ESTIMATION IN SEMIPARAMETRIC REGRESSION MODELS

Fikri AKDENİZ*

ABSTRACT

In this paper we consider the semiparametric regression model, $y = X\beta + f + \varepsilon$. We introduce a Liu-type estimator (LTE) in a semiparametric regression model. We obtained the semiparametric restricted ridge regression and Liu-type estimators for the parametric component in semiparametric regression model. We also introduced difference-based ridge regression and Liu-type estimators in semiparametric regression model. Difference-based estimator and difference-based Liu-type estimator are compared in the sense of mean-squared error criterion.

Keywords: Difference-based estimator, Differencing matrix, Liu-type estimator, Ridge estimator, Multicollinearity, Semiparametric regression model.

1. INTRODUCTION

Let us consider the following semiparametric regression model (partially linear model):

$$y_i = x_i^T \beta + f(t_i) + \varepsilon_i, \qquad i = 1, 2, \dots, n$$
(1)

where x_i (p-dimensional vectors) and t_i (real numbers) are known. In matrix form, it can be written as:

$$y = X\beta + f(t) + \varepsilon, \tag{2}$$

where $X = (x_1, ..., x_n)^T$ is an nxp-dimensional matrix, $\beta = (\beta_1, ..., \beta_p)^T$ is p-dimensional parameter vector, f is unknown function and to be estimated, ε is independent and identically distributed (i.i.d.) error terms with $E[\varepsilon | X, t] = 0$, $Var[\varepsilon | X, t] = \sigma_{\varepsilon}^2 I$. Without loss of generality, data is rearranged so that $0 \le t_1 \le t_2 \le ... \le t_n \le 1$. The explanatory variables are represented separately in two parts: the nonparametric part f(t) and the parametric linear part $(X\beta)$.

Semiparametric regression model (SPRM) generalizes both the parametric linear regression model $y = X\beta + \varepsilon$ and nonparametric regression model $y = f(t) + \varepsilon$ which correspond to the cases f = 0 and $\beta = 0$, respectively. Some of the relations are believed to be of certain parametric form while others are not easily parameterized. SPRM is more flexible than the standard linear regression model since it combines both parametric and nonparametric components. Due to its flexibility, SPRM has been widely used in econometrics, finance, biology, sociology and so on. It allows easier interpretation of the effect of each variable compared to a completely nonparametric regression.

^{*} Prof. Dr., Department of Statistics, Faculty of Arts and Sciences, Çukurova University, Adana, Turkey, e-posta: akdeniz@cu.edu.tr

This article was presented in the 7th Statistics Days Symposium organized by METU during 28-30 June 2010 and it has not been published in any other publications.

Fikri AKDENİZ

All the existing approaches for the partially linear model are based on different nonparametric regression procedures (see Härdle et al. (2000) and Härdle et al. (2004)). There have been several approaches for estimating β and f. Among the most important approaches are the spline methods used by Engle et al. (1986); Rice (1986); Speckman (1988); Green and Silverman (1994); Eubank et al. (1998) and Eubank (1999). Speckman (1988) introduced the idea of a profile least squares method and studied local constant smoother.

The paper is organized as follows. In Section 2, we introduce penalized least squares and Speckman's methods in semiparametric regression models under multicollinearity. Section 3 gives restricted estimators in semiparametric regression models. The semiparametric restricted Liu-type estimator is given in Section 4. In Section 5, the differencing estimator proposed by Yatchew (2003) is given. The difference-based Liu estimator is explained in Section 6. Section 7 compares the difference based Liu-type estimator and difference-based estimator in terms of the mean squared error.

2. ESTIMATION IN SEMIPARAMETRIC REGRESSION MODELS UNDER **MULTICOLLINEARITY**

The problem of multicollinearity and its statistical consequences on a linear regression model are very well-known in statistics. A popular numerical technique to deal with multicollinearity is the use of ridge regression estimator: $\hat{\beta}(k) = (X'X + kI)^{-1}X'v, k > 0$ (see Hoerl and Kennard (1970)). An alternative is the use of Liu-type estimator (or Liu estimator): $\hat{\beta}_d = (X^T X + I)^{-1} (X^T y + \eta \hat{\beta}_{OLS}), 0 < \eta < 1$ (see Liu (1993); Akdeniz and Kaçıranlar (1995)).

