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MULTIFRACTAL BEHAVIOUR IN OIL PRICES BY USING MF-DFA AND WTMM METHODS

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─Abstract ─

This study analyses the multifractal properties of the most prominent oil-related derivative which is ''WTI'' since the West Texas Intermediate grade of crude oil for delivery at Cushing, Oklahoma. To be able to test multifractality of the WTI prices, we used two different methodologies which are multifractal detrended fluctuation analysis (MF - DFA) and wavelet transform modulus maxima (WTMM).

Key Words: *Multifractal, MF-DFA, WTMM, Oil Price, WTI*
JEL Classification: G17 Financial Forecasting and Simulation

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1. INTRODUCTION

Over the past 20 years oil has become the biggest commodity market in the world and it has evolved from a primarily physical activity into a sophisticated financial market with trading horizons now extending to over 10 years forward.

In parallel, the volatility of the oil price and the hedging needs this created for industry participants triggered the development of a financial sphere of derivative contracts (Futures, forwards, swaps and options) which now dominate the process of worldwide oil price formation. However, since oil is an inherently nonstandard commodity, the industry has had to choose a small number of ‘‘reference’’ or ‘‘marker’’ grades of crude oil and refined products that provide the physical basis for a much larger derivatives market and attract most of the liquidity. The most prominent oil-related derivative is without doubt the Futures contract on light sweet crude that is quoted on the New York Mercantile Exchange (NYMEX); it is usually known as ‘‘WTI’’ since the West Texas Intermediate grade of crude oil for delivery at Cushing, Oklahoma still underpins the market (despite the introduction of fungible delivery grades for physical settlement of the contract in recent years). (German,2005:224)

Fractals, one of the most useful discoveries in mathematics, were first introduced into the Finance area by Mandelbrot (Mandelbrot,1963:394). A fractal is an object in which the parts are in some way related to the whole. Self-similarity, an invariance with respect to scaling, is an important characteristic of fractals. It means that the object or process is similar at different scales. Each scale resembles the other scales, but is not identical. For example, individual branches of a tree are qualitatively self-similar to the other branches, but each branch is also unique. A self-similar object appears unchanged after increasing or shrinking its size. (Gencay,2002:586)

Theoretically, financial time series have random walk behaviour and have no multifractality properties. However, multifractality has been a ‘‘stylized fact’’ which widely exists in financial time series (Cont,2001:223) . In general, there are two main reasons of multifractality in time series that are long-range correlations of small and large fluctuations and fat tail distributions. (Matia,2003:422)

2. METHODS TO TEST MULTIFRACTAL BEHAVIOUR

2.1. Multifractal Detrended Fluctuation Analysis (MF-DFA)

To be able to test multifractality of timeseries, we have used Multifractal Detrended Fluctuation Analysis (MF-DFA) method. In this method, we use the logarithmic return of the time series for each step u.

$$r(u) = \ln(p_t) - \ln(p_{t-1}) = \ln(p_t/p_{t-1}) \quad (u=1,2,....N) \qquad (1)$$

The deviation in return from the mean of the return is

$$y(u) = \sum_{u}^{n} (r(u) - \bar{r}) \quad (u=1,2,....N) \qquad (2)$$

The length N of the time series is partitioned into n segments, each of length s, N = ns. A least squares method can be used to identify trends in running deviation over each segment k by a polynomial $g(u)$. The average fluctuation $F_{k(s)}$ in each subregion k is

$$(F_{k(s)})^2 = \frac{1}{s} \sum_{u=(k-1)s+1}^{ks} \left((y_k(u) - g(u))^2 \right) \qquad (3)$$

The average moment of the fluctuation of order q over n segments of the time series is

$$F_{q(s)} = \left\{ \frac{1}{n} \sum_{k=1}^{n} (F_{k(s)})^{n} \right\}^{\frac{1}{q}} \qquad (4)$$

