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EVENT RECONSTRUCTION BY INVERSE METHODS

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Abstract- *This work deals with an inversion technique that was developed to reconstruct a pulse after it has propagated along a pipe; a complex pulse that is progressively distorted as explained. The technique developed makes use of the theory of inverse problems.*

Keywords: Pulse, event, inverse method, deconvolution, sensor,

INTRODUCTION

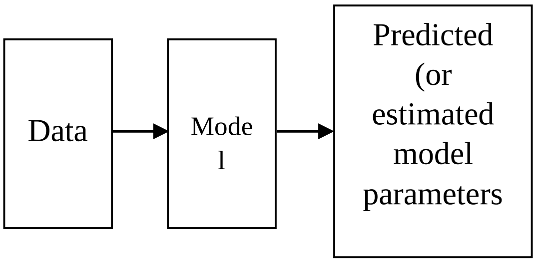
An inverse problem is one that occurs in many branches of science and mathematics where the values of some model parameter(s) must be obtained from the observed data. Inverse methods can be basically considered as an approach for interpolating or smoothing a data set in space and time where a model acts as a dynamical constraint [1].

An inverse problem involves determining the parameters of a model that describes or explains a set of observed data.
In geophysics, inverse problems require an understanding of the "forward process" that relates the model to its geophysical response. They also require knowledge of the statistical reliability of the observed data. Aspects that must be considered in inverse problems include the formulation and parameterization of the problem, the existence, uniqueness and resolution of solutions, strategies for dealing with over-determined and under-determined model parameters, and strategies for introducing independent constraints into the solutions [2].

FORWARD THEORY

The (mathematical) process of predicting data based on some physical or mathematical model with a given set of model parameters (and perhaps some other appropriate information, such as geometry, etc.).
As an example, consider a two-way vertical travel time t of a seismic wave through M , *layers* of thickness h_i and velocity v_i . Then t is given by: $t = 2\sum_{i=1}^{M} \frac{h_i}{v_i}$.

The forward problem consists of predicting data (travel time) based on a (mathematical) model of how seismic waves travel. Suppose that for some reason thickness was known for each layer (perhaps from drilling). Then only the M velocities would be considered model parameters. One would obtain a particular travel time t for each set of model parameters one chooses. Schematically, one might represent this as follows:



As an example, one might invert the travel time t above to determine the layer velocities. Note that one needs to know the (mathematical) model relating travel time to layer thickness and velocity information. Inverse theory should not be expected to provide the model itself.
The work as carried out by this research is made more difficult as there are no possibilities of making repeat trials, but fortunately this is made possible because the pulses propagate in a one-dimensional wave guide rather than the three dimensional interior of the Earth.

THE INVERSE THEORY

Inverse problems may be described as problems where the solutions are known, but not the causes. Alternatively, where the results, or consequences of the problem are known but not what must have caused it. Inverse theory therefore requires knowledge of a forward model capable of predicting data if the model parameters are already known. In an inverse problem measurements are taken of these effects and calculations made to establish what caused them. This requires a description of the data; in most inverse problems the data are simply a table of numerical values, of which a vector provides a convenient means of representing them.

Inverse theory is inherently mathematical and as such does have its limitations. It is best suited to estimating the numerical values of model parameters for some known or assumed mathematical model. It is good for extracting the model parameters that best fit the data.
The basic theory of inverse methods is fully explained by Menke [3]. Briefly, it can be summarised as follows. Suppose in the course of an experiment N measurements are obtained, these numbers may be considered as the elements of the vector **d** of length N. Also, the model parameters can be represented as the elements of the vector **m**, of length M.
Thus, we can write,

data: $\mathbf{d} = \left[d_1, d_2, d_3 ..., d_N\right]^T$ (1)

model parameters:
$\mathbf{m} = \left[m_1, m_2, m_3, ..., m_M\right]^T$ (2)

In the statement of an inverse problem there is a relationship between the model parameters and the data. This relationship is referred to as the model. The model usually takes the form of one or more formulas that the model parameters and data are anticipated to follow. For example, in trying to determine the resistance of a wire by measuring its voltage and current, there will be two data sets, voltage $\mathbf{d}_1$ and current $\mathbf{d}_2$ respectively, and one unknown model parameter, resistance (m_1). The model statement would be the resistance times the current equals voltage, which can be represented compactly by vector equation (3),

