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MULTIFRACTAL BEHAVIOUR IN GOLD PRICES BY USING MF-DFA AND WTMM METHODS

Feleknaz Dilek Terzi

İstanbul Gelişim University and Yeditepe University

Title: Instructor and Phd Candidate

E-mail: dterzi@gelisim.edu.tr

Dr. Gazanfer Ünal

Yeditepe University

Title : Professor

E-mail: gunal@yeditepe.edu.tr

—Abstract —

In this paper, we investigate the multifractal features of a gold market which is known as the safe harbour in the face of political and economic chaos. We performed two different methodologies which are Multifractal Detrended Fluctuation Analysis (MF-DFA) and the Wavelet Transform Modulus Maxima (WTMM) in order to investigate the multifractality of Gold spot price/ounce. After given some brief introduction, then we explain the particular implementation of the above methods and compare their effectiveness. Finally, we conclude that gold price series are multifractal by these methods.

Key Words: Gold, Multifractal, MF-DFA, WTMM

JEL Classification: G17 Financial Forecasting and Simulation

1. INTRODUCTION

Gold is known as a precious metal which has the characteristics of monetary and commodity assets, because of its some particular features such as hard currency and its constant original value. Therefore, central banks reserve plenty of gold to against inflation and hedge the risk of exchange rate. However, there is some reason which affects the gold price. First, the exchange rate of US dollars directly affects the gold price which is labeled as dollars per ounce. Namely, an appreciation of the US dollar can cause a decrease of the gold price and a depreciation of the US dollar can cause an increase of the gold price. Second, the geopolitical events again directly affect the gold price. People worry about the depreciation of local currency caused by the war and buy enough gold to hedge, when the social situation is unstable. As everybody has noticed that the large gold demand increases the gold price during the period of the war. From 2011 to 2013, geopolitical events broke out usually in the area of the Middle East so the gold price increased greatly because of the most people chose to keep gold, not to US dolar. Third, the gold price increases, because of the financial crisis. People have chosen to invest in gold for its constant original value because of the financial system in the US and European countries have shown a high instability in the recent years. Fourth, under the environment of a high inflation rate, the gold price is always very high. A very high inflation rate implies the low purchasing power of local currency. Then, the local currency would be less attractive and more people tend to choose gold to preserve the value. (Yudong Wang, Yu Wei, Chongfeng Wu, 2011:817). Finally, the oil shock which can drive up the inflation rate can also bring an increase of the gold price. All of the reasons above shows that it is essential to analyze the dynamics of gold prices due to the important role of gold and the high sensitivity of its price to many market local and global factors.

This paper is focused on two competitive methods to analyze multifractality of Gold Prices such as Multifractal Detrended Fluctuation Analysis (MF-DFA) (Kantelhardt,2002:87, M.Ignaccolo,2004:595) and Wavelet Transform Modulus Maxima (WTMM) (Muzzy,1994:245) which are commonly used to eliminate trends and concentrate on the analysis of fluctuations.

In this paper, it is aimed to empirically test whether returns on the Gold spot prices exhibit long-range correlations and multifractal patterns. The term fractal was coined by Mandelbrot (Mandelbrot,1982:15) to characterize a rough or fragmented geometric shape that displays a large degree of self similarities within its own fractional dimensions. Self-similarity, an invariance with respect to scaling, is an important characteristic of fractals. It means that the object or process is similar at different scales. Each scale resembles the other scales, but is not identical. For example, individual branches of a tree are qualitatively self-similar to the other branches, but each branch is also unique. A self-similar object appears unchanged after increasing or shrinking its size. (Gencay,2002:586)

Theoretically, financial time series have random walk behaviour and have no multifractality properties. However, multifractality has been a ‘‘stylized fact’’ which widely exists in financial time series (Cont,2001:223) . In general, there are two main reasons of multifractality in time series that are long-range correlations of small and large fluctuations and fat tail distributions. (Matia,2003:422)

2. METHODOLOGY

2.1. Multifractal Detrended Fluctuation Analysis

Through the multifractal detrended fluctuation analysis, we estimate the generalized Hurst and the Renyi exponents for price fluctuations. By deriving the singularity spectrum from the above exponents, we quantify the multifractality of a financial time series and compare the multifractal properties.(W. Kantelhardt,2002:A316). The MF-DFA procedure consists of five steps in these papers;(J.W. Kantelhardt, S.A. Zschiegner, E. Koscielny-Bunde, S. Havlin, A. Bunde, H.E. Stanley, Physica A 316 (2002) 87–114.)

In general, the exponent h_q may depend on q . For stationary time series, h_2 is identical to the well-known Hurst exponent H. Thus, we will call the function h_q generalized Hurst exponent.