As q - > 0 $$\ln F_{0(s)} = \frac{1}{n}\sum_{k=1}^{n} \square F_{k(s)} \ln\left(F_{k(s)}\right) \cdot \qquad (5)$$

The power-law dependence of the q-th order moment of the fluctuation Fq(s) in interval s of the time series provides an estimate of the Hurst exponent h(q), i.e.,

$$F_q\,(s) \sim s^{h(q)} \;. \qquad (6)$$

In general, the exponent h_q may depend on q . For stationary time series, $h(2)$ is identical to the well-known Hurst exponent H. Thus, we will call the function h_q generalized Hurst exponent. The family of Generalized exponents h_q can be obtained by observing the slope of log_log plot of $F_q\,(s)$versus (s) through the method of least squares.

When h_q is constant for all q , the time series are mono-fractal. Otherwise, the series are multifractal.Positive values of q are used for magnifying the effects of large price variations in the scaling analysis, and negative values of q are used for magnifying the effects of small price variations.

Specifically, when $h_q > 0.5$ the kinds of fluctuations related to are persistent. An increase (decrease) is always followed by another increase (decrease). When $h_q < 0.5$, the kinds of fluctuations related to are anti-persistent. An increase (decrease) is always followed by another decrease (increase). However, $h_q = 0.5$ the, the kinds of fluctuations related to display random walk behavior.

h_q

The richness in multifractality is associated with high variability of and th degree can be quantified as

$$\Delta h = h(q_{min}) - h(q_{max}) \qquad (7)$$

h_q

A typical characteristic for multifractal time series is that varies with . As large h_q fluctuations are characterized by smaller scaling exponent than small h_q fluctuations, for are larger than those for , and Δh is positively defined. Multifractality degree can be used to measure the efficient extent of a finance market. When multifractality degree is weaker, for all value, generalized Hurst exponents are closer to 0.5. This shows that no matter the fluctuation is big or small, its change of state is closer to random walk, so the market is more efficient.

The analytical relationship between generalized Hurst exponents based on MF-DFA and Renyi exponent τ_q is,

$$\tau_q = q h_q - 1 \quad (8)$$

The exponent τ_q represents the temporal structure of the time series as a function of the various moments q , or τ reflects the scale dependence of smaller fluctuations for negative values of q , and larger fluctuations for positive values of q . If τ_q increases nonlinear with q , then the series is multifractal.
Via a Legendre transform, another important variable set $\alpha - f(\alpha$) is defined by

$\alpha -$

$$f(\alpha) = q(\alpha - h_q) + 1 \quad (9)$$

Here, α is the Holder exponent or singularity strength which characterizes the singularities in a time series. Singularity basically points at the rapid changes in the time series values for small changes in time. In the multifractal case, the

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different parts of the dataset are characterized by different values of α , or the singularity spectrum . (Kantelhardt,2002:2)

2.2. Wavelet transform modulus maxima method (WTMM)

The wavelet transform modulus maxima (WTMM) is a method for detecting the fractal dimension of a signal. More than this, the WTMM is capable of partitioning the time and scale domain of a signal into fractal dimension regions, and the method is sometimes referred to as a "mathematical microscope" due to its ability to inspect the multi-scale dimensional characteristics of a signal and possibly inform about the sources of these characteristics.

The wavelet transform of $\mathbf{f(t)} = \mathbf{P(t)}$ is defined as:

$$\mathbf{W(\tau, a)} = \int_{-\infty}^{+\infty} \mathbf{f(t)\psi_{\tau,a}t\,(dt)} \qquad (10)$$

where the analyzing wavelet ψ is a function with local support, centered around zero and the family of wavelet vectors is obtained by translation τ and dilatation a. The modulus maxima (largest wavelet transform coefficients) are found at each scale a as the suprema of the computed wavelet transforms such that:

$$\frac{\partial \mathbf{W}\,(\tau, \mathbf{a})}{\partial \tau} = 0 \qquad (11)$$

The WTMM method uses continuous wavelet transform rather than Fourier transforms to detect singularities singularity – that is discontinuities, areas in the signal that are not continuous at a particular derivative.In particular, this method is useful when analyzing multifractal signals, that is, signals having multiple fractal dimensions.