$\mathbf{d}_1 = \mathbf{d}_2 m_1$ (3)

In more realistic situations the relationship between the data and model parameter is more complicated. In the most general case, the data and model parameters are related by one or more implicit equation such as in equation (4),

$$l_1(\mathbf{d}, \mathbf{m}) = 0$$
$$l_2(\mathbf{d}, \mathbf{m}) = 0$$
$$\vdots \qquad (4)$$
$$l_N(\mathbf{d}, \mathbf{m}) = 0$$

where, N is the number of equations.
In the above problem concerning the measurements of the resistance, $N = 1$ and $d_2 m_1 = d_1$ would constitute one equation of the form

$$l_1(d, m) = 0 \qquad (5)$$

These implicit equations, which can be compactly written as the vector equation **l** (**d**, **m**) = 0, summarize what is known about how the measured data and the unknown model parameter are related. The goal of inverse theory, therefore, is to solve, or invert, these equations for the model parameters.

THE LINEAR INVEERSE PROBLEM

The simplest form of a linear inverse problem as described by Menke [3], is given by,

d = Gm (6)

where,
d = measured data, **G** = data kernel **m** = model parameters
The data and model parameters are functions $d(x)$ and $m(x)$, in which χ is some independent variable. Again using the problem of determining the resistance of a wire it is possible to formulate an inverse problem. Supposing that N voltage measurements V_j are made at current i_j in a circuit, then, the data form a vector **d** of N measurements of voltage, where,

$$\mathbf{d} = [V_1, V_2, V_3, \ldots, V_N]^T \qquad (7)$$

The current i_j provides auxiliary information that describes the geometry of the experiment. If we assume a model in which the voltage is a linear function of the current;

$$V = a + bi \qquad (8)$$

The intercept *a* and the slope b form the two model parameters of the problem, m = $[a, b]^T$. According to this model, each voltage observation must satisfy $V = a + bi$:

$$V_1 = a + bi_1$$
$$V_2 = a + bi_2$$
$$\vdots$$
$$V_N = a + bi_N \qquad (9)$$

These equations can be arranged as the matrix equation **d** = **Gm**:

$$\underbrace{\begin{bmatrix} V_1 \\ V_2 \\ . \\ . \\ . \\ V_N \end{bmatrix}}_{d} = \underbrace{\begin{bmatrix} 1 & i_1 \\ 1 & i_2 \\ . & . \\ . & . \\ . & . \\ 1 & i_N \end{bmatrix}}_{G} \underbrace{\begin{bmatrix} a \\ b \end{bmatrix}}_{m} \qquad (10)$$

For any real set of measurements with experimental errors equation (8) will not be satisfied exactly, but equation 10 can still be used for a least square solution to determine the model parameters *a* and *b*.

INVERSE METHOD BASED ON LEAST SQUARES

The two most common vectors that are concerned with inverse problems are the data-error or misfit vector and the model parameter vector [3]. The methods based on data-error or misfit give rise to classic least squares solutions, while methods based on the model parameter give rise to what is called minimum length solutions.

The improvements over simple least squares and the minimum length solutions include the use of information about noise in the data and a fore-knowledge about the model parameters.

The most important application of these vectors is in data fitting. The best fit in the least square sense minimizes the sum of the squared residuals, a residual being the difference between an observed value and the fitted value provided by a model.

Minimizing the Misfit-Least Squares (after Menke [3])

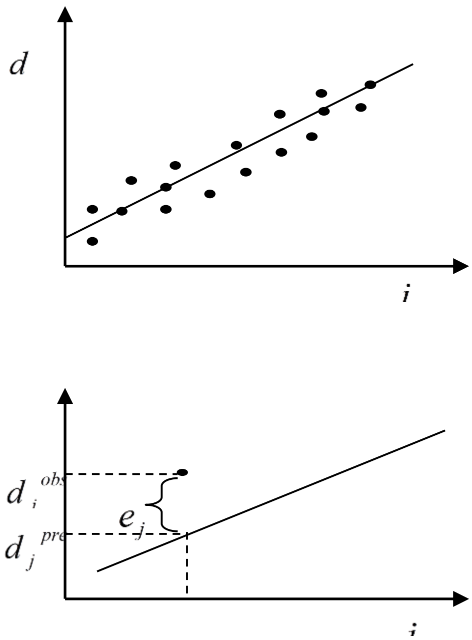


Figure 1 Least squares fitting of a straight line to (i, d) pairs

The error e_j for each observation is the difference between the observed and predicted datum:

$$e_j = d_j^{obs} - d_j^{pre} \tag{11}$$

The jth predicted datum d_j^{pre} for the straight line problem is given by

$$d_j^{pre} = m_1 + m_2 i_j \tag{12}$$

where the two unknowns, m_1 and m_2, are the intercept and the slope of the line and i_j is the value along the i axis where the j th observation is made.

