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Araştırma Makalesi / Research Article

Design and Analyse of Structured H-infinity Controller for Level Control of Nonlinear Quadruple Tank Systems

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Abstract

Industrial process systems operate by interacting with each other to fulfil an essential industrial task. The majority of processes have nonlinear dynamics and multiple-input multiple-output systems which make them even more difficult to design control schemes. In industrial process applications, level control is one of the important problems. This study computes a structured H-infinity controller for the level control of a nonlinear quadruple tank system. This study aims to design a simple (low-order) robust controller and compare its performance to the performance of the classical Proportional-Integral-Derivative (PID) controller. PID controllers are mostly used in industrial processes due to their simplicity of implementation. However, PID controllers have some disadvantages. One of these disadvantages is that PID controllers offer lower robustness than robust control schemes when the industrial process operates in the presence of disturbances. Simulations are conducted in MATLAB\Simulink environment. Furthermore, the performance of the proposed controller is compared with the PID controller in terms of error-dependent performance indices and time-domain specifications. The simulation results have shown that the proposed controller provides robust set-point tracking, good disturbance rejection, and handling of the parametric uncertainty properties for nonlinear quadruple tank systems in industrial processes.

Keywords: Low-order robust controller, structured H-infinity control, PID, quadruple tank systems, multiple-input multiple-output systems

Doğrusal Olmayan Dörtlü Tank Sistemlerinin Seviye Kontrolü için Yapılandırılmış H-sonsuz Denetleyici Tasarımı ve Analizi

Öz

Endüstriyel süreç sistemleri, temel bir endüstriyel görevi yerine getirmek için birbirleriyle etkileşime girerek çalışır. Süreçlerin çoğu, kontrol şemalarını tasarlamayı daha da zorlaştıran, doğrusal olmayan dinamiklere ve çok girdili çok çıktılı sistemlere sahiptir. Endüstriyel süreç uygulamalarında seviye kontrolü önemli problemlerden biridir. Bu çalışma, doğrusal olmayan bir dörtlü tank sisteminin seviye kontrolü için düşük dereceli gürbüz bir denetleyici tasarlamaktadır. Bu çalışmanın amacı, basit (düşük dereceli) dayanıklı denetleyici tasarlamak ve performansını klasik PID denetleyicinin performansıyla karşılaştırmaktır. PID denetleyiciler, uygulama kolaylığı nedeniyle çoğunlukla endüstriyel süreçlerde kullanılmaktadır. Ancak PID kontrolörlerin bazı dezavantajları vardır. Bu dezavantajlardan biri, endüstriyel süreç bozucu etkilere maruz kaldığında, PID denetleyici gürbüz denetleyiciden daha düşük dayanıklılık sunmasıdır. Benzetimler MATLAB\Simulink ortamında gerçekleştirilmiştir. Önerilen denetleyicinin performansı, klasik Oransal-İntegral-Türev (PID) kontrolör ile hataya bağlı performans indeksleri ve zaman-alanı özelikleri açısından karşılaştırılmıştır. Simülasyon sonuçları, endüstriyel süreçte doğrusal olmayan dörtlü tank sistemi için önerilen denetleyicinin, dayanıklı istek takibi, iyi bir bozucu etki reddetme ve parametre belirsizliklerine karşı dayanıklılık yeteneklerine sahip olduğunu göstermiştir.

Anahtar Kelimeler: Düşük derece gürbüz denetleyici, yapılandırılmış H-sonsuz denetleyici, PID, dörtlü tank sistemleri, çok-girdili çok-çıktılı sistemler

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1. Introduction

Multi-tank processes are used in the majority of industries such as petrochemical, food, paper-making, etc. industries. The multi-tank process is Multi input-Multi output (MIMO), complex and nonlinear system. A tank process comprises of pumps, tanks, valves, and sensors. While the industrial process is operating, the aperture of the valves and/or liquid type in the tanks can change. Furthermore, the power supply of the pump and valve coefficients might be uncertain in some cases which results in a highly nonlinear and varying behaviour of the tank systems. Hence, it is a difficult task to obtain the dynamic model of a tank system. From the aspect of the control law design, the liquid level control of the quadruple tank process is a difficult problem due to high nonlinearity, unknown disturbance, parameter uncertainty, and strong coupling. Hence, it is very important to develop effective control methods for multivariable quadruple tank processes for industries. To develop and test different control methods for multivariable systems, the quadruple tank system given by Johansson (2000) is a well-known benchmark.

Control techniques have been proposed to improve the level control performance of tank systems. Here, some of the recent papers in level control are reviewed. An adaptive PID controller was proposed in (Mizumoto et al., 2010) for the water level control of a three-tank system. Son (2020) proposed a hybrid method that consists of an inverse evolutionary neural model and a PID controller to eliminate the tracking error of the water level position. Furthermore, Osman et al., (2021) developed an adaptive sliding mode controller and an adaptive state feedback controller based on the pole placement method for the level control of a quadruple tank system. Shah and Patel, (2019) designed a sliding mode controller with a time delay compensator to overcome the problem of parametric uncertainty and process delays. Mehri and Tabatabaei (2021) developed an adaptive fractional-order sliding mode controller for a quadruple tank system in the case of time-varying pump coefficients. For the time delay problem in a quadruple-tank system, Naami et al., (2022) designed an H-infinity observer-based controller using Takagi-Sugeno fuzzy modelling method. Bennani et al., (2018) designed an observer-based H-infinity controller for the level problem in case of parameter uncertainties. Wei et al., (2013) designed an adaptive nonlinear controller to reject disturbances and fluctuations. To maintain closed-loop stability, Thamallah et al., (2019) proposed a predictive controller employing a fuzzy modelling method and particle swarm optimization algorithm. Pradhan and Ghosh (2022) proposed a periodic controller for a quadruple tank system with a non-minimum phase zero to improve robustness. A dynamic sliding surface controller was proposed by Zare et al., (2022) for a system with uncertainties, disturbances, and control input constraints. Vijay Anand and Manoharan (2022) proposed a robust controller to tackle parameter uncertainties in a quadruple tank system. Meng et al., (2021) designed an active disturbance rejection control that consists of a feedback linearization method and extended state observer to eliminate disturbances on a quadruple tank system.

The standard H-infinity controllers have practical limitations which have slowed their implementation in the industry. In this work, the structured H-infinity controller is proposed to overcome the limitations of standard H- infinity controllers. We have designed a structured H-infinity controller for the liquid level of the quadruple tank process. The developed control law is simulated in the time-varying set points, in the presence of a disturbance and uncertainty of a parameter. As a commonly used approach to quadruple tank control, a classical PID controller is employed to analyse the capacity of the proposed controller in the simulation. Time-domain parameters and error-dependent performance indices are calculated to analyse the performance of the proposed controller with respect to the performance of the PID controller.

