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Prospective Elementary Mathematics Teachers' Abilities of Using Geometric Proofs in Teaching of Triangle

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Abstract – In this study, it was aimed to reveal prospective elementary mathematics teachers' abilities of using geometric proofs in triangle teaching. The data of this study which was based on the techniques of qualitative research was obtained from students attending third class of elementary mathematics teacher education program at a public university in the north of Turkey, in 2013-2014 academic year. As data colletion tool, five open-ended questions which were prepared by the researchers taking the opinions of experts were used and the obtained data was analyzed through content analysis. The results show that proving abilities of prospective teachers are low. Three volunteer prospective teachers were made a semi-structured interviews with the intent of examining their proofs in detail. The obtained interview data have exposed that prospectice teachers usually attempt to produce geometric proofs using elementary, concrete and practical solution ways in terms of their own perpectives.

Key words: Geometric proof, triangles, prospective teachers.

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Introduction

Geometry is a domain which is based on reasoning and includes various concepts and the system of demonstrations (Battista, 2007). It provides to make connection between real life and mathematical world enabling to concretize and reflect the mathematical concepts

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(Clements, 1998). Because engaging in geometry helps development of the skills such as critical thinking, deductive reasoning and logical deduction, it makes students good problem solvers (Driscoll, 2007; Van de Walle, 2004). According to many research, the place of proof is quite important in geometry teaching (Hanna, 2000; Martin & Harel, 1989; Moutsios-Rentzos & Spyrou, 2015). Van Hiele defends that geometry teaching is a process composing of five stages which are vizualization, identification, abstraction, deduction and realization of relationships (Van de Walle, 2004). Students are expected to progress from identification to producton of informal proof in this process (Salazar, 2012). In parallel, althoug the objectives such as defending the accuracy of the arguments and making generalizations are among the objectives of geometry curriculum in our country (MEB, 2013), the evaluations (ÖBBS, 2009; PISA, 2009; TIMMS, 2007) show that Turkish students are unsuccessful in this domain (Kılıç, 2013). The difficulties in proving partially explain the reasons behind being not successful in geometry including objectives based on justifying (Mason, 1997).

Mathematical proving is a presentation of justifying and reasoning (National Council of Teachers of Mathematics [NCTM], 2000). It involves explaining the reasons of these as well as indicating the accuracy or inaccuracy of an argument (Bell,1976; Hersh, 1993). In other words, proving, one of the essential part of mathematics, is a tool which is used for presenting the validity of the mathematical statements (McCrone&Martin, 2004). Besides, proving is a whole of effective ways which provides to develop and to express the mathematical insights (NCTM, 2000). Geometric proving is a particular written discourse (Pimm & Wagner, 2003) and requires to have the abilities that are difficult to learn (Wong, Yin, Yang & Cheng, 2011). It firstly includes understanding the existence of geometric elements, recognizing the principles and features, using them while proving, realizing the logical link among principles, features and assumptions in proof and transmitting it to different situations (Lin & Yang, 2007).

According to NCTM (2000), proving should be an inseperable component of all mathematics from preschool to senior year of high school. However, students generally encounter with proof in geometry lesson in high school level (Stylianides, 2007; Stylianou, Blanton & Knuth, 2009) and have few experiences related to this subject (NCTM, 2000). At this period, due to the fact that proving has been taught as a formula which is needed to follow, it is not able to become meaningful for students and this situation causes perceptions of mathematics as body of rules of which answers are previously known (Middleton, 2009). As a natural consequence of this, various challanges of students in proving and understanding proofs come out (Chazan 1993a; Harel & Sowder 2007; Healy & Hoyles, 2000; Riley, 2004). Many research show that high school students' abilities of producing algebraic and geometric proofs (Healy & Hoyles, 2000; Kahan, 1999; Senk, 1985) and understanding a proved generalization are low (Fischbein & Kedem, 1982; Healy & Hoyles, 2000; Kahan,1999; Porteous, 1991; Schoenfeld, 1988; Vinner, 1983). Riley (2004), in a study conducted with prospective teachers, found that very few of prospective teachers produced valid proofs and emphasized that prospective teachers who are not sufficient in understanding and consructing proofs will be teachers who have difficulties in teaching of proving in future education system. Yet, teachers who will constitute needed structure for comprehending proving in class and present various opportunities of learning, firstly, must understand better this concept (Ko, 2010; Riley, 2004).

The results of the studies related to proving which are conducted with prospective elementary mathematics teachers, prospective secondary mathematics teachers, elementary and secondary mathematics teachers reveal that teachers consider empirical inferences, that is, arguments based on examples and numerical calculations as proving (Goulding, Rowland, & Barber, 2002; Knuth, 2002; Ma, 1999; Martin & Harel, 1989; Morris, 2002; Simon & Blume, 1996). Almeida (2001) has also stated that pupils' views of proof have been generally empirical, and they may have difficulties in justifying their results due to lack of their knowledge. Since these perceptions of teachers may lead students to regard empirical demonstrations as proving, this situation become a threat in terms of learning to prove (Stylianides, 2007). Hence, most of the student do not know the differences between empirical and deductive arguments (Chazan, 1993b). Therefore, arising of the difficulties in geometry and the other domains in terms of students is inevitable (Martin & Harel, 1989). Besides, teachers' limited recognitions of proving cause the composition of students' misconceptions and lead to hold these misconceptions (Stylianides & Stylianides, 2009). Thus, future teachers need to be able to read, understand and produce proofs (Salazar, 2012).

Students need the help of their teachers for improvement of proving skills because they have complex structure. The roles of teachers in teaching of proving are presenting whether an argument is a proof or not and what constitutes a proof by choosing appropriate activities. The most important factor which affects these teacher roles is the knowledge of teacher about proving (Stylianides, 2007), namely, teachers' knowledge have an impact on the development of students' geometric thinking and the frequency of use of proof in lessons. Therefore, it is important to investigate knowledge and proving processes of teachers, particularly, prospective teachers who will be future teachers in order to determine the deficiencies and make precautions. However, the examination of literature shows that there are few studies related to proving towards prospective elementary mathematics teachers (Harel, 2002; Knuth, 2002; Morris, 2002; Movshovitz-Hadar, 1993; Stylianides, Stylianides, & Philippou, 2004, 2007), the studies were further conducted with prospective high school mathematics teachers or focused on a particular proving method such as induction (Stylianides & Stylianides, 2009). Proof is important due to the fact that it provides to make sense of logical structure which the result based on and reveal mathematical resoning explicitly (Coe & Ruthven, 1994), hence, it strengthens geometrical thinking and the comrehension of geometric concepts. To be succesful in geometry teaching, proof should be integrated into this lesson (Harel, 2008). Although there are sort of research including the difficulties in proving (Chazan, 1993a; Hart, 1994; Martin & Harel, 1989; Senk, 1985), the studies focusing on learning of geometric proofs are limited (Herbest, 2002; McCrone & Martin, 2004). From the scarcity of the studies related to proof in geometry and towards prospective elementary mathematics teachers, the aim of this study is to determine the abilities of prospective elementary mathematics teachers in geometric proving in the subject of triangle. In this direction, the reseach questions are as the following:

1. How are proving abilities of prospective elementary mathematics teachers in geometry ?
2. How are proving processes of prospective elementary mathematics teachers in geometry?

Methodology

Research Design

In this study, case study which is one of the qualitative research designs was used. Case study is an approach which is based on detailed investigation of a case or cases by using resources that provide to obtain rich data such as observations, interviews and documents (Creswell, 2006). Due to the fact that the aim of the study was to examine the geometric proofs of prospective elemenrary mathematis teachers about triangles in detailed, this approach was preferred.

Participants

The research data was obtained from 86 prospective teachers attending third class of a elementary mathematis teacher educatiom program at a public university in the North of

Turkey, in 2013-2014 academic year. Besides, after collection of data, semi-structured interviews were conducted with three, two girls and one boy, volunteer prospective teachers. In determination of these participants for interview, maximum variation sampling method was preferred in order to define whether there were common and shared aspects among various cases and reveal the different dimensions of the problem in accordance with them (Yıldırım & Şimşek, 2005). At this process, the students' solutions were examined and three of them who were evaluated as low, average and high level students were interviewed.