For N points we have a system of N such equations that can be written in matrix form as:

$$\begin{bmatrix} d_1 \\ . \\ . \\ . \\ d_N \end{bmatrix} = \begin{bmatrix} 1 & i_1 \\ . & . \\ . & . \\ . & . \\ 1 & i_N \end{bmatrix} \begin{bmatrix} m_1 \\ m_2 \end{bmatrix} \tag{13}$$

or

$$\underset{(N\times 1)}{\mathbf{d}} = \underset{(N\times 2)}{\mathbf{G}} \underset{(2\times 1)}{\mathbf{m}} \tag{14}$$

The total misfit E is given by

$$E = \mathbf{e}^{\mathrm{T}}\mathbf{e} = \sum_{j=1}^{N} \left[d_j^{obs} - d_j^{pre} \right]^2 \tag{15}$$

$$= \sum_{j=1}^{N} \left[d_j^{obs} - (m_1 + m_2 i_j) \right]^2$$

(16)

Dropping the "obs" in the notation for the observed data, we have

$$E = \sum_{j}^{N} \begin{bmatrix} d_j^2 - 2d_j m_1 - 2d_j m_2 i_j + \\ 2m_1 m_2 i_j + m_1^2 + m_2^2 i_j^2 \end{bmatrix} \tag{17}$$

Taking the partial derivatives of E with respect to m_1 and m_2, and equating them to zero yields:

$$\frac{\partial E}{\partial m_1} = 2Nm_1 - 2\sum_{j=1}^{N} d_j + 2m_2 \sum_{j=1}^{N} i_j = 0 \qquad (18)$$

And

$$\frac{\partial E}{\partial m_2} = -2\sum_{j=1}^{N} d_j i_j + 2m_1 \sum_{j=1}^{N} i_j + 2m_2 \sum_{j=1}^{N} i_j^{\,2} = 0 \qquad (19)$$

Rewriting equations (5.18) and (5.19) above

$$Nm_1 + m_2 \sum_j i_j = \sum_j d_j \qquad (20)$$

and

$$m_1 \sum_j i_j + m_2 \sum_j i_j^{\,2} = \sum_j d_i i_j \qquad (21)$$

Combining the two equations in matrix notation in the form **Am** = **b** gives

$$\begin{bmatrix} N & \sum i_j \\ \sum i_j & \sum i_j^{\,2} \end{bmatrix} \begin{bmatrix} m_1 \\ m_2 \end{bmatrix} = \begin{bmatrix} \sum d_j \\ \sum d_j i_j \end{bmatrix} \qquad (22)$$

or, simply

$$\underset{(2\times 2)}{\mathbf{A}} \ \underset{(2\times 1)}{\mathbf{m}} = \underset{(2\times 1)}{\mathbf{b}} \qquad (23)$$

Equation (23) above shows that the problem has been reduced from one with N equations to two unknowns (m_1 and m_2) in **Gm** = **d** to one with two equations in the same unknowns as in **Am** = **b**.

The matrix equation **Am** = **b** can also be rewritten in terms of the original **G** and **d** when it is observed that the matrix **A** can be factored as:

$$\begin{bmatrix} N & \sum i_j \\ \sum i_j & \sum i_j^{\,2} \end{bmatrix} = \begin{bmatrix} 1 & 1 & \ldots & 1 \\ i_1 & i_2 & \ldots & i_N \end{bmatrix} \begin{bmatrix} 1 & i_1 \\ 1 & i_2 \\ . & . \\ . & . \\ . & . \\ 1 & i_N \end{bmatrix} = \mathrm{G}^{\mathrm{T}}\mathrm{G}$$

(24)

Also, **b** above can be written similarly as

$$\begin{bmatrix} \sum d_j \\ \sum d_j i_j \end{bmatrix} = \begin{bmatrix} 1 & 1 & \ldots & 1 \\ i_1 & i_2 & \ldots & i_N \end{bmatrix} \begin{bmatrix} d_1 \\ d_2 \\ . \\ . \\ . \\ d_N \end{bmatrix} = \mathrm{G}^{\mathrm{T}}\mathrm{d} \qquad (25)$$

