INTERNATIONAL JOURNAL OF ECONOMICS AND FINANCE STUDIES
Vol 3, No 1, 2011 ISSN: 1309-8055 (Online)

RISK MANAGEMENT WITH TAIL COPULAS FOR EMERGING MARKET PORTFOLIOS

Svetlana Borovkova
Vrije Universiteit Amsterdam
Faculty of Economics and Business Administration
De Boelelaan 1105, 1081 HV Amsterdam, The Netherlands
E-mail: sborovkova@feweb.vu.nl

─Abstract ─

We study the tail dependence of emerging markets in South-East Asia and we show that this tail dependence increased during the financial crisis of 2008-2010. After applying ARMA-GARCH models to individual markets, we fit various copulas to the pairs of market returns and find that in most cases tail copulas such as the *t*-copula and Symmetrised Joe-Clayton provide the best fit. During the crisis, nonlinear dependence measures (such as rank correlations) and the tail dependence coefficients typically increased by tenfold or even more. We apply our method to portfolio Value-at-Risk estimation and show that the copula-based Value-at-Risk performs remarkably well for South-East Asian market portfolios.

Key Words: *emerging markets, copulas, tail dependence, Value-at-Risk*
JEL Classification: C51, C52, C14, G17

1. INTRODUCTION

1.1. Motivation

The co-movements of financial markets are greatly amplified in times of distress, as the recent financial crisis has shown. This phenomenon is referred to as the *tail dependence*. This is particularly true for emerging markets, where tail dependence has also been observed in non-crisis periods. In this paper we study the tail dependence of emerging markets in South-East Asia and investigate how this tail dependence evolved during the financial crisis of 2008-2010.

The dependence between returns on financial assets is traditionally measured by the linear correlation. However, the linear correlation fully describes the dependence only in the case of multivariate normal distribution. It is well-documented that the multivariate normal distribution cannot adequately describe returns on financial assets (especially in the tails of the distribution). Clearly, a univariate normal distribution has too light tails to model extreme returns occurring in reality. A serious drawback of a *multivariate* normal distribution (often overlooked in literature and in practice) is its asymptotic independence in the tails of the distribution. This contradicts the observed phenomenon that the dependence between returns are greatly amplified during stressed market conditions, i.e., at the extremes of the joint distribution. Another empirically observed phenomenon is an asymmetry between the lower and the upper tails of the returns distribution: it is often the case that lower tail dependence (i.e., where simultaneous big *losses* occur) is much greater than the upper tail dependence. A symmetric distribution such as normal is unable to capture this feature.

Dealing with heavier (than normal) tails in one return series is a thoroughly studied and well-understood subject. For example, a GARCH model with *t*-distributed errors can often successfully model heavy tails and volatility clustering. Alternatively, extreme value distributions such as Generalized Pareto can be used for modelling the tails of the returns' distribution. However, dealing with tail dependence and multivariate extremes is a more complicated issue. Modelling a joint distribution with a copula is a flexible approach that is rapidly becoming the industry standard in risk management applications. A copula function offers a convenient way to separate the marginal behaviour from the joint distribution. Various copulas (in particular, the so-called *tail copulas*) are able to deal with tail dependence and lack of symmetry between the left and right tail.

The tail dependence in emerging markets is a subject of several papers, but mostly in relationship to the Asian crisis of 1997 (Mendes (2005), Rodriguez (2007)). In this paper we study the effect of the most recent financial crisis of 2008-2010 on the tail dependence between South East Asian stock markets. We fit a variety of copulas to market returns and find those that provide the best fit, for the entire historical sample as well as for non-crisis and crisis periods separately. For these periods, we compare various dependence measures such as rank correlations and tail dependence coefficients, to see how they evolved during the financial crisis. We take the currency risk into account by considering a viewpoint of a European investor and converting all returns into Euro-based returns using historical exchange rates.

1.2. Copulas and tail dependence

Copulas are parametrically specified joint distributions with given marginals. The most attractive property of copulas is that they separate the marginal behaviour of random variables from their joint distribution. A copula is a multivariate distribution function $C: [0,1]^d \to [0,1]$ with standard uniform margins. The celebrated *Sklar's Theorem* relates any joint distribution to a copula and provides a constructive way of generating a joint distribution from a collection of marginal distributions and a copula function. In essence, it states that, if *F* is a joint distribution function with marginals $F_1, \ldots, F_d$, then there exist a copula function *C*, such that for all $x_1, \ldots, x_d$ we have $F(x_1,..., x_d) = C(F_1(x_1),..., F_d(x_d))$. Conversely, if *C* is a copula and $F_1, \ldots, F_d$ are univariate distribution functions, then *F* defined above is a multivariate distribution with margins $F_1, \ldots, F_d$.