Data Collection Tools

With the purpose of investigating how third grade elementary mathematics teachers produce geometric proofs in triangle subject, five open-ended questions related to proving were asked. These questions include foundation proofs such as the demonstration of that the sum of interior angles of a triangle is 180˚, the proof of Pythagorean Theorem and the expression of the formula of triangle area. In addition, it involves unfamiliar proofs such as the demonstration of that the edge across bigger angle is bigger than the edge across smaller angle if the measurements of those both angles are not equal and the proof showing that the sum of any two interior angles is larger than third interior angle in a triangle. These questions were determined by the researchers considering essential proofs so as to investigate geometric proofs regarding triangles of prospective teachers and determine mistakes of them. The final form was constituted by taking the opinions of the experts. In order to examine the geometric proving process of prospective teahers in detailed and elaborate how they think while producing geometric proofs, semi structured interviews were made with three prospective teachers. They were required to clarify their proofs with reasons on the basis of each question.

Data Analysis

The data obtained from 5 open-ended questions were analyzed through content analysis method. Content analysis is an technique which provides to analyze, understand, organize, identify and interpret verbal and written data objectively and systematically (Holsti, 1969; Sommer & Sommer, 1991). In this direction, the solutions of participants were analyzed, various categories were formed, the concepts composing categories were defined and interpreted. It was benefited from the system of classification and scoring which used by some researchers (Soylu & Soylu, 2006; Yeşildere, 2006; Şimşek, Şimşek & Dündar, 2013) in this analysis. They generally used the categories of correct, partly correct, incorrect and unanswered to classify their data. In addition to these categories, the topic of mathematical language was included in this study because there were answers which reflected only symbolic representations of mathematical expressions without any justification. The followings show the categories which were used in this study and explanations of them:

Valid Proof: It includes expression of reasoning through correct mathematical representations and symbols, indication of mathematical concepts which justification is based on and presentation of acceptable solution. It is based on using appropriate principles, properties and assumptions, understanding logical connection between them and transferring to different cases.

Incomplete Proof: It includes making partly correct solution but not being able to present whole answer. Solution in this category shows that individual knows something about the subject but the proof is not complete because of insufficient knowledge and not being able to associate necessary mathematical concepts.

Invalid Proof: It includes presentation of incorrect or irrelevant solutions. Individual exhibits mathematical approach which is not correct and uses invalid mathematical representations.

Mathematical Language: It includes symbolic representations of mathematical expressions without any justification. The answer does not include solution or connection between mathematical concepts.

Empty: It does not include any solution or mathematical expression. It is the question which individual prefer not to answer.

The answers of students were classified for each question according to these categories. The percentage and frequency values on the basis of category were given in the table. In this process, the data were coded by both researchers seperately and the percentage of conformity was found to be %90. Researchers reached a consensus on different points after a meeting, discussed what kind of answer should be under which category and matched the responses and categories. In order to provide validity and relability, the data were evaluated by two researchers objectively and interviews were conducted as well as written documents to strengthen the interpretations. Besides, some quatotions from solutions and dialogs of intervews were presented.

Findings

The obtained data was presented in the light of research questions: the categories related to geometric proofs of prospective teachers and the processes of proving.

The Categories Related to Geometric Proofs of Prospective Teachers

The answers of prospective teachers to five open-ended questions about proving in triangle subject were examined and these were stated under five categories through the values of frequency (*f*) and percentage (%). If the student made formal proving by following the expected logical steps, this answer was accepted as a valid proof. If the student began and proceeded proving properly, yet could not obtain the result, this answer was incorporated into the category of incomplete proof. In case the student's answer included illogical and irrelevant mathematical demonstrations, this answer was accepted as invalid proof. Some students only wrote the same expression in the question using mathematical language and symbols and did not take any step for proving. These kind of answers were evaluated under the category of mathematical language. If the student did not attempt to solve the question, it was described as empty.

Table 1 The Values of Frequency and Percentage With Regard to Prospective Elemetary Mathematics Teachers' Geometric Proofs

Categories	***1. Question***		***2. Question***		***3. Question***		***4. Question***		***5. Question***	
	f	%	*f*	%	*f*	%	*f*	%	*f*	%
Valid Proof	50	58	0	0	20	23	1	1	45	52
Incomplete Proof	26	30	9	10	4	5	5	6	1	1
Invalid Proof	5	6	28	33	17	20	23	27	2	2
Mathematical Language	5	6	14	16	12	14	32	37	38	45
Empty	0	0	35	41	33	38	25	29	0	0

When the values in Table 1 were examined in terms of each question and category, for the first question requesting demonstration of that the sum of interior angles of a triangle is 180˚, %58 of participants produced valid proofs whereas proofs of %6 were invalid. This situation reveals that the most of the students are succesful in showing this proof. Besides %30 of the students did not complete their proofs while %6 of them only expressed desired proof as mathematical language and could not continue on it. There were no students who had no attempt for proving this question. In below, solution examples of students for each category of first question were given.

In First question including to show that the sum of interior angles of triangle is 180 ˚, %58 of the solutions were valid proofs and %26 of them were incomplete. There were no students who leaved empty. Besides, students wrote the given statement in the form of mathematical language were in proportion of %6 as well as solutions which were invalid.

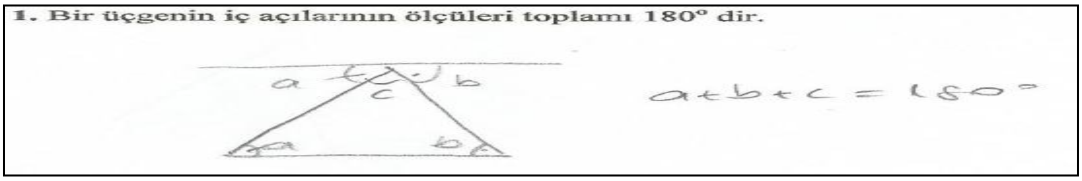

Figure 1 Student Solution Example Related to Valid Proof Category

In Figure 1, student correctly showed that the sum of interior angles of a triangle is 180˚ transforming angles side by side to form straight angle and benefiting from the concepts of parallelism, alternate interior angle and alternate exterior angle.

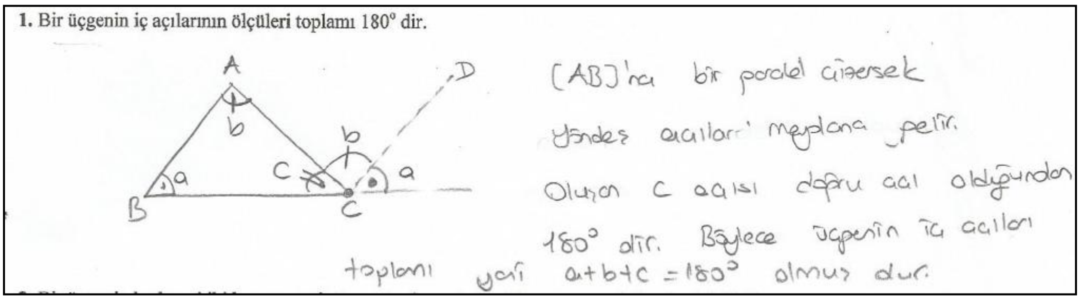

Figure 2 Student Solution Example Related to Incomplete Proof Category

In the solution example of Figure 2, student propably tried to justify through proving way which he was familiar from lessons, yet could not internalize the reasons and showed it without explanation of carrying angles based on parallelism.

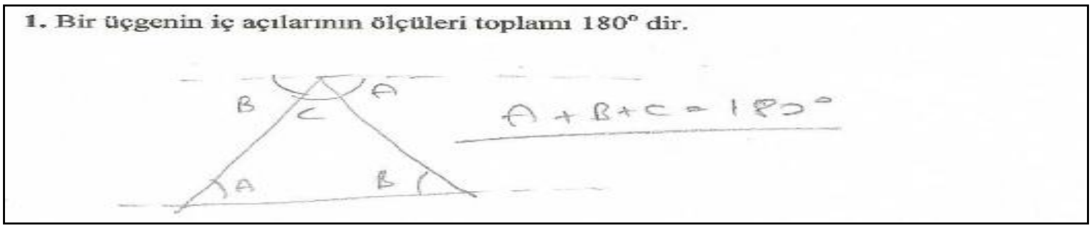

Figure 3 Student Solution Example Related to Invalid Proof Category

In the answer of Figure 3, it can be seen that student did not metion the necessary conditions for this kind of solution such as parallelism as well as confuse the concepts of corresponding angle and alternate interior angle. Therefore, student transported the angles incorrectly and produced invalid proof.