Positive values of q are used for magnifying the effects of large price variations in the scaling analysis, and negative values of q are used for magnifying the effects of small price variations. (Yudong Wang, Yu Wei, Chongfeng Wu, 2011:817). When $h_q > 0.5$ the kinds of fluctuations related to are persistent. An increase (decrease) is always followed by another increase (decrease). When $h_q < 0.5$, the kinds of fluctuations related to q are anti-persistent. An increase (decrease) is always followed by another decrease (increase). However, $h_q = 0.5$, the kinds of fluctuations related to q display random walk behavior. The richness in multifractality is associated with high variability of h_q and the degree can be quantified as .

A typical characteristic for multifractal time series is that h_q varies with q . When h_q is constant for all q, the time series are mono-fractal. As large fluctuations are characterized by smaller scaling exponent h_q than small fluctuations, h_q for $q < 0$ are larger than those for $q > 0$, and Δh is positively defined. Multifractality degree can be used to measure the efficient extent of a finance market. When multifractality degree is weaker, for all q value, generalized Hurst exponents are closer to 0.5. This shows that no matter the fluctuation is big or small, its change of state is closer to random walk, so the market is more efficient. The analytical relationship between generalized Hurst exponents based on MF-DFA and Renyi exponent τ_q

$$\tau_q = q h_q - 1$$

is, .The exponent τ_q represents the temporal structure of the time series as a function of the various moments q , or τ reflects the scale dependence of smaller fluctuations for negative values of q , and larger fluctuations for positive values of q . If τ_q increases nonlinear with q , then the series is multifractal. Via a Legendre transform, another important variable set $\alpha - f(\alpha$) is defined by

$\alpha =$ $$f(\alpha) = q\left(\alpha - h_q\right) + 1 \quad (1)$$

Here, α is the Holder exponent or singularity strength which characterizes the singularities in a time series. Singularity basically points at the rapid changes in the time series values for small changes in time. In the multifractal case, the different parts of the data set are characterized by different values of α, or the singularity spectrum $f(\alpha)$. (Kantelhardt,2002:2)

2.2. Wavelet transform modulus maxima method (WTMM)

As proven by Mallat and Hwang (1990:549), multifractal formalism based on wavelet transform modulus maxima (WTMM) allows us to determine the whole singularity spectrum directly from any experimental signal. Muzy et al. (1991:3515) define the scaling behavior of partition functions $Z(q,a)$ from the WTMM. The slope of the partition function determines the scaling $\tau(q)$of moments of the distribution. Linearity of the scaling function suggests monofractal behavior of the time series, (all moments exhibit the same H scaling with time). The procedures of calculating the multifractal singularity spectrum based on WTMM is described in Yalamova (2003). Wavelet transform has proved to be a particularly efficient tool for measuring the local regularity of a function. The wavelet transform of $f(t) = P(t)$ is defined as:

$$W(\tau, a) = \int_{-\infty}^{+\infty} f(t)\psi_{\tau,a} t(dt) \quad (2)$$

where the analyzing wavelet ψ is a function with local support, centered around zero and the family of wavelet vectors is obtained by translation τ and dilatation a. The modulus maxima (largest wavelet transform coefficients) are found at each scale a as the suprema of the computed wavelet transforms such that:

$$\frac{\partial W(\tau, a)}{\partial \tau} = 0 \qquad (3)$$

The originality of the WTMM method is in the calculation of the partition function $Z(q, a)$ from these maxima lines. The space-scale partitioning given by the wavelet tiling or skeleton defines the particular Gibb's partition function:

$$Z(q, a) = \sum_{\tau, a} \sup_{a} |W(\tau, a)|^{q} \qquad (4)$$

The WTMM method uses continuous wavelet transform rather than Fourier transforms to detect singularities – that is discontinuities, areas in the signal that are not continuous at a particular derivative. Another interesting property of the wavelet transform is that the coefficients at these maxima-which are a small fraction of the total number of coefficients-are enough to encode the information contained in the signal (Muzzy,1994:245), Moreover, as one follows a maxima line from the lowest scale to higher and higher scales, one is following the same singularity. This fact allows for the calculation of h_q by a power law fit to the coefficients of the wavelet transform along the maxima line (Struzik,2000:163).

Wavelet Skeleton is an aggregate of all Local Maxima Lines (LML) on each scale of Wavelet coefficient matrix. The idea of Skeleton matrix construction is to remove all wavelet coefficients in absolute wavelet coefficients matrix that are not maximal. Skeleton matrix is a scope of all local maxima points that exist on each scale a. If scaling exponential function is everywhere convex that indicates multifractal behaviour of the signal. It assumes that the signal does not have some decent fractal measure, but is characterized by the scope of fractal measures. In case of monofractal behaviour, the scaling exponential function is line. (Puckovs, 2012:83)

3. DATA ANALYSIS

The Gold Spot prices data were taken from Bloomberg (http://www.bloomberg.com/markets/commodities/futures/metals). We choose the daily closing data of gold prices traded in spot over the period of Jan 3th, 2000-March 3th, 2013, with 3453 observations in Figure1. In our analyses we used log return of the spot prices illustrated as Figure 2.