2. Materials and Methods

2.1. Quadruple Tank System

This section gives the nonlinear dynamics of a quadruple tank system. The quadruple tank system is illustrated in Figure 1. The control objective is to adjust the level in the lower two tanks using pumps in the presence of disturbances and parametric uncertainties. The quadruple tank system has two inputs ($v_1(t), v_2(t)$ input voltages to the pumps) two pumps, and two outputs ($y_1(t), y_2(t)$) voltages from level measurement devices. The differential equations are derived using mass balances and Bernoulli's law as follows (Johansson, 2000):

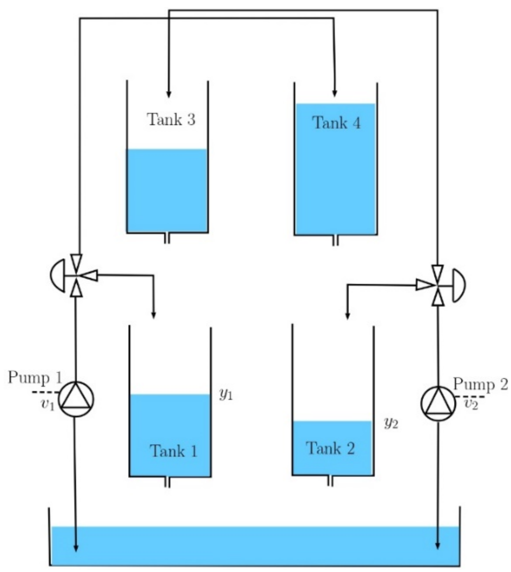


Figure 1. Schematic of Quadruple tank system.

$$\begin{aligned}
\dot{h}_1(t) &= -\frac{a_1}{A_1}\sqrt{2gh_1(t)} + \frac{a_3}{A_1}\sqrt{2gh_3(t)} + \frac{\gamma_1 k_1}{A_1} v_1(t) \\
\dot{h}_2(t) &= -\frac{a_2}{A_2}\sqrt{2gh_2}(t) + \frac{a_4}{A_2}\sqrt{2gh_4}(t) + \frac{\gamma_2 k_2}{A_2} v_2(t) \\
\dot{h}_3(t) &= -\frac{a_3}{A_3}\sqrt{2gh_3}(t) + \frac{(1-\gamma_2)k_2}{A_3} v_2(t) \\
\dot{h}_4(t) &= -\frac{a_4}{A_4}\sqrt{2gh_4}(t) + \frac{(1-\gamma_1)k_1}{A_4} v_1(t)
\end{aligned} \tag{1}$$

Where h_i is the level in tank i, a_i is the outlet cross-sectional of area tank i, A_i is the cross-sectional area of tank i and g is the acceleration of gravity. The voltage $v_i\ (t)$ is applied to pump i that results in the flow $k_i\ v_i$. The parameters γ_1 and γ_2 belong to values (0,1) presenting the portion of the flow. The flow to tanks 1 and 4 are $\gamma_1\ k_1\ v_1$ and $(1-\gamma_1)k_1\ v_1$ respectively and similar flows to tanks 2 and 3.

Table 1. The quadruple tank parameters (Johansson, 2000).

Parameters	Values
A_1, A_3 (cm^2)	28
A_2, A_4 (cm^2)	32
a_1, a_3 (cm^2)	0.071
a_2, a_4 (cm^2)	0.057
k_c (V/cm)	0.5
g (cm/s^2)	981

The quadruple tank system has minimum and non-minimum phase properties. Control laws will be designed at the minimum phase in this study. This operating point has following the parameter values:

$$\begin{aligned}
&[h_1^0, h_2^0, h_3^0, h_4^0] = [12.4, 12.7, 1.8, 1.4] \\
&[k_1, k_2] = [3.33, 3.35] \\
&[\gamma_1, \gamma_2] = [0.70, 0.60] \\
&[v_1^0, v_2^0] = [3, 3]
\end{aligned} \tag{2}$$

Define the state variable $x_i := h_i - h_i^0$ and the input variable $u_i := v_i - v_i^0$. Then the linearized quadruple-tank system is:

$$\begin{aligned}
\dot{x}(t) &= A\,x(t) + \mathrm{B}u(t) \\
y(t) &= Cx(t)
\end{aligned} \tag{3}$$

where $A = \begin{bmatrix} -\frac{1}{\beta_1} & 0 & \frac{A_3}{A_1\beta_3} & 0 \\ 0 & -\frac{1}{\beta_2} & 0 & \frac{A_4}{A_2\beta_4} \\ 0 & 0 & -\frac{1}{\beta_3} & 0 \\ 0 & 0 & 0 & -\frac{1}{\beta_4} \end{bmatrix}$, $B = \begin{bmatrix} \frac{\gamma_1 k_1}{A_1} & 0 \\ 0 & \frac{\gamma_2 k_2}{A_2} \\ 0 & \frac{(1-\gamma_2)k_2}{A_3} \\ \frac{(1-\gamma_1)k_1}{A_4} & 0 \end{bmatrix}$ and $C = \begin{bmatrix} k_c & 0 & 0 & 0 \\ 0 & k_c & 0 & 0 \end{bmatrix}$

with $\beta_i = \frac{A_i}{a_i}\sqrt{\frac{2h_i^0}{g}}$, $i = 1, \dots, 4$

2.2. Structured H- Infinity Controller Design

The H-infinity synthesis is built on the standard closed-loop feedback system of Figure 2 where the signals (z; w; y; u) represent, respectively, the output objectives, the exogenous inputs such as disturbance and reference, and the measured outputs and control inputs.

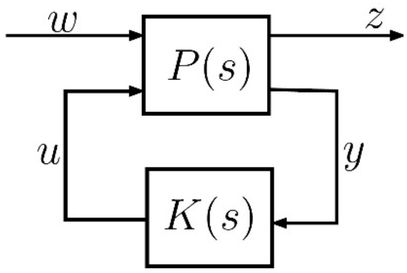


Figure 2. Standard feedback system.

The transfer function from w to z is

$$\mathcal{F}_l(P,K) = P_{11} + P_{12}K(I - P_{22}K)^{-1}P_{21} \tag{4}$$

The open-loop system $P(s)$ of Figure 4, which is a general plant model that includes weighting functions and a plant model, is given as

$$P(s) = \begin{vmatrix} A & B_1 & B_2 \\ C_1 & D_{11} & D_{12} \\ C_2 & D_{21} & D_{22} \end{vmatrix} \tag{5}$$

where $A \in R^{n\times n}, D_{11} \in R^{p_1\times m_2}, D_{21} \in R^{p_2\times m_1}$ and the rest of the matrices possess compatible dimensions. The controller K is realized in state-space form as

$$K(s) = \begin{vmatrix} A_c & B_c \\ C_c & D_c \end{vmatrix} \tag{6}$$

in which $A_c \in R^{n_c\times n_c}$ and matrices B_c, C_c, D_c possess compatible dimensions with A_c. The objective of the H –infinity control design is $\inf_K \|\mathcal{F}_l(P,K)\|_\infty$. The structured H-infinity controller is computed by *the hinfstruct* algorithm and here a low (fixed) order diagonal structured controller is

$$K(s) = \begin{vmatrix} k_1(s) & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & k_N(s) \end{vmatrix} \tag{7}$$

where $k_1(s), \dots, k_N(s)$ are linear time-invar systems. The *hinfstruct* is a deterministic approach that does not involve any randomization apart from extra starting points. The *hinfstruct* algorithm automatically executes multiple optimizations, setting random starting points. This improves the speed at which parameter values that meet the design requirements (for more details see (Gahinet and Apkarian, 2011; Gahinet and Apkarian, 2012).

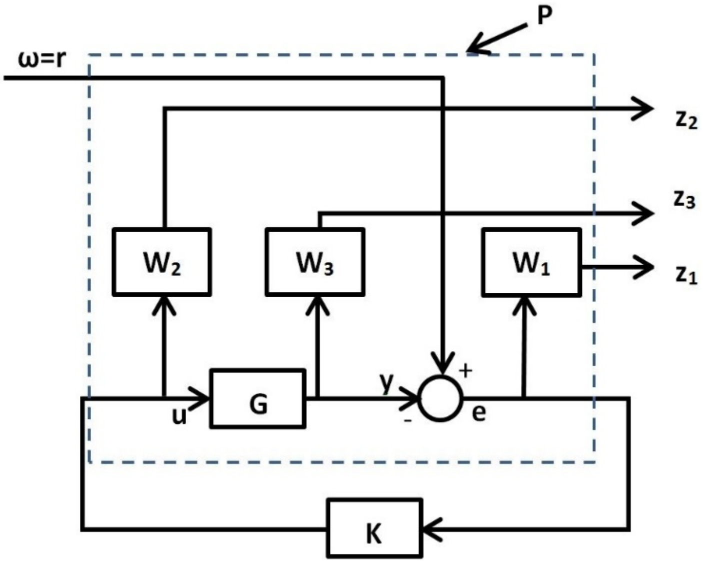


Figure 3. The augmented plant including weighting functions in standard form.

