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# OPTIMALITY ISSUES IN A SERIES SYSTEM FROM MANUFACTURER'S PERSPECTIVE CONSIDERING BURN-IN AND WARRANTY PERIODS

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***Abstract-*** *Reliability engineers generally have to deal with systems that consist of some components in series and others in parallel. Reliability of a series system can be calculated by multiplying the reliability of individual elements in that system. Failure rate of many deteriorating systems shows a bathtub shape curve. The aim of this paper is to find the average total cost of a series system, from a manufacturer's point of view, during the first two phases of its life; considering optimality issues for burn-in and warranty periods. Numerical illustration is provided to show the applicability of the model.*

***Keywords:*** *Series System, deteriorating system, renewing warranty, burn-in, optimization*

## 1. INTRODUCTION

Nowadays, increase in consumers' knowledge and expectations has forced manufacturers to produce high quality products at reasonable prices. This necessitates manufacturers to either employ various techniques such as a burn-in test in order to manufacture highly reliable products; or satisfy their customers by offering different types of warranties. Two types of extended warranty policies from manufacturer's point of view and their relevant cost and profit models have been proposed by Su and Shen (2012). Burn-in is a process in which the product is run under conditions like the real field conditions for a specified period of time in the manufacturer's place so as to detect problems and filter out defective items. After successfully passing burn-in period a product is considered to be of a quality that can be marketed. In order to maximize reliability, Kim and Kuo (2009) established an exchange between component reliabilities during system burn-in period and developed an optimal burn-in time for repairable non-series system.

When selling a product, warranty is offered by the producer to the customer for two simple but important reasons: be in the safe side and increase customer's satisfaction. Generally warranties are in two classes, namely renewing and non-renewing warranties. Renewing warranty means that if a sold product faces a failure during its warranty period of length $W$, it will be replaced by a new product with a new warranty period of length $W$. Jung *et al*. (2010) considered the maintenance policy of a system under the renewing warranty during its wear out period. In another study, Vahdani *et al*. (2011) developed a replacement repair model in order to investigate a renewing free replacement warranty for a class of multi-state deteriorating repairable products. In a non-renewing warranty policy, however, the failed item at age $t$ is replaced with a new item with a warranty period of length $W$-$t$. Recently, Jung *et al*. (2012) studied the optimal maintenance policy for non-renewing warranty by formulating the average cost rate per unit time from the consumer's viewpoint.

Generally for deteriorating products, the rate of failure is a high value during two periods of their life; first when they are completely new. During this period the failure rate decreases with a sharp slope. This period is known as burn-in or infant mortality period. The second period, where the rate of failure is a high value but has increasing rate, is known as post warranty or wear out period. Between these two periods, the failure rate is approximately constant and known as useful life period which overlaps the warranty period. Failure rate curves of such systems based on their failure rate functions usually depict a bathtub shape curves (see Figure 1).

The purpose of this work is to find the average of the total cost during the first two periods (burn-in and useful life) of product life by considering optimum values of burn-in and warranty periods from the manufacturer's point of view. Considering both burn-in and maintenance policy at the same

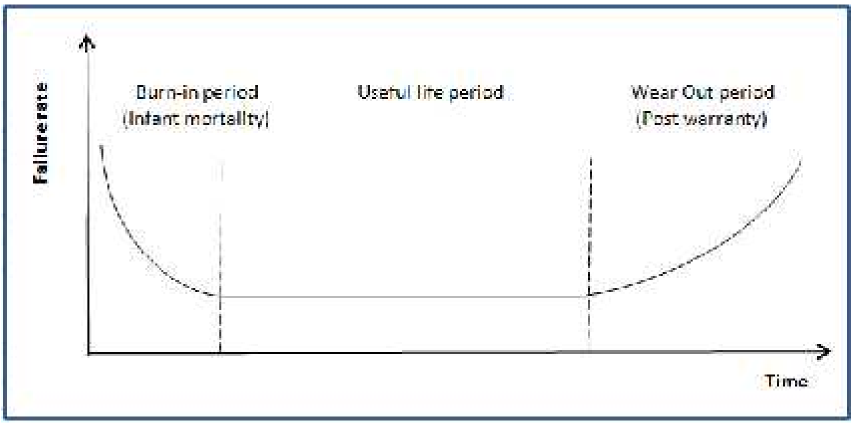


Figure 1: Bathtub failure rate curve

time to study optimum burn-in with random minimal repair cost has been presented by Kwon et al. (2010). Moghimihadji and Rangan (2012) in their recent work, investigated some optimality issues based on the total average cost of a parallel system over its entire useful life from the perspective of both consumers and manufacturers.

