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THE VALUE OF SOCIAL NETWORK AS FUNCTION OF NUMBER OF USERS

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─Abstract ─

At the presence of network externality, the value of a product or service is dependent on the number of others using it. This means that the more social network has subscribers, the greater its value. Metcalfe's law states that network value is proportional to the number of users in square. Reed's law takes into account the possibility of groups' formation. In this case, network value scales exponentially with the size of the network. Odlyzko is much more moderate. He argues that the incremental value of adding one person to network of n people is approximately the n-th harmonic number. From early ages of television Sarnoff stated that the value of a broadcast network is proportional to the number of viewers.
This paper deals with the problem of determining the value of such social networks as Facebook, Twitter, LinkedIn etc. as function of the number of users and other parameters.

Key Words: *social networks, Metcalfe's law, network externality*
JEL Classification: D59, D62

1. INTRODUCTION

This paper deals with the question of dynamics of social networks. These networks are self-organizing, emergent and complex. These patterns become more apparent as network size increases. The main purpose of this article is to determine the dependency of network value on number of users or, in other words, the functional form. The analysis begins with the adoption of Bass diffusion model and latter estimation made to reveal the corresponding law. Research is based on the data from US region.

2. THE MODEL FORMULATION

Bass (1969) introduced new product growth model. According to it, sales obey to this differential equation:

$$\frac{dy}{dt} = \left(p + q\,y(t)\right) * \left(1 - y(t)\right), \quad (1)$$

here y – product sales, t – time, p – the coefficient of innovation and q – the coefficient of imitation. The solution to this equation is this:

$$y(t) = \frac{1-e^{-(p+q)t}}{1+\frac{q}{p}e^{-(p+q)t}}\,. \quad (2)$$

Let m be the potential size of the network then it is possible to transform above equation in to this:

$$N_t = N_{t-1} + p(m - N_{t-1}) + q\frac{N_{t-1}}{m}(m - N_{t-1}), \quad (3)$$

where $N_t = m\,y(t)$. This is how social network growing. The growth is influenced by the network externality. First significant works in this area are made by Katz and Shapiro (1985), Farrell and Saloner (1985). Their research was extended later by Economides and Himmelberg (1995), Liebowitz and Margolis (1994). Network externality means that one user of a social network has influence on the value of that service to other people. In other words, the value of social network service is dependent on the number of others using it. Metcalfe law is based on this property. This law is formulated by George Gilder in 1993 but is based on Robert Metcalfe insight made in 1980 by exploring Ethernet network (Shapiro & Varian, 1999). Mathematically it can be expressed as the triangular number:

$$T \sim \frac{n(n-1)}{2} \quad (4)$$

where T – network value, n – number of users (or devices in original view). There are some other views. For example, Reed (2001) proposes that network value grows exponentially:

$$T \sim 2^n - n - 1 \quad (5)$$

He takes in to account a formation of groups and subgroups. More reasonable view is proposed by Odlyzko et al. (2006). It is argued that the incremental value of adding one person to network of n people is approximately the n-th harmonic number. This means network value is proportional to:

$$T \sim n\ln(n). \quad (6)$$

Sarnoff's law (Gunasekaran & Harmantzis, 2007) states that the value of a broadcast network is proportional to the number of viewers:

$$T \sim n. \tag{7}$$

From theoretical point of view it is quite reasonable to say that network type defines corresponding law. In case of "one way" network, such as television or radio, the value of network is ruled by Sarnoff's law. In case of "two way" network, such as phones or faxes, the value of network is ruled by Metcalfe's or Odlyzko's law. In case of the most sophisticated network type, such as social network, where can be many types of connections one may guess it is described by Reed's law. But this study of social network does not approve this hypothesis.
According to Swann (2002), individual utility u is

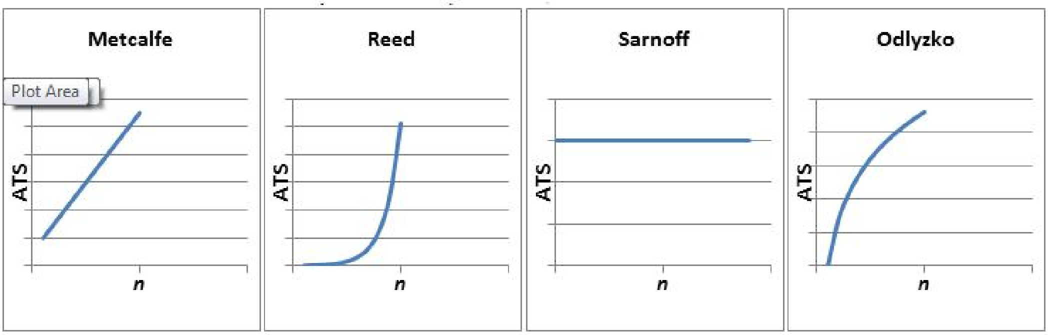
$$u \sim \frac{\partial T}{\partial n}. \tag{8}$$

In case of social networks one can make the assumption that individual utility is proportional to average time on site ATS:

$$u \sim ATS. \tag{9}$$

The increase of individual utility causes longer time spent on site. This implies four different approaches which are displayed next.

