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Assessing the Resilience of Turkey's Economy during the Covid-19 Pandemic with its 2050 Projections

Kovid-19 Pandemisi Sırasında Türkiye Ekonomisinin Dayanıklılığını 2050 Öngörülerıyla Değerlendirmek

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ÖZ

Bu makale, Türkiye'nin 2050 reel GSYİH tahminlerini, pandemi öncesinde 2019'un sonuna kadar ve pandemi sırasında (2021'in sonuna kadar) tarihsel veriler ile spektral analiz kullanarak sunmaktadır. Makalenin amacı, 2020, 2021'de küresel ekonomiyi vuran ve 2022'ye yayılan Covid-19 pandemiden kaynaklanan ekonomik şokun ardından Türkiye ekonomisinin direncini değerlendirmektir. Türkiye'nin 2050 tahminleri, Euro bölgesi (19 ülke)'nin 2050 GSYİH tahminleriyle kıyaslanmaktadır. Üç aylık Reel GSYİH büyüme oranları zaman serileri, tek boyutlu ayrık dalgacık analizi çerçevesinde, tahminler ve ayrıntılar olarak bilinen basit sinyallere ayrıştırılır. Sonuçlar, 2019'un ikinci çeyreği ile 2021'in dördüncü çeyreği arasında, Euro bölgesi ekonomisinin (19 ülke) üç aylık Reel GSYİH büyüme oranının (yıllıklandırılmış) çoğu zaman Türkiye'nin altında olduğunu ve Euro için ortalama %0,94 büyüme oranı olduğunu gösteriyor. Türkiye için bu oran %6,57'dir. Bu nedenle Türkiye, Covid-19 salgını sırasında Euro bölgesine göre daha dirençliydi. Sonuç olarak, yazarlar Türk ekonomik ve siyasi liderlerini son ekonomik gelişmeler ışığında Avrupa Birliği'ne katılmayı yeniden düşünmeye davet etmektedir.

ABSTRACT

This paper presents Turkey's 2050 real GDP forecasts using historical data before the pandemic up to the end of 2019 and during the pandemic (up to the end of 2021) using spectral analysis. The objective of the paper is to assess the resilience of Turkey's economy following the economic shock from the Covid-19 pandemic that hit the global economy in 2020, 2021 and extending in 2022. Turkey's 2050 forecasts are benchmarked to the 2050 GDP forecasts of the Euro Area (19 countries). The quarterly Real GDPs growth rates time series are decomposed into simple signals known as approximations and details within the framework of the one-dimensional discrete wavelet analysis. The results show that between Q2 2019 and Q4 2021, the quarterly Real GDP growth rate (annualized) of the Euro area economy (19 countries) was most of the time below the one of Turkey with an average growth rate of 0.94% for the Euro area versus 6.57% for Turkey. Therefore, Turkey was more resilient during the Covid-19 pandemic than the Euro area. In conclusion, the authors invite Turkish economic and political leaders to reconsider joining the European Union in the light of recent economic developments.

1. Introduction

This paper presents Turkey's 2050 real GDP forecasts

before (up to 2019) and during the pandemic (up to 2021) by using spectral analysis. The objective of the paper is to assess the resilience of Turkey's economy during the Covid-

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19 pandemic with its 2050 projections. These projections are benchmarked to the 2050 GDP forecasts of the Euro Area (19 countries). The Euro Area or Eurozone is the monetary union of 19 out of 28 European Union member states, all of which have adopted the Euro as their common currency and sole legal tender. The monetary authority of the Eurozone is the Eurosystem. Eurozone members are Austria, Belgium, Finland, France, Germany, Ireland, Italy, Luxembourg, Netherlands, Portugal, Spain, Greece, Slovenia, Cyprus, Malta, Slovakia, Estonia, Latvia, and Lithuania. The other nine members of the European Union continue to use their own national currencies, although the majority of them have undertaken to adopt the Euro in the future.

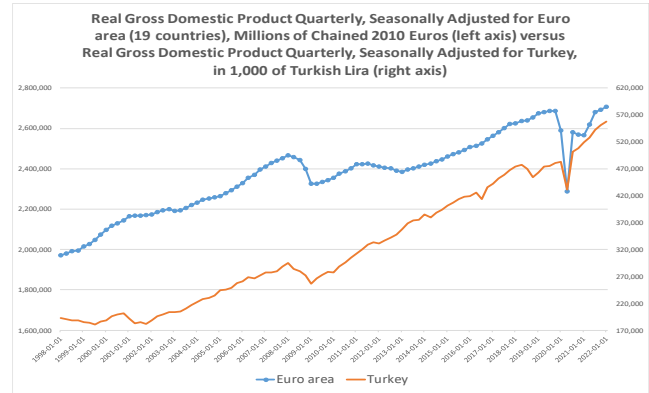
The assumption in this research is that real GDPs propagate through time in waveforms. Wavelet analysis capture the dynamics of these waves. 'Wavelet analysis expands functions in terms of wavelets generated in the form of translations and dilations of a fixed function called the mother wavelet. The resulting wavelets have special scaling properties, localized in time and frequency, permitting a closer connection between the represented function and their coefficients. Greater numerical stability in reconstruction and manipulation is ensured' (Lee and Yamamoto, 1994, p. 44). Extending the analysis to the complex-behavior of economic signals, the originality of this paper lies in the application of wavelet analysis to economic variables subject to common dynamics such as real GDP time series. Rostan and Rostan have previously applied wavelet analysis to the forecasts of fossil fuel prices (2021a) and to the forecasts of the Spanish (2018c), Greek (2018d), Austrian (2020), Saudi Arabian (2021b) and Persian Gulf (2022) economies. This paper is an extension of their work.

On the COVID-19 virus pandemic issue, present since late 2019 and having spread to the five continents in 2020, killing people by the millions and plunging the world economy into severe recession, this unexpected and dramatic event has forced governments to introduce unprecedented measures such as lockdowns of populations to contain its spread. By August 5, 2022, the recorded number of Coronavirus Cases in the world was 587,110,053 people with a Coronavirus death toll of 6,431,557 (Worldometers, 2022). The lockdowns have paralyzed economies across the five continents, shutting down factories and bringing manufacturing to a halt, with service sectors contracting on a massive scale, forcing millions of workers to leave the labor force. Globally, the economic activity has contracted at a rapid pace and put economies into recession.