## 2. NOTATIONS USED

$f_j(t)$: Failure rate density of jth component

$h_j(t)$: Bathtub shape failure rate curve of jth component

b: Burn-in period length

W: Warranty period length

m: Number of components in the series system

N(b): Number of failed systems until for the first time one system can survive burn-in period of length b

$c_{0j}$: Purchasing and installation cost of jth component in the series system

$c_{1j}$: Operating cost of jth component during burn-in period

$C_2$: Cost of installation of the whole system

$x_i$: Lifetime of ith failed system during burn-in before completing burn-in period b

$C_3$: Cost of replacing failed system in customer's place

$N_b(W)$: Number of failed burnt-in systems during warranty period until for the first time one burnt-in system can survive warranty period of length W

## 3. THE MODEL

In the proposed model, we consider a series system with a bathtub failure rate curve. This system consists of *m* independently distributed elements with bathtub shape failure rate curve $h_j(t)$ and failure rate density $f_j(t)$. The manufacturer puts the system under a specified burn-in process of length *b*. If during this period the system fails, the manufacturer replaces the failed system with a new one.

***Expected cost during burn-in period (0, b]***

Related costs during this period are the *purchasing* and *installation* cost of each component of the series system, the *operating* cost of each component per unit time during burn-in period in manufacturer's place, and finally the *installation* cost of the whole system to do the burn-in test.

$$C_b = N(b) * \sum_{j=1}^{m} c_{0j} + \sum_{j=1}^{m} c_{1j} * \left(b + \sum_{i=1}^{N(b)-1} x_i\right) + C_2 * [N(b) - 1] \quad (1)$$

Let

$$C_0 = \sum_{j=1}^{m} c_{0j} \quad (2)$$

and $C_1 = \sum_{j=1}^{m} c_{1j}$. (3)

Now

$$P_r[N(b) = k] = G(b)^{k-1}\bar{G}(b) \quad (4)$$

where $G(b)$ and $\bar{G}(b)$ are the distribution and survivor functions of the series system and it can be seen that $N(b)$ has a geometric distribution function. Thus,

$$E[N(b)] = \frac{1}{G(b)} \quad (5)$$

Using Wald's identity

$$E\left(\sum_{i=1}^{N(b)-1} x_i\right) = E[N(b)].E(x_1) - E[x_{N(b)}] = \frac{\int_0^b G(t)dt}{G(b)} - b \quad (6)$$

Hence, the expected cost during burn-in period is given by

$$E(C_b) = \frac{C_0}{G(b)} + C_1\frac{\int_0^b G(t)dt}{G(b)} + C_2\frac{G(b)}{G(b)} \quad (7)$$

***Expected cost during warranty period (b, b+W]***

Based on definition, systems that successfully pass the burn-in period are burnt-in systems and of the quality to reach the market. At this phase, the burnt-in system is sold in the market along with a warranty period of length *W*. The cost elements in this period are the cost of having a burnt-in system ($C_b$) and the cost of changing a failed system with a new one in customer's place ($C_3$). Thus,

$$C_W = (C_b + C_3)[N_b(W) - 1] \quad (8)$$

Again using Wald's identity

$$E[N_b(W) - 1] = \frac{G_b(W)}{\bar{G}_b(W)} \quad (9)$$

where $G_b(W)$ and $\bar{G}_b(W)$ are distribution and survivor functions of a burnt-in system, respectively. These functions are given by

$$G_b(W) = \int_0^W g_b(t)dt = \int_0^W \frac{g(b+t)}{\bar{G}(b)}dt = \frac{\int_b^{b+W} g(t)dt}{\bar{G}(b)} \quad (10)$$

and

$$\bar{G}_b(W) = \frac{\bar{G}(b+W)}{\bar{G}(b)} = \frac{\int_{b+W}^{\infty} g(t)dt}{\bar{G}(b)}. \quad (11)$$

Thus, the expected cost during warranty period is given by

$$E(C_W) = (C_b + C_3)\frac{G_b(W)}{\bar{G}_b(W)} \quad (12)$$

At the end of warranty period the total age of the system is *b*+*W* (it is obvious that *b*<*W*) and the average total cost of the system up to this point is the summation of equations (7) and (12) over the total age of the system.

$$ATC = \frac{E(C_b) + E(C_W)}{b+W} \quad (13)$$

## 4. NUMERICAL EXAMPLE

In order to illustrate the application of this model for real cases, we consider a simple series system, which contains only two components. It should be emphasized that only two components are considered in this system for the sole purpose of keeping the calculations simple. Now, suppose that each component has a bathtub failure rate curve

given by

$$h(t) = KC\lambda t^{C-1} + b(1-K)\beta t^{b-1}e^{\beta t^b}. \quad (14)$$

This function was introduced for the first time by Dhillon (1979). By changing the values of shape parameters *b* and *C* in this five-parameter failure rate function; one can generate different shapes of failure rate curve for different types of component. In this illustration, we fix these five parameters as follows: $\beta = 1, \lambda = 2, K = 0.5, b = 1.5$, and $C = 0.3$ for the first component, and for the second component we chang only *b* and *C* as follows: $b = 1.4$, and $C = 0.4$. Since the mean value of lifetime of this system is about 0.3587, we define the range of burn-in period from 0.025 to 0.375 by step size 0.025. In addition, we define the range of warranty period from 0.05 to 1.1 by step size 0.05. Table 1 shows the amount of average total cost for different pair values of burn-in and warranty periods.