Substituting equations (24) and (25) into equation (22), gives the equations for the least squares problem:

$\mathbf{G}^{\mathrm{T}}\mathbf{G}\mathbf{m} = \mathbf{G}^{\mathrm{T}}\mathbf{d}$ (26)

The least squares solution $\mathbf{m}_{\mathbf{LS}}$ is then obtained as

$\mathbf{m}_{\mathbf{LS}} = [\mathbf{G}^{\mathrm{T}}\mathbf{G}]^{-1}\mathbf{G}^{\mathrm{T}}\mathbf{d}$ (27)

assuming that $[\mathbf{G}^{\mathrm{T}}\mathbf{G}]^{-1}$ exists.

This solution implies that the forward problem as in equation (13) can be used to obtain an explicit relationship between the model parameters (m_1 and m_2) and a measurement of the misfit to the observed data E. The value E is then minimized by taking the partial derivatives of the misfit function with respect to the unknown model parameters, equating the partial derivatives to zero, and solving for the model parameters [4].

APPLICATION OF LINEAR INVERSE TO PULSE PROPAGATION IN PIPELINES

Consider again the pipeline illustrated in Figure 2 To formulate the problem into a least-squares inverse problem, we can use the rule governing the attenuation between sensors 2 and 3: that the attenuation coefficient defining propagation is not only frequency dependent, but is proportional to frequency squared [5], and then work backwards from sensor 2 to the event site. Though pulse propagation in a pipeline is in reality non-linear, it is only weakly so if the dispersion effect is not too strong. In this section the assumption of local linearity is made by neglecting dispersion, and so that the problem can be approached by the least–squares inverse method.

The technique developed works in the frequency domain. The pulse signals at the sensor locations 2 and 3 are first transformed into the frequency domain using the fast fourier transform (FFT). Assuming exponential attenuation proportional to the square of each frequency component a best-fit coefficient for the attenuation between locations 2 and 3 is found. Applying this to the pulse signal from sensor 2 the original pulse at the event site is reconstructed in the frequency domain, and finally transformed back into the time domain using the inverse FFT.

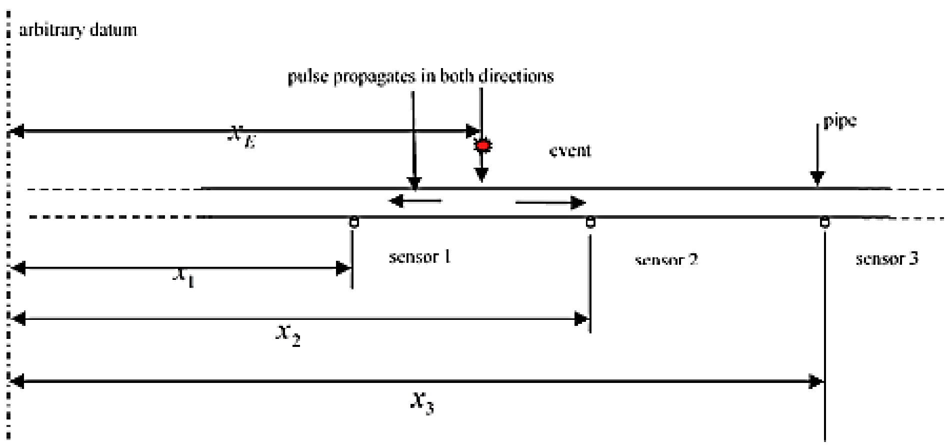


Figure 2 Schematic representation of sensors on a pipeline

Pulses propagating in fluid filled pipelines can be expressed in the form:

$$p(x,t) = p_o(t)\exp(-\beta x) \tag{28}$$

where p is the description of the pulse which is a function of time and distance along the pipe, p_o is the function defining the pulse at $x = 0$, β is frequency dependent attenuation factor, proportional to frequency squared. This formulation is general and is not restricted to any particular pulse shape.