Figure 1 illustrates the historical quarterly Real GDP time series of the Eurozone economy (19 countries) and Turkey from 1/1/1998 to 1/1/2022. It shows two almost identical patterns at the start of 2020 when the two economies have entered into recession following the economic shock from the Covid-19 pandemic that hit the global economy. Since 1998, the Real GDP of Turkey seems to have grown at a more rapid pace with a steeper slope than the Real GDP of

the Eurozone economy between 1998 and 2020, the two economies seem to have dived at an identical rate in 2020 and Turkey seems to have recovered at a more rapid pace in 2021 with a steeper slope.

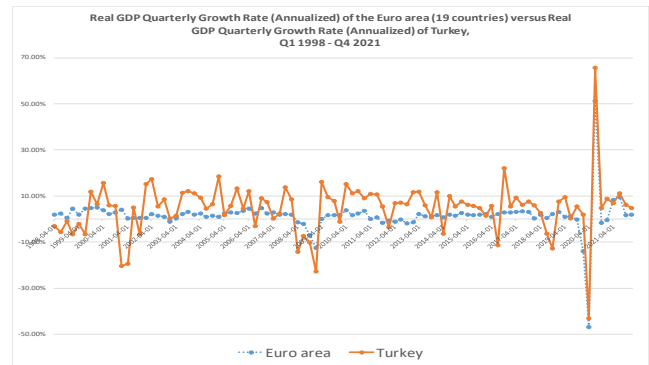
Figure 1: Quarterly Real GDPs time series of the Eurozone economy (19 countries) and Turkey from 1/1/1998 to 1/1/2022



Sources: Authors' own elaboration. Real Gross Domestic Product for Euro area (19 countries) [https://fred.stlouisfed.org/series/CLVMNACSCAB1GQEA19] and Turkey [https://fred.stlouisfed.org/series/NGDPRSAXDCTRQ], retrieved from FRED, Federal Reserve Bank of St. Louis, August 5, 2022.

Figure 2 illustrates the historical quarterly Real GDP growth rate (annualized) time series of the Eurozone economy (19 countries) and Turkey from 1/1/1998 to 1/1/2022.

Figure 2: Quarterly Real GDP growth rate (annualized) time series of the Eurozone economy (19 countries) and Turkey from Q1 1998 to Q4 2021

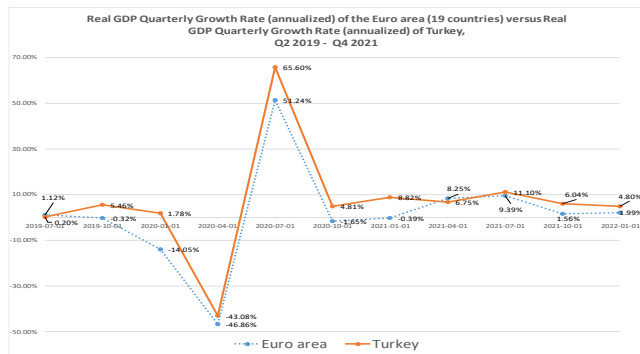


Sources: Authors' own elaboration. Real Gross Domestic Product for Euro area (19 countries) [https://fred.stlouisfed.org/series/CLVMNACSCAB1GQEA19] and Turkey [https://fred.stlouisfed.org/series/NGDPRSAXDCTRQ], retrieved from FRED, Federal Reserve Bank of St. Louis, August 5, 2022.

As illustrated in Figure 2, between Q1 1998 and Q3 2019, the quarterly Real GDP growth rate (annualized) of the Euro area economy (19 countries) was most of the time below the one of Turkey with an average growth rate of 1.43% for the Euro area versus 4.33% for Turkey.

Figure 3 is a zoom of Figure 2 between Q3 2019 and Q4 2021. It illustrates the historical quarterly Real GDP growth rate (annualized) time series of the Eurozone economy (19 countries) and Turkey from Q2 2019 to Q4 2021.

Figure 3: Quarterly Real GDP growth rate (annualized) time series of the Eurozone economy (19 countries) and Turkey from Q2 2019 to Q4 2021.



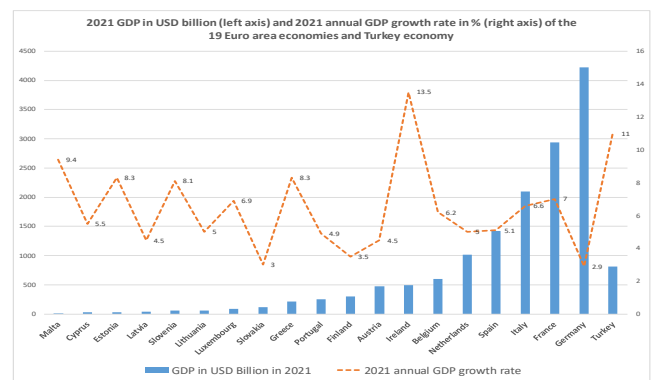
Sources: Authors' own elaboration. Real Gross Domestic Product for Euro area (19 countries) [https://fred.stlouisfed.org/series/CLVMNACSCAB1GQEA19] and Turkey [https://fred.stlouisfed.org/series/NGDPRSAXDCTRQ], retrieved from FRED, Federal Reserve Bank of St. Louis, August 5, 2022.

As illustrated in Figure 3, between Q2 2019 (2019-07-01) and Q4 2021 (2022-01-01), the quarterly Real GDP growth rate (annualized) of the Euro area economy (19 countries) was most of the time below the one of Turkey with an average growth rate of 0.94% for the Euro area versus 6.57% for Turkey. It confirms the historical relationship between the 2 economies observed between 1998 and 2019.