Figure-1: Theoretical relation between number of visitors and average time on site

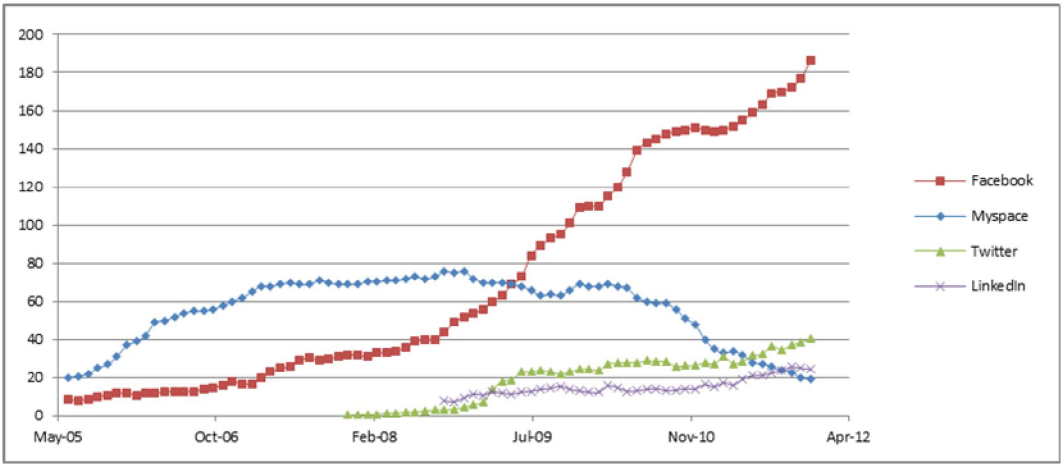


This figure presents the dependency between average time on site and number of visitors in different views. In case of Metcalfe law the linear dependency takes place. In case of Reed the exponential dependency occurs. If average time on site does not depend on number of visitors then Sarnoff law takes place. Last one is Odlyzko case, where logarithmic dependency occurs. Finding dependency between number of visitors and average time on site reveals the actual law that governs social networks.

2. EMPIRICAL RESULTS

In this analysis four social networks (Facebook, Myspace, Twitter, LinkedIn) will be analyzed. One of them is rapidly growing and one is rapidly shrinking. The dynamics of these networks is presented in next figure:

Figure-2: The dynamics of social networks



Data source: Compete, The Nielsen Company, Hitwise, Silicon Alley Insider.

Vertical axis represents monthly unique visitors in United States. Figure confirms proposition that Facebook is growing as Twitter and LinkedIn do. Myspace is contracting the third year.
Adapting the Bass model and (3) equation one can estimate this stochastic equation:

$$N_t = C_1 + C_2 N_{t-1} + C_3 N_{t-1}^2 + C_4 IPO + AR(1) + \epsilon_t, \qquad (10)$$

Here AR(1) means autoregressive process, which is used to solve the autocorrelation problem. In case of LinkedIn the dummy variable IPO is used. It accounts initial public offering that take place in January 2011. The results are presented in next table.

Table 1: Estimation results of diffusion

Social network	Estimated equation	C_2	C_3	C_4	AR(1)	R^2	Observations
Facebook	$N_t = C_2 N_{t-1} + C_3 N_{t-1}^2 + AR(1)$	1.071 (0.019)	-0.000305 (0.00013)	-	0.4	0.998	69
Myspace	$N_t = C_2 N_{t-1} + C_3 N_{t-1}^2$	1.153 (0.031)	-0.0021 (0.00046)	-	-	0.977	34
Twitter	$N_t = C_2 N_{t-1} + C_3 N_{t-1}^2$	1.141 (0.06)	-0.00387 (0.0021)	-	-	0.979	48
LinkedIn	$N_t = C_2 N_{t-1} + C_3 N_{t-1}^2 + C_4 IPO$	1.113 (0.053)	-0.00745 (0.0034)	1.66 (0.67)	-	0.925	38