2.1 **Penalized Least Squares Method**

We consider a penalized least squares method. Suppose we have the model provided in (2), then we estimate β and f by minimizing with respect to β and f simultaneously

$$S(f,\beta) = (y - X\beta - f)^{T}(y - X\beta - f) + \alpha f^{T}Kf$$
(3)

(Buja *et al.* 1989), where K is a symmetric non-negative definite matrix, $\alpha K = S^{-1} - I$, $S = (I + \alpha K)^{-1}$, smoother matrix, and α is smoothing parameter. For model (2) the penalized least squares solution is given as below:

$$\hat{\beta} = \left[X^{T} (I - S) X \right]^{-1} X^{T} (I - S) y,$$

$$\hat{f} = S(y - X \hat{\beta}),$$
and
$$\mu_{p} = X \hat{\beta} + \hat{f} = H_{p} y$$
for
$$H_{p} = S + (I - S) X \left[X^{T} (I - S) X \right]^{-1} X^{T} (I - S)^{T},$$
provided that $X^{T} (I - S) X$ is invertible (Buia et al. 1989, pp. 500)

veruble (Buja et al. 1989, pp. 500).

2.2 Speckman's Method

If we wish to fit a SPRM given in (2) to data, we may discuss an alternative approach proposed by Speckman (1988) which minimizes

$$(\tilde{y} - \tilde{X}\beta)^{T} (\tilde{y} - \tilde{X}\beta) = \left\| (I - S)(y - X\beta) \right\|^{2}$$
(5)

where $\tilde{y} = (I - S)y$, $\tilde{X} = (I - S)X$ and *S* is an arbitrary smoother matrix. We obtain a solution different from (4), namely the solution is now given in (6).

$$\tilde{\beta} = \left[X^{T} (I-S)^{T} (I-S)X \right]^{-1} X^{T} (I-S)^{T} (I-S)y,$$

$$\tilde{f} = S(y-X\tilde{\beta}),$$

$$\mu_{s} = X\tilde{\beta} + \tilde{f} = H_{s}y$$
for
$$H_{s} = S + (I-S)X \left[X^{T} (I-S)^{T} (I-S)X \right]^{-1} X^{T} (I-S)^{T} (I-S).$$
(6)

As it can be seen, the estimates are obtained by regression on partial residuals. $\hat{\beta}$ and $\tilde{\beta}$ are identical for symmetric and idempotent *S* matrix. The Speckman approach was investigated by Buja et al. (1989); Eubank et. al. (1998) and Schimek (2000).

a) Consider the following objective function:

$$L^* = (\tilde{y} - \tilde{X}\beta)^T (\tilde{y} - \tilde{X}\beta) + k\beta^T \beta$$
(7)

The first order condition of this objective function minimized by the vector β is

$$\frac{\partial L^*}{\partial \beta} = 0.$$
(8)

From condition (8) we obtain the following equation:

$$\tilde{\beta}_k = (\tilde{X}^T \tilde{X} + kI)^{-1} \tilde{X}^T \tilde{y}, \quad k > 0$$
(9)

We call it a ridge regression estimator of the semiparametric regression model.

b) Consider the following conditional objective function:

$$F(\beta) = \arg\min_{\beta} \left\{ (\tilde{y} - \tilde{X}\beta)^T (\tilde{y} - \tilde{X}\beta) + (\eta\tilde{\beta} - \beta)^T (\eta\tilde{\beta} - \beta) \right\}$$
(10)

By differentiating function F with respect to β , we obtain

$$\frac{\partial F}{\partial \beta} = 0.$$

The solution of this system is $\tilde{\beta}_{\eta} = (\tilde{X}^T \tilde{X} + I)^{-1} (\tilde{X}^T \tilde{y} + \eta \tilde{\beta}), \ 0 < \eta < 1,$ (11)

Fikri AKDENİZ

where η is a biasing parameter. Since there is a formal resemblance between (11) and the Liu estimator of the linear model, we call it a *Liu-type estimator of the semiparametric regression model*.

Combining $\tilde{\beta}_{\eta}$ and $f = S(y - X\beta)$, we obtain Liu-type estimator of f: $\tilde{f} = S(y - X\tilde{\beta}_{\eta})$.