The WTMM is then capable of producing a "skeleton" that partitions the scale and time space by fractal dimension. Wavelet Skeleton is an aggregate of all Local Maxima Lines (LML) on each scale of Wavelet coefficient matrix. The idea of

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Skeleton matrix construction is to remove all wavelet coefficients in absolute wavelet coefficients matrix that are not maximal. Skeleton matrix is a scope of all local maxima points that exist on each scale a. In general, the skeleton function shows the scalability of the signal.

If scaling exponential function is everywhere convex that indicates multifractal behaviour of the signal. Multifractal behaviour of signal assumes that the signal does not have some decent fractal measure, but is characterized by the scope of fractal measures. In case of monofractal behaviour, the scaling exponential function is line. (Puckovs,2012:83)

3. METHODS TO TEST MULTIFRACTAL BEHAVIOUR

3.1. Data Analysis

W e used the WTI Spot Price FOB (Dollars per Barrel) data from January 02, 1986 to March 25, 2013 and data are taken from U.S. Energy Information Administration web page. (http://www.eia.gov/)

We have 6868 observations which are the close price of the WTI spot prices and trend of the WTI spot prices can be seen in the Figure 1.

In our analysis, we have used the log return of the WTI prices which can be seen in Figure 2.

Figure 1 : WTI Spot Price

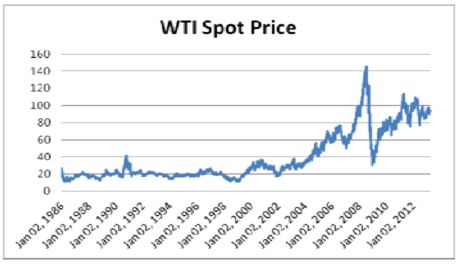
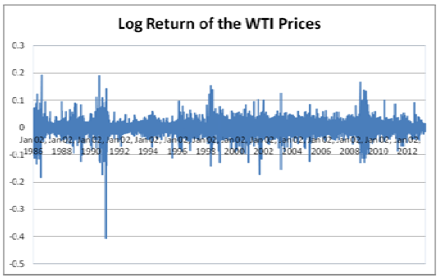


Figure 2 : Log Return of the WTI Spot Price



3.2. Emprical Results

3.2.1. MF - DFA

We have used matlab codes to implement MFDFA on log-return data of WTI prices. (Ihlen,2012:3) By using multifractal detrended fluctuation analysis (MFDFA), the generalized Hurst exponent can be estimated. The generalized Hurst exponents for time scales can be seen in the Figure 3. When varies from -5 to 5, h_q decreases from 0.58151 to -0.45507. h_q is not a constant, therefore, WTI prices has multi fractal properties.

For stationary time series, $h(2)$ is identical to the well-known Hurst exponent H. If H = 0.5, the system displays ‘‘Markovian’’ behavior and there is no long-term correlation or memory. If H < 0.5, the system displays fractional Brownian motion and anti-correlation and finally, if H > 0.5, there is a positive long-term correlation or memory exist in the series. Our data shows almost “Markovian” behaviour because of H values. Multifractality degree can be used to measure the efficient extent of a finance market. When multifractality degree is weaker, for all value, generalized Hurst exponents are closer to 0.5. This shows that no matter the fluctuation is big or small, its change of state is closer to random walk, so the market is more efficient. Therefore, according to our data analysis, WTI support the market efficiency.