The sensitivity transfer function and the complementary sensitivity transfer function are defined respectively $S(s) = (I + GK)^{-1}$ and $T = I - S(s)$. Here the aim is to obtain a stabilizing controller such that

$$\left\| \begin{matrix} W_1 S \\ W_2 KS \\ W_3 T \end{matrix} \right\|_\infty < 1 \tag{8}$$

Then the frequency domain requirements will be satisfied. Weighting function W_2 is selected as a constant and weigh functions W_1 and W_3 will be selected as follows:

$$W_1(s) = \frac{\frac{s}{M_s} + w_{BS}}{s + w_{BS} A_s} \tag{9}$$

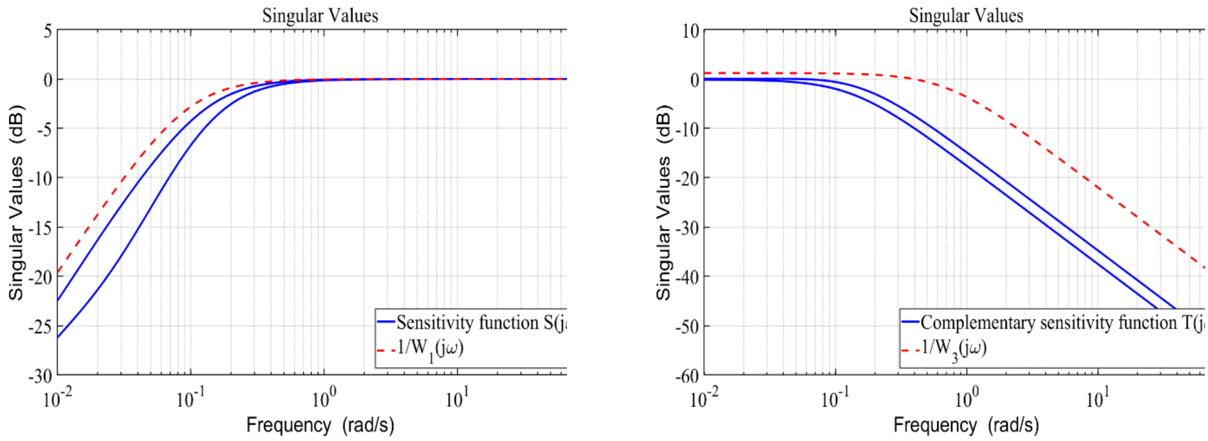
$$W_3\ (s) = \frac{\frac{s}{w_{BT}} + 1/M_T}{\frac{A_T s}{w_{BT}} + 1} \tag{10}$$

where M_S and M_T determine limitations on maximum peak values. M_S and M_T are chosen less than 2 to obtain sufficient gain and phase margins. A_S and A_T are selected as nearly zero to obtain integral action but this results in numerical problems due to internal stability. A_S and A_T will be selected as small positive numbers. The closed-loop bandwidth is determined by w_{BS}. If the maxımum singular value of S is made small over the frequency of disturbance, then the disturbance will be rejected (Skogestad and Postlethwaite, 2007).

The weighting functions are found by using equations (8) and (9). The controller obtained is 2nd order and the best-achieved H-infinity norm of the closed-loop system is 1.15. The matrices of the controller, K in equation (5) are computed as follows:

$$A_C = \begin{vmatrix} 0.2820 & -.0113 \\ 0.7269 & -0.2914 \end{vmatrix} \tag{11}$$

$$B_c = \begin{vmatrix} 4.8839 & -236.09 \\ 45.3627 & -595.6572 \end{vmatrix}$$
$$C_c = \begin{vmatrix} -0.0088 & 0.0035 \\ 0.0076 & -0.0031 \end{vmatrix}$$
$$D_c = \begin{vmatrix} 4.3984 & -0.0859 \\ 0.0351 & 4.3577 \end{vmatrix}$$



(a) (b)

Figure 4. Singular value of the sensitivity function (a), complementary sensitivity function (b), and inverse of the weighting functions.

Figure 4 shows the closed-loop frequency responses. It can be seen from Figure 4a that the 2^{nd} order controller (standard H infinity controller is 8^{th} order) will attenuate the disturbance until the frequency is 5 rad/s. Furthermore, from the plot of the complementary sensitivity function (Figure 4b), it can be said that there is good command tracking until the frequency of 0.04 rad/s. If we associate a single bandwidth for the MIMO system, then we consider the worst-case direction and the closed-loop bandwidth is about 0.131 rad/s (measured at -3dB). Gain margin (GM) =Inf dB and phase margin (PM)= 114.0139 degrees are computed with command *margin* in MATLAB.

3. Simulatıon Results and Discussion

The structured H-infinity controller design is given in Section 3 and implemented in the simulation. The simulation block structure of the proposed controller with the quadruple tank system is given in Figure 5. To investigate the performance of the developed controller, the classical PID level control of the quadruple tank system is also studied in simulation. The simulation study of the quadruple tank system with the classical PID control law is displayed in Figure 6. The control law of the decentralized PID controller is

$$u_{PID}(t) = K_p e(t) + K_i \int_0^t e(t)dt + K_d \frac{de(t)}{dt} \tag{12}$$

where $u_{PID}(t)$ and $e(t)$ represent the control input signal and error signal respectively. Simulations are run for 350 seconds at scenarios which are demand variation, disturbance, and parametric uncertainty to analyse the performance of the controllers. The parameters of PID controllers are found as $K_p = [2 \;\; 1.7], K_I = [0.2 \;\; 0.08]$, and $K_D = [4 \;\; 5\,]$ using MATLAB/ PID tuner toolbox.

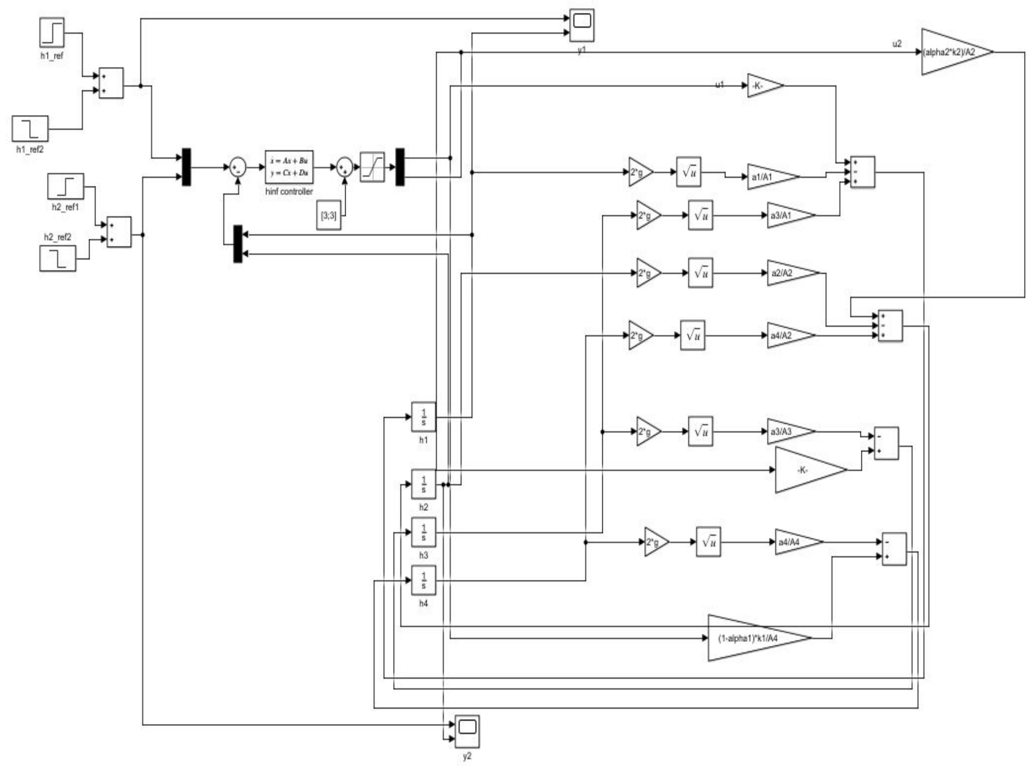


Figure 5. Simulation block scheme of the quadruple tank system with the structured H-infinity controller.