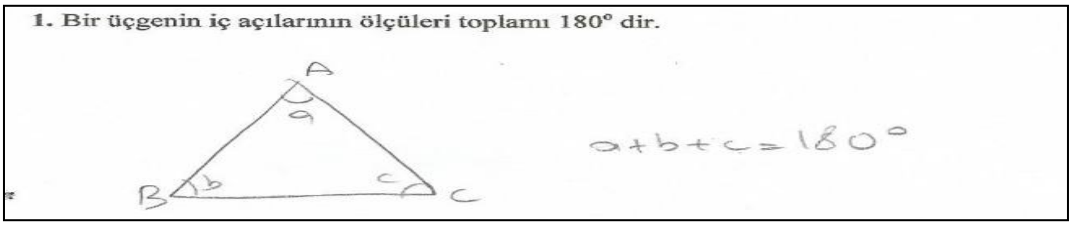

Figure 4 Student Solution Example Related to Mathematical Language Category

From the demonstration in Figure 4, it can be understood that student only wrote the given verbal expression in the form of mathematical language and did not follow any step so as to prove.

In the second question including to show that the sum of any two interior angles is larger than third interior angle, none of the students could do desired proving correctly. %33 of the solutions were invalid proofs and %10 of solutions were incomplete although they involved some correct steps. In addition, students wrote the given statement as mathematical in proportion of %14 whereas %41 of them colud not present any answer and preferred to leave empty. It is seen that this rate is obviously high.

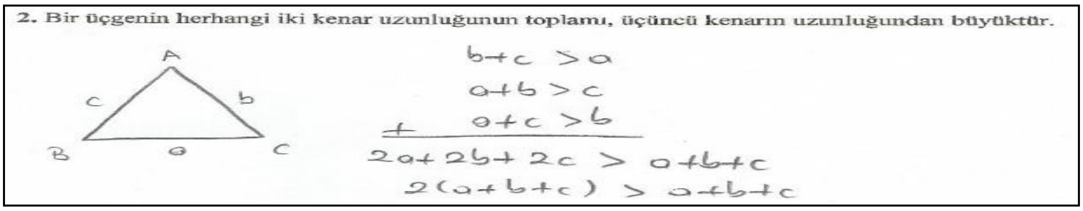

Figure 5 Student Solution Example Related to Incomplete Proof Category

In the solution of Figure 5, it can be understood that student made a kind of justification based on the given expression, however, she could not complete and obtain valid proof.

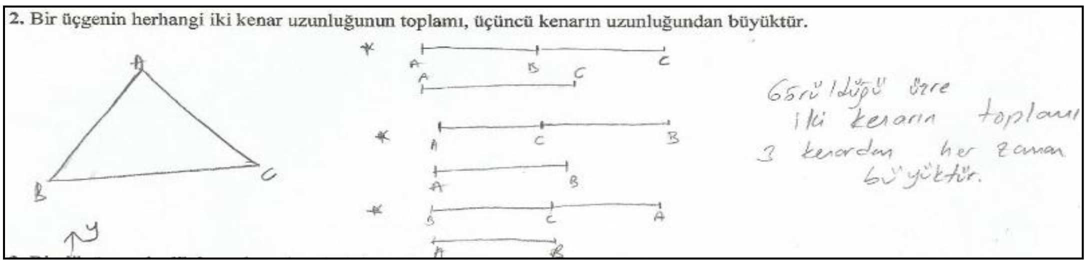

Figure 6 Student Solution Example Related to Invalid Proof Category

In Figure 6, the solution revealed that student reflected the edges of triangle drawing them relatively and thought that this statement could be proved through comparision in this way.

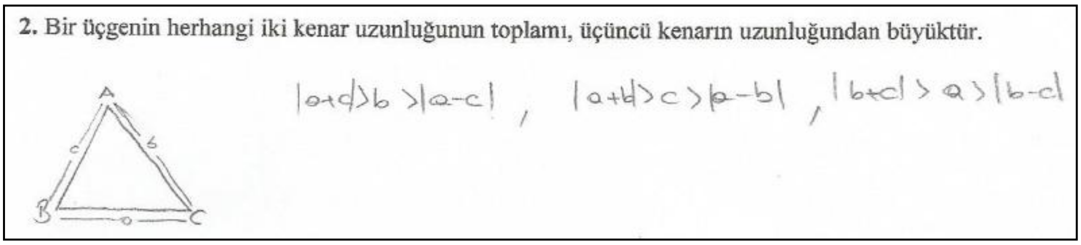

Figure 7 Student Solution Example Related to Mathematical Language Category

The presentation in Figure 7 shows that student wrote the expression that was requested to be proven using mathematical language and did not continue to work on it.

When third question which desired the proof of pythagorean theorem is tackled, %23 of the students produced valid proofs, %5 of them presented incomplete proofs and the proofs of %20 were invalid. Besides, it was concluded that %14 of the students expressed the theorem as mathematical and %38 of them had no attempt to solve the question.

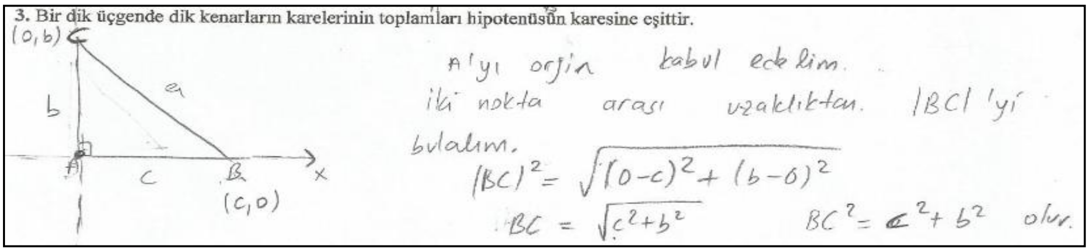

Figure 8 Student Solution Example Related to Valid Proof Category

In Figure 8, student showed the justification of the statement using own knowledge about finding the distance between two points in analytic geometry.

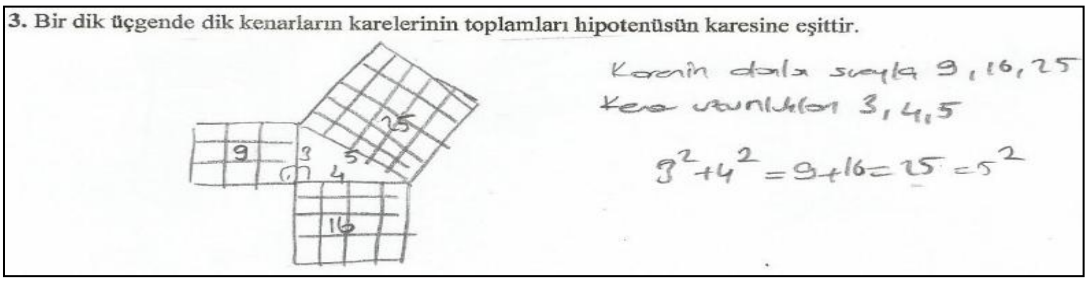

Figure 9 Student Solution Example Related to Invalid Proof Category

The solution in Figure 9 showed that student attempted to prove the Pythagorean Theorem based on an example using numeric values. It revealed that student had an incorrect preception like that only one example supporting the accraucy of the statement was enough for proving.

Figure 10 Student Solution Example Related to Incomplete Proof Category

The solution in Figure 10 showed that students became familiar to the proof of Pythagorean Theorem from lessons or textbooks. However, because of that they could not precisely make sense of it, they were not aware of that their demostration reflected only a part of proof and the steps for generalization were lack.