Copulas obtained from the Sklar's theorem using multivariate Normal or *t*-distributions are called respectively Normal and *t*-copulas. Of a particular interest for us are the so-called *tail copulas*: these copulas exhibit dependence in one or both tails of a joint distribution. The tail copulas we shall consider here are: Clayton, Gumbel, symmetrised Joe-Clayton (SJC) and *t*-copulas. The *t*-copula is symmetric, while Clayton and Gumbel copulas are asymmetric, describing the tail dependence in respectively lower and upper tails of the distribution. The symmetrised Joe-Clayton copula is a convenient copula which models both upper and lower tail dependence; however these can be of different magnitude. To allow for tail independence, we also include Frank copula (a symmetric copula not exhibiting any tail dependence) into our analysis. For the sake of brevity we do not give copula expressions and their numerical properties here; for a good overview of copulas, see McNeil et al. (2005).

Copulas are invariant under strictly increasing (possibly nonlinear) transformations of the random variables. Several dependence measures, such as Kendall's tau and Spearman's rho, also possess this property, offering a convenient alternative to the linear correlation. These dependence measures are often referred to as *rank correlations* or *measures of concordance* and they capture nonlinear dependence between random variables: something linear correlation is unable to do. For definitions and properties of these measures we refer to McNeil et al. (2005).

Tail dependence coefficients measure concordance in the tails, or extreme values of random variables *X* and *Y*. Geometrically, they measure the dependence between *X* and *Y* in the upper-right and lower-left quadrant of the joint distribution function. The parameter of asymptotic lower tail dependence, denoted by λ_L, is the conditional probability (in the limit) that *X* takes a very low value, given that *Y* also takes a very low value. The upper tail dependence coefficient λ_U is defined similarly; both coefficients can be deduced from a copula function.
Tail dependence coefficients are independent of the marginal distributions of the random variables and are also invariant under strictly increasing transformations of *X* and *Y*. The variables *X* and *Y* are said to be asymptotically independent if $\lambda_U(X,Y) = \lambda_L(X,Y) = 0$. If λ_L>0, large losses tend to occur simultaneously. For Gaussian copula, both tail dependence coefficients are zero, for e.g., Student-*t* copula, both are greater than zero, indicating lower and upper tail dependence.

1.3.Data description

We focus on the dependence between emerging markets of the South East Asia region, as the tail dependence and asymmetric behaviour has been observed for these markets' returns (see e.g., Rodriguez (2007)). We choose the following countries to represent this economic region: Indonesia, Malaysia, Pakistan, Philippines and South Korea. Arguably, South Korea is not an emerging market, but it is an important player in this region, so we included it into our analysis.

For each country, we use the main stock market index as the representation for the equity prices. We use daily closing prices over a period of 10 years, ranging from January 3, 2001 to January 3, 2011. Unlike in Rodriguez (2007), the start of the financial crisis is given exogenously and not determined by the data. We set the beginning of the financial crisis at July 2007, based on the observation that the first effects of the credit crisis outside USA were seen in the summer of 2007. We take the exchange risk into account by taking the viewpoint of a European investor and expressing all stock market indices in Euros; we use daily closing spot exchange rates for the Euro vs. each respective country's currency.

There is a remarkable feature for Pakistan stock market: during the crisis, the trading on Karachi Stock Exchange was suspended for a several prolonged periods at the end of 2008 (five non-trading periods each lasting one to three weeks), leading to artificially zero returns. However, this does not represent a problem if we model Euro-based returns: a position in Karachi SE 100 Index would have non-zero returns during non-trading periods, due to fluctuations of the local currency against the Euro. We do expect significantly lighter tails in Pakistani returns series and lower tail dependence for pairs that include Pakistan during the crisis period, due to lack of extreme returns which would have occurred had the Exchange not been closed.