In brackets standard errors of coefficients are presented. These coefficients corresponds to equation (3): $C_1 = p\,m$, $C_2 = 1 - p + q$ ir $C_3 = -\frac{q}{m}$.
Assumption is made that $C_1 = 0$. It based on the fact that in three of four cases null hypothesis ($C_1 \neq 0$) at 0.05 significance level cannot be rejected (probabilities that $C_1 = 0$ are $P_{Facebook} = 0.72$; $P_{Myspace} = 0.02$; $P_{Twitter} = 0.25$; $P_{LinkedIn} = 0.1$). This implies that social networks exhibits logistic distribution which has cumulative distribution functions *F*:

$$F(x, \mu, s) = \frac{1}{1 + e^{-(x-\mu)/s}}. \qquad (11)$$

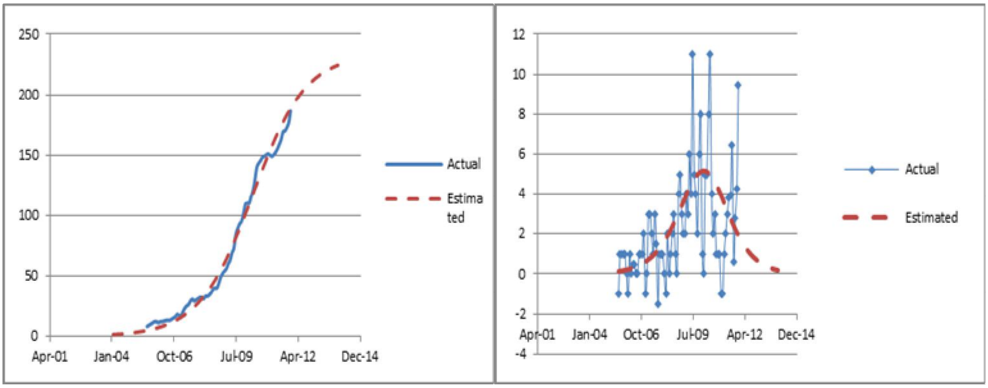
Here μ is mean and s corresponds to standard deviation. This distribution resembles the normal distribution in shape but has heavier tails (higher kurtosis).
Based on results from table 1 it possible to estimate potencial of social networks *m* and coefficient *q* which is called the coefficient of imitation, internal influence or word-of-mouth effect.

Table 2: Social networks estimated characteristics

Social network	*m*	*q*
Facebook	232.8	0.071
Myspace	72.9	0.153
Twitter	36.4	0.141
LinkedIn	15.2	0.113

Here *q* value are quite low compared to other researches. The average value of *q* has been found to be 0.38, with a typical range between 0.3 and 0.5 (Mahajan, Muller, & Bass, 1995). The other estimated value is the potential size of social network. In case of Twitter and LinkedIn it is quite low and in case of Facebook it is quite high compared to the number of US population, which is approximately 313 mln. (US Census Bureau, 2012).

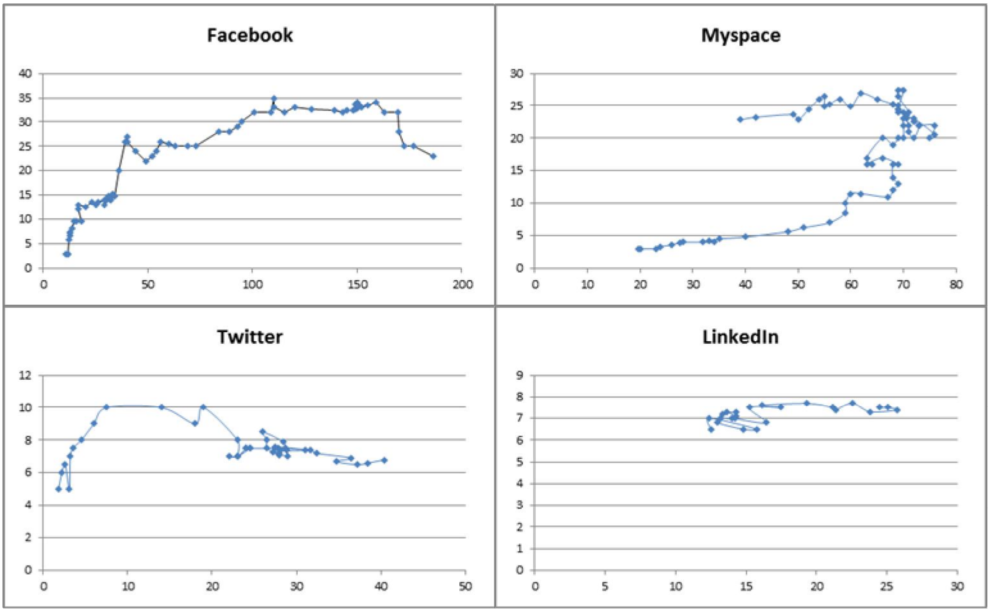
Figure-3: Actual and estimated Facebook diffusion



On the left side of the figure there are presented estimated (dotted line) and actual (not dotted line) monthly unique visitors and on the right side the change of visitors, respectively.