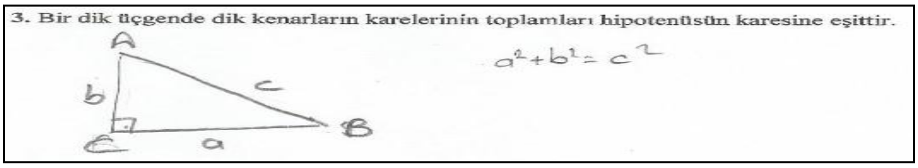

Figure 11 Student Solution Example Related to Mathematical Language Category

Figure 11 showed that student wrote Pythagorean Theorem as transforming it into mathematical language.

In fourth question including the demonstration of that the edge across bigger angle is bigger than the edge across smaller angle if the measurements of those both angles are not equal, only %1 of the students could produce valid proof. On the basis of question, it was seen that the ratio of correct geometric proving was notably low. Students were produce invalid proofs in proportion of %27 whereas %6 of them could not complete their proofs. Besides %37of them wrote the expression in the form of mathematical language and %29 of them left empty the question.

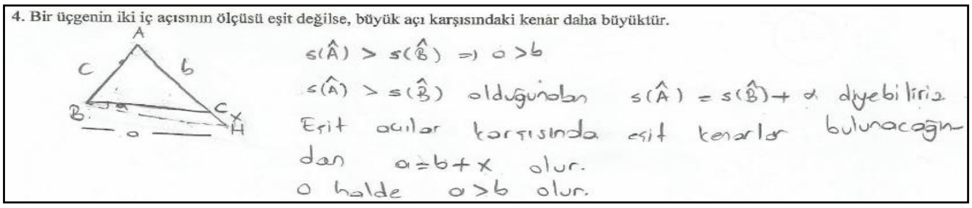

Figure 12 Student Solution Example Related to Valid Proof Category

In the solution of Figure 12, student validly justified the statement based on making an assumption about it and using logical connections between mathematical concepts.

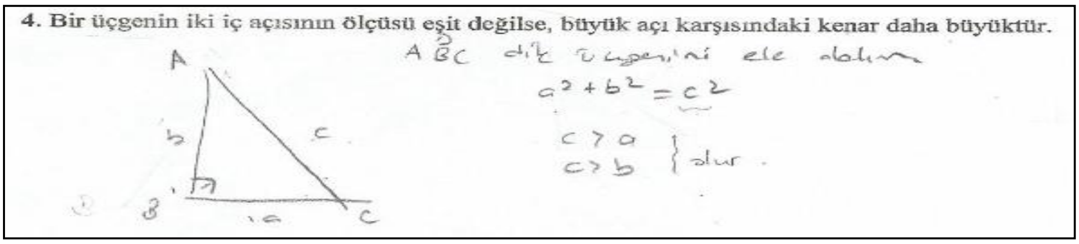

Figure 13 Student Solution Example Related to Incomplete Proof Category

In Figure 13, it can be seen that student used expressions which were correct mathematically utilizing a right triangle and pythagorean theorem, yet he did not focus on the other necessary conditions for proving.

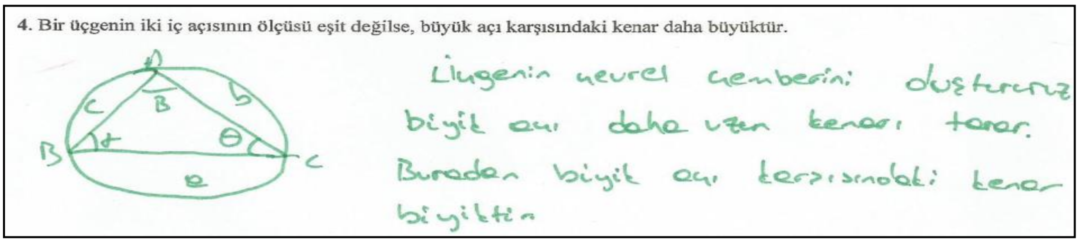

Figure 14 Student Solution Example Related to Invalid Proof Category

In Figure 14, the answer of the student was evaluated as invalid since meaningless explanation was made as verbally without using mathematical expression.

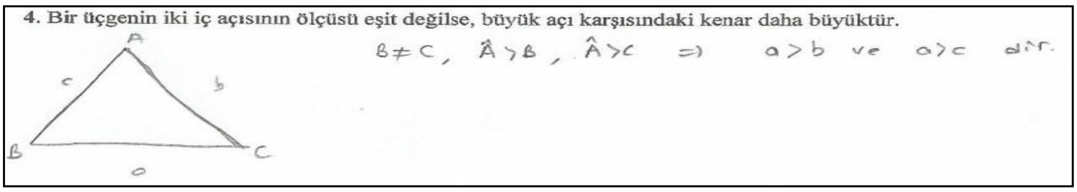

Figure 15 Student Solution Example Related to Mathematical Language Category

Figure 15 showed that student did not maintain proving after wrote the verbal statement in the form of mathematical language.

In fifth question including the expression of the formula of triangle area, all students tried to solve this question. In this direction, %52 of the students produced valid proofs, %2 of them presented invalid proofs and %1 of them could not complete proving. Besides %45 of the students could not reflect the reason behind of the formula and only used mathematical language.

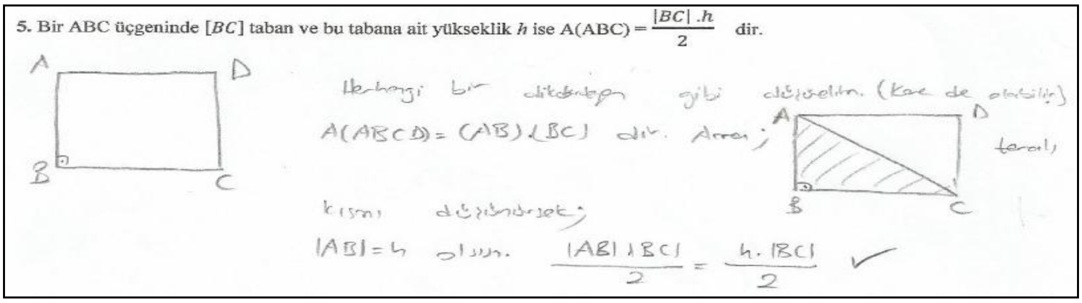

Figure 16 Student Solution Example Related to Valid Proof Category

In Figure 16, it can be seen that student showed the accuracy of the statement benefiting from the knowledge related to the area of rectangle and the relationship between rectangle and triangle.

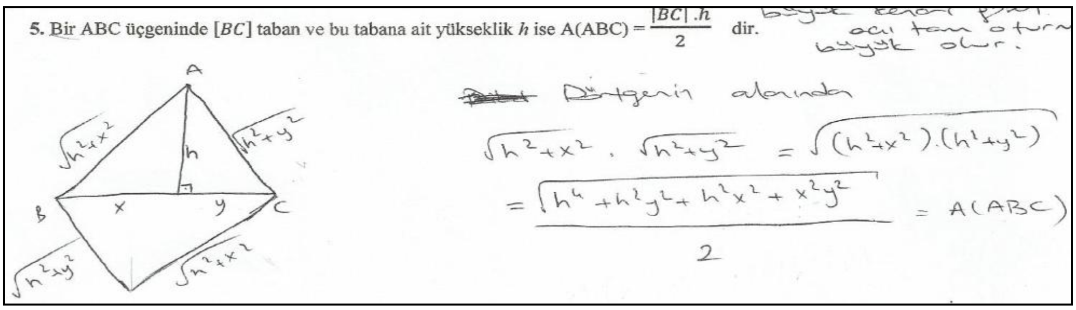

Figure 17 Student Solution Example Related to Invalid Proof Category

Figure 17 showed that student presented invalid proof following meaningless mathematical steps rather than operations which would facilitate to reach expected proof.

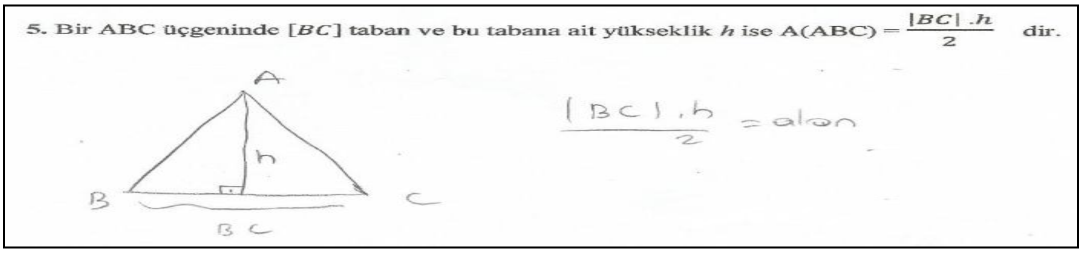

Figure 18 Student Solution Example Related to Mathematical Language Category

In Figure 18, it is seen that student wrote the given statement through mathematical language and did not present any diffent attempt.