In conclusion, the 2050 projections of these 2 economies are expected to respect this relationship, displaying a stronger and more resilient Turkish economy than the Euro area economy in terms of Real GDP growth rate. However, we need to keep in mind that the size of the Euro area economy (19 countries) was about 18 times the size of the Turkish economy in 2021. In the Euro area, the growth of its economies varies widely. For example, based on Statista (2022) data, the top 5 economies based on the GDP annual growth rate in 2021 (year to year) were Ireland (+13.5%), Malta (+9.4%), Greece (+8.3%), Estonia (+8.3%) and Slovenia (+8.1%) when the 5 laggards were Germany (+2.9%), Slovakia (+3%), Finland (+3.5%), Austria (+3.5%) and Latvia (+4.5%). The average annual GDP growth rate for the 19 economies of the Euro area was 6.22% in 2021. The annual GDP growth rate for Turkey was 11% in 2021 (Turkstat Corporate, 2022). It is interesting to mention that Turkey's economy comforts its situation of leader by almost doubling up the Euro area annual growth rate in 2021. Besides, as illustrated in Figure 4, the largest economy of the Euro area, Germany, was the worst performer in 2021 with +2.9% of annual growth rate. The smallest economy of the Euro area, Malta, was the second top performer in terms of 2021 annual growth rate (+9.4%). The 13 smallest

economies from Malta to Ireland ranked in increasing GDP had an average 2021 annual growth rate of 6.56%, when the 6 top largest economies from Belgium to Germany had an average 2021 annual growth rate of 5.46%. The correlation coefficient between the size of the 2021 GDP and the growth rate of the 19 countries is -20%. It is not a strong relationship between the two variables but it is still negative and it shows that small economies have copped better with the Covid19 pandemic crisis than large economies of the Euro area. However, this rule does not apply to Turkey, since the size of its economy (815 USD billions) is almost equal to the average of Belgium and Netherlands economies (809 USD billions) when the average of their 2021 annual growth rates was only +5.6% or half the growth of Turkey's economy (+11%).

Figure 4: 2021 GDP in USD billion (left axis) and 2021 annual GDP growth rate in % (right axis) of the 19 Euro area economies and Turkey's economy



Sources: Authors' own elaboration. Annual gross domestic product growth rate forecast in selected European countries in 2021 from https://www.statista.com/statistics/686147/gdp-growth-europe/ Gross Domestic Product (GDP) in 2021 from https://data.tuik.gov.tr/Bulten/Index?p=Quarterly-Gross-Domestic-Product-Quarter-IV:-October-December,-2021-45548&dil=2

GDP | Europe in USD Billion in 2021 from https://tradingeconomics.com/country-list/gdp?continent=europe

This paper presents Turkey versus Euro area 2050 real GDP forecasts before (up to 2019) and during the pandemic (up to 2021) by using wavelets analysis. The following section will discuss the meaning of wavelets in signal processing and explore the ways signal processing has been applied in the literature.

2. Literature Review

2.1. Spectral analysis versus traditional economic forecasting methods

Traditional economic forecasting methods include causal methods (regression analysis, logit, probit), time series methods (moving average, exponential smoothing, trend and seasonal decomposition, Box-Jenkins ARIMA used as a

benchmark in this paper) and qualitative methods (Delphi Method, Jury of Executive Opinion, Sales Force Composite, Consumer Market Survey) (FHI, 2019). Signal processing used in this paper to forecast the Eurozone's real GDPs belongs to time series methods. Signal processing, a field of physics, focuses on the analysis, synthesis, and modification of signals. The basic assumption of this paper is that economic time series behave like signals propagating through time instead of propagating through space as do the phenomena studied by physics such as audio, video, speech, geophysical, sonar, radar, medical and musical signals (IEEE, 2019). Wavelet analysis is a tool of signal processing. In physics, wavelets assume the practical applications of modeling physical phenomena such as electrical, audio or seismic signals which propagate through space in waveforms. Wavelets have specific properties that mimic signals, which makes them useful for signal processing. Signal processing focuses on the analysis, synthesis, and modification of signals. Spectral (or spectrum) analysis focuses on the data analysis of signals. More specifically (Stoica and Moses, 2005), from a finite record of a stationary data sequence, spectral analysis estimates how the total power is distributed over frequency. In meteorology, astronomy and other fields, spectral analysis may reveal 'hidden periodicities' in data, which are to be associated with cyclic behavior or recurring processes.

Regarding wavelet analysis, forecasters have focused on the Discrete Wavelet Transform (DWT, explained at step three of the methodology), directing attention to several non-tractable properties of continuous wavelet transform (CWT) such as highly redundant wavelet coefficients (Valens, 1999), the infinite number of wavelets in the wavelet transform and the absence of analytical solutions for many functions of the wavelet transforms. A wavelet-based forecasting method using the redundant "à trous" wavelet transform and multiple resolution signal decomposition was presented in Renaud et al. (2002). Challenges involved in forecasting day-ahead electricity prices based on the wavelet transform and ARIMA models has been detailed in Conejo et al. (2005). Schlüter and Deuschle (2010), capturing seasonalities with time-varying period and intensity, incorporated the wavelet transform to improve forecasting methods. Tan et al. (2010) proposed a price forecasting method based on wavelet transform combined with ARIMA and GARCH models. Kao et al. (2013) integrated wavelet transform, multivariate adaptive regression splines (MARS), and support vector regression (SVR called Wavelet-MARS-SVR) to address the problem of wavelet sub-series selection and to improve forecast accuracy. Ortega and Khashanah (2013) proposed a wavelet neural network model for the short-term forecast of stock returns from high-frequency financial data. Kriechbaumer et al. (2014) showed the cyclical behavior of metal prices using wavelet analysis to capture the cyclicity by decomposing a time series into its frequency and time domain. They presented a wavelet-autoregressive integrated moving average (ARIMA) approach for forecasting monthly prices of aluminum,

copper, lead and zinc. He et al. (2014) proposed an entropy optimized wavelet-based forecasting algorithm to forecast the exchange rate movement. Berger (2016) transformed financial return series into its frequency and time domain via wavelet decomposition to separate short-run noise from long-run trends and assess the relevance of each frequency to value-at-risk (VaR) forecast. Rostan and Rostan (2018a) illustrated the versatility of wavelet analysis to the forecast of financial time series with distinctive properties. Choosing two market indices with divergent properties of their time series—the S&P 500 Composite Index being nonstationary and the VIX (volatility) index being stationary—they proved that using wavelet analysis combined with the Burg model offers high accuracy in terms of forecasts of their time series, thus demonstrating the versatility of this model. Rostan et al. (2015) appraised the financial sustainability of the Spanish pension system, and Rostan and Rostan (2018b) did the same regarding the Saudi pension system using spectral analysis. With a refined methodology using multiscale principal component analysis to take into account the co-dynamics of age groups, Rostan and Rostan (2017) forecasted European and Asian populations with signal processing which resulted in original outcomes that might be contrasted with those of the more conformist population projections of the United Nations. In addition, Rostan and Rostan (2019) identified when the European Muslim population will become a majority in Europe. Rostan and Rostan applied wavelet analysis to the forecasts of Spanish (2018c), Greek (2018d), Saudi (2020b), Austrian (2020c) and Persian Gulf (2022) economies.

