsim} = \{x_5, x_6\}$

$[x_7]_A^{\gtrsim} = \{x_4, x_7\}$

$[x_8]_A^{\gtrsim} = \{x_8\}$

$U_A^{\gtrsim} = \{\{x_1, x_4, x_5\}, \{x_2\}, \{x_3, x_5\}, \{x_4\}, \{x_5\}, \{x_5, x_6\}, \{x_4, x_7\}, \{x_8\}\}$

Therefore, $L_A(X) = \{x_2, x_4, x_7, x_8\}$, and $B_A(X) = \{x_1\}$.

Step 3 The basis of the nano-topology over U is as follows:

$$\beta_A^{\gtrsim}(X) = \{U, \emptyset, \{x_2, x_4, x_7, x_8\}, \{x_1\}\}$$

Step 4 For $a_1 \in A$, $U_{A-\{a_1\}}^{\gtrsim} = \{\{x_1, x_4, x_5\}, \{x_2\}, \{x_3, x_5\}, \{x_4\}, \{x_5\}, \{x_5, x_6\}, \{x_4, x_7\}, \{x_8\}\}$, $L_{A-\{a_1\}}^{\gtrsim}(X) = \{x_2, x_4, x_7, x_8\}$, and $B_{A-\{a_1\}}^{\gtrsim}(X) = \{x_1\}$

For $a_2 \in A$, $U_{A-\{a_2\}}^{\gtrsim}(X) = \{\{x_1, x_2, x_4, x_5\}, \{x_2, x_4\}, \{x_3, x_4, x_5, x_7\}, \{x_4\}, \{x_4, x_5\}, \{x_4, x_5, x_6\}, \{x_4, x_7\}, \{x_4, x_7, x_8\}\}$

$L_{A-\{a_2\}}^{\gtrsim}(X) = \{x_2, x_4, x_7, x_8\}$, and $B_{A-\{a_2\}}^{\gtrsim}(X) = \{x_1, x_3, x_5, x_6\}$

For $a_3 \in A$, $U^{\geq}_{A-\{a_3\}}(X) = \{\{x_1, x_4, x_5\}, \{x_2, x_3, x_5\}, \{x_5\}, \{x_5, x_6\}, \{x_1, x_2, x_3, x_4, x_5, x_7\}, \{x_2, x_3, x_5, x_8\}\}$

$L^{\geq}_{A-\{a_3\}}(X) = \emptyset$, and $B^{\geq}_{A-\{a_3\}}(X) = \{x_1, x_2, x_3, x_4, x_7, x_8\}$

Step 5 For $\beta^{\geq}_{A-\{a_1\}}(X) = \{U, \emptyset, \{x_2, x_4, x_7, x_8\}, \{x_1\}\}$, $\beta^{\geq}_{A}(X) = \beta^{\geq}_{A-\{a_1\}}(X)$

For $\beta^{\geq}_{A-\{a_2\}}(X) = \{U, \emptyset, \{x_2, x_4, x_7, x_8\}, \{x_1, x_3, x_5, x_6\}\}$, $\beta^{\geq}_{A}(X) \neq \beta^{\geq}_{A-\{a_2\}}(X)$

For $\beta^{\geq}_{A-\{a_3\}}(X) = \{U, \emptyset, \{x_2, x_4, x_7, x_8\}, \{x_1, x_2, x_3, x_4, x_7, x_8\}\}$, $\beta^{\geq}_{A}(X) \neq \beta^{\geq}_{A-\{a_3\}}(X)$

Step 6 $CORE(A) = \{a_2, a_3\}$

Consequently, the attribute a_1 is ineffective over the success of the considered students according to the SODIS.

RESULTS And RECOMMENDATIONS

In the current era of advanced technology and information, the need for efficient data processing methods is crucial in every field. This allows for better organization and analysis of the vast amounts of available data to make accurate inferences. The use of rough set methods has become increasingly popular due to their effectiveness. In this study, we employed basic concepts of rough set theory to evaluate the performance of 5th-grade students. By doing so, we avoided relying solely on a single exam result which may not fully capture the students' abilities. It is important to note that exams limited by time constraints and specific subject material may not be sufficiently comprehensive for assessing overall success in a course.

Thanks to the rough set theory, the characteristics that will affect the success of the course were determined. Features are grouped according to the factors in the emergence of the features. Students were monitored during a lecture period to see if the students had these characteristics. The degree of presence of these features in the student has been determined. Table 1 and Table 2 were created by adding the end-of-term course grades and exam results to these features.