The Processes of Prospective Teachers in Proving

In this section, the dialogs of semi-structured interviews which were made with 3 prospective teachers were presented. The interview conducted with the first student is as follows:

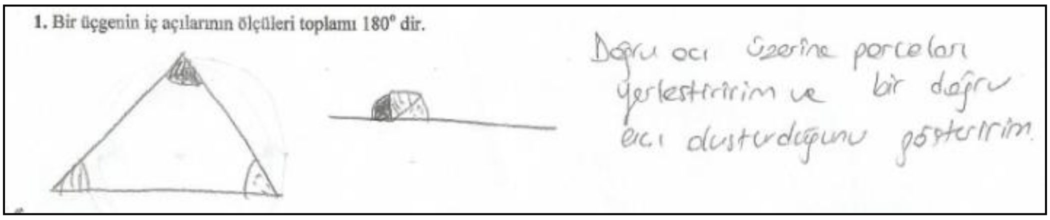

Figure 19 S1 The Solution Example for First Interview Question

R: Let's start with first question. How did you think while proving in this question?

S1: First, I thought at a level of elementary. When we were in first class of university, we prepared a geometry file and we made proving with the help of materials. When we cut the interior angles of a triangle and combined on a straight line, I saw that we obtained a straight angle at every turn. If I proved this expression through this simple method in the future, I would did in this way, therefore, I would show that we obtain a straight line by cutting the angles and combining on a line so that it is 180° with the help of compasses.

R: Why did you prefer this solution way?

S1: There is certainly another theoretical proof, yet this way is easy for me. I preffered because of that it is easy and concrete.

R: Ok. Are you saying like that considering in terms of yourself or students?

S1: Students.

R: What is proof according to you?

S1: Proof is demonstrating that any statement is firmly correct or incorrect by setting out in full. Namely, I can also say describing it well.

R: What kind of features should a statement include to be a proof, namely, can we say every accuracy that we showed is a proof?

S1: No, we cannot. We may not prove accuracy but we can show inaccuracy with an example. While we are trying to prove the accuracy do we consider all worked population? In another word, it must include all population to be a proof, one correct example is not enough.

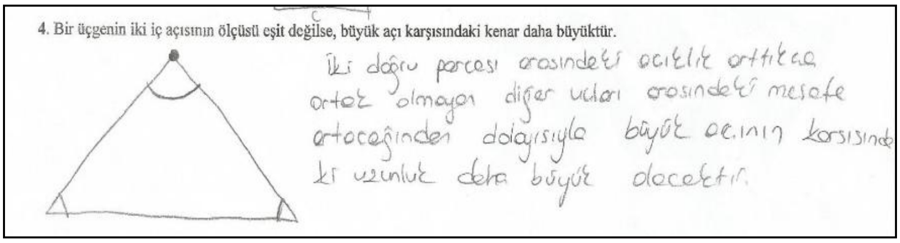

Figure 20 S1 The Solution Example for Fourth Interview Question

First student could not prove 2. ve 3. questions.

R: If we look at fourth question, how did you think while proving this question?

S1: When I first saw this, actually, I directly accepted it, that is, I sad that it has already be like this. However, I think proof of this kind of statement is harder. I already considered this as correct so I did not feel the need of proving. When we think in this respect, the proofs of essential things are more diffcult. This statement is one of them, therefore, I wrote a simple explanation. If we think that we combine the lines on a point and increase the angle between these two lines gradually, the distance between the other points also increases gradually. This is a situation which secondary school children or people who have high level education will be able to understand. As increasing the distance, the lenght across the big angle will increase. Yet there is no clear proof of this but it is true like this.

R: According to you, is your explanation not a proof?

S1: Yes. I am aware of that it is not a proof. However, is it has to be proven? Namely, it is true, it appears that everybody can sense of this.

R: What did you do in last question?

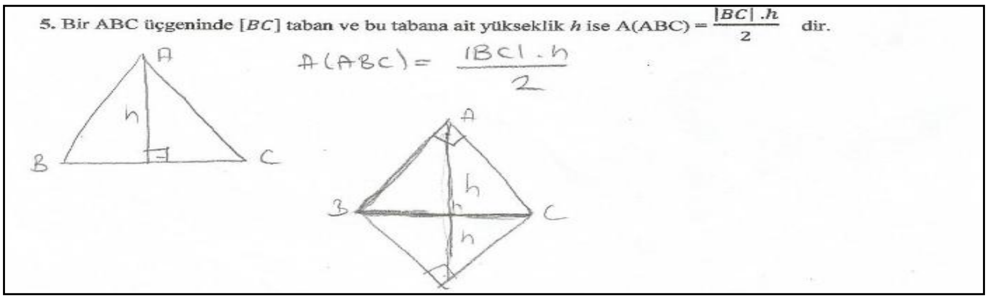

Figure 21 S1 The Solution Example for Fifth Interview Question

S1: To find the area of a triangle, base times heighth divided by two. I could not prove this question as well. I do not know but I thought that if we start by looking at a square, we see that it is divided by a diagonal and two triangles occur. This diagonal will be the base of the other triangle, that is, BC. Similarly, when we draw the other diagonal, it will be height, namely, h. The area of these triangles will be the half of the area of the square. Therefore, I tried to show the area of triangle utilizing the area of the square.

R: Why did you feel the need of proving through square?

S1: Because there is a expression of half in formula, it brings cutting into half of the area in my mind. The half of what? So I thought that it is the half of a square or a rectangle.

R: If we consider the place of the area in geometry, would you use this kind of proof while teaching area in triangle?

S1: No, I would not. If I knew well, I might use but I do not know well now. I do not prefer to give proof of this statement.

R: How can you tell this subject?

S1: I can give the area formula. The essential proofs should be given but I do not know.

R: In general, is there anything else you want to add about proof or using of it in mathematics teaching?

S1: To what extent is proof important in mathematics teaching? Actually, I do not know also. I did not have an education related to proof. If I learned proofs, I might say that it is useful. However, as a prospective teacher, I think that I am insufficient and also most of my friends.

Because we do not feel the need of proving the simple essential concepts. However, we should recognize that we do advanced proofs using these essential concepts.

The findings of first student showed that geometric proving abilities of prospective teacher was weak and she was aware of this. The student presented a proof which was simple and concrete for her and based on solution of an example. She did not prefer theoretical proof. As is also understood from her answer, choosing this solution way results from encountering these kinds of notations in the lessons. Whereas the student preferred to show no attempt in second and third questions, she emphasized the correctness of the expression by giving verbal answer in forth question. However, she was aware of that her explanation did not have the characteristics of proof. In fifth question, she tried proving based on her knowledge about the concepts such as square and rectangle. Although she logically expressed her proof, she was not sure about her proof. In general, we can say that she comprehends the meaning of proof concept but she is not able to internalize proving.

The interview conducted with the second student is as follows:

R: Could you clarify your solution in first question? How did you start the solution? How did you think?

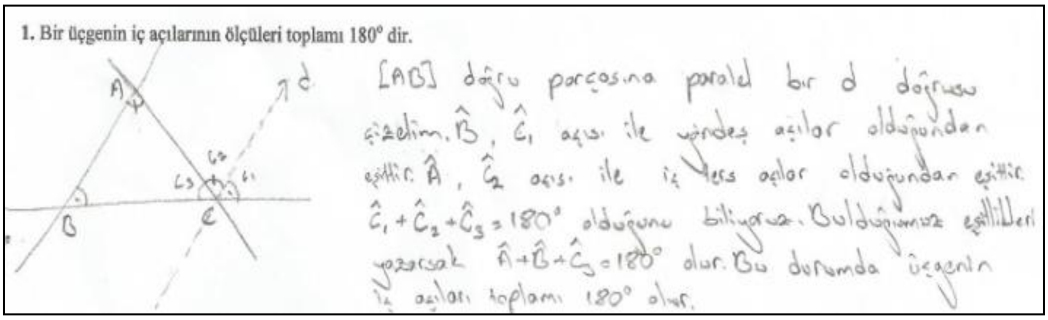

Figure 22 S2 The Solution Example for First Interview Question

S2: I started building a triangle and I thought what I can do after. At first, I put the interior angles in triangle. I had already knowledge about the parallelism of the lines and transporting the angles. In the direction of my previous knowledge, I decided forming an appropriate parallel line determining an edge. I attempted to draw together the interior angles on a line transporting the angles according to being corresponding or alternate interior angles. I found a corresponding angle and an alternate interior angle. After gathering angles together on a line, I found that the sum of these three interior angles is 180˚.