3. RESTRICTED ESTIMATORS IN SEMIPARAMETRIC REGRESSION MODELS

We will call an estimator $\beta^*(y)$ for β a restricted estimator with respect to $R\beta = r$, if it satisfies $R\beta^*(y) = r$ for all $n \times 1$ vectors y (Gross, 2003).

If the linear restrictions $R\beta = r$ are assumed to hold, we shall give the restricted estimator of β in semiparametric regression model. Here, *R* is an mxp known matrix with rank m < p and *r* is an mx1 known vector.

Theorem 1. If the smoother matrix S is symmetric, then the estimator of β obtained by using the penalized least-squares criterion, under the restriction $R\beta = r$, is

$$\hat{\beta}_{R} = \hat{\beta} + D^{-1}R^{T}(RD^{-1}R^{T})^{-1}(r - R\hat{\beta})$$
(12)

with $D = X^T (I - S) X$ and $\hat{\beta} = D^{-1} X^T (I - S) y$.

Proof: (see Akdeniz and Tabakan (2009))

Theorem 2. If S is an arbitrary smoother matrix, then the estimate of β , under $R\beta = r$, is

$$\tilde{\beta}_{R} = \tilde{\beta} + Z^{-1}R^{T}(RZ^{1}R^{T})^{-1}(r - R\tilde{\beta})$$
(13)

with $Z = X^T (I - S)^T (I - S) X$ and $\tilde{\beta} = (\tilde{X}^T \tilde{X})^{-1} \tilde{X}^T \tilde{y}$. Let $\tilde{y} = (I - S) y$, $\tilde{X} = (I - S) X$.

Proof: (see Akdeniz and Tabakan (2009))

4. RESTRICTED LIU-TYPE ESTIMATORS IN SEMIPARAMETRIC REGRESSION MODELS

The semiparametric restricted ridge regression estimator for the parametric component in semiparametric regression model is established by Akdeniz and Tabakan (2009). We shall give the restricted Liu-type estimator $\tilde{\beta}_{R\eta}$ of β in semiparametric regression model such that condition $R\beta = r$ is satisfied. Przystalski and Krajewski (2007) presented a method of estimation of treatment effects and its application to hypothesis testing in a semiparametric model. Assuming the linear restrictions $R\beta = r$ hold, we

shall give the restricted estimator of β in the semiparametric regression model (2) in the following two theorems.

Theorem 3. If S is an arbitrary smoother matrix, then the estimate of β in semiparametric model under the penalized least squares criterion such that condition $R\beta = r$ is satisfied, is equal to

$$\tilde{\beta}_{R\eta} = \tilde{\beta}_{\eta} + \tilde{Z}^{-1} R^T (R \tilde{Z}^{-1} R^T)^{-1} (r - R \tilde{\beta}_{\eta}).$$
(14)

with

$$\tilde{\beta}_{\eta} = (\tilde{X}^T \tilde{X} + I)^{-1} (\tilde{X}^T \tilde{y} + \eta \tilde{\beta}); \qquad \tilde{Z} = \tilde{X}^T \tilde{X} + I = X^T (I - S)^T (I - S) X + I \qquad and$$
$$\tilde{\beta} = (\tilde{X}^T \tilde{X})^{-1} \tilde{X}^T \tilde{y} = \left[X^T (I - S)^T (I - S) X \right]^{-1} X^T (I - S)^T (I - S) y.$$

Proof: (Akdeniz and Akdeniz Duran (2010))

Theorem 4. If the smoother matrix S is symmetric, then the estimate of β in semiparametric model, obtained by using the penalized least squares criterion

$$\left\|y - X\beta - f\right\|^{2} + \alpha f^{T} K f + (\eta \hat{\beta}_{\eta} - \beta)^{T} (\eta \hat{\beta}_{\eta} - \beta)$$
(15)

satisfying condition $R\beta = r$, is equal to

$$\hat{\beta}_{R\eta} = \hat{\beta}_{\eta} + V^{-1} R^T (R V^{-1} R^T)^{-1} (r - R \hat{\beta}_{\eta}).$$
(16)

Proof: (Akdeniz and Akdeniz Duran (2010)) Let us assume that the smoother matrix *S* is symmetric and we want to find an estimate of β in semiparametric model satisfying condition $R\beta = r$ using the criterion (15). For this purpose we construct the following Lagrange function

$$L(\beta, f, \lambda) = \left\| y - X\beta - f \right\|^2 + \alpha f^T K f + \left\| \eta \hat{\beta} - \beta \right\|^2 - 2\lambda^T (R\beta - r).$$

By differentiating function L with respect to β , f and λ we obtain

$$\hat{\beta}_{R\eta} = \hat{\beta}_{\eta} + V^{-1} R^T (R V^{-1} R^T)^{-1} (r - R \hat{\beta}_{\eta}).$$
(17)

This estimator satisfies $R\hat{\beta}_{R\eta} = r$. Thus $\hat{\beta}_{R\eta}$ is the restricted Liu-type estimator of β in semiparametric regression model.