| Warranty \ Burn-in | 0.025 | 0.050 | 0.075 | 0.100 | 0.125 | 0.150 | 0.175 | 0.200 | 0.225 | 0.250 | 0.275 | 0.300 | 0.325 | 0.350 | 0.375 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.05 | 2509.1 | 2107.0 | 1890.6 | 1770.1 | 1705.5 | 1686.5 | 1695.0 | 1728.7 | 1785.3 | 1863.8 | 1961.5 | 2089.6 | 2240.5 | 2421.0 | 2635.3 |
| 0.10 | 1609.3 | 1588.7 | 1529.5 | 1506.5 | 1510.9 | 1537.4 | 1584.8 | 1649.5 | 1734.0 | 1838.9 | 1965.2 | 2117.5 | 2297.4 | 2502.8 | 2760.7 |
| 0.15 | 1376.2 | 1352.9 | 1353.8 | 1374.6 | 1412.9 | 1467.8 | 1539.3 | 1628.3 | 1735.9 | 1861.4 | 2016.7 | 2196.2 | 2407.6 | 2656.6 | 2950.4 |
| 0.20 | 1319.3 | 1297.4 | 1267.0 | 1413.4 | 1374.1 | 1449.9 | 1541.9 | 1651.5 | 1781.6 | 1934.6 | 2114.4 | 2325.4 | 2573.9 | 2867.0 | 3214.0 |
| 0.25 | 1142.1 | 1181.2 | 1233.2 | 1298.2 | 1377.3 | 1471.7 | 1582.4 | 1714.6 | 1868.7 | 2049.2 | 2261.2 | 2510.5 | 2804.7 | 3153.3 | 3565.1 |
| 0.30 | **1112.4** | 1168.5 | 1246.8 | 1418.5 | 1411.1 | 1578.6 | 1661.5 | 1817.1 | 1999.3 | 2213.1 | 2464.8 | 2762.0 | 3114.6 | 3534.9 | 4035.8 |
| 0.35 | 1116.5 | 1187.5 | 1271.6 | 1370.2 | 1485.7 | 1620.6 | 1778.4 | 1963.1 | 2180.0 | 2435.6 | 2737.8 | 3097.1 | 3526.3 | 4042.1 | 4665.8 |
| 0.40 | 1148.5 | 1234.8 | 1345.7 | 1453.5 | 1591.0 | 1751.6 | 1940.9 | 2161.2 | 2427.3 | 2741.8 | 3100.5 | 3542.2 | 4054.6 | 4720.5 | 5507.9 |
| 0.45 | 1207.2 | 1310.5 | 1421.0 | 1571.7 | 1726.2 | 1929.1 | 2156.3 | 2424.9 | 2744.0 | 3123.2 | 3583.4 | 4137.5 | 4812.5 | 5640.7 | 6665.3 |
| 0.50 | 1294.0 | 1411.8 | 1567.3 | 1731.6 | 1940.4 | 2165.1 | 2444.3 | 2774.8 | 3172.1 | 3651.3 | 4233.2 | 4945.0 | 5822.5 | 6914.7 | 8282.7 |
| 0.55 | 1413.3 | 1562.6 | 1737.8 | 1944.1 | 2185.1 | 2478.2 | 2823.2 | 3242.6 | 3748.1 | 4364.6 | 5122.3 | 6051.8 | 7216.4 | 8719.2 | 10610.4 |
| 0.60 | 1572.2 | 1755.0 | 1960.4 | 2225.9 | 2530.8 | 2896.8 | 3338.9 | 3876.9 | 4536.3 | 5351.2 | 6362.4 | 7645.8 | 9251.0 | 11359.0 | 14077.9 |
| 0.65 | 1783.8 | 2010.1 | 2259.7 | 2602.3 | 2991.7 | 3464.1 | 4041.9 | 4754.0 | 5629.4 | 6730.4 | 8158.2 | 9961.6 | 12297.4 | 15360.3 | 19423.7 |
| 0.70 | 2061.4 | 2350.9 | 2695.5 | 3112.9 | 3633.3 | 4248.6 | 5025.5 | 5997.8 | 7236.6 | 8795.9 | 10822.6 | 13471.7 | 16939.5 | 21689.4 | 28108.9 |
| 0.75 | 2441.6 | 2812.6 | 3264.3 | 3818.6 | 4504.6 | 5361.1 | 6440.8 | 7816.1 | 9587.3 | 11896.3 | 14944.2 | 19023.7 | 24565.4 | 32214.5 | 42935.8 |
| 0.80 | 2957.5 | 3450.9 | 4060.1 | 4819.3 | 5773.9 | 6987.3 | 8547.8 | 10572.1 | 13239.8 | 16800.1 | 21622.0 | 28256.5 | 37512.0 | 50779.6 | 70050.8 |
| 0.85 | 3670.2 | 4353.8 | 5205.0 | 6281.7 | 7661.9 | 9451.7 | 11802.3 | 14931.1 | 19157.0 | 24934.8 | 33045.5 | 44541.9 | 61206.3 | 85883.1 | 123252.4 |
| 0.90 | 4717.4 | 5678.1 | 6907.7 | 8498.3 | 10552.3 | 13349.3 | 17078.1 | 22179.3 | 29279.0 | 39335.7 | 53876.0 | 75334.9 | 107719.4 | 157952.7 | 237850.4 |
| 0.95 | 6261.4 | 7683.0 | 9541.5 | 12004.1 | 15313.8 | 19831.5 | 26102.8 | 34965.4 | 47737.4 | 66326.2 | 94846.6 | 138554.7 | 207584.3 | 321130.5 | 512182.8 |
| 1.00 | 8619.6 | 10853.2 | 13808.6 | 17833.7 | 23407.3 | 31265.1 | 42500.1 | 59140.7 | 84041.4 | 122373.9 | 182993.1 | 281702.8 | 447650.8 | 736559.3 | 1259036.2 |
| 1.05 | 12218.7 | 16122.0 | 21307.7 | 28116.2 | 38166.5 | 52883.4 | 74928.5 | 108792.7 | 162147.2 | 248806.7 | 394036.8 | 646110.6 | 1100422.6 | 1934040.6 | 3632878.2 |
| 1.10 | 19136.9 | 25423.4 | 34117.9 | 47558.4 | 67194.5 | 97266.5 | 141566.3 | 231157.3 | 349170.8 | 570652.9 | 963666.3 | 1711089.5 | 3175053.8 | 6181513.9 | 12734742.1 |