Relating this to the Fourier spectrum in the frequency domain,

$$P_j^{\,1} = P_j^{\,0} e^{-\beta x_{0,1}} \tag{29}$$

or

$$P_j^{\,1} = P_j^{\,0} e^{-a f_j^{\,2} x_{0,1}} \tag{30}$$

Where $P_j^{\,0}$ and $P_j^{\,1}$ are the pulse functions transformed into the frequency domain, the subscripts denoting the index of the Fourier spectrum components and the superscripts the location of the pulse, 0 being the event location and 1, 2, 3 being locations of the sensors as defined in Figure 2.

f_j = the frequency of the jth Fourier component,

$x_{0,1}$ = distance between the event 0 and sensor 1.

a = a proportionality constant to be determined

Considering now, the Fourier spectrums of the pulse signals at sensors 2 and 3 along the pipeline in Figure 2,

$$P_j^{\,3} = P_j^{\,2} e^{-a f_j^{\,2} x_{2,3}} \tag{31}$$

Taking the natural logarithm of both sides of the expression in equation 31 makes this into a linear inverse problem of the form,

$$LnP_j^{\,2} = LnP_j^{\,3} + a f_j^{\,2} x_{2,3} \tag{32}$$

In matrix form, this becomes:

$$\underbrace{\begin{bmatrix} LnP_1^{\,2} \\ LnP_2^{\,2} \\ . \\ . \\ . \\ LnP_N^{\,2} \end{bmatrix}}_{d} = \underbrace{\begin{bmatrix} LnP_1^{\,3} & x_{2,3} f_1^{\,2} \\ LnP_2^{\,3} & x_{2,3} f_2^{\,2} \\ . & . \\ . & . \\ . & . \\ LnP_N^{\,3} & x_{2,3} f_N^{\,2} \end{bmatrix}}_{G} \underbrace{\begin{bmatrix} 1 \\ a \end{bmatrix}}_{m} \tag{33}$$

which is of the same basic form as equation 13.

Forming the matrix products

G

$$\mathbf{G}^{\mathrm{T}}\mathbf{G} = \begin{bmatrix} LnP_1^{\,3} & LnP_2^{\,3} & \dots & LnP_N^{\,3} \\ x_{2,3} f_1^{\,2} & x_{2,3} f_2^{\,2} & \dots & x_{2,3} f_N^{\,2} \end{bmatrix} \begin{bmatrix} LnP_1^{\,3} & x_{2,3} f_1^{\,2} \\ LnP_2^{\,3} & x_{2,3} f_2^{\,2} \\ . & . \\ . & . \\ . & . \\ LnP_N^{\,3} & x_{2,3} f_N^{\,2} \end{bmatrix}$$

$$= \begin{bmatrix} \sum (LnP_j^{\,3})^2 & \sum (LnP_j^{\,3} \times x_{2,3} f_j^{\,2}) \\ \sum (x_{2,3} f_j^{\,2} \times LnP_j^{\,3}) & \sum x_{2,3}^{\,2} f_j^{\,4} \end{bmatrix} \tag{34}$$

and

G^{T}d=

$$\begin{bmatrix} LnP_1^{\,3} & LnP_2^{\,3} & \cdots & LnP_N^{\,3} \\ x_{2,3} f_1^{\,2} & x_{2,3} f_2^{\,2} & \cdots & x_{2,3} f_N^{\,2} \end{bmatrix} \begin{bmatrix} LnP_1^{\,2} \\ LnP_2^{\,2} \\ \cdot \\ \cdot \\ \cdot \\ LnP_N^{\,2} \end{bmatrix} = \begin{bmatrix} \sum (LnP_j^{\,3} \times LnP_j^{\,2}) \times x_{2,3} f_j^{\,2} \\ \sum x_{2,3} f_j^{\,2} LnP_j^{\,2} \end{bmatrix} \quad (35)$$

$$\mathbf{m}_{LS} = [\mathbf{G}^T\mathbf{G}]^{-1}\mathbf{G}^T\mathbf{d} = \begin{bmatrix} \sum (LnP_j^{\,3})^2 & \sum (LnP_j^{\,3} \times x_{2,3} f_j^{\,2}) \\ \sum (x_{2,3} f_j^{\,2} \times LnP_j^{\,3}) & \sum x_{2,3}^{\,2} f_j^{\,4} \end{bmatrix}^{-1} \begin{bmatrix} \sum (LnP_j^{\,2} \times LnP_j^{\,3}) \times x_{2,3} f_j^{\,2} \\ \sum x_{2,3} f_j^{\,2} LnP_j^{\,2} \end{bmatrix} \quad (36)$$