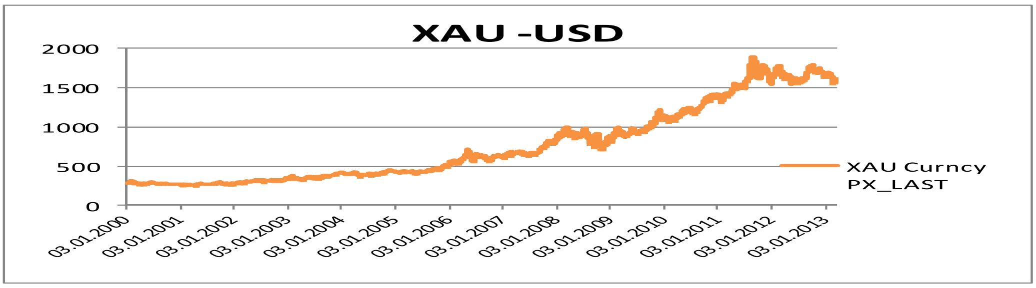

Figure 1 – Gold Spot Price

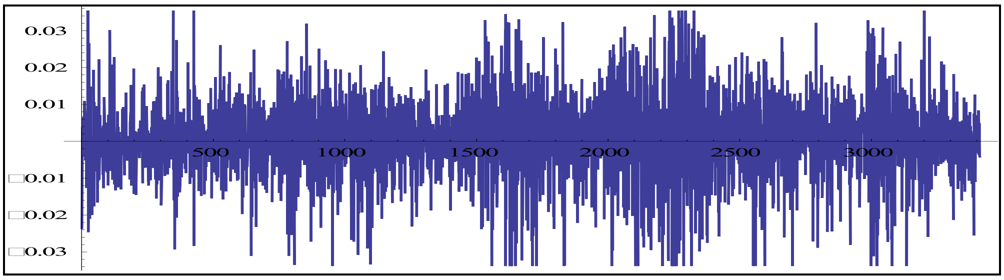

Figure 2: Log Return Of Spot Price

4. EMPRICAL RESULTS

4.1. MDF-FA Method

We have used matlab codes to implement MF-DFA on log-return data of Gold Spot prices. (Ihlen,2012:3) We estimate the generalized Hurst and the Renyi exponents for price fluctuations. Deriving the singularity spectrum from the exponents, we quantify the multifractality of a financial time series. In Figure 3

(Upper Right Corner), The generalized Hurst exponents for time scales is given. When q varies from -5 to 5, h_q decreases from 0.56695 to 0.34598. h_q is not a constant, indicating multifractality in time series. Due to H < 0.5, (as H value for h_2 is between 0.4 and 0.5) the system displays fractional Brownian motion and anti-correlation and finally, the antipersistent behavior and deviations of one sign generally followed by deviations with the opposite sign is an indication of the degree of gold prices nervousness and uncertainty. It means that the analysis of these events doesn't give support to the Efficiency Market Hypothesis.

There are two factors contributing to multifractal properties, namely long-range temporal correlations for small and large fluctuations and the fat-tailed probability distributions of variations. (K.Matia, 2003:422) As shown in Figure 3 (Lower Left Corner), multifractal scaling function almost linear with q for negative moments, but show significant non-linearities for positive moments. This means that the temporal structure of the larger fluctuations play an important role in the multifractality.

The spectrum, in Figure 3 (Lower Right Corner) as an upside-down parabola, peaks at f_{max} and stretches from min to max. $\Delta\alpha = \alpha_{max} - \alpha_{min}$ conventionally quantifies the degree of multifractality which is the width of singularity spectrum, while f(α) tells how frequently events with α scaling exponent occur. The width of the fractal spectrum, which shows the distinction between the maximum probability and the minimum probability. The larger the value of $\Delta\alpha$, the more uneven is the distribution of time series, and thus the stronger is the multifractality. The long-range and short correlations lead to a relatively narrow width of the spectrum (lower risk) or vice versa. In our case, the width of spectrum 0.64891-(0.23432) = 0.41459 reveals that there is the multifractality in Gold Spot daily returns.