The resulting information systems and decision tables formed the basis of the rough set theory and topology synthesis. In other words, it helped to provide all the conditions for the creation of nano-topology.

In the educational literature, data processing and decision-making studies have been carried out by means of many rough set studies. In our work, an educational study has been put in place through the rough set theory and nano-topology that emerges from the synthesis of topology, which will be an alternative to these studies.

Our study is essential in terms of alternative assessment and evaluation methods in education. In general, the assessment concept is included in the literature by focusing on alternative assessment tools, but studies on alternative assessment methods are thought to be insufficient in terms of many respects. The alternative assessment is a process that includes the evaluator's decision-making processes such as analysis, synthesis, and decision-making, which cannot be reduced only to the results of alternative measurement tools. Measurement, which provides convenience in many works in our daily life, is to express the properties of an object with numbers or adjectives (Thivagar & Richard, 2014). In this context, tools that provide these qualities are called measurement tools. Expressing the results of observations made quantitatively and qualitatively for any object based on a criterion is the evaluation (Lashin & Medhat, 2005). However, it should be noted that the concept of criteria meant here has a narrow meaning, and the evaluation process has an algorithmic nature that includes making decisions according to certain criteria. In today's education system, equalizing assessment and evaluation in general poses serious problems in terms of reaching more holistic and realistic evaluations.

For example, the Descriptive Branched Tree is an assessment tool that allows students to conclude by placing the propositions about the concepts in a particular subject on the tree diagram, by answering these propositions as true or false, and thus, aiming to identify the information patterns and misconceptions in students' mental structures (Yaşar, 2011; Bahar, Nartgün, Durmuş, & Bıçak, 2015). Since it is an assessment tool, it includes evaluation at a basic level, but it does not show the holistic features revealed by the alternative evaluation approach. It is necessary to analyse more than one measurement source in the evaluation process and synthesize them according to an algorithm. Alternative assessments, also called performance tests or authentic assessments, determine what students can and cannot do instead of what students know or not know. In other words, an alternative assessment focuses on competence more than assessing the information. Typical examples of alternative assessments include portfolios, project work, and other activities requiring a rubric. The essence of performance assessment is that students are allowed to do one or more of the following:

- Show their abilities
- Performing a meaningful task
- Get feedback from a qualified person on relevant and defensible criteria

In short, the purpose of using alternative assessments is to assess students' competence in performing complex tasks that are directly related to learning gains. Although it is generally recommended to make more holistic evaluations by employing more than one assessment tool in the literature, it is seen that there is not much scientific research on which algorithm and according to which logic these assessments will be made, therefore the evaluation side is neglected to some extent. For this reason, it can be stated that this study contributes to the literature by presenting a mathematical model to the evaluation process in education. To be more precise, we can say that this study focused on developing an evaluation algorithm based on alternative assessment and evaluation principles in the assessment and evaluation process, as shown in Fig. 2 below.

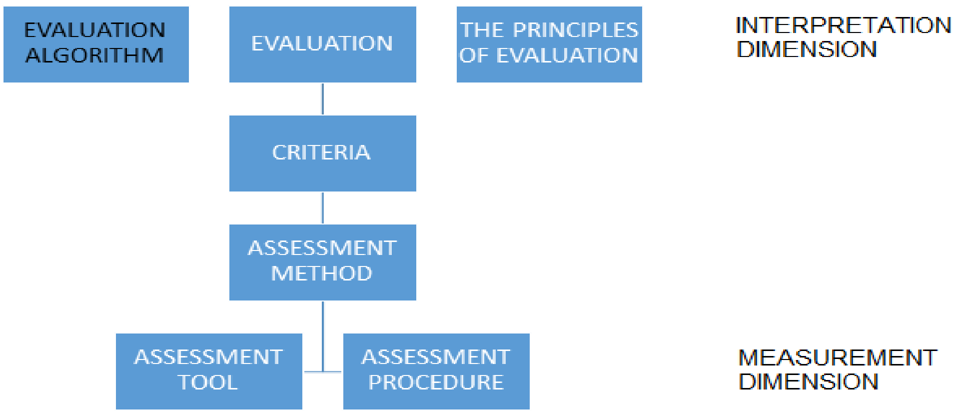

Fig. 2 Dimensions of assessment and evaluation

The relationship between success in mathematics lessons and exam grades cannot be fully understood without considering the factors that impact learning during the lesson. Information systems and decision tables, created using rough set theory, were used to identify these factors. Nano-topology was then utilized to analyze and interpret these systems. The analysis was based on a reduced set of attributes obtained through the application of nano-topology to information systems. This study employed this approach to evaluate the mathematics achievement of 5th-grade students. However, more detail is needed on how the rough set theory and nano-topology were applied in practice and how they specifically contributed to the analysis and interpretation of the relevant data.