2.2. Assessing the resilience of the Turkish economy during the Covid-19 pandemic

As mentioned by the World Bank (2021), 'COVID-19 (coronavirus) has taken a heavy toll on Turkey. Turkey's economic growth performance in 2020 was strong compared to other countries, but poverty spread, and unemployment became more predominant. Turkey's GDP grew by 1.8 percent in 2020, which was in 2020 the fastest amongst G20 countries, aside from China. This was on the back of one of the strongest economic rebounds in the second half of 2020, largely achieved in Q3, but with positive growth continuing into Q4. Despite measures to support labour market jobs, households and poverty, the situation nonetheless deteriorated. Indicators of domestic demand showed some signs of economic growth cooling early in 2021, although supply side measures such as industrial output remained robust. Turkey responded to COVID-19 with a large economic stimulus program, focused on credit channels. In fiscal terms, Turkey's COVID-19 stimulus package amounted to nearly 12 percent of GDP when including tax deferrals and contingent liabilities. This is larger than the average for emerging market, G20 countries, and is similar in size to the stimulus packages of the United States, Australia and Canada.'

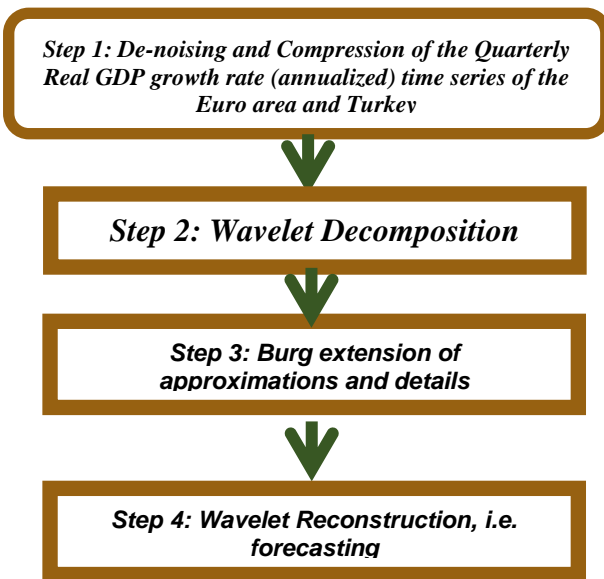
Çakmaklı et al. (2021) confirmed that the fiscal stimulus adopted in Turkey has been partially responsible for the economic crisis that hit Turkey and especially the lower income groups. ‘While the expansionary monetary policy that was adopted at the onset of the pandemic was the proper policy to support the economy, the risks of excessive credit growth should have been better evaluated and a larger portion of the money that was injected into the economy could have been used in the form of direct transfer payments to society. Çakmaklı et al. noted that monetary policy operates through monetary injections with no distributive power. It is the fiscal policy that is responsible for the allocation of the stimulus package to the neediest agents. Given the asymmetric nature of the crisis that hit the lower income groups the hardest, transfer payments were critical to provide much-needed income relief and revive demand.’

Section 3 presents the methodology of the paper. Section 4 gathers the results and section 5 concludes.

3. Methodology

The objective of the paper is to assess using spectral analysis the resilience of Turkey’s economy following the economic shock from the Covid-19 pandemic that hit the global economy in 2020, 2021 and extending in 2022.

Figure 5: Flowchart of the methodology from step 1 to 4.



Source: Authors’ own elaboration.

3.1. Step 1: De-noising and Compression of the Quarterly Real GDP growth rate (annualized) time series of the Euro area and Turkey

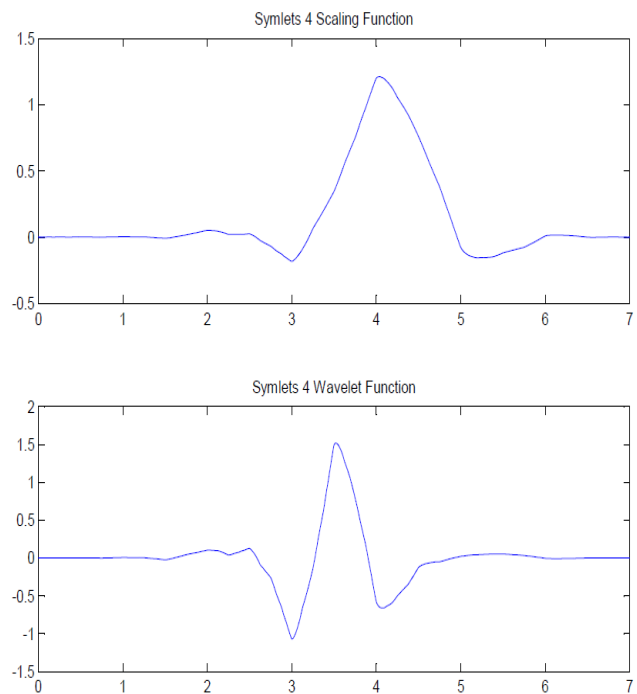
Each series is de-noised using a one-dimensional de-noising and compression-oriented function using wavelets. The function is called ‘wdencomp’ in Matlab (Misiti et al., 2015). The underlying model for the noisy signal is of the form:

$$s(n) = f(n) + \sigma e(n) \tag{1}$$

where time point n is equally spaced, $e(n)$ is a Gaussian white noise $N(0,1)$ and the noise level σ is supposed to be equal to 1. The de-noising objective is to suppress the noise part of the signal s and to recover f . The de-noising procedure proceeds in three steps: 1) Decomposition. Choose the wavelet sym4, and choose the level 2-decomposition. Wavelet analysis breaks a signal down into its constituent parts for analysis, in this case with a level 2-decomposition. The decomposition method is explained in section 3.2, step 2-Wavelet Decomposition.

Wavelet analysis is the breaking down of a signal into shifted and scaled versions of the original mother wavelet. Sym4 is a Symlets wavelet of order 4 used as the mother wavelet for decomposition and reconstruction. It is a nearly symmetrical wavelet belonging to the family of Symlets proposed by Daubechies (1994). The scaling and wavelet functions of Symlets 4 are illustrated in Figures 6. Wavelets are defined by the wavelet function, also naming the mother wavelet and the scaling function, the latter also named the father wavelet in the time domain. The wavelet function is in effect a band-pass filter and scaling that for each level halves its bandwidth (Mallat, 2009).