Equation (36) gives a linear solution to the least square inverse problem obtained based on the general form of the solution set out, from which the estimate of the model parameter *a*, which defines the frequency dependent attenuation in $\mathbf{m}_{LS} = [1\ a]^T$ is determined. This value of the estimated model parameter *a* in $\mathbf{m}_{LS}$ can then be applied to the Fourier spectrum of the pulse signal at sensor 2 to compute the estimated Fourier spectrum of the form of the pulse to be reconstructed at the start of the event using equation (33)

$$\underbrace{\begin{bmatrix} LnP_1^{\,0} \\ LnP_2^{\,0} \\ \cdot \\ \cdot \\ \cdot \\ LnP_N^{\,0} \end{bmatrix}}_{d} = \underbrace{\begin{bmatrix} LnP_1^{\,2} & x_{0,2} f_1^{\,2} \\ LnP_2^{\,2} & x_{0,2} f_2^{\,2} \\ \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ LnP_N^{\,2} & x_{0,2} f_N^{\,2} \end{bmatrix}}_{G} m_{LS} \quad (37)$$

Since, the Fourier spectrum of the pulse signal at sensor 2 is known, this allows for the computation of the log Fourier spectrum of the pulse at the event that is to be reconstructed, and from which it is a simple matter to convert to the time domain using the inverse FFT.

RECONSTRUCTION OF SIMPLE PULSE BY THE INVERSE METHOD

This involved the use of the simple pulse data described containing a single frequency component of 53 Hz.

- Case 1: close sensor spacing of 110 m from the event site to sensor 2 and 300 m between sensors 2 and 3.

Figures 3 and 4 show the reconstructed pulses obtained using the inverse method.

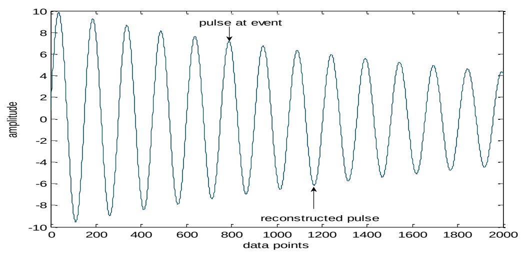
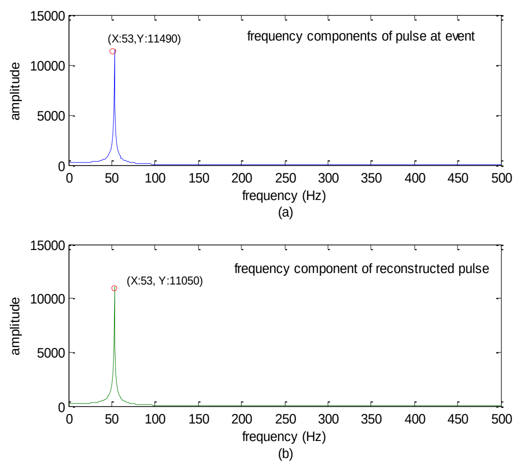


Figure 3 Reconstructed simulated pulse at event by inverse method (single frequency with sensors close to the event) in time domain



(a) original pulse (b) reconstructed pulse

Figure 4 Reconstructed simulated pulse at event by inverse method (single frequency with sensors close to the event) in time domain

The reconstructed pulse in Figure 3 shows a good fit to the original pulse. This impression is confirmed by the Fourier spectra of the pulses in Figure 4, which have the same fundamental frequency of 53 Hz and differ by only 4% in magnitude compared with the 7% using the deconvolution filter method.

Case 2: wide sensor spacing of 800 m from the event site to sensor 2 and 1000 m between sensors 2 and 3

Figures 5 and 6 shows the simple pulse reconstructed by the inverse method in the time and frequency domains, respectively.