R: Why did you prefer this way? Could you solve the question in a different way?

S2: It could be solved differently, yet, when I first looked at this question, the first way was this. Because I thought that it might be the most practical way, namely, using a parallel.

R: According to you, what is proof? What does a solution need to be a proof?

S2: It must not cause hesitation in terms of plenty of aspects. That is, it must not create question mark in the minds of people so that it can be a proof. When I reframe the question, it must not lead to different result. I thougt my solution was proof since I had exact data. I believed that I proved through well proven knowledge. However, there are more than one solutions.

The student did not answer second and third question. He was asked whether he want to comment on these questions.

S2: I could not determine which method I should use. There are lots of ways but I realized that I do not completely know none of them. Because I learned them by rote and did not query. Then, when I learned, I think that I put them in short term memory so that I forgot quickly and could not remmember.

R: Let`s look at the next question.

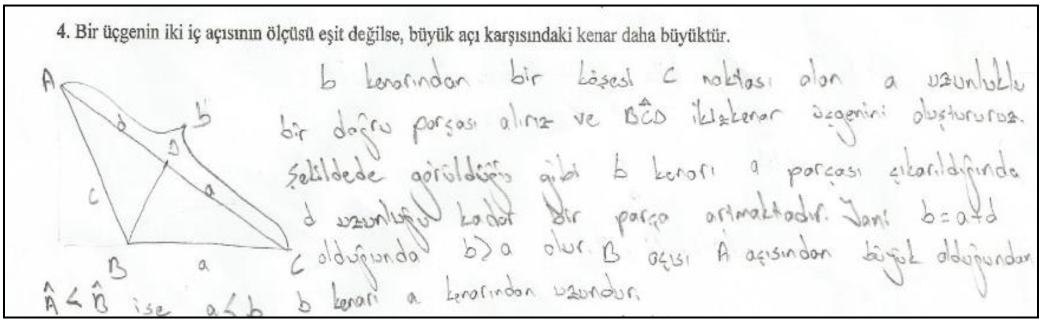

Figure 23 S2 The Solution Example for Forth Interview Question

S2: In this question, I do not believe that I produced a proof since I thought that I did not prove the accuracy of the statement. According to me, there were open points. I preferred to decompose the angle. Firstly, I constituted a triangle and showed that B angle was larger than A angle. Then, I did not do anything on C and I tried to form an isosceles triangle by drawing a line from B angle to AC edge. Morever, I previously said that there was the edge as a across A angle. Therefore, I formed the edge on AC line and named it as a. When I looked at the figure, b was a plus d. Thus, I saw that b edge was larger than a edge. In this direction, I decided that the edge across the bigger angle is larger than the edge across the smaller angle. However, I do not believe that it is a proof.

R: What kind of way did you follow to solve next question?

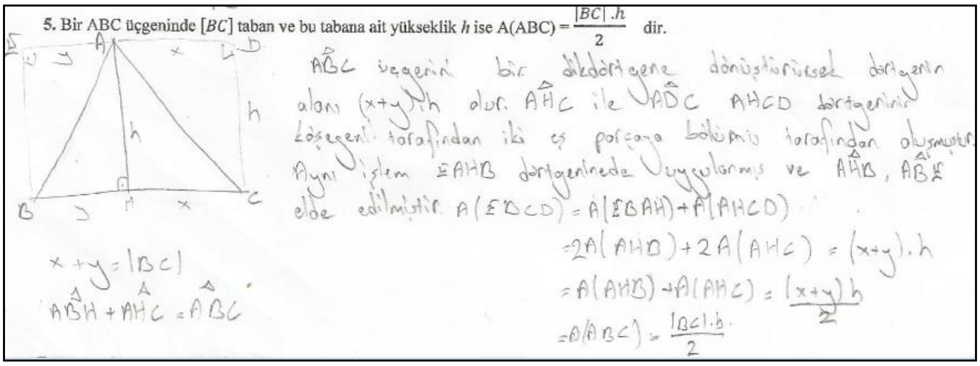

Figure 24 S2 The Solution Example for Fifth Interview Question

S2: I preferred to create a rectangle of which one side was h and the other side was BC. Then, I drew AH and two rectangles occurred. I drew the diagonals of these two rectangles. I already knew that the area of rectangle is the multiplication of its two sides and diagonal seperates a rectangle into two equal part. At this situation, I had four triangles. I reached the desired by summing up the areas of all triangles.

In first question, prospective teacher produced proof using the concepts parallelism, corresponding angle and alternate interior angle and explaning the followed steps with reasons. Besides it was seen that he was sure about his proof. Whereas the students could not solve second question, his explanations about third question actually reflect the current situation which is prevalent for most of prospective teachers. Due to the fact that students learn by rote, they have difficulties to remmember the knowledge and use it correctly. In forth question, the student formed an isosceles triangle in the big triangle through correct reasoning. He mathematically showed that b edge across B angle is larger than a edge across A angle. However, he only assumed that B angle is larger than A angle, expressed verbally and did not show mathematically. Since the student felt this deficiency in his proof, it is likely that he thought his proof was incorrect. In last question, he could logically conduct geometric proof using his knowledge about rectangle. The way he expressed of his proof show that he had positive thoughts on it. Although this prospective student had unattainable geometric proofs, it can be said that the student is conscious about proving and is able to transfer his current knowledge into proving and also show better performance than the other students.

The interview conducted with third student is as follows:

R: How did you think while solving the question?

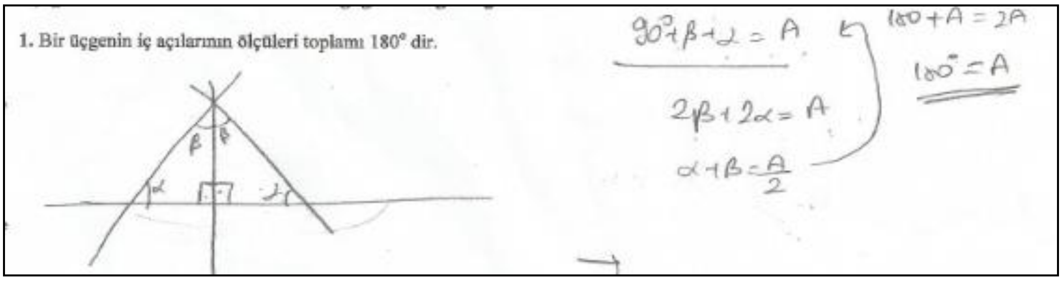

Figure 25 S3 The Solution Example for First Interview Question

S3: First, I drew a coordinate system and formed the symmetric angles of 90°, α and β on it. Then, I wrote the sum of the interior angles of right triangle as 90°+ α + β = A. For big triangle which includes two symmetric triangles in it, I wrote the sum of the interior angles as 2α + 2β = A.

R: Why did you equalize the sum of interior angles of both triangles to A?

S3: Because both were triangles. One was right triangle and the other was big symmetric triangle. Since the sum of the interior angles both triangles were equal, I benefited from equations here. In big triangle, I found α + β in terms of A. Then, I wrote this expression in the first equation and obtained the value of A. Therefore, I showed that the sum of the interior angles of a triangle is 180°.

R: Why did you prefer this solution way?

S3: It was easy for me. I though that I could prove sooner.

R: According to you, what is proof?

S3: Showing the trueness of the statement with examples.

R: What does a solution need to have to be accepted as proof?

S3: It must be logical, appropriate for operation and convincing. Also, when we find an incorrect example while proving, we see that proof is not valid. Therefore, it must not be falsifiable like that.