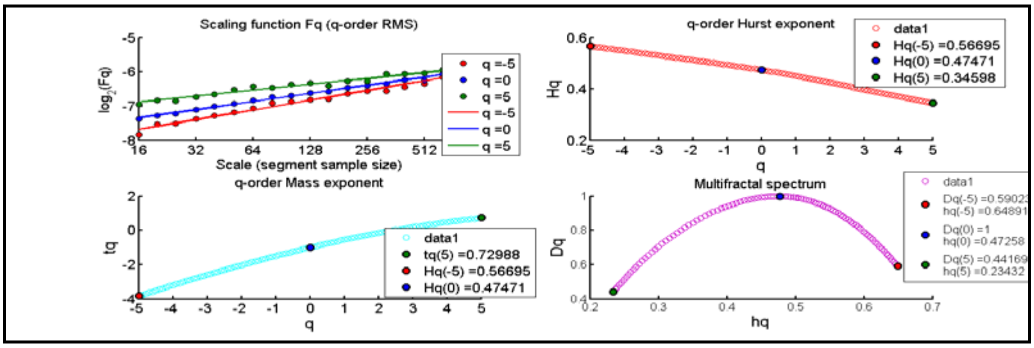

Figure:3 Generalized Hurst Exponent, Reyni Exponents, Multifractal Spectrum

4.2. Wavelet Transform Modulus Maxima (WTMM)

We draw the Wavelet Skeleton by using Last Wave Program and Mathematica. Time shifting coefficients (b) are drawn on x axis, Scales (a) are drawn on Y axis. Local maxima lines are constructed using Wavelet coefficient matrix, selecting local maxima points on each scale parameter (Figure 4). The scope of all local maxima lines builds the so called Skeleton function. This function illuminates periodicity of the signal on decent scales. Dark colours correspond to lower absolute wavelet coefficient values. Light colours indicate higher absolute wavelet coefficient values. Continuous maxima line from small scales to large scales determine the time of the singularity at different scales. According to WTMM methods, we see that scaling exponential function is convex which shows that Gold spot daily returns have multifractal properties. Unlike Multifractal, If we look at the scaling function, the Monofractal price series' behaviour is observed to be less deviated from the trend line (Figure 7). And also fat tails in both ends of the Local Holder Exponents indicates multifractality of the series (Figure 5).

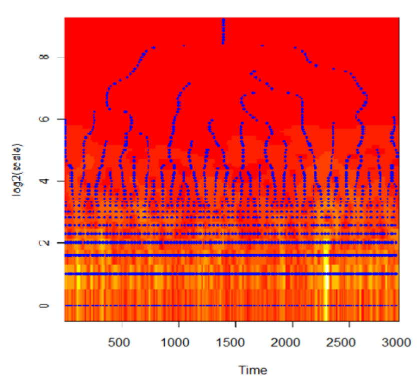

Figure 4:Wavelet Sceleton

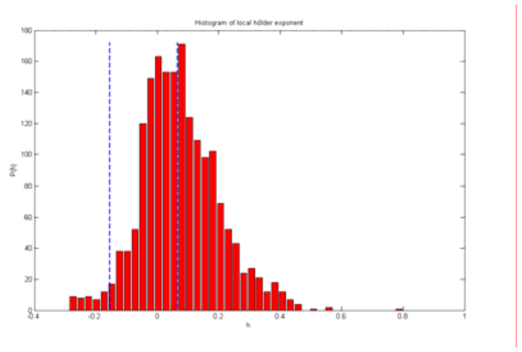

Figure 5:Histogram Of Local Holder Exponent

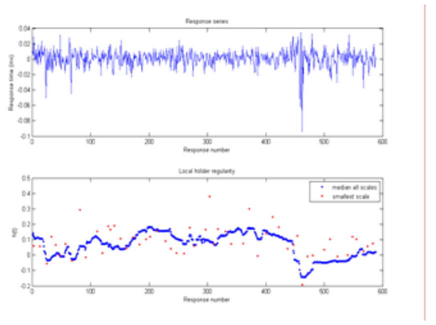

Figure 6: Local Holder Regularity

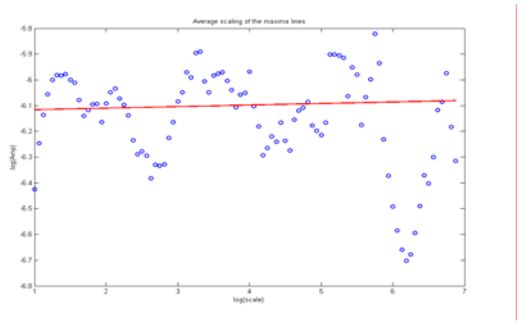

Figure 7. Average Scaling of Maxima Lines

5. CONCLUSION

Using the multifractal detrended fluctuation analysis (MF-DFA) together with WTMM, this paper explores multifractal properties of the Gold Price returns. We find that gold data exhibits multifractal patterns. Further, through comparing Δh and $\Delta\alpha$, we verify that the multifractality degree of returns is mostly due to different long-range correlations for small and large fluctuations. The fat-tailed distributions (probability distribution of returns), also contribute to the multifractal behaviour of the time series.

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