5. DIFFERENCE-BASED ESTIMATORS

Remember that the model is given as the following:

$$y_i = x_i^{T} \beta + f(t_i) + \varepsilon_i, \qquad i = 1, 2, ..., n$$

In the matrix form, we can write

$$y = X\beta + f(t) + \varepsilon, \qquad \dim(t) \le 3$$
 (18)

where $X = (x_1, ..., x_n)^T$ with $x_i, i \in (1, n)$ non-stochastic, p-dimensional vector; $\beta = (\beta_1, ..., \beta_p)^T$ is parameter vector; f is smooth function whose first derivative is bounded by a constant; t has compact support, i.e. [0,1]; ε is i.i.d. with $E[\varepsilon | X, t] = 0$, $Var[\varepsilon | X, t] = \sigma_{\varepsilon}^2 I$. t_i are equally spaced on the unit interval. Data is rearranged so that $0 \le t_1 \le t_2 \le ... \le t_n \le 1$.

Applying the differencing matrix to model (18) permits direct estimation of the parametric effect. In particular, take

$$Dy = DX\beta + Df(t) + D\varepsilon \doteq DX\beta + D\varepsilon$$

or

$$\ddot{y} \doteq \ddot{X}\beta + \ddot{\varepsilon} \tag{19}$$

where $\ddot{y} = Dy$, $\ddot{X} = DX$ and $\ddot{\varepsilon} = D\varepsilon$. We obtain the simple differencing estimator of the parameter β in semiparametric regression model as

$$\hat{\boldsymbol{\beta}}_{diff} = \left[(DX)^T DX \right]^{-1} (DX)^T Dy$$

$$= (\ddot{X}^T \ddot{X})^{-1} \ddot{X}^T \ddot{y}.$$
(20)

where differencing matrix D is $(n-m) \times n$ matrix

and *m* order d_0, d_1, \ldots, d_m differencing weights minimize

$$\min_{d_0,\dots,d_m} \delta = \sum_{k=1}^m (\sum_{j=1}^{m-k} d_j d_{k+j})^2 \text{ such that } \sum_{j=0}^m d_j = 0 \text{ and } \sum_{j=0}^m d_j^2 = 1 \text{ (Yatchew, 2003).}$$

5.1 What is the advantage of differencing?

An important advantage of differencing procedures is their simplicity. Increasing the order of differencing as sample size increases, the estimator of the linear component becomes asymptotically efficient (Yatchew 2003, p.72). Differencing allows one to perform inferences as if there were no nonparametric component f in the model (Yatchew, 2003). The *m*-th order differencing estimator of the residual variance is

$$S_{diff}^{2} = \frac{1}{n} (Dy - DX \hat{\beta}_{diff})^{T} (Dy - DX \hat{\beta}_{diff}).$$

6. DIFFERENCE-BASED RIDGE AND LIU-TYPE ESTIMATORS

As an alternative to (20) we propose the use of

$$\hat{\beta}_{D}(k) = \left[(DX)^{T} DX + kI \right]^{-1} (DX)^{T} Dy,$$

$$= (\ddot{X}'\ddot{X} + kI)^{-1} \ddot{X}'\ddot{y}$$
(21)

where *I* is $p \times p$ identity matrix, $k \ge 0$ is biasing parameter. We call it a differencebased ridge estimator of semiparametric regression model (Tabakan and Akdeniz, (2010)). If k=0, then $\hat{\beta}_D(k) = \hat{\beta}_{diff}$.