As can be understood from this study, several factors such as readiness to learn, behaviour, interest, in-class performance, and responsibility are important indicators of mathematics achievement for 5th-grade students. The study found that parental awareness was not always necessary for student success. Additionally, the algorithm and CORE(A) were found to be effective in reducing features for evaluating student achievement, while exam grades showed a linear relationship with mathematics achievement. However, the study also highlighted that exams alone should not be the sole measure of success. Lastly, nano-topology was identified as a potential alternative evaluation method for 5th-grade mathematics achievement.

An alternative assessment and evaluation is a student-centred approach and focuses on the real-life application of knowledge and skills, taking into account the individual characteristics of the students. While traditional assessment and evaluation only deal with cognitive domain behaviours, the alternative approach observes emotional and psychomotor behaviour developments. Portfolios, projects, performance assignments, concept maps, structured grids, descriptive branched trees, word association, self-assessment and peer-assessment are accepted as alternative assessment and evaluation tools (Kepek, 2019) . As Hancock (1994) supported the use of alternative assessment, which fosters autonomy on the grounds that teachers have broader evidence to judge students' competencies, language programs become more sensitive to individual differences, and students are equipped with lifelong skills. Therefore, we used nano-topology when interpreting the mathematics achievements of 5th-grade students. The advantages and disadvantages of the evaluation algorithm we use can be given as follows:

Advantages:

1) They provide a way to assess valuable skills that cannot be directly assessed with traditional tests and assessments.
2) They provide a more realistic approach to student performance and achievement than traditional tests.
3) They focus on student performance and the quality of the work performed by the students.
4) They can be easily matched with established learning gains.

Disadvantages:

1) The process can be costly in terms of time, effort, equipment, materials, facilities or funds.
2) The grading process can sometimes be more subjective than traditional exams.

CONCLUSION

There are many measurement and evaluation methods in education. Thanks to these methods, the success and achievement levels of the students can be measured and evaluated more objectively. In this study, we focused on an alternative method that we can use to evaluate the students' success levels in education and interpret it more deeply. Generally, topology addresses many different areas and deals with many different situations. At the same time, the importance of the analysis of topologically based methods, which serves as a bridge between mathematics and science, is increasing in the education field. On the other hand, the rough set theory is an effective mathematical tool in the interpretation and information extraction of systems used to organize the data and make it suitable for analysis in the presence of incomplete, insufficient and uncertain data. The synthesis of rough set theory and topology will be the basis of our method.

In this study, the characteristics that will affect the 5th-grade students' mathematics achievements and their success levels in these lessons were examined. These features having such great importance have been revealed with the help of information systems, and decision tables were created using the features revealed in the information system. We have tried to reach a result by synthesizing the data obtained in decision tables by using topology. The topology we use in this study is called nano-topology. The kernel of the features that will affect the success was found by applying an algorithm and reducing the feature to the nano-topology we obtained. However, it has been revealed that the main characteristics of the students can determine the achievements or success of the students.

CONFLICT OF INTEREST

The authors declare no conflict of interest.

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APPENDIX

See Table 3 for subjects, achievements, and evaluation of the final exam

The exam held at the end of the term was composed of the first five subjects of the 5th-grade mathematics course. Points are equally distributed according to the number of questions regarding the achievements in the subject. Topics and achievements related to the subjects are given below.

1. Natural Numbers

- ($K_{1.1}$): Reads and writes at most nine-digit natural numbers.
- ($K_{1.2}$): Specifies the divisions and digits of up to nine-digit natural numbers and the digit values of the numbers.
- ($K_{1.3}$): Forms the required steps of the number and figure patterns given the rule.

2. Operations with Natural Numbers I

- ($K_{2.1}$): Performs the addition and subtraction of natural numbers with up to five digits.
- ($K_{2.2}$): Does the multiplication of two natural numbers with three digits at most.
- ($K_{2.3}$): Divides a four-digit natural number at most by a two-digit natural number.

3. Operations with Natural Numbers II

• $(K_{3.1})$: Represents the square and cube of a natural number as an exponential expression and calculates its value.

• $(K_{3.2})$: Finds the result of bracketed expressions that contain up to two types of operations.