By approaching the main objective of this paper let's move to the analysis of dependency between network value and number of users. Below graphically presented data that displays average time on site in minutes against number of monthly unique visitors:

Figure-4: Average time on site and number of monthly unique visitors



Here vertical axis corresponds to average time on site and horizontal axis to the monthly unique visitors. Visually it can be identified some growth as number of users increases. In case of Facebook and Twitter on some point average time on site begins to decrease. Very strange dynamics is visible in Myspace case. At the beginning average time on site increases exponentially but at some point it begins to decrease. This point corresponds to the shrinking of Myspace in time. That is what graphical analysis reveals. In table below the results of regression analysis are presented.

Table 3: The results of functional form estimation

Social network	**Law**	**Estimated equation**	**Adjusted R^2**	C_2	**Probability** $(C_2 > 0)$
Facebook	Metcalfe	$ATS_t = C_1 + C_2 n_t + C_3 ATS_{t-1}$	0.9763	-0.011 (0.0064)	0.048
Facebook	Reed	$ATS_t = C_1 + C_2 2^{n_t} + C_3 ATS_{t-1}$	0.9761	-1.69E-56 (1.15E-56)	0.070
Facebook	Odlyzko	$ATS_t = C_1 + C_2 \ln n_t + C_3 ATS_{t-1}$	0.9755	-0.505 (0.692)	0.243

Facebook	Sarnoff	$ATS_t = C_1 + C_3ATS_{t-1}, C_2 = 0$	0.9757	-	-
Myspace	Metcalfe	$ATS_t = C_1 + C_2n_t + C_3ATS_{t-1}$	0.9836	-0.02 (0.012)	0.054
Myspace	Reed	$ATS_t = C_1 + C_22^{n_t} + C_3ATS_{t-1}$	0.9825	2.32E-24 (9.9E-24)	0.408
Myspace	Odlyzko	$ATS_t = C_1 + C_2\ln n_t + C_3ATS_{t-1}$	0.9831	-0.93 (0.56)	0.052
Myspace	Sarnoff	$ATS_t = C_1 + C_3ATS_{t-1}, C_2 = 0$	0.9827	-	-
Twitter	Metcalfe	$ATS_t = C_1 + C_2n_t + C_3ATS_{t-1}$	0.6205	-0.019 (0.0089)	0.020
Twitter	Reed	$ATS_t = C_1 + C_22^{n_t} + C_3ATS_{t-1}$	0.5778	-1.5E-13 (4.4E-13)	0.368
Twitter	Odlyzko	$ATS_t = C_1 + C_2\ln n_t + C_3ATS_{t-1}$	0.6079	-0.21 (0.12)	0.043
Twitter	Sarnoff	$ATS_t = C_1 + C_3ATS_{t-1}, C_2 = 0$	0.5871	-	-
LinkedIn	Metcalfe	$ATS_t = C_1 + C_2n_t + C_3ATS_{t-1}$	0.4814	0.001 (0.016)	0.275
LinkedIn	Reed	$ATS_t = C_1 + C_22^{n_t} + C_3ATS_{t-1}$	0.4805	2.28E-09 (3.95E-09)	0.285
LinkedIn	Odlyzko	$ATS_t = C_1 + C_2\ln n_t + C_3ATS_{t-1}$	0.4796	0.17 (0.31)	0.296
LinkedIn	Sarnoff	$ATS_t = C_1 + C_3ATS_{t-1}, C_2 = 0$	0.4970	-	-

In this table sixteen estimations are presented. Each social network corresponds to four cases. These cases are subject to the functional forms or the law of network value growth. Last column in table displays probability that social network value is not govern by Sarnoff law.

The procedure of choosing the functional form is quite simple. It consists of two stages. In first step the selection of the biggest adjusted determination coefficient is made. In next step the null hypothesis ($C_2 > 0$) is tested. It is one-tailed hypothesis.

Table shows that biggest adjusted R^2 is in Matcalfe case. In Facebook and Twitter cases null hypothesis with 5 percent significance level can be rejected. This means these two social networks values correspond to Sarnoff law. In Myspace and LinkedIn cases the null hypothesis with 5 percent significance level cannot be rejected although the first one probability is near this level.

4. CONCLUSION

This paper models social networks dynamics by applying Bass diffusion model. Estimation results show with high probability of no so called innovators. Other estimated parameter is market potential. In case of Facebook the value is very

high. It is equal to 233 mln. monthly unique visitors or 74% of all US population. The Internet users as percentage of population in US is 80% in 2010.
The main purpose of this article is to reveal the dependency between the value of social network and the number of users. Based on the result of estimation it is arguable to say that network value is liner proportional to the number of visitors. This means that Sarnoff law governs social networks.

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