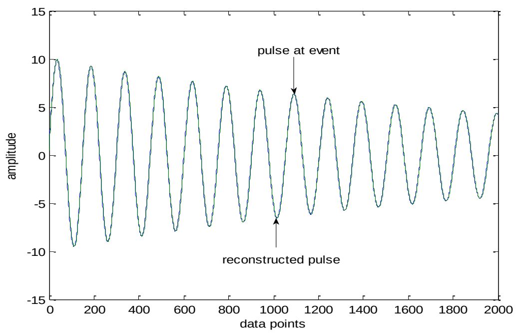
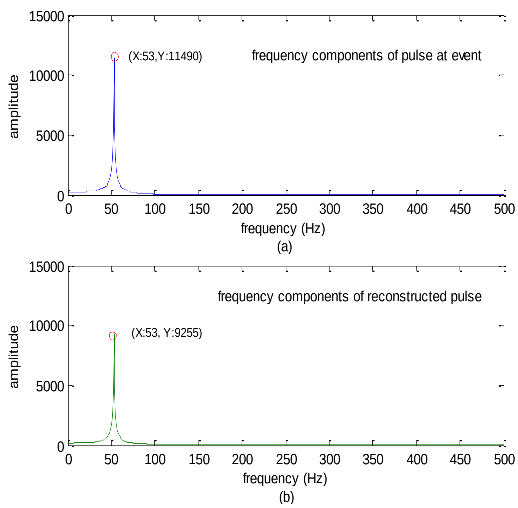


Figure 5 Reconstructed simulated pulse at event by inverse method (single frequency with sensors widely spaced) in time domain



(a) original pulse (b) reconstructed pulse

Figure 6 Reconstructed simulated pulse at event by inverse method (single frequency with sensors widely spaced) in frequency domain

The fit of the reconstructed pulse with the original looks better using the inverse method, though this impression is not confirmed by Figure 5 where the height of the 53Hz peak is 20% different from the original, compared with 15% using the deconvolution filter.

COMPLEX PULSE RECONSTRUCTION BY INVERSE METHOD

This entailed the reconstruction of a complex pulse with eight frequency components as described under the same conditions of cases 1 and 2.

- Case 1: closely spaced sensors

Figures 7 and 8 shows the reconstructed pulses obtained using the inverse method under the conditions of case 1.

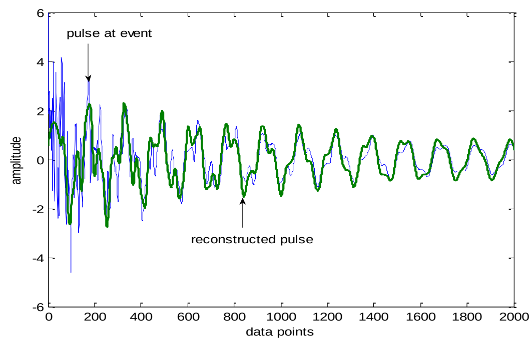
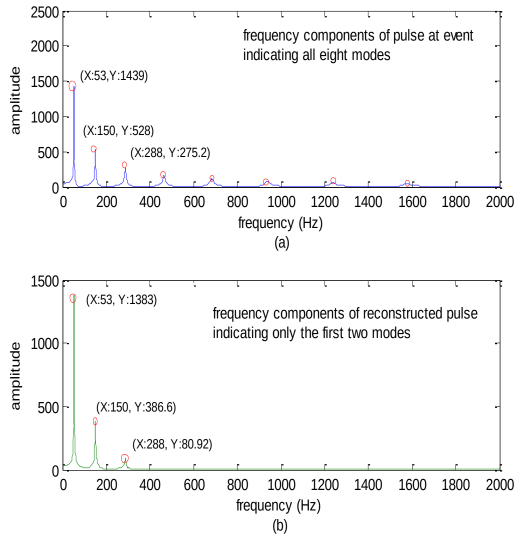


Figure 7 Reconstructed simulated pulse at event by inverse method (multiple frequencies with sensors closer to the event) in time domain

The reconstructed pulse in Figure 7 using the inverse method shows a better approximation to the original pulse than that obtained using the deconvolution filter. This is confirmed in Figure 8 where the first three peaks reduce in a similar progression. The magnitude of these three low modes are all underestimated by this inverse reconstruction technique, and this effect is progressively more marked in the higher frequency modes so that the fourth and subsequent modes are not reconstructed at all. As in the deconvolution filter reconstruction method, this is because these components attenuate fast and so did not reach the second sensor, so they would not have appeared in the inverse calculation.



(a) original pulse (b) reconstructed pulse

Figure 8 Reconstructed simulated pulse at event by inverse method (multiple frequencies with sensors closer to the event) in frequency domain

- Case 2: widely spaced sensors

Reconstruction of the same complex pulse with the widely spaced sensors is shown in Figures 9 and 10 in the time and frequency domains.