R: According to you, when is a proof convincing?

S3: As I said, if it is appropriate for operation and we can not show the contrary of the statement, it is convincing.

R: Ok, let's think that I gave lots of examples which show the accuracy of the statement not the contary of it. In this situation, can we say that I produced a proof?

S3: There is no one method to prove. Yet, until we find an example which show the contrary of the statement, it is correct.

Prospective teacher could not prove the second question. Then, looked at third question.

R: How did you think in third question?

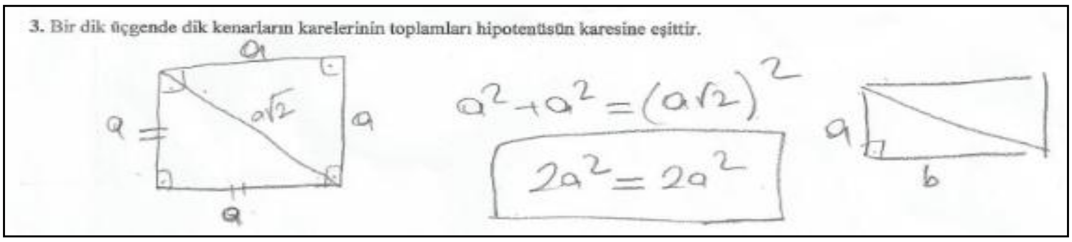

Figure 26 S3 The Solution Example for Third Interview Question

S3: Here, I previously tried to utulize square. I formed a square of which edge is a. Then, I drew the diagonal and I obtained two symmetrical right triangles of which both edges are equal and diagonal is √2 times of edge.

R: Could you explain your solution clearer?

S3: If we resolve the equation, is a^2+a^2 square of $a\sqrt{2}$? $2a^2$ will be equal to $2a^2$, therefore, we can obtained this based on equation.

R: What kind of solution way did you follow to prove forth question?

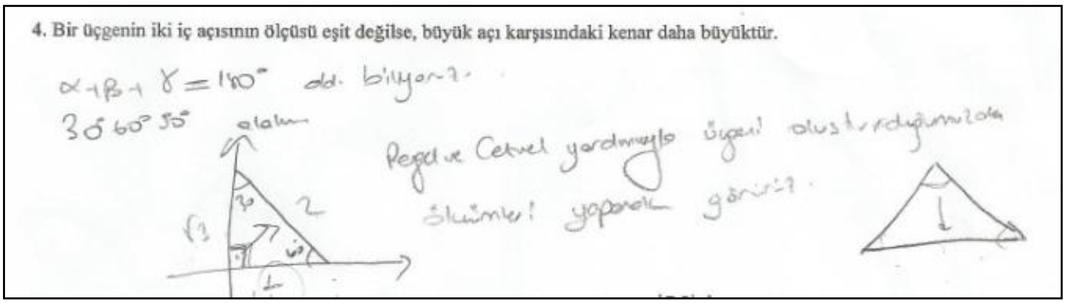

Figure 27 S3 The Solution Example for Forth Interview Question

S3: I said that we know the sum of three angles such as α, β, γ is 180°. Using the known again, I chose the angles as 30°, 60°, 90°. When we put the triangle on coordinate system, we are able to locate the angle of 90°directly. When we locate 30° and 60°angles through measurements with the help of ruler and compasses, triangle will occur. In this way, I thought we can see what the statement indicates.

R: How did you continue after that?

S3: I wrote 1 across the angle of 30° and √3 across the angle of 60°, if we consider it numerically, we can say it has a value between 1 and 2. There will be 2 across the angle of 90°. When we compare all of them, we can see that the edge across 90° is the biggest.

R: If you want to show whether the statement is correct or not to your student, what kind of way do you prefer?

S3: I draw any triangle. When we look at the angles in triangle we can understand which edge is bigger visually (student showed her opinion as drawing a triangle). Then, I show this to students using a triangle which I knew its angles and edges. I try to prove like this.

R: How did you think in fifth question?

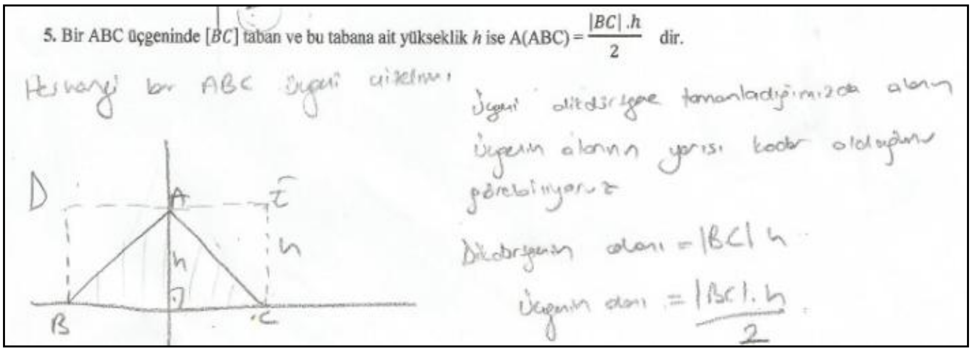

Figure 28 S3 The Solution Example for Fifth Interview Question

S3: I formed a triangle on the coordinate system as symmetrical. I named it as ABC and drew h. Then, I thought if I want to complete this triangle to form a rectangle since I already knew the area of rectangle. I drew lines upward as far as h from the points of B and C and linked them so that I constituted DEBC rectangle. I thought that triangle fills the half of the area of rectangle. I know that the area of rectangle will be BC times h. Because the area of triangle will be the half of the area of rectangle I divided it with two and I showed the accuracy of the statement.

In first question, prospective teacher attempted to prove based on mathematical equations. It was inferred that prospective teacher rely on proving based on examples and has an opinion that proving includes not showing the inaccuracy of the statement, only accuracy of it. The expressions of the student revealed that her sense of proof had some mistakes and deficiencies. Besides the student tends to prefer the way which is easy and practical in terms of herself. In second question, prospective teacher crosschecked the statement controlling whether both sides of the equation are equal or not rather than proving. In forth question, student tried to show the accuracy of the statement with only one example using a private triangle which she knew the angles and edges of it. She could not prove but believed that she showed the accurcy of it. Also, prospective teacher believe that visual structure without any mathematical expression could be used for comparing and proving. This situation shows that the sense of that mathematical decision cannot be reached by looking at only a drawing was not internalized by the student. For only last question, prospective teacher was able to present logical explanations.

The interview data obtained show that prospective teachers have difficulties in producing geometric proofs and various defciencies in sense of proof. They are not also capable of using their current knowledge for proving.

Discussion, Conclusion and Suggestions

The written data obtained shows that the prospective teachers produce more valid proofs in the questions which they are familiar including to prove the sum of interior angles of a triangle, area of it and Pythagorean Theorem. At this point, it can be said that the students give better answers in statements involving proofs which they made it practice in lessons as against which they did not. It shows that there is a direct proportion between experiences towards proving and the abilities of proving. However, the students both begin to have these kinds of experiences in late periods and get less chance to experience (Knapp, 2005; Moore, 1994; NCTM, 2000; Stylianides, 2007; Stylianou, Blanton & Knuth, 2009; Usiskin, 1987). On the other hand, although the number of valid proof towards Pythagorean Theorem is larger than the number of valid proofs in which questions they are not familiar, it is lower than expected level. In general, students uncomprehendingly use a demonstration based on the areas of three squares as they learned in lessons for proving of this theorem or they try to prove through the special triangle of 3-4-5. In addition, although they utilize Pythagorean Theorem so as to solve many kind of questions in geometry, a considerable amount of them are not able to put forward an idea about proving of this theorem. It is found that students mainly produce invalid proofs in the questions which they are not familiar with their proofs from lessons. Riley (2004) also finds that the number of prospective teachers who are able to produce valid proofs is low. This situation indicates that activities which aim to internalize the concept of geometric proof of students and to develop their proving skills must be more included in class (DeGroot, 2001; Stylianides & Ball, 2008). Therefore, students become familiar with geometric proving, feel more comfortable while proving and better comprehend the concept of proof (Ball & Bass, 2003; Carpenter, Franke, & Levi, 2003; Lampert, 1990; Maher & Martino, 1996; Reid, 2002; Stylianides, 2007; Wood, 1999; Yackel & Cobb, 1996; Zack, 1997).