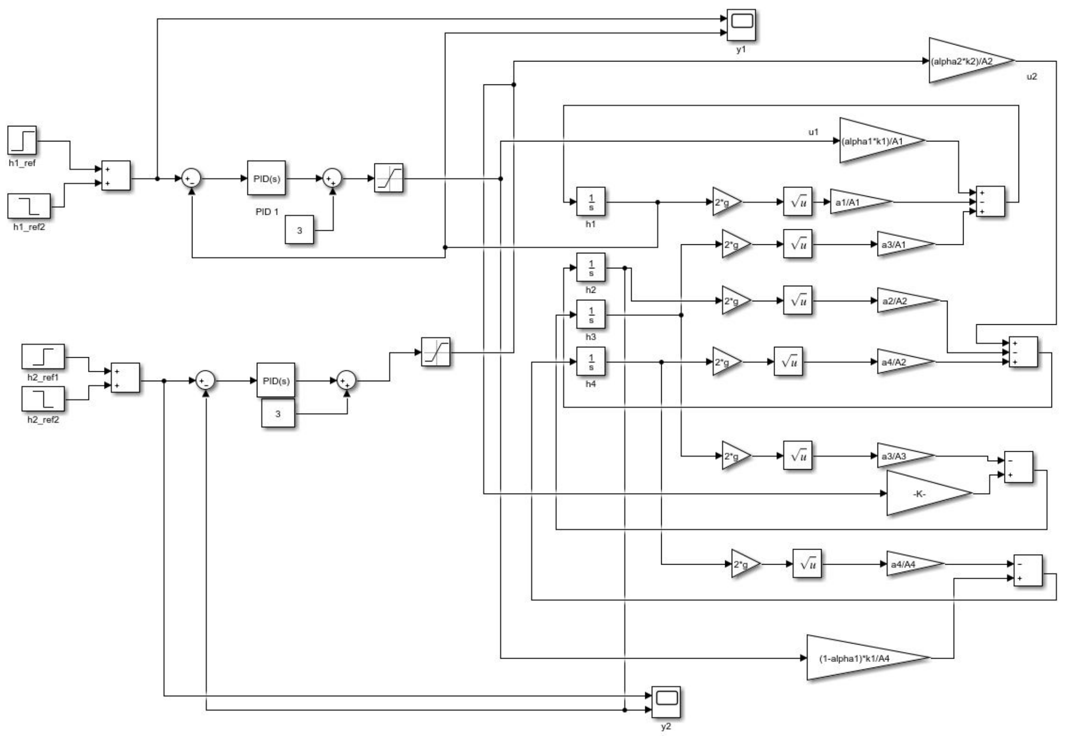


Figure 6. Simulation block scheme of the quadruple tank system with the PID controller.

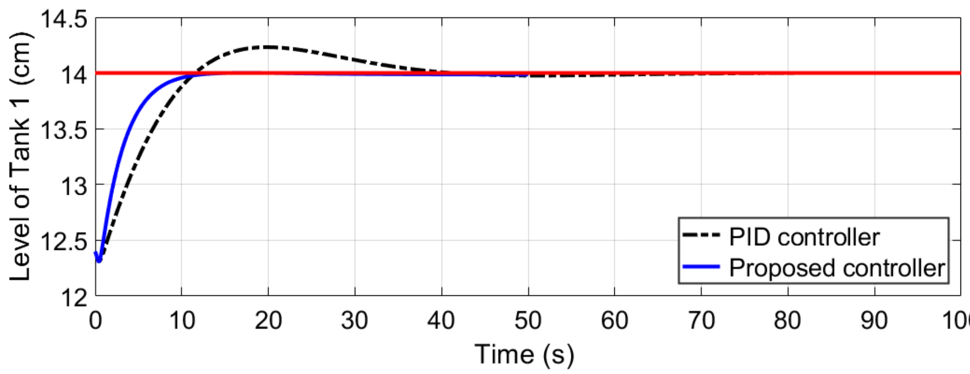
To analyse the performance of the controlled four-tank system, performance indices such as integral square error (ISE), integral absolute error (IAE), integral of time-weighted absolute error (ITAE) and mean-square-error (MSE) are computed whose formulates are given as follows:

$$ISE = \int_0^\infty e(t)^2 dt \tag{13}$$

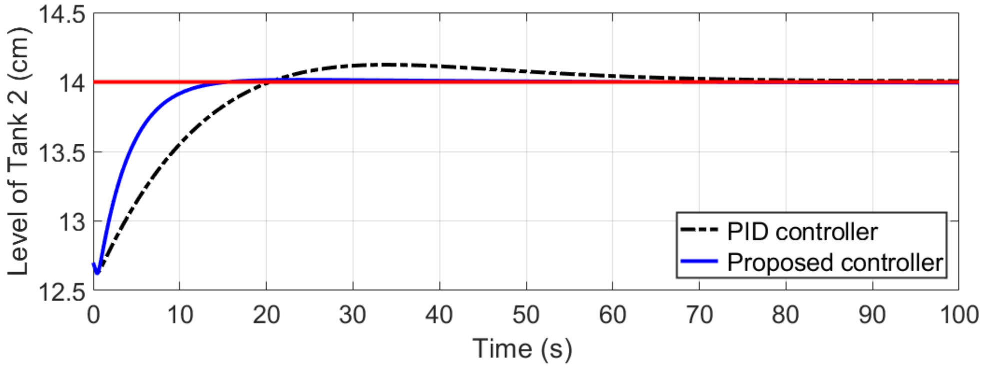
$$IAE = \int_0^\infty |e(t)|\, dt \tag{14}$$

$$ITAE = \int_0^\infty t\, |e(t)| dt \tag{15}$$

$$MSE = \frac{1}{N} \int_0^\infty e(t)^2\, dt \tag{16}$$



(a)



(b)

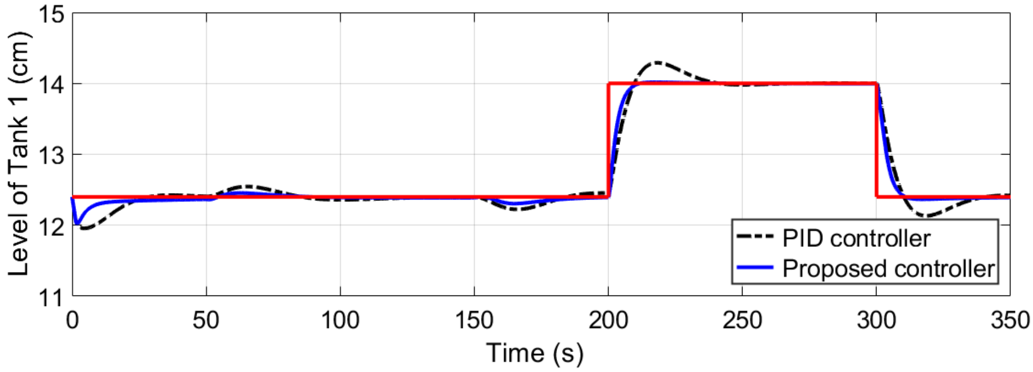
Figure 7. The step responses of controllers to evaluate time-domain parameters.