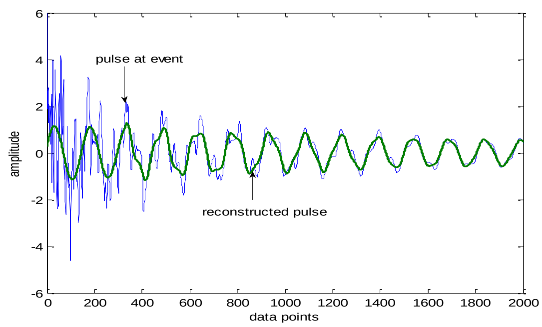
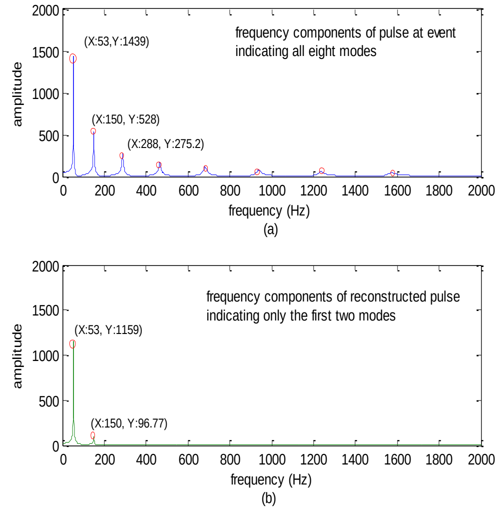


Figure 9 Reconstructed simulated pulse at event by inverse method (multiple frequencies with sensors far apart to the event) in time domain



(a) original pulse (b) reconstructed pulse
Figure 10 Reconstructed simulated pulse at event by inverse method (multiple frequencies with sensors far apart to the event) in frequency domain

From Figure 9 it can be seen that virtually all the high frequency components in the original simulated pulse at the event are not seen in the reconstructed pulse, and this is confirmed in Figure 10 where only the first two modal peaks of the reconstructed pulse are visible, and the second is very small. Nevertheless, even with this very wide sensor spacing and the very limited signal produced at the more remote sensor the reproduced pulse does still contain useful information about the magnitude and decay rate of the original pulse. From these test results obtained using the developed pulse propagation model, it is evident that the inverse method of pulse reconstruction gives better results than the deconvolution filter method. However, it should be noted that the inverse method consistently underestimated the pulse magnitude, whereas the deconvolution filter overestimated it, so there could be a case for using both methods in a practical application to obtain the best possible estimate.

EXPERIMENTAL WORK AND RESULTS

To investigate the use of inverse methods in the reconstruction of the pulse at the event site, the event to be reconstructed was similarly taken as the pulse as it passes through the first sensor. As before, the pressure pulse data at the first sensor was recorded but not used in the subsequent calculations. The same pressure pulse measurements obtained from sensors 2 and 3, and , in the previous section were used and later transformed into the frequency domain using the fast Fourier transform (FFT) function in Matlab to obtain their respective Fourier spectra. Based on the Fourier spectrum of the measured pressure pulses, a solution to the least square inverse problem was obtained to determine the estimate of the required model parameter mLS following the procedures outlined. This value of the estimated model parameter was then used to calculate the Fourier spectrum of the pressure pulse to be reconstructed. The exponential of the log Fourier spectrum was then taken
and transformedinto the time domain using the inverse fast Fourier transform (IFFT) function in Matlab

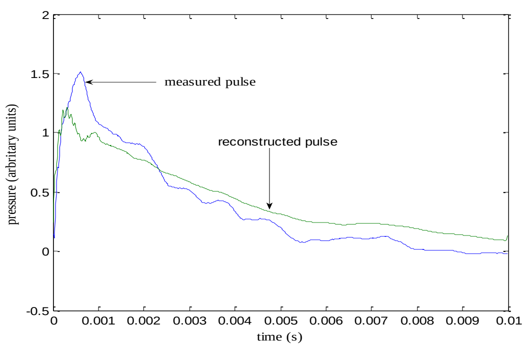


Figure 11 Reconstructed pulse at sensor 1 by inverse method

From Figure 11 the shapes of the reconstructed and measured original pulse at sensor 1 agree quite well. The magnitude of the reconstructed pulse in this case can be seen to have underestimated the measured pulse by 20%. This result is typical of the fifteen repeat tests, in which the underestimate ranged between 20 % and 22 %.
These experimental results are consistent with the model results which gave a similar level of underestimation.

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