2. EMPIRICAL STUDY

2.1. Modelling the marginal behaviour

Historical log-returns series for all considered countries exhibit high kurtosis (from 6 for Pakistan to 17 to Philippines) and negative skewness. The normality is strongly rejected by the Jarque-Bera test for all series. Also volatility clustering is present in all series, as confirmed by Ljung-Box test. So to model individual log-return series, we shall use the class of ARMA-GARCH models with Student-*t* distributed residuals, as it allows for heavier tails.

Estimation of the ARMA-GARCH model is done by the conditional likelihood approach. The model selection is done on the basis of Akaike Information Criterion. This leads to the models shown in Table 1. After the model is selected and estimated, we can retrieve the standardized residuals, which will be used to model the dependence structure.

Country	**Full-sample**	**Crisis**	**Non-crisis**
Indonesia	AR(1)-GARCH(1,1)	GARCH(1,1)	ARMA(5,5)-GARCH(1,1)
Malaysia	AR(1)-GARCH(1,1)	GARCH(1,1)	ARMA(1,1)-GARCH(1,1)
Pakistan	ARMA(1,1)-GARCH(1,1)	AR(1)-GARCH(1,1)	ARMA(5,5)-GARCH(1,1)
Philippines	AR(1)-GARCH(1,1)	GARCH(1,1)	ARMA(1,1)-GARCH(1,1)
Korea	ARMA(1,1)-GARCH(1,1)	GARCH(1,1)	ARMA(3,2)-GARCH(1,1)

Table 1: Selected models for the marginal variables

Table 2 shows the parameter estimates of the GARCH part of the models for the full sample (the results for non-crisis and crisis periods are similar). The last column shows the estimated number of degrees of freedom for the *t*-distribution, which is quite low for all countries, indicating heavier than Normal tails.

Remarkably, for most countries, the number of degrees of freedom is comparable for non-crisis and crisis periods, indicating that the crisis did not lead to significantly heavier tails in the returns distribution. Only for Philippines and Korea, this number is lower for the crisis period (5.7 vs. 7.7 for Philippines and 5.6 vs. 6.6 for Korea), but due to rather large standard errors, we cannot conclude that these differences are significant. This is in line with a general view that emerging countries' markets withstood the financial crisis quite well. In the next section we will show, however, that the tail dependence has increased significantly during the crisis.

Country	α_1 **(s.e.)**	β_1 **(s.e.)**	ν **(s.e.)**
Indonesia	0.116 (0.028)	0.829 (0.043)	4.583 (0.407)
Malaysia	0.078 (0.032)	0.874 (0.066)	6.158 (0.748)
Pakistan	0.141 (0.019)	0.820 (0.023)	6.745 (0.838)
Philippines	0.103 (0.025)	0.807 (0.051)	6.760 (0.856)
Korea	0.067 (0.014)	0.922 (0.016)	6.201 (0.766)

Table 2: GARCH parameters of estimated models, full sample

2.2.Modelling the dependence structure by copulas

Here we only consider pairwise dependence, so we model bivariate distributions of returns for computational tractability. Although this is certainly a restriction, it will give us a good indication of the tail dependence features and allow for visualization of this dependence. To each pair of countries, we fit the following copulas: Normal, *t*, Clayton, Gumbel, Symmetrised Joe-Clayton and Frank. We fit the copulas to the pairs of standardized residuals, obtained after fitting ARMA-GARCH models to each returns series, transformed to uniform distribution by their empirical distribution functions. The copula selection is done on the basis of AIC. Table 3 shows the optimal copulas (above the diagonal) and their estimated parameters (below the diagonal) for the full sample. We also fit copulas to non-crisis and crisis periods separately. Tables 4 and 5 show the optimal copulas for these two periods.

The first remarkable observation from Tables 3-5 is the clear presence of tail dependence for all pairs except those involving Pakistan, as expected. Second, we notice some shifts in the optimal copulas between non-crisis and crisis. For example, for the pair Indonesia-Philippines we observe the shift from *t*-copula for non-crisis period to SJC copula for the crisis. This indicates the asymmetry in the tail dependence during the crisis; in particular, a shift towards the lower tail dependence. In many cases, the *t*-copula provides the best fit. There are big differences in estimated copula parameters between the non-crisis and crisis. For example, the pair Indonesia-Malaysia exhibits much greater tail dependence during the crisis, which is seen from the lower number of degrees of freedom (5 vs. 14.5) and is illustrated in Figure 1.