Figures 7a and 7b illustrate the step responses of controllers. In order to compare the performance of the controllers, time-domain performance specifications such as the overshoot percentage (M_p), the rise time (T_r), the settling time (T_s) and steady-state error (E_{ss}) are given in Table 2. In the table, the best values are highlighted in bold text. The structured H-infinity controller is more successful than the PID controller in terms of time-domain performance specifications.

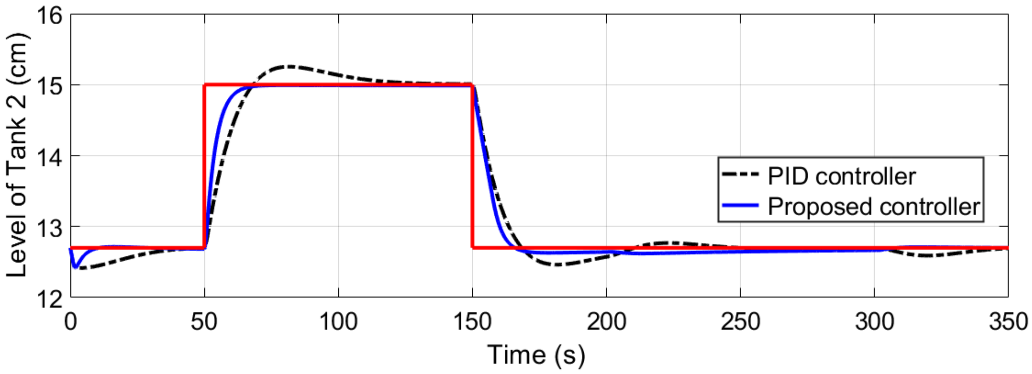
Table 2. Comparison of transient performance of controllers.

	Tank 1		Tank 2	
	PID	Structured H-infinity controller	PID	Structured H-infinity controller
M_p %	14,375	**0.06**	9.23	**1.15**
T_r (s)	9.6	**7**	16.2	**8.7**
T_s (s)	51	**40**	108	**82**
E_{ss} (cm)	0.02	**0.01**	0.006	**0.003**

Figures 8a and 8b display the robust stability and tracking performances of the quadruple tank system with the structured H-infinity controller and the classical PID controllers while the reference demand varies from 12.4 to 14 cm (Tank 1) and from 12.7 to 15 cm (Tank 2). A comparison of the voltages required by the pumps is given in Figure 9. It can be seen from Figure 9 that the structured H-infinity controller scheme requires a higher voltage than the classical PID controller does. The maximum value and energy, $E_{cs} = \int_0^\infty u^2\,(t)dt$ of control signals are calculated and reported in Table 3. In the table, the best values are highlighted in bold text. A comparison of the transience performances with the controllers is reported in Table 4. Each performance index indicates that the classical PID controller obtains higher errors than the structured H-infinity controller does.



(a)



(b)

Figure 8. The level control of Tanks 1 and 2 under the demand variations.

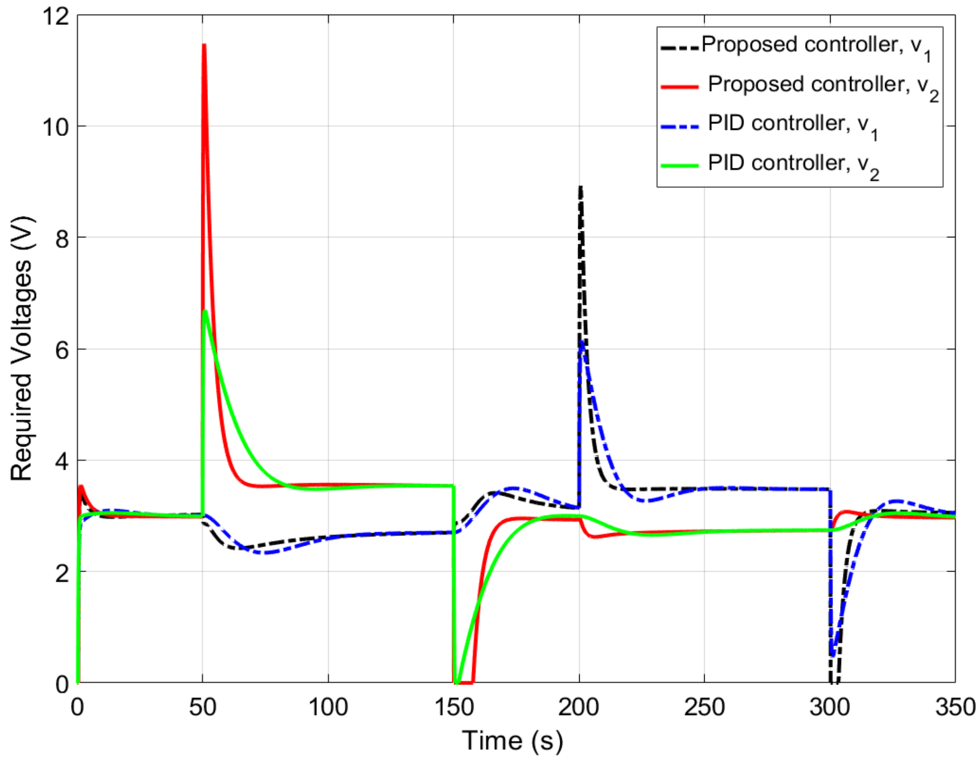


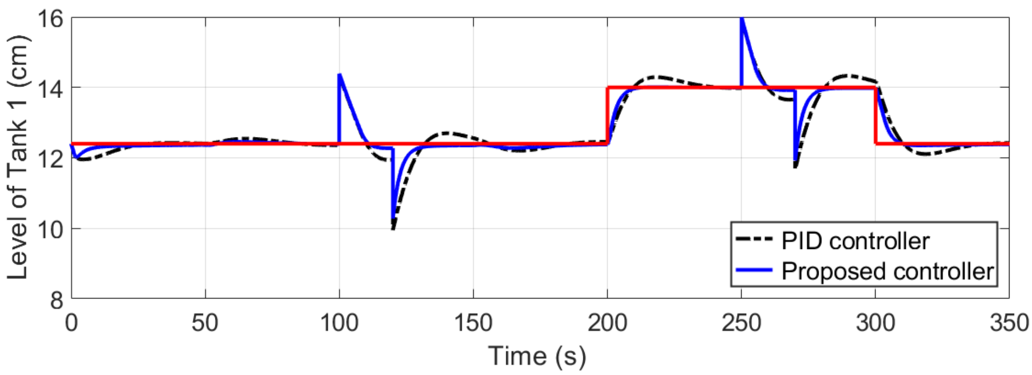
Figure 9. Comparison of control signals corresponding to Figure 6.

Table 3. The energy and maximum values of the control signals under the demand variations.

	Tank 1		Tank 2	
	PID	Structured H-infinity controller	PID	Structured H-infinity controller
Maximum values	**6.14**	8.92	**6.69**	11.47
Energy	**34454**	34724	**35152**	36232

Table 4. Performance indices of the PID and the proposed controllers under the demand variations.