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There are two main factors contributing to multifractal properties, namely long-range temporal correlations for small and large fluctuations and the fat-tailed probability distributions of variations. Another way to characterize the multifractality behavior is presented in Fig. 4 with the multifractal scaling function τ_q of the data calculated from the power-law relation between $F_q(s)$ and . As shown in figure 3, multifractal scaling function has non-linearity. This means that the temporal structure of the larger fluctuations play a role in the multifractality.

Figure 3 : Generalized Hurst Exponent

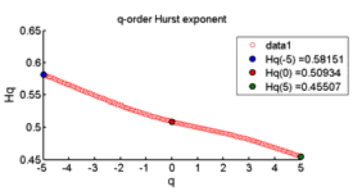


Figure 4 : Renyi Exponent

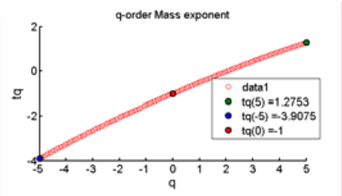
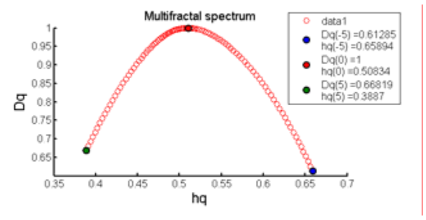


Figure 5 : Multifractal Spectrum



The width of the fractal spectrum, which shows the distinction between the maximum probability and the minimum probability (i.e. $\Delta\alpha = \alpha_{max} - \alpha_{min}$). The larger the value of $\Delta\alpha$, the more uneven is the distribution of time series, and thus the stronger is the multifractality. In Figure 5 the width of

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spectrum 0.65894-0.3887=0.6200 reveals that there is a multifractility in WTI prices.

3.2.2. WTMM

We draw the Wavelet Skeleton by using wmtsa library in R. The result is shown in Figure 6. Time shifting coefficients (b) are drawn on x axis, Scales (a) are drawn on Y axis. Local maxima lines are constructed using Wavelet coefficient matrix, selecting local maxima points on each scale parameter. The scope of all local maxima lines builds the so called Skeleton function. This function illuminates periodicity of the signal on decent scales.In figure, Dark colours correspond to lower absolute wavelet coefficient values. Light colours indicate higher absolute wavelet coefficient values.

Figure 6 : WTMM

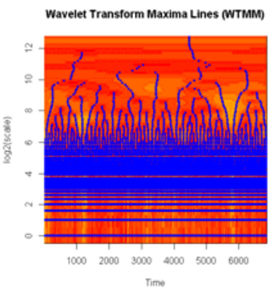
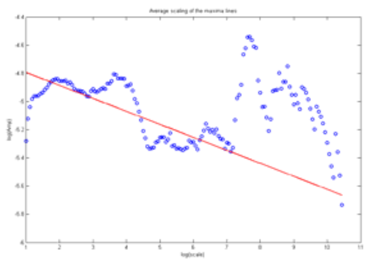
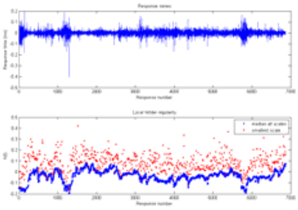


Figure 7 : Average Scaling of Maxima Lines



According to WTMM methods, we see that scaling exponential function is convex which shows that WTI spot price has multifractal properties. As one can see in the figure 7, there is a serious deviation from trend-line which is shows the multifractality and there is less deviation for monofractal series.

Figure 8 : Local Holder Regularity



4. CONCLUSION

In contrast to theoretical assumption which is time series with random walk behaviour has no multifractal properties, we would like to analyse the oil prices regarding multifractal properties. Although, there are many methods to analyse fractal properties of time series, we choose two of them which are mainly preferred in recent researches. Applying both MF-DFA and WTMM approaches on to the most prominent oil-related derivative "WTI", we realized that WTI price has multifractality properties. The reasons of multifractality are different long-range correlations for small and large fluctuations and fat-tailed distributions.

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