Table 1: Average total cost for different pair values of burn-in and warranty periods.

As it can be seen from Table 1, the minimum average total cost holds when the burn-in time is 0.025 and warranty period is 0.3. Indeed, it can be seen that when warranty period is very small, by increasing the burn-in time, first the average total cost decreases and then it starts to increase. When the amount of warranty period reaches 0.2 or more, the average total cost becomes a strictly increasing function of burn-in period. Same thing happens when we fix burn-in period at a very small value and then increase warranty period. As in the previous case, first the average total cost decreases and after that it increases. From b = 0.2745, the average total cost is an increasing function of warranty period. Figure 2 depicts the mesh diagram of average total cost for this example.

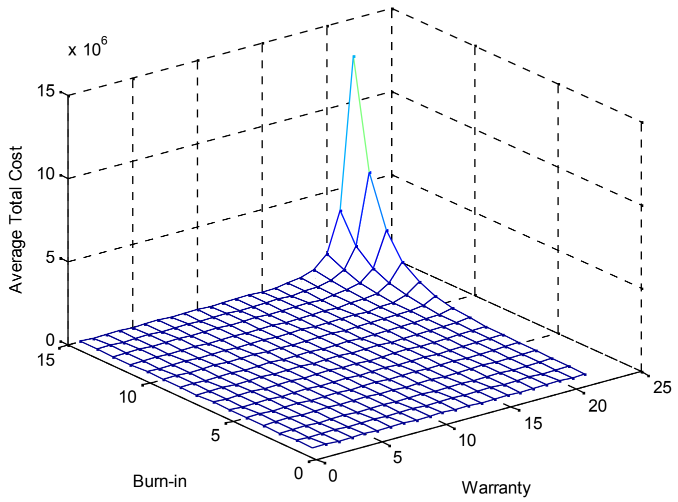


Figure 2: Mesh diagram of average total cost.

## 5. CONCLUSION

Nowadays, in light of fierce competition along with prudent and diligent customers, is it difficult to survive in the market without possessing the ability of producing high quality products that lure new customers while preserving the old ones. Hence, using techniques such as burn-in tests and offering suitable warranties along with selling the product are very important. In this research, we discussed some optimality issues like optimum burn-in and warranty periods in a series system from producer's point of view. Next step after this work can be to consider the maintenance cost during post warranty period, where the system maintenance and its relevant costs are borne by the consumer.

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