Figures 6: Scaling Function and Wavelet Function of Symlets 4



Source: Authors’ own elaboration using Matlab

Wavelets are mathematical functions that cut up data into different frequency components and then study each component with a resolution matched to its scale (Graps, 1995). We compute the wavelet decomposition of the signal s at level 2. 2) Detail coefficients thresholding. For each

level from 1 to 2, we select a threshold and apply soft thresholding to the detail coefficients. 3) Reconstruction. We compute wavelet reconstruction based on the original approximation coefficients of level 2 and the modified detail coefficients of levels from 1 to 2.

Like de-noising, the compression procedure contains three steps: 1) Decomposition. 2) Detail coefficient thresholding. For each level from 1 to 2, a threshold is selected and hard thresholding is applied to the detail coefficients. 3) Reconstruction. The difference with the de-noising procedure is found in step 2. The notion behind compression is based on the concept that the regular signal component can be accurately approximated using a small number of approximation coefficients (at a suitably selected level) and some of the detail coefficients.

The de-noising technique works in the following way: ‘When a data set using wavelets is decomposed, filters act as averaging filters and others that produce details. Some of the resulting wavelet coefficients correspond to details in the data set. If the details are small, they might be omitted without substantially affecting the main features of the data set. The idea of thresholding, then, is to set to zero all coefficients that are less than a particular threshold. These coefficients are used in an inverse wavelet transformation to reconstruct the data set’ (Graps, 1995, p.12).

3.2. Wavelet Decomposition

Wavelet analysis breaks a signal down into its constituent parts for analysis. Signals are decomposed after being differentiated, de-noised and compressed at step 2. The signals, i.e., the quarterly time series of Turkey and the Euro Area GDPs, are decomposed into decomposed signals cAs named approximations and cDs named details. To understand this process, a quick review of wavelet theory is presented.

A wavelet dictionary (Mallat, 1999) is constructed from a mother wavelet ψ of zero mean:

$$\int_{-\infty}^{+\infty} \psi(t) dt = 0 \tag{2}$$

To analyze a non-stationary signal, wavelet analysis identifies the correlation between the time and frequency domains of this signal (Wavelet.org, 2019). The wavelet transform allows exceptional localization in both the time domain via translations of the mother wavelet, and in the scale domain, also called frequency domain via dilations. The translation and dilation operations applied to the mother wavelet are performed to calculate the wavelet coefficients, which represent the correlation between the wavelet and a localized section of the signal. The wavelet coefficients are calculated for each wavelet segment, giving a time-scale function relating the wavelet correlation to the signal.

The mother wavelet ψ represented by equation 2 is dilated with a scale parameter b , and translated by a :

$$D = \left\{ \psi_{a,b}(t) = \frac{1}{\sqrt{b}} \psi\left(\frac{t-a}{b}\right) \right\}_{a \in \mathbb{R}, b > 0} \tag{3}$$

The present methodology uses Sym4, symlets wavelet of order 4, as the mother wavelet ψ for decomposition and reconstruction. It is a nearly symmetrical wavelet belonging to the family of Symlets proposed by Daubechies (1994) and illustrated in Figures 6. We tested many other wavelets including the ones belonging to the Daubechies family with equal or lower performance.

The discrete form of the wavelet (Mallat, 1999) is defined as:

$$\psi_{j,n}(t) = \frac{1}{\sqrt{s_0^j}} \psi\left(\frac{t - n\tau_0 s_0^j}{s_0^j}\right) \tag{4}$$

with j and n integers, $s_0 > 1$ is a fixed dilation step and the translation factor τ_0 depends on the dilation step.

The continuous wavelet transform of a signal s at any scale b and position a is the projection of s on the corresponding wavelet atom:

$$Ws(a,b) = \langle s, \psi_{a,b} \rangle = \int_{-\infty}^{+\infty} s(t) \frac{1}{\sqrt{b}} \psi\left(\frac{t-a}{b}\right) dt \tag{5}$$

The reconstruction of the original signal $s(t)$ is obtained by inverse wavelet transform (Mallat, 1999, p.111):

$$s(t) = \frac{1}{C_\psi} \int_0^{+\infty} \int_{-\infty}^{+\infty} ws(a,b) \psi_b(t-a) \frac{db}{b^2} da \tag{6}$$

The scaling function and the wavelet function of a discrete wavelet transform (DWT) are defined as:

$$\phi(2^j t) = \sum_{i=1}^n h_{j+1}(i) \phi(2^{j+1} t - i) \tag{7}$$

$$\psi(2^j t) = \sum_{i=1}^n g_{j+1}(i) \phi(2^{j+1} t - i) \tag{8}$$

The signal $s(t)$ is expressed as:

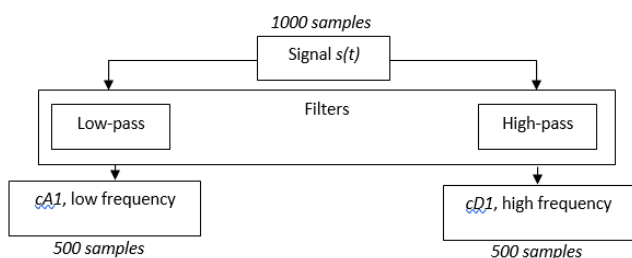
$$s(t) = \sum_{i=1}^n \lambda_{j-1}(i) \phi(2^{j-1} t - i) + \sum_{i=1}^n \gamma_{j-1}(i) \psi(2^{j-1} t - i) \tag{9}$$

The discrete wavelet transform (DWT) is evaluated by passing the signal through lowpass and highpass filters (Corinthios, 2009), dividing it into a lower frequency band and an upper band. Each band is subsequently divided into

a second level lower and upper bands. The process is repeated, taking the form of a binary, or “dyadic” tree. The lower band is referred to as the approximation cA and the upper band as the detail cD . DWT decomposes the signal into mutually orthogonal set of wavelets.

Misiti et al. (2015) illustrated the filtering process with a simple diagram (Figure 7).

Figure 7: Diagram of a one-level decomposed signal $s(t)$ using one-dimensional discrete wavelet analysis—illustration of the process of downsampling from 1,000 to 500.