Full sample	Indonesia	Malaysia	Pakistan	Philippines	Korea
Indonesia		t	Frank	t	t
Malaysia	(0.532, 8.405)		t	t	t
Pakistan	(1.383)	(0.314, 110.358)		Frank	Frank
Philippines	(0.437, 8.805)	(0.505, 12.346)	(1.492)		SJC
Korea	(0.468, 8.326)	(0.508, 9.444)	(1.224)	(0.127, 0.305)	

Table 3: Optimal copulas for full sample

Non-crisis	Indonesia	Malaysia	Pakistan	Philippines	Korea
Indonesia		T	t	t	SJC
Malaysia	(0.461, 14.438)		t	t	t
Pakistan	(0.224, 17.679)	(0.317, 63.229)		Frank	SJC
Philippines	(0.398, 10.568)	(0.453, 11.910)	(1.553)		t
Korea	(0.178, 0.299)	(0.463, 10.014)	(0.019, 0.092)	(0.385, 18.704)	

Table 4: Optimal copulas for non-crisis period

Crisis	Indonesia	Malaysia	Pakistan	Philippines	Korea
Indonesia		t	Frank	SJC	t
Malaysia	(0.644, 4.985)		Frank	t	t
Pakistan	(1.276)	(2.057)		Frank	Frank
Philippines	(0.197, 0.406)	(0.584, 12.570)	(1.495)		SJC
Korea	(0.543, 10.357)	(0.589, 11.726)	(1.269)	(0.135, 0.399)	

Table 5: Optimal copulas for crisis period

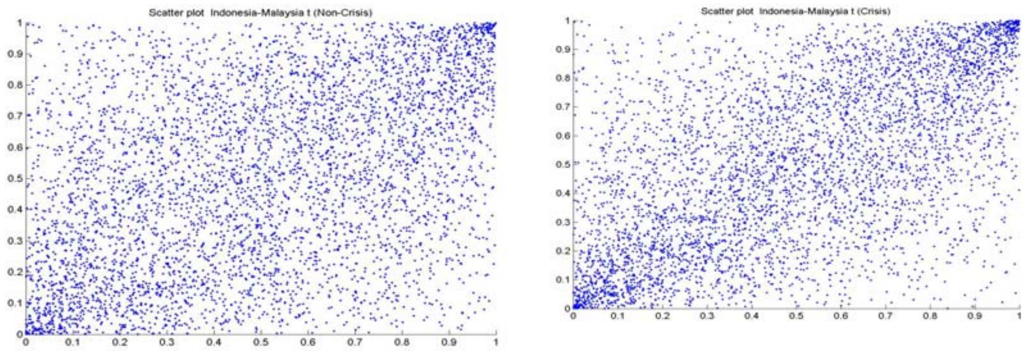

Figure 1: Samples from copulas for Indonesia-Malaysia, left: non-crisis, right: crisis period.

Table 6 shows the rank correlations and Table 7 – upper and lower tail dependence coefficients. In all tables, the estimates for the non-crisis period are given below and for the crisis period – above the diagonal. For all pairs except those containing Pakistan, the rank correlations are higher for the crisis than for non-crisis. Also most crisis tail dependence coefficients are higher, in some cases by one or more orders of magnitude. For example, for Indonesia-Malaysia pair, λ_L increased from 0.03 to 0.3 during the crisis, and for Philippines-Korea pair – from 0.008 to 0.4. Note that the lower dependence coefficients are always higher (or equal) than the upper ones, indicating that large losses have more tendency to occur together than large gains.

Kendall	Indonesia	Malaysia	Pakistan	Philippines	Korea
Indonesia	1.000	0.445	0.139	0.319	0.366
Malaysia	0.305	1.000	0.220	0.397	0.401
Pakistan	0.144	0.205	1.000	0.162	0.139
Philippines	0.260	0.300	0.169	1.000	0.298
Korea	0.262	0.306	0.122	0.251	1.000
Spearman	Indonesia	Malaysia	Pakistan	Philippines	Korea
Indonesia	1.000	0.626	0.208	0.455	0.526
Malaysia	0.445	1.000	0.325	0.566	0.571
Pakistan	0.215	0.304	1.000	0.242	0.207
Philippines	0.382	0.437	0.251	1.000	0.428
Korea	0.379	0.446	0.181	0.370	1.000

Table 6: Kendall's tau and Spearman's rho.