• $(K_{3.3})$: Solves problems involving four operations.

4. Fractions

• $(K_{4.1})$: Understands that a compound number is the sum of a natural number and a simple fraction. Converts an integer fraction to a compound fraction and a compound fraction to an integer fraction.

• $(K_{4.2})$: Understands that simplification and expansion will not change the value of the fraction, and it creates fractions that are equivalent to a fraction.

• $(K_{4.3})$: Sorts the numerators or denominators equal fractions.

5. Operations with Fractions

• $(K_{5.1})$: Calculates the desired simple fraction of a multiplicity and a simple fraction of a whole given multiple using unit fractions.

• $(K_{5.2})$: Makes the addition and subtraction of two fractions whose denominators are equal or whose denominator is a multiple of the other's denominator and makes sense.

• $(K_{5.3})$: Solves and sets up problems requiring addition and subtraction with fractions whose denominators are equal or whose denominator is a multiple of the denominator of the other.

The evaluation results of the exam are as in Table 6. In Table 6, the students are symbolized by indexing s_i. The questions answered correctly by the students are indicated with the positive (+) symbols, and the questions made incorrectly by the negative (-) symbols. Question sequence numbers are added under the topics and learning gains in the table. The learning gains symbolized by $(K_{1.1})$, $(K_{2.2})$, ..., etc., as stated above. The exam has been evaluated over 100 points. Points are equally distributed according to the number of questions regarding the gains in the subject. In other words, 20 points given for questions on each subject are equally distributed among the gains.

Table 3 Analysis of the exam held at the end of the term

	Naturel Numbers					**Operations with Natural Numbers I**				**Operations with Natural Numbers II**				**Fractions**					**Operations with Fractions**		
	$(K_{1,1}$	$(K_{1,1}$	$(K_{1,2}$	$(K_{1,3}$	$(K_{1,3}$	$(K_{2,1}$	$(K_{2,1}$	$(K_{2,2}$	$(K_{2,3}$	$(K_{3,1})$	$(K_{3,2}$	$(K_{3,2}$	$(K_{3,3}$	$(K_{4,1})$	$(K_{4,1}$	$(K_{4,2}$	$(K_{4,2}$	$(K_{4,3}$	$(K_{5,1})$	$(K_{5,2}$	Points
No	17	10	13	11	2	7	20	1	15	16	6	3	14	19	12	4	8	18	5	9	
s_1	+	+	+	+	+	+	+	-	+	+	-	+	+	+	+	-	+	+	+	+	86
s_2	+	+	+	+	-	+	-	-	+	+	+	+	-	+	+	-	-	+	+	-	63
s_3	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	100
s_4	+	+	+	-	-	+	-	-	+	+	-	+	-	-	+	-	-	-	+	-	46
s_5	+	+	+	+	+	+	-	+	-	+	+	-	+	+	+	+	+	+	+	+	85
s_6	+	+	+	-	-	+	-	-	+	-	-	+	+	+	+	-	+	+	+	-	58
s_7	+	-	-	-	+	+	-	-	+	-	+	+	+	+	+	-	-	+	+	-	55
s_8	+	+	-	-	-	+	-	-	+	-	-	-	+	-	-	-	-	+	+	-	38
s_9	+	+	-	-	-	+	-	-	+	-	-	+	-	+	-	-	-	-	+	-	38
s_{10}	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	100
s_{11}	+	+	+	+	+	+	+	-	+	+	-	+	+	+	+	-	+	+	+	-	76
s_{12}	+	+	-	-	+	+	+	+	-	+	-	+	+	+	-	-	-	+	+	-	60
s_{13}	+	+	-	+	+	+	-	+	+	+	-	+	+	+	-	-	-	-	+	+	65
s_{14}	-	-	-	-	-	+	-	-	-	-	-	-	+	+	-	-	+	+	-	-	21
s_{15}	-	+	+	-	-	+	+	-	+	+	+	+	-	+	-	-	-	+	+	-	55
s_{16}	+	+	+	-	+	+	-	-	-	-	-	+	+	-	-	-	+	-	+	-	45
s_{17}	+	+	+	+	-	+	+	+	+	+	+	+	+	+	+	+	-	+	+	+	90
s_{18}	+	+	+	+	+	+	+	-	+	+	-	+	+	+	-	-	-	-	+	-	65
s_{19}	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	-	+	+	+	+	86
s_{20}	+	+	+	+	+	-	-	+	+	-	+	+	+	-	-	+	-	+	+	+	63