Source: Misiti et al. (2015)

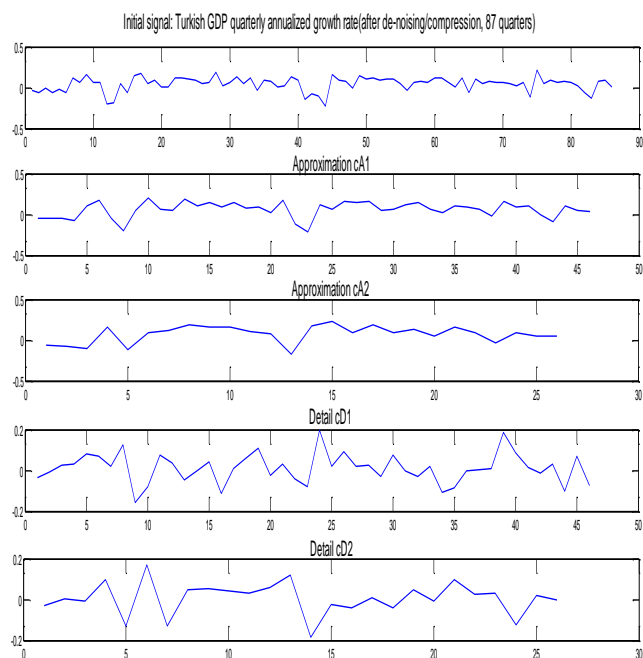
The model produces two sequences called cA and cD , which are downsampled.

The signal is decomposed after being differentiated, de-noised and compressed. The signal, i.e. for the 1998-2019 period the 87-quarter (for the 1998-2021 period the 96-quarter) time series of Turkish GDP quarterly annualized growth rate transformed at step 1, is decomposed into decomposed signals cA s named approximations and cD s named details. The Discrete Wavelet Transform is a kind of decomposition scheme evaluated by passing the signal through lowpass and highpass filters (Corinthios, 2009), dividing it into a lower frequency band and an upper band. Each band is subsequently divided into a second level lower and upper bands. The process is repeated, taking the form of a binary, or “dyadic” tree. The lower band is referred to as the approximation cA and the upper band as the detail cD . The two sequences cA and cD are downsampled. The downsampling is costly in terms of data: with multilevel decomposition, at each one-level of decomposition the sample size is reduced by half (in fact, slightly more than half the length of the original signal, since the filtering process is implemented by convolving the signal with a filter. The convolution “smears” the signal, introducing several extra samples into the result). Therefore, the decomposition can proceed only until the individual details consist of a single sample. Thus, the number of levels of decomposition will be limited by the initial number of data of the signal. Figure 5 illustrates the 2nd-level decomposition of Turkish GDP quarterly annualized growth rate (after de-noising/compression, 87 or 96 quarters). We observe in Figure 8 that details cD s are small and look like high-frequency noise, whereas the approximation $cA2$ contains much less noise than does the initial signal. In addition, the higher the level of decomposition, the lower the noise

generated by details. For a better understanding of signal decomposition using discrete wavelet transform, refer to the methodology section of Rostan and Rostan (2018a).

Figure 8: 2nd-level decomposition of Turkish GDP quarterly annualized growth rate (after de-noising/compression, 87 quarters from 1998 to 2019) using one-dimensional discrete wavelet analysis

Source: Authors’ own elaboration using Matlab.



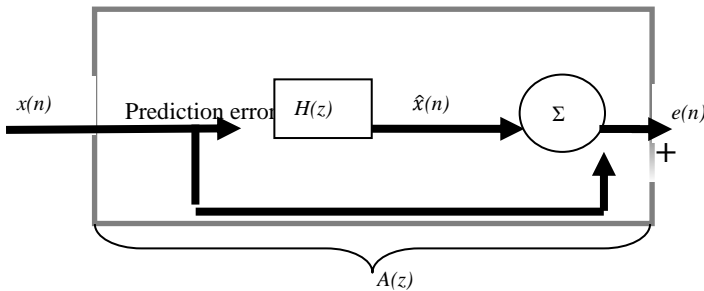
3.3. Step 3: Burg extension of approximations and details

We apply Burg extension to cA and cD as presented in Figure 8. To run the Burg extension, we apply an autoregressive p th order from historical data, in this paper we choose a p th order equal to the longest available order when forecasting. For instance in 2019, when forecasting Turkish Real GDP returns for the subsequent 31 years until 2050 ($4 \times 31 = 124$ quarters), the longest p th order available is 86 out of 87 historical data. Given x the decomposed signal (which is cA or cD), a vector a of all-pole filter coefficients is generated that models an input data sequence using the Levinson-Durbin algorithm (Levinson, 1946; Durbin, 1960). The Burg (1975) model is used to fit a p th order autoregressive (AR) model to the input signal, x , by minimizing (least squares) the forward and backward prediction errors while constraining the AR parameters to satisfy the Levinson-Durbin recursion. x is assumed to be the output of an AR system driven by white noise. Vector a contains the normalized estimate of the AR system parameters, $A(z)$, in descending powers of z :

$$H(z) = \frac{\sqrt{e}}{A(z)} = \frac{\sqrt{e}}{1 + a_2 z^{-1} + \dots + a_{(p+1)} z^{-p}} \quad (10)$$

Since the method characterizes the input data using an all-pole model, the correct choice of the model order p is important. In Figure 9, the prediction error, $e(n)$, can be viewed as the output of the prediction error filter

Figure 9: Prediction error filter to run the Burg extension



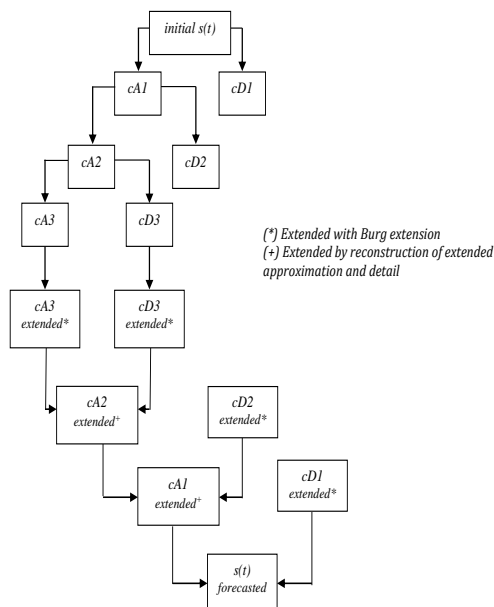
Source: Matlab.

In a last step, the Infinite Impulse Response (IIR) filter extrapolates the index values for each forecast horizon. IIR filters are digital filters with infinite impulse response. Unlike finite impulse response (FIR) filter, IIR filter has the feedback (a recursive part of a filter) and is also known as recursive digital filter.

3.4. Step 4: Wavelet Reconstruction

We recompute/reconstruct the forecasted signals after Burg extension using the methodology illustrated in Figure 5. We present the 3rd-level decomposition/reconstruction diagram in Figure 10. In our paper, we use a second-level decomposition/reconstruction that is most of the time the optimal level confirmed in the literature.