From the least squares perspective, the coefficients β are chosen to minimize

$$(Dy - DX\beta)'(Dy - DX\beta).$$
⁽²²⁾

Adding to least squares objective (22), a penalizing function of the squared norm $\left\|\eta\hat{\beta}_{diff} - \beta\right\|^2$ for the vector of regression coefficients yields a conditional objective:

$$L(\beta) = \arg\min_{\beta} \left[(\ddot{y} - \ddot{X}\beta)^{T} (\ddot{y} - \ddot{X}\beta) + \left\| \eta \hat{\beta}_{D} - \beta \right\|^{2} \right].$$
(23)

Minimizing L(β) by vector β , we obtain another alternative estimator $\hat{\beta}_D(\eta)$ to $\hat{\beta}_{diff}$ in (20):

$$\hat{\beta}_D(\eta) = (\ddot{X}^T \ddot{X} + I)^{-1} (\ddot{X}^T \ddot{y} + \eta \hat{\beta}_{diff})$$
(24)

where $0 < \eta < 1$ is a biasing parameter for $\eta = 1$, $\hat{\beta}_D(\eta) = \hat{\beta}_{diff}$. We call $\hat{\beta}_D(\eta)$ differenced-based Liu-type estimator.

7. MSEM SUPERIORITY OF DIFFERENCE-BASED LIU-TYPE ESTIMATOR, $\hat{\beta}_{_D}(\eta)$ OVER DIFFERENCING ESTIMATOR, $\hat{\beta}_{_{diff}}$

7.1 Bias and Covariance of $\hat{\beta}_D(\eta)$

Putting $F_{\eta} = (\ddot{X}^T \ddot{X} + I)^{-1} (\ddot{X}^T \ddot{X} + \eta I)$ and observing F_{η} and $(\ddot{X}^T \ddot{X})^{-1}$ are commutative, we may write $\hat{\beta}_D(\eta)$ as

$$\hat{\beta}_{D}(\eta) = F_{\eta}\hat{\beta}_{diff} = F_{\eta}(\ddot{X}^{T}\ddot{X})^{-1}\ddot{X}^{T}\ddot{y} = (\ddot{X}^{T}\ddot{X})^{-1}F_{\eta}\ddot{X}^{T}\ddot{y}.$$
(25)

The expected value of $\hat{\beta}_D(\eta)$ can be written as

$$E[\hat{\beta}_{D}(\eta)] = \beta - (1 - \eta)(\ddot{X}^{T}\ddot{X} + I)^{-1}\beta.$$
(26)

The bias of the $\hat{\beta}_{_D}(\eta)$ estimator is given as

Bias
$$[\hat{\beta}_{D}(\eta)] = -(1-\eta)(\ddot{X}^{T}\ddot{X}+I)^{-1}\beta.$$
 (27)

Setting $S = (D^T \ddot{X})^T (D^T \ddot{X})$ and $U = (\ddot{X}^T \ddot{X})^{-1}$ we may write $\text{Cov}[\hat{\beta}_D(\eta)]$ as

$$\operatorname{Cov}[\hat{\beta}_{D}(\eta)] = \sigma^{2} U F_{\eta} S F_{\eta}^{T} U .$$
⁽²⁸⁾

Similarly, we may obtain $\text{Cov}[\hat{\beta}_{diff}]$ as

$$\operatorname{Cov}[\hat{\beta}_{diff}] = \sigma^2 U S U. \tag{29}$$

7.2 Variance Comparison

Theorem 5. The sampling variance of the difference-based Liu-type estimator, $\hat{\beta}_D(\eta)$ is always less than or equal to that of differencing estimator, $\hat{\beta}_{diff}$ if and only if

$$\lambda_{\max}\left(\frac{1}{\eta+1}M^{-1}N\right) \leq 1.$$

Proof: (Akdeniz Duran (2010))

When is difference-based Liu-type estimator superior to the differencing estimator in MSEM sense?

Theorem 6. Consider two competing estimators $\hat{\beta}_D(\eta)$ and $\hat{\beta}_{diff}$ of β . Let $W = \left(\frac{1+\eta}{1-\eta}\right)(M+tN)$ be a positive definite matrix. Then, the biased estimator $\hat{\beta}_D(\eta)$

is MSEM- superior over the $\hat{\beta}_{diff}$ if and only if

$$\beta' W^{-1}\beta \leq \sigma^2.$$

Proof: (Akdeniz Duran (2010))

8. CONCLUSION

In this paper, we presented Liu-type Speckman and Liu-type penalized least squares estimators for the semiparametric regression model when linear restrictions on the parameter vector are assumed to hold. We also investigated ridge and Liu-type difference based regression estimators.