	Tank 1		Tank 2	
	PID	Structured H-infinity controller	PID	Structured H-infinity controller
MSE	0.0560	**0.0360**	0.1496	**0.1006**
ISE	18.9852	**10.8552**	51.1585	**32.3378**
IAE	38.7173	**19.6930**	58.5066	**33.2943**
ITAE	7242.9	**3594.6**	6899	**4521.1**



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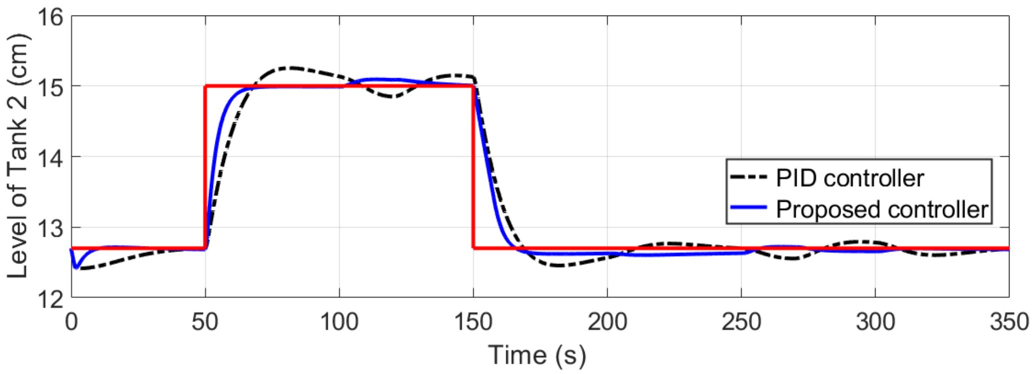


(b)

Figure 10. The performance of the controlled liquid level of Tanks 1 and 2 under the demand variations and output disturbance.

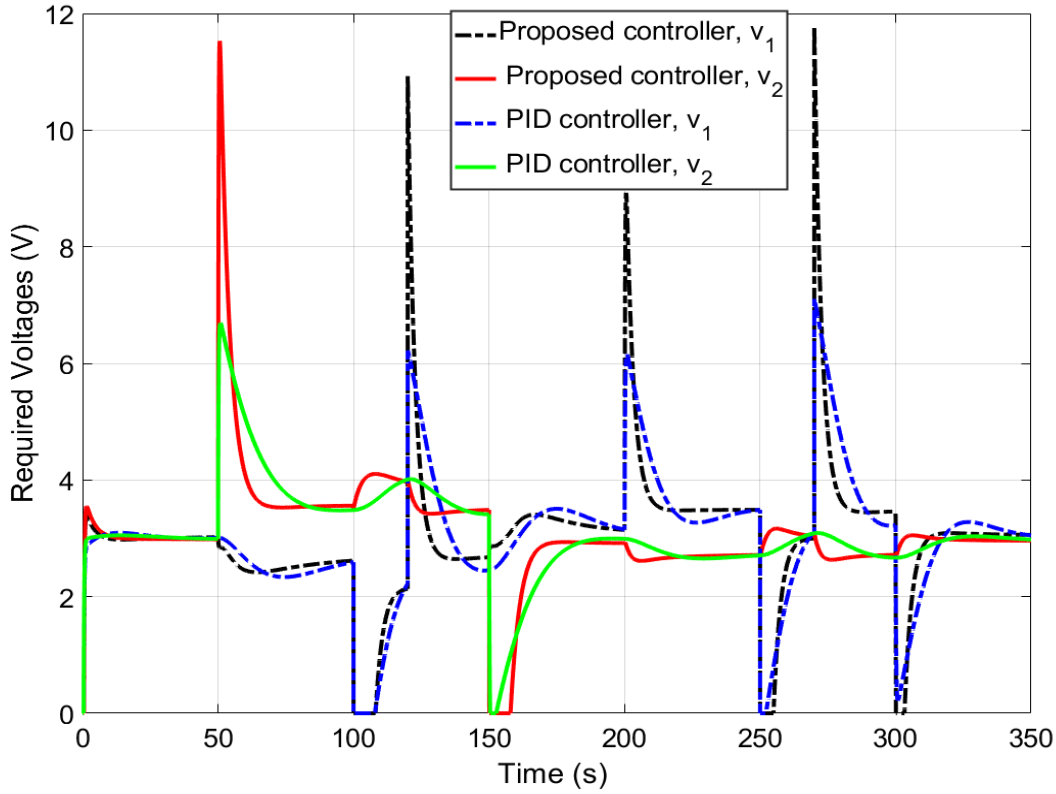


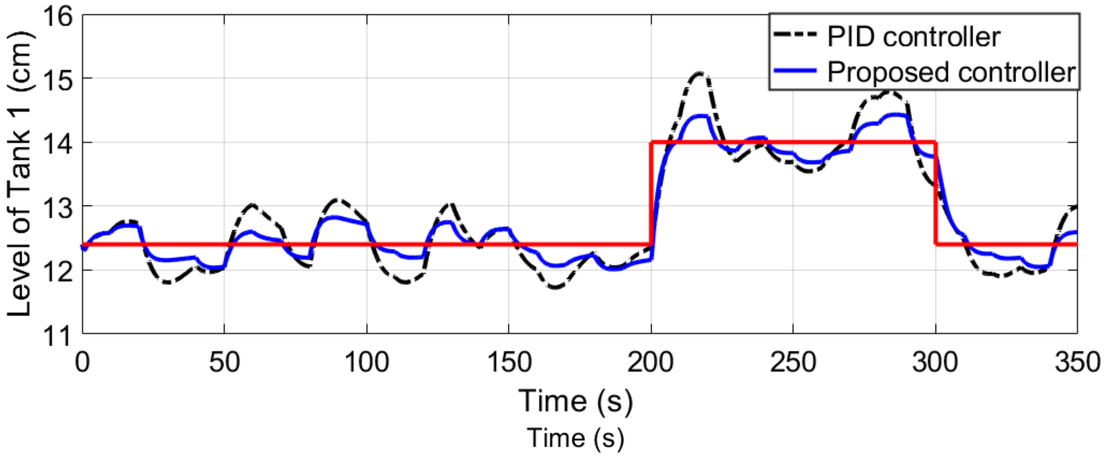
Figure 11. Control signals corresponding to Figure 10.

Table 5. The energy and maximum values of the control signals under the demand variations and the output disturbance.

	Tank 1		Tank 2	
	PID	Structured H-infinity controller	PID	Structured H-infinity controller
Maximum value	**7.1075**	11.7893	**6.694**	11.5294
Energy	**36236**	37175	**36063**	37167

Table 6. Performance indices of the PID and the proposed controllers under the demand variations and the output disturbance.

	Tank 1		Tank 2	
	PID	Structured H-infinity controller	PID	Structured H-infinity controller
MSE	0.2347	**0.1423**	0.1667	**0.1028**
ISE	78.7344	**46.0058**	55.1167	**33.1783**
IAE	93.2287	**52.6756**	66.2362	**36.1870**
ITAE	17790	**9569.3**	8457.2	**4925.4**



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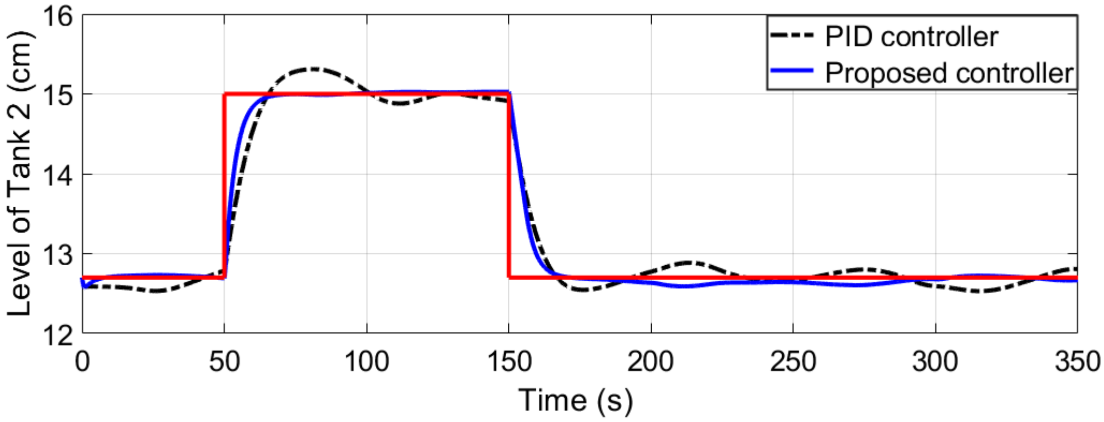


(b)

Figure 12. The performance of the controlled liquid level of Tanks 1 and 2 under the demand variations and the parameter uncertainty.

