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EXAMINATION OF KRAENKEL-MANNA-MERLE SYSTEM BY SINE-GORDON EXPANSION METHOD

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Abstract

In this study, Kraenkel-Manna-Merle (KMM) system is discussed. Sine-Gordon expansion method (SGEM), which is one of the solution methods of nonlinear evolution equations (NLEEs), has been applied to this system. Thus, by applying this method for the first time, some dark soliton, bright soliton, and dark-bright soliton solutions of the KMM system have been obtained. In addition, by giving specific values to the achieved solutions, 2D and 3D graphics of the solutions were plotted by way of the Wolfram Mathematica 12 program.

Keywords: Kraenkel-Manna-Merle System, mathematica, SGEM

KRAENKEL-MANNA-MERLE SİSTEMİNİN SGEM YOLUYLA İNCELENMESİ

Özet

Bu çalışmada, Kraenkel-Manna-Merle sistemi ele alınmıştır. Doğrusal olmayan evrim denklemlerinin çözüm yöntemlerinden biri olan sinüs-Gordon açılım yöntemi bu sisteme uygulanmıştır. Böylece ilk kez bu yöntem uygulanarak KMM sisteminin bazı dark soliton, bright soliton ve dark-bright soliton çözümleri elde edilmiştir. Ayrıca elde edilen çözümlere belirli değerler verilerek Wolfram Mathematica 12 programı aracılığıyla çözümlerin 2 boyutlu ve 3 boyutlu grafikleri çizilmiştir.

Anahtar Kelimeler: Kraenkel-Manna-Merle Sistemi, mathematica, SGEM

Cite

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1. Introduction

NLEEs have very important practices in many fields such as biology, chemistry, optic, physics, fluid dynamics, hydromagnetic waves, and many others. Recently, various methods have been developed and implemented in order to obtain NLEEs solutions, which have such important areas of use [1-11].

In recent years, intensive information and data have emerged thanks to the rapid developments in computer and information technologies. The transportation, storage, and processing of the obtained big data have also been an important subject of study. Today, many researchers are working in the field of ferromagnetic materials due to the important areas of use in technologies such as storing and processing large-capacity data [12-19].

Solitons are a special type of solitary waves. Solitary waves in integrable equations are called solitons. Therefore, hyperbolic function solutions are soliton solutions if the equation is integrable. In addition, the obtained soliton solutions can be of different types, such as dark soliton, bright soliton and dark-bright soliton [20].

In a study by Kraenkel et al. in 2000, short-wave propagation in saturated ferromagnetic materials was investigated from Maxwell and Landau-Lifshitz-Gilbert equations, and they have obtained the system of equations given below [21],

$$\begin{aligned} &-\nabla(\nabla.P)+\nabla^2 P=\frac{\partial^2}{\partial t^2}(P+K),\\ &\frac{\partial}{\partial t}K=-K\wedge P_{eff}+\frac{\sigma}{m}K\wedge\frac{\partial}{\partial t}K. \end{aligned} \tag{1}$$

From the notations here, P shows dimensionless magnetic induction and K represent magnetization density. m and σ are constants and exemplify dimensionless saturation magnetization and Gilbert-damping parameter respectively, ∇ is divergence for vector field. Nguepjouo et al. introduced a combination of coordinate transformations and expansion series of the magnetization density in recent years, and thus changed the (1) system to the following form [22],

$$\begin{aligned} &u_{xt} - uv_x + su_x = 0, \\ &v_{xt} + uu