In general, the written data related to proving shows that geometric proving skills of prospective teachers are weak (Healy & Hoyles, 2000; Kahan, 1999; Senk, 1985), they have difficulties in proving (Chazan 1993a; Harel & Sowder 2007; Hart, 1994; Healy & Hoyles, 2000; Martin & Harel 1989; Riley, 2004; Senk, 1985), they have misconceptions like that only an example which shows accury of the statement or a numeric demonstration is enough for proving (Goulding, Rowland, & Barber, 2002; Knuth, 2002; Ma, 1999; Martin & Harel, 1989; Morris, 2002; Simon & Blume, 1996) and they are not able to transfer their current knowledge into the process of proving.

The interview data obtained reveals that prospective teachers are tend to prove using solution ways which are simple, concrete and practical in terms of them and which they better make sense. According to Chen (2008), due to the fact that informal demonstrations are easier and more flexible than formal proofs, students generally prefer these kind of solution ways. The most general opinions related to the concept of proof of students are further based on demonstration of the statement certainly and logically (Goulding, Rowland & Barber, 2002; Knuth, 2002; Martin & Harel, 1989; Morris, 2002). Besides it is concluded that students do not need to prove the statement since they believe that its accuracy seems very clear (Harel & Sowder, 1998), their awareness about whether their own demonstrations are proofs is changeable (Chazan, 1993b), they attempt to prove through verbal expressions, they believe that they can prove a statement with a few examples (Weber, 2001) and desire towards proving is low since proving is hard for them.

In general, the interview data related to proving shows that prospective teachers do not know how to use their current knowledge (Weber, 2001), therefore, have difficulties in understanding and producing proofs (Güven, Çelik & Karataş, 2005; Harel & Sowder, 1998; Jones, 2000; Martin & Harel, 1989; Moore, 1994), they are not able to decide from where to start proving (Sarı, Altun, Aşkar, & 2007; Weber, 2001) and have difficulties in remembering knowledge later because of memorization (Condradie & Firth, 2000; Galindo, 1998).

When prosective teachers' preceptions, comprehensions, skills and difficulties regarding proof are considered, their awareness and abilities in proving may be enriched through giving trainings towards proving and increasing these trainings. Thus, an opportunity which they may utilize when they become teachers can be provided (Herbst, 2002; Mariotti, 2000). If we take in consideration that they are not capable of proving the most essential statements, a sequence from basic to complicated proof can be followed. Because it is difficult to transfer into different situations without understanding the reasons behind the foundation statements, it is known that the habit of memorization occurs. Besides students may be asked to describe what kind of solutions are proofs and what kind of solutions are not, they can be made to understand this difference through various demonstrations and the elements which constitute proof can be emphasized in lessons (Yackel & Hanna, 2003).

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İlköğretim Matematik Öğretmen Adaylarının Üçgenlerin Öğretiminde Geometrik İspatları Kullanabilme Becerileri

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Özet – Bu çalışmada ilköğretim matematik öğretmen adaylarının üçgenlerin öğretiminde geometrik ispatları kullanabilme becerilerini ortaya koymak amaçlanmışır. Nitel araştırma teknikleri temel alınarak desenlenen bu araştırmanın verileri 2013-2014 eğitim-öğretim bahar yarıyılında bir devlet üniversitesinin İlköğretim Matematik Öğretmenliği Programı' nda öğrenim gören üçüncü sınıf öğrencilerinden elde edilmiştir. Veri toplama aracı olarak araştırmacılar tarafından hazırlanan, uzman görüşleri alınarak son şekli verilen ispata yönelik 5 tane açık uçlu soru kullanılmış ve veriler içerik analizi ile çözümlenmiştir. Elde edilen sonuçlar öğretmen adaylarının ispat yapabilme becerilerinin düşük olduğunu göstermektedir. Gönüllü olan 3 öğretmen adayı ile yaptıkları ispatları daha detaylı biçimde incelemek amacıyla yarı yapılandırılmış mülakatlar yapılmıştır. Elde edilen mülakat verileri, öğretmen adaylarının genellikle basit, somut ve kendileri açısından pratik olan çözüm yollarını kullanarak ispat yapmaya çalıştıklarını göstermiştir.

Anahtar kelimeler: Geometrik ispat, üçgenler, öğretmen adayları.

Özet

Amaç

Bu çalışmanın amacı ilköğretim matematik öğretmen adaylarının üçgenlerin öğretiminde geometrik ispatları kullanabilme becerilerini ve ispat yapma süreçlerini ortaya koymaktır.

Yöntem

Bu çalışmada nitel araştırma desenlerinden örnek olay yöntemi kullanılmıştır. Araştırma verileri 2013-2014 eğitim-öğretim bahar yarıyılında bir devlet üniversitesinin

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İlköğretim Matematik Öğretmenliği Programının üçüncü sınıfında öğrenim gören toplam 86 öğrenciden elde edilmiştir. Araştırmacılar tarafından geliştirilen ve uzman görüşleri alınarak son şekli verilen ispata yönelik 5 tane açık uçlu soru sorulmuştur. Veriler toplandıktan sonra gönüllü olan 3 öğretmen adayı ise yarı yapılandırılmış mülakat çalışma grubunu oluşturmuştur. Çalışmada açık uçlu sorulardan elde edilen veriler içerik analizi yöntemi kullanılarak analiz edilmiştir.

Bulgular

Üçgenler konusunda ispat yapmaya yönelik sorulan 5 açık uçlu soruya verilen cevaplar incelenmiş ve bunlar geçerli ispat, eksik ispat, geçersiz ispat, matematiksel ifade ve boş olmak üzere 5 kategori altında toplanmıştır. Birinci soruda, öğrencilerin %58' i geçerli ispat, %6' sı geçersiz ispat, %30' u eksik ispat ve %6' sı matematiksel ifade şeklinde ispat yapmıştır. İkinci soruda, çözümlerden %33' ü geçersiz ispat, %10' u eksik ispat, %14' ü matematiksel ifade ve %41' i boş ispat kategorisinde yer almaktadır. Üçüncü soru incelendiğinde, öğrencilerin %23' ünün geçerli ispat, %5' inin eksik ispat, %20' sinin geçersiz ispat, %14' ünün matematiksel ifade kullandıkları ve %38' inin ise soruyu boş bıraktıkları görülmüştür. Dördüncü soruda, öğrencilerin sadece %1' i geçerli ispat, %27' si geçersiz ispat ve %6' sı eksik ispat yaparken, %37' si matematiksel ifade sunmuş, %29' u ise soruyu boş bırakmıştır. Beşinci soruda öğrencilerin %52' si geçerli ispat, %2' si geçersiz ispat, %1' i eksik ispat yaparken, %45' i ifadenin arkasındaki gerekçeyi yansıtamamış sadece matematiksel dil olarak ifade edebilmişlerdir.

Tartışma ve Sonuç

Genel olarak ispata yönelik yazılı veriler, öğretmen adaylarının geometrik ispat yapma becerilerinin zayıf olduğunu, ispat yapmakta zorlandıklarını, doğruluğu gösteren tek bir örneğin ya da sayısal gösterimin ispat için yeterli olduğuna yönelik yanlış algılarının bulunduğunu ve mevcut bilgilerini ispat sürecine aktaramadıklarını göstermektedir. Elde edilen mülakat verilerinde, öğretmen adaylarının öncelikle kendilerinin iyi bir şekilde anlamlandırdıkları, genellikle basit, somut ve kendileri açısından pratik olan çözüm yollarını kullanarak ispat yapmaya çalıştıkları sonucuna ulaşılmıştır. Öğrencilerin, verilen bilgilerin doğruluğu kendilerine çok açık geldiği için ispatlama ihtiyacı duymadıkları, yaptıklarının ispat olup olmadığına yönelik farkındalıklarının değişkenlik gösterdiği, ispatı sözel ifadelerle yapmaya çalıştıkları, birkaç örnekle ispatın yapılabilir olduğuna inandıkları ve kendileri ispat yapmakta zorlandıkları için ispat yapma isteklerinin düşük olduğu sonucuna varılmıştır.