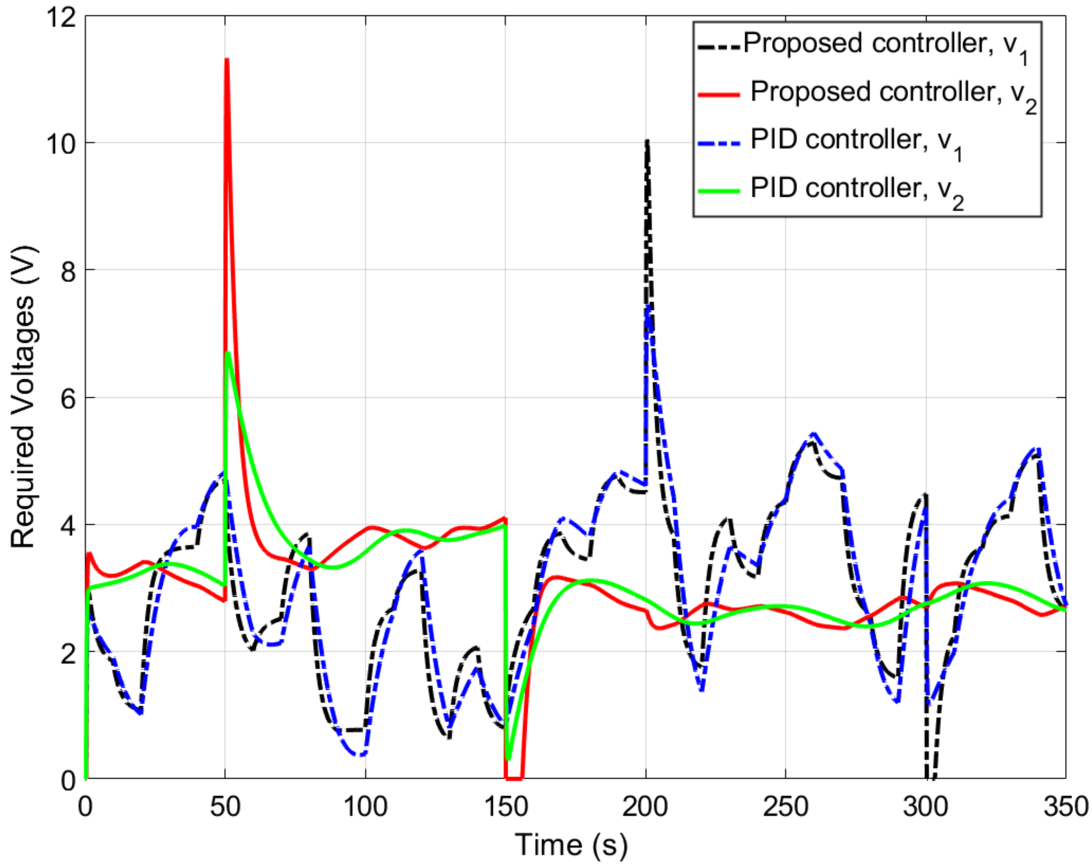


Figure 13. Control signals corresponding to Figure 12.

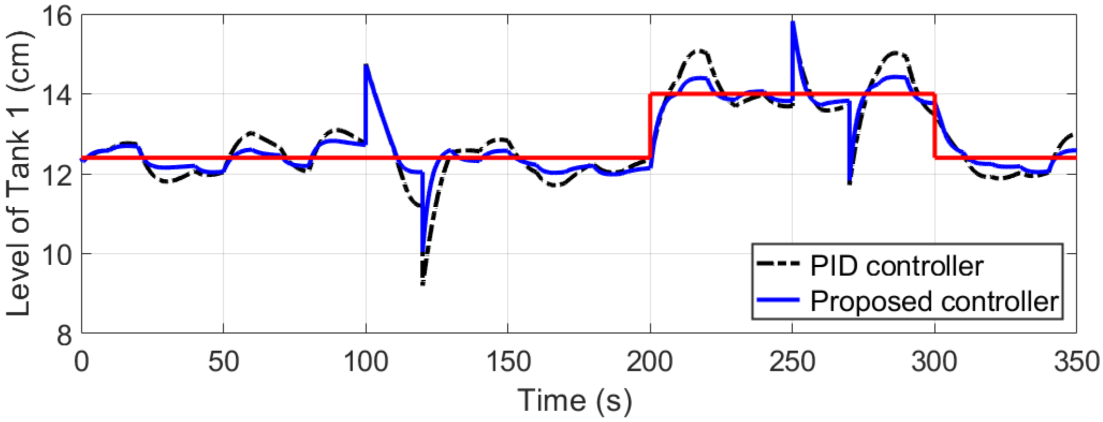
The simulation results of the disturbance rejection test are given in Figures 10a and 10b, which verify that the structured H-infinity controller can guarantee robustness and enhance the performance of the controlled system in the presence of disturbance. Here, the magnitude of the disturbance is 2 cm which is added to the output y_1 at t=100-120 s and t=250-270 s. Figure 11 shows the outputs of the PID controller and the structured H-infinity controller in the presence of disturbance. The maximum value and energy of control signals are reported in Table 3, which indicates that introducing the disturbance into the system causes an increase in energy and maximum value of control signals. In Table 6, the errors of the closed-loop quadruple tank system are presented. Once again, the structured H-infinity controller achieves a lower error than the PID controller does.

Table 7. The energy and maximum values of the control signals under the demand variations and the parametric uncertainty.

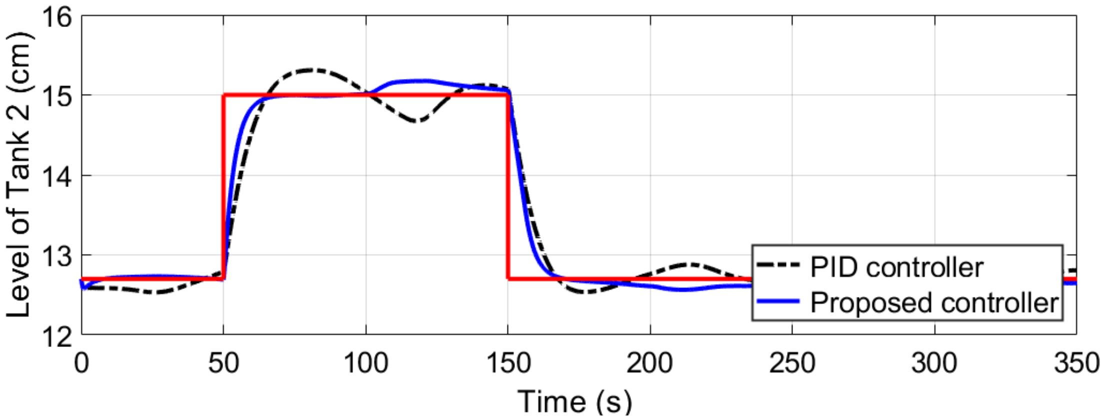
	Tank 1		Tank 2	
	PID	Structured H-infinity controller	PID	Structured H-infinity controller
Maximum value	**7.4501**	10.0451	**6.7136**	11.3249
Energy	39977	**39647**	**35880**	36858

Table 8. Performance indices of the PID and the proposed controllers under the demand variations and the parametric uncertainty.