Acknowledgements

The author wish to thank to Esra Akdeniz Duran for her helpful suggestions and comments

9. REFERENCES

Akdeniz D. E., 2010. Ph D. Thesis, Gazi University, Ankara (unpublished).

Akdeniz, F., Kaçıranlar, S., 1995. On the almost unbiased generalized Liu estimator and unbiased estimation of the Bias and MSE. Commun. Statist.-Theory and Meth. 24(7), 1789-1797.

Akdeniz, F., Tabakan, G., 2009. Restricted ridge estimators of the parameter in semiparametric regression model. Communications in Statistics- Theory and Methods 38(11):1852-1869.

Akdeniz, F., Akdeniz D. E., 2010. Liu-type estimator in semiparametric regression models. Journal of Statistical Computation and Simulation 80(8):853-871.

Akdeniz D. E., Akdeniz, F., 2010. Efficiency of Modified Jacknife Liu-type estimator. Statistical Papers DOI: 10.1007/s00362-010-0334-5).

Akdeniz D, E., Akdeniz, F., Hongchang Hu., 2011. Efficiency of estimators in semiparametric regression models. Journal of Computational and Applied Mathematics 235:1418-1428.

Buja, A., Hastie, T., Tibshirani, R., 1989. Linear smoothers and additive models. The Annals of Statististics, 17(2):453-555.

Engle, R. F., Granger, C. W. J., Rice, J., Weiss, A., 1986. Semiparametric estimates of the relation between weather and electricity sales. J. Amer, Statist. Assoc. 81:310-320.

Eubank, R. L., 1999. Nonparametric regression and Spline Smoothing. Marcel Dekker, New York.

Eubank, R. L., Kambour, E. L., Kim, J. T., K.Klipple, K., Reese, C. S., Schimek, M. G., 1998. Estimation in partially linear models. Computational Statistics and Data Analysis 29:27-34.

Green, P. J., Silverman, B. W., 1994. Nonparametric Regression and Generalized Linear Models. Chapman and Hall, New York.

Gross, J., 2003. Restricted ridge estimation. Statistics and Probability Letters 65:57-64.

Härdle, W., Liang, H., Gao, J., 2000. Partially Linear Models Springer-Verlag Co., New York.

Härdle, W., Müler, M., Sperlich, S., Werwatz, A., 2004. Nonparametric and Semiparametric Models. Springer, New York.

Hoerl, A. E., Kennard, R.W., 1970. Ridge regression: biased estimation for orthogonal problems. Technometrics 12:55-67.

Liu, K. J., 1993. A new class of biased estimate in linear regression. Communications in Statistics–Theory and Methods 22:393-402.

Przystalski, M., Krajewski, P., 2007. Constrained estimators of treatment parameters in semiparametric models. Statistics and Probability Letters 77(10):914-919.

Rice, J. A., 1986. Convergence rates for partially spline models. Statistics and Probability Letters 4:203-208.

Schimek, M. G., 2000. Estimation and inference in partially linear models with smoothing splines. Journal of Statistical Planning and Infrence 91:525-540.

Speckman, P., 1988. Kernel somoothing in partial linear models. J. Roy, Statist.Soc. Ser. B. 50(3):413-436.

Tabakan, G., Akdeniz, F., 2010. Difference –based ridge estimator of parameters in partial linear model. Stat. Papers, 357-368.

Yatchew, A., 2003. Semiparametric Regression for the Applied Econometrician. Cambridge University Press.

YARI PARAMETRİK REGRESYON MODELLERİNDE TAHMİN ETME

ÖZET

Bu makalede $y = X\beta + f + \varepsilon$ yarı parametrik regresyon modeli düşünüldü; yarı parametrik regresyon modelinde Liu-tip tahmin edici (LTE) önerildi. Ayrıca, yarı parametrik regresyon modelinde parametrik bileşen için yarı parametrik kısıtlı ridge regresyon ve Liu-tip tahmin edicileri de tanıtıldı. Farka dayalı tahmin edici ve farka dayalı Liu-tip tahmin edici hata kareleri ortalaması ölçütüne göre karşılaştırıldı.

Anahtar Kelimeler: Farka dayalı tahmin edici, Fark alma matrisi, Liu-tip tahmin edici, Ridge tahmin edici, Çoklu içilişki, Yarı parametrik regresyon modeli.