Figure 10: Diagram of a 3rd-level wavelet decomposition/reconstruction tree to forecast the initial signal $s(t)$.



Source: Authors' own elaboration using Matlab.

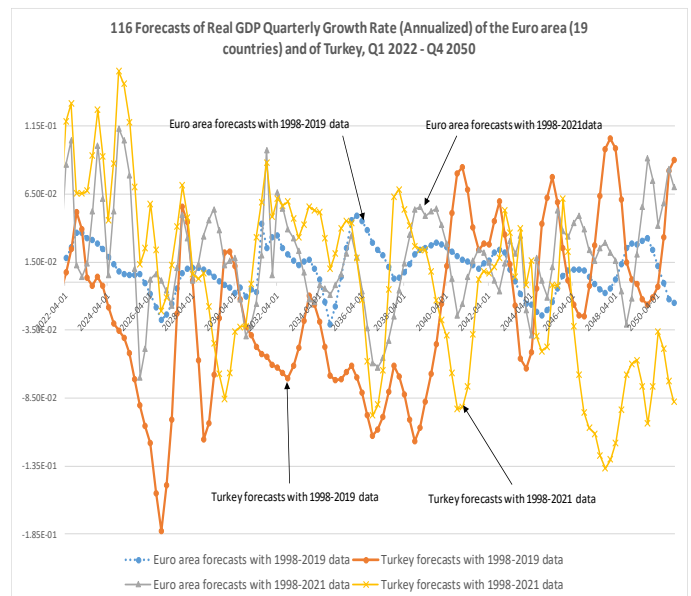
4. Results

The objective of the paper is to assess the resilience of Turkey's economy following the economic shock from the Covid-19 pandemic that hit the global economy in 2020, 2021 and extending in 2022. The 2050 projections are benchmarked to the 2050 GDP forecasts of the Euro Area (19 countries). This paper presents Turkey and Euro area quarterly annualized real GDP growth rates forecasts with spectral analysis up to 2050 using the historical data before the pandemic from 1998 up to the end of 2019 and during the pandemic (up to the end of 2021). The 2 sets of forecasts (pre- and during-pandemic) are then compared to check which economy between Turkey and the Euro area was more resilient to the Covid-19 pandemic.

4.1. Forecasts of Q1 2022 to Q4 2050 of Turkey and Euro area quarterly annualized real GDP growth rates

Figure 11 illustrates 116 forecasts with spectral analysis of Turkey and Euro area quarterly annualized real GDP growth rates from Q1 2022 to Q4 2050.

Figure 11: 116 forecasts with spectral analysis of Turkey and Euro area quarterly annualized real GDP growth rates from Q1 2022 to Q4 2050



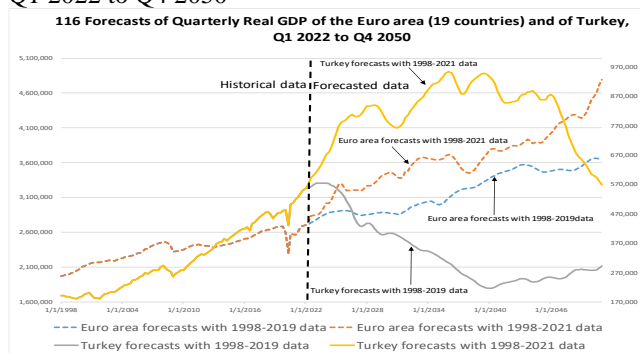
Source: Authors' own elaboration using Matlab.

Based on the 116 forecasts for the period 2022-2050, Turkey forecasts are more pessimistic than the Euro area, with an average quarterly (annualized) growth rate of -2.17% with the 1998-2019 data versus +0.12% with the 1998-2021 data when the Euro area forecasts have an average quarterly (annualized) growth rate of +1.03% with the 1998-2019 data versus +2.00% with the 1998-2021 data. It shows that the 2021 recovery of both economies has a positive impact on the forecasts. Why the Euro area has a better 2022-2050 average forecast than Turkey, +1.03% versus -2.17% with the 1998-2019 data and +2.0% versus +0.12% with the

1998-2021 data? The wavelet analysis forecasting model used in this paper is sensitive to the volatility of historical data. The historical data from 1998 to 2021 have a higher volatility for Turkey than the Euro area, 11.44% versus 7.67%. Thus, the volatility of the 116 forecasts with the 1998-2021 data is respectively 6.47% for Turkey versus 3.69% for the Euro area. This observation may explain the better projected performance of the Euro area compare to Turkey. In addition, wavelet analysis forecasting model is sensitive to the most recent data. Figure 12 illustrates 116 forecasts with spectral analysis of Turkey and the Euro area quarterly Real GDP from Q1 2022 to Q4 2050. It clearly appears that the rebound of both economies in Q2 2020 (refer to Figure 3), +65% in Turkey and +51% in the Euro area, following the huge contraction in Q1 2020, -43% in Turkey and -46% in the Euro area, clearly explains the divergent trend of the forecasts with the 1998-2019 historical data and with the 1998-2021 historical data for both Turkey and the Euro area since the model is sensitive to the most recent data.

4.2. Forecasts of Q1 2022 to Q4 2050 of Turkey and Euro area quarterly real GDP

Figure 12: Historical data and 116 forecasts with spectral analysis of Turkey and Euro area quarterly Real GDPs from Q1 2022 to Q4 2050



Source: Authors' own elaboration using Matlab.

4.3. Assessing the resilience of Turkey's economy following the economic shock from the Covid-19 pandemic

To recall, the objective of the paper is to assess the resilience of Turkey's economy following the economic shock from the Covid-19 pandemic that hit the global economy in 2020, 2021 and extending in 2022.

To answer this question, by subtracting the forecasted 2022-2050 average Euro area quarterly growth rate (annualized) obtained with the 1998-2021 data, +2%, by the one obtained with the 1998-2019 data, +1.03%, the difference is +0.97%, when with Turkey the difference is $0.12\% - (-2.17\%) = +2.30\%$. Turkey shows therefore a higher resilience to the Covid-19 pandemic (+2.30%) than the Euro area (+0.97%). But, the authors pointed out that the forecasted quarterly (annualized) growth rate average of the Euro area is

expected to be +2.00% with the 1998-2021 data whereas it is expected to be only +0.12% for Turkey.