	Tank 1		Tank 2	
	PID	Structured H-infinity controller	PID	Structured H-infinity controller
MSE	0.2005	**0.0837**	0.1221	**0.0897**
ISE	70.0578	**28.0933**	41.5997	**28.8869**
IAE	133.8407	**79.5947**	58.4112	**32.5866**
ITAE	24313	**14553**	79143	**47404**



(a)



(b)

Figure 14. The performance of liquid level controllers under the demand variations and the output disturbance and the parameter uncertainty.

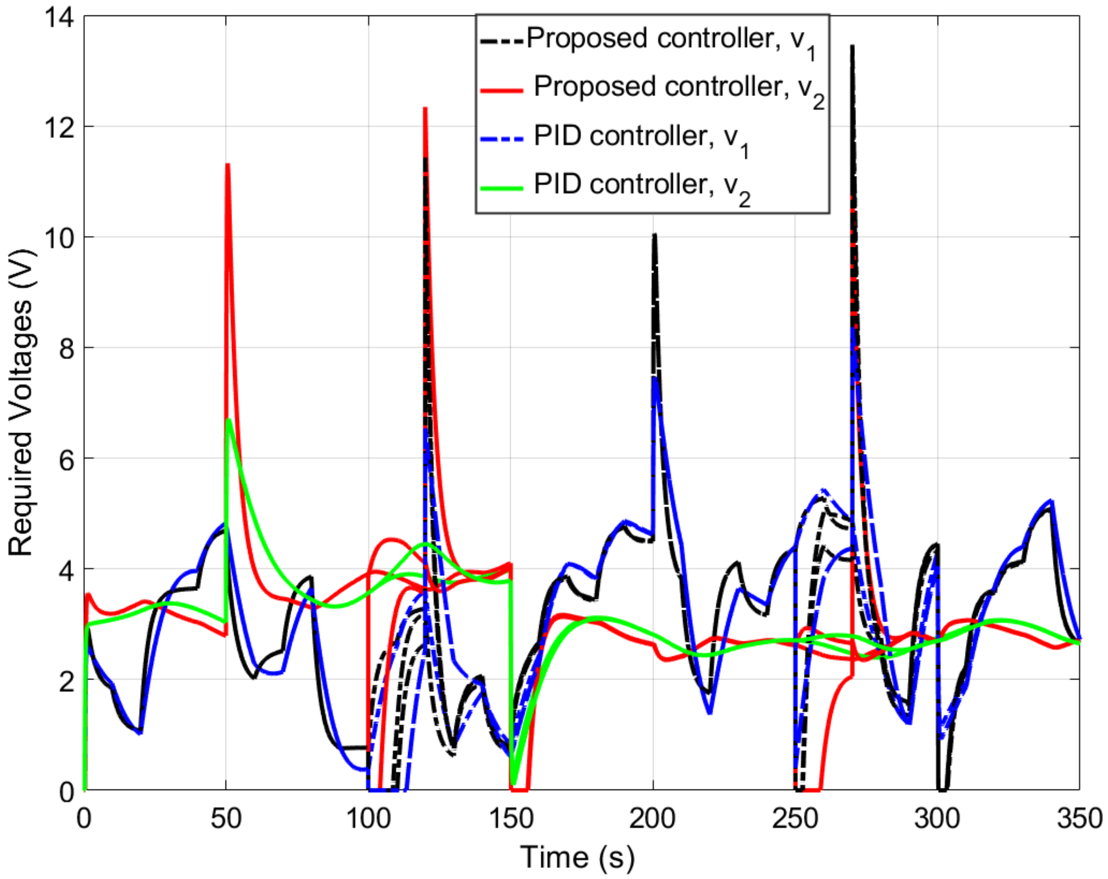


Figure 15. Control signals corresponding to Figure 14.

Table 9. The energy and maximum values of the control signals under the demand variations, the output disturbance and the parameter uncertainty.

	Tank 1		Tank 2	
	PID	Structured H-infinity controller	PID	Structured H-infinity controller
Maximum value	**8.3661**	12.9513	**6.7136**	11.3249
Energy	**40217**	41026	**36855**	37868

Table 10. Performance indices of the PID and the proposed controllers under the demand variations, the output disturbance and the parameter uncertainty.

	Tank 1		Tank 2	
	PID	Structured H-infinity controller	PID	Structured H-infinity controller
MSE	0.3819	**0.1888**	0.1354	**0.0950**
ISE	130.4221	**62.4211**	46.1843	**30.7704**
IAE	163.8281	**100.1702**	62.8946	**39.6472**
ITAE	29069	**17613**	82341	**58202**

In robustness to the parametric uncertainty test, ±50% of uncertainty in coefficient a_1 $(\text{cross} - \text{sectional area of the outlet pipe from Tank 1})$ was considered using inform random input. Figures 12a and 12b illustrate the transience performances of the quadruple tank system with the structured H-infinity controller and the PID controller. The structured H-infinity controller handles the parametric uncertainty better than the PID controller does. Figure 13 is given to display the outputs of the PID controller and the structured H-infinity controller. The maximum value and energy of control signals are reported in Table 7 under the demand variations and the parametric uncertainty. $\pm$50% of uncertainty in coefficient a_1 causes an increase in energy and maximum values of control signals for Thank 1. Performance indices for the scenario of the parametric uncertainty are given in Table 8.

The robustness test under the demand variations, the output disturbance and the parametric uncertainty together is taken into consideration and the simulation results are given in Figures 14a and 14b. Simulation results verify that the structured H-infinity controller ensures robust stability and improves the performance of the controlled system in comparison with the standard PID controller. Figure 15 is given to compare the outputs of the PID and the structured H-infinity controller under the demand variations, the output disturbance and the parametric uncertainty together. In Table 9, the maximum value and energy of control signals are reported. The energies of control signals increased to 40217 and 41026 to compensate effects of the disturbance and the parametric uncertainty for Tank 1. Performance indices for the last scenario are given in Table 10. It can be seen that the PID controller is less successful than the structured H-infinity controller in terms of error-dependent performance indices. To increase the performance of the PID controller, an intelligent controller might be employed as discussed in (Hilmi, 2019).

4. Conclusion

The quadruple tank system is a challenging multivariable industrial process comprising four interconnected tanks, two level sensors and two input pumps. This paper presents the design and analysis of a structured H-infinity controller for such a quadruple tank system. The second-order structured H-infinity controller (its order is quite lower than the order of the standard H-infinity controller) is computed using the structured H-infinity control method. A comparison with a classical PID controller is also carried out to demonstrate the superiority of the structured H-infinity controller despite its simple implementation. The tracking performances of controllers under the demand variations, the output disturbance and the parametric uncertainty are illustrated in extensive simulation studies. The simulation results show that the proposed controller achieves a good level of position control on the quadruple tank system and disturbance rejection and robustness to parametric uncertainty. The closed-loop system with the structured H-infinity controller provides better time-domain parameters and error-dependent performance indices than the classical PID controller does. Future work will be the implementation of the proposed controller on a real system.

Statement of Research and Publication Ethics

The author declares that this study complies with Research and Publication Ethics.

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