λ_U	Indonesia	Malaysia	Pakistan	Philippines	Korea
Indonesia	1.000	0.298	0.000	0.197	0.093
Malaysia	0.030	1.000	0.000	0.081	0.093
Pakistan	0.003	0.□00	1.000	□.000	0.000
Philippines	0.046	0.046	0.000	1.000	0.135
Korea	0.178	0.069	0.019	0.008	1.000

λ_L	Indonesia	Malaysia	Pakistan	Philippines	Korea
Indonesia	1.000	0.298	0.000	0.406	0.093
Malaysia	0.030	1.000	0.000	0.081	0.093
Pakistan	0.003	0□000	1.000	0.000	0.000
Philippines	0.046	0.046	0.000	1.000	0.399
Korea	0.299	0.069	0.092	0.008	1.000

Table 7: Upper and lower tail dependence.

3. IMPLICATIONS FOR RISK MEASUREMENT

The main application of modelling the joint returns distribution is the portfolio Value-at-Risk estimation. We performed an extensive VaR study (estimation and out-of-sample testing) for all pairwise portfolios and compared the performance of the copula-based method to historical, Normal and Student-*t* variance-covariance and EWMA methods. The copula model fitted to the full sample performs for nearly all portfolios, this is especially pronounced for higher percentile VaR. The performance of VaR estimation methods is illustrated in Figure 2 for the pair Indonesia-Malaysia.

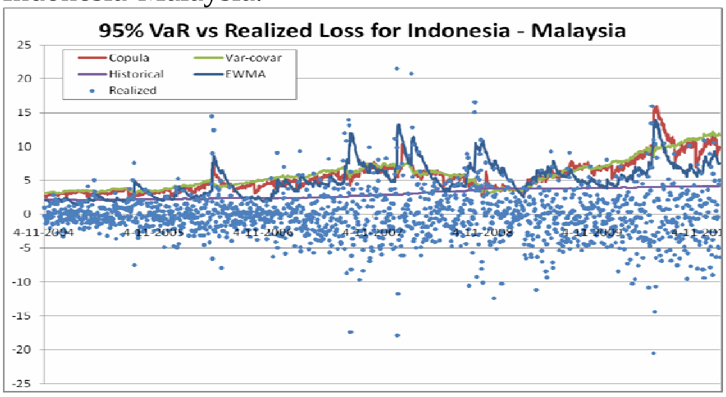

Figure 2: 95% VaR and realized losses for portfolio of Indonesian and Malaysian stocks

The figure shows that the copula method adapts well to the changing market volatility, but provides smoother VaR estimates than the EWMA method. Testing VaR methods for the non-crisis period leads to similar conclusions; however, out-of-sample testing of VaR methods during the financial crisis is problematic due to relatively short series of historical data.

4. CONCLUSIONS AND FUTURE RESEARCH

In this paper we model the dependence between market returns of South East Asian countries by means of copula functions. We provide evidence for significant tail dependence, which dramatically increased during the most recent financial crisis. We show that the copula-based Value-at-Risk model performs remarkably well for stock portfolios of South East Asian markets, and certainly better than other commonly used models.

Our analysis has several important implications for risk and portfolio managers. Financial contagion is clearly present in emerging markets, as seen on the South East Asian example here, and it increases at times of crisis. The diversification benefits of emerging markets portfolios decrease during periods of turmoil in financial markets – exactly at times when these benefits are particularly needed. Failure to take into account the increasing likelihood of simultaneous extreme losses can lead to inadequate risk estimates for emerging markets portfolios.

Here we modelled pairwise dependencies with bivariate copulas. A logical next step is to apply multivariate copulas, to assess risks of larger portfolios. Finally, here we restricted our analysis to one particular region: South East Asia. Extending our analysis to other regions (Latin America, Eastern Europe) can provide us with more insight into regional and inter-regional dependencies and help constructing more robust and diversified emerging markets portfolios.

BIBLIOGRAPHY

McNeil, Alexander, Rudiger Frey and Paul Embrechts (2005), Quantitative Risk Management, Princeton University Press.

Mendes, Beatriz (2005), “Asymmetric extreme interdependence in emerging equity markets”, *Appl. Stochastic Models Bus. Ind.,* No. 21, pp. 483-498.

Rodriguez, Juan Carlos (2007), “Measuring financial contagion: A Copula approach”, *Journal of Empirical Finance*, Vol. 14, pp. 401-423.