5. Conclusion and Discussion

This paper presents Turkey's 2050 real GDP and growth rate quarterly forecasts before (up to 2019) and during the pandemic (up to 2021) by using spectral analysis. The objective of the paper is to assess the resilience of Turkey's economy during the Covid-19 pandemic with its 2050 projections. These projections are benchmarked to the 2050 GDP forecasts of the Euro Area (19 countries). Spectral analysis is able to analyze changing transient physical signals. Extending the analysis to complex-behavior economic signals, the originality of this paper is to apply spectral analysis to economic variables subject to common dynamics such as real GDP time series. The forecasts cover 116 quarters from Q1 2022 to Q4 2022 and derive from historical quarterly data extending from Q1 1998 to Q4 2021.

Spectral analysis methodology follows four steps that lead to real GDP quarterly (annualized) growth rate forecasts: the Quarterly Real GDP growth rate (annualized) time series of the Euro area and Turkey are de-noised and compressed, then decomposed in simpler signals called approximations and details in the framework of the one-dimensional discrete wavelet analysis. Third, the decomposed series are extended with the Burg (1975) model which fits a p th order autoregressive (AR) model to the input signal by minimizing (least squares) the forward and backward prediction errors while constraining the AR parameters to satisfy the Levinson-Durbin recursion. Finally, the series are reconstructed, the extensions being the forecasts.

To recall, the objective of the paper is to assess the resilience of Turkey's economy following the economic shock from the Covid-19 pandemic that hit the global economy in 2020, 2021 and extending in 2022. As illustrated in Figure 3, between Q2 2019 (2019-07-01) and Q4 2021 (2022-01-01), the quarterly Real GDP growth rate (annualized) of the Euro area economy (19 countries) was most of the time below the one of Turkey with an average growth rate of 0.94% for the Euro area versus 6.57% for Turkey. Therefore, Turkey was more resilient during the Covid-19 pandemic than the Euro area. Using the 2022-2050 forecasts of both economies, by subtracting the forecasted 2022-2050 average Euro area quarterly growth rate (annualized) obtained with the 1998-2021 data, +2%, by the one obtained with the 1998-2019 data, +1.03%, the difference is +0.97%, when with Turkey the difference is $+2.30\%$ [$0.12\% - (-2.17\%)$]. Turkey shows therefore a higher resilience to the Covid-19 pandemic (+2.30%) than the Euro area (+0.97%) based on 2022-2050 forecasts. However, the authors pointed out that the forecasted 2022-2050 quarterly (annualized) growth rate average of the Euro area is expected to be +2.00% with the 1998-2021 data whereas it is expected to be only +0.12% for Turkey.

Another question that has already stirred the mind of Turkish politicians is "Is it worth it for Turkey to join the European

Union (EU) of 27 members?”. In 1987, Turkey applied to join what was then the European Economic Community, and in 1999 it was declared eligible to join the EU (European Commission, 2022). Turkey is a key strategic partner of the EU on issues such as migration, security, counter-terrorism, and the economy, but has been backsliding in the areas of democracy, rule of law and fundamental rights. In response, the General Affairs Council decided in June 2018 that accession negotiations with Turkey were effectively frozen. Turkey has to make some progress in the above-mentioned areas in order to join EU. However, Turkey has a powerful lever on the European Commission regarding refugees. Irregular migration from Turkey to the EU involving smugglers and thousands of refugees moving to the EU every year has forced the EU to implement a EU Facility for Refugees in Turkey to stem migratory flows. In June 2021, the European Commission proposed a total package of €3 billion to be allocated in Turkey for refugee support and retention (European Commission, 2021). The Facility for Refugees, a key component of the 2016 EU-Turkey Statement, marked a significant scale-up of EU support to refugees in Turkey. Thus, Turkey has means to negotiate its accession to EU. The main advantage for Turkey of joining the EU is the waving of tariffs between member countries, the three main disadvantages are 1) that it is costly, member states providing billions in support to the European Union every year, 2) members are constrained by economic and political decisions taken outside their countries by the most influential members such as Germany and 3) members of the Euro area (using Euro currency as sole legal tender) lose flexibility in their monetary policy. As mentioned in ECB Economic Bulletin (2020) ‘the Covid-19 pandemic has put unprecedented burdens on euro area countries’ economies and government finances. In response to the dramatic COVID-19 shock, all euro area countries implemented packages of fiscal measures. These packages consist of discretionary fiscal stimulus measures, state guarantees for loans to firms and other liquidity support measures. An important component of the discretionary measures relates to support for firms, in particular to preserve employment.’ In short, globally central banks including the European Central Bank (ECB) and TCMB (the Turkish Central Bank) have printed billions of their domestic currencies to fix the impact of the Covid-19 pandemic on their economies by implementing large economic stimulus programs. Printing money means inflation on the short run by increasing the money supply. Inflation exploded everywhere, the Euro area annual inflation is expected to be 8.9 % in July 2022, up from 8.6 % in June 2022 (Eurostat, 2022). In Turkey, annual inflation rate accelerated for the 14th consecutive month to a skyrocketing 79.6 percent in July of 2022 (Trading Economics, 2022a). The Turkish lira currency depreciated past 17.94 per USD in August 2022 from 13.32 per USD at the start of 2022 (-25%), extending its decline amid large negative interest rates and soaring consumer prices. ‘Meanwhile, the latest current account figure (i.e. imports and exports of goods and services, payments made to foreign investors, and transfers such as foreign aid) pointed in 2022

to a deficit of \$6.5 billion and the export order index from the central bank tanked to a 15-month low, contradicting President Erdogan’s pledge that Turkey would consolidate a strong current account surplus position’ (Trading Economics, 2022b). In addition, the Russian invasion of Ukraine, which started in February 2022, hit economies globally including Euro area and Turkish economies with soaring food and energy prices. Coming back to the question: “Is it worth it for Turkey to join the European Union of 27 members?”, the authors believe that the current Turkey economic crisis would have been lessened for Turkey with the status of European Union member, the European Union organization supporting lagging members. In addition, using the Euro currency as legal tender would have avoided the situation related to the sharp depreciation of Turkish lira. During the first 8 months of 2022, Euro depreciated against USD by 8.5%, when Turkish lira depreciated by 25%. Less volatility of the domestic currency reassures international investors. Finally, the not so optimistic 2022-2050 forecasts of quarterly growth rate of Turkey with an average of +0.12% versus +2.00% for the Euro area point out the benefit of Turkey to join the European Union in coming years. Further research may focus on additional economic indicators of Turkey to identify the areas of strengths and weaknesses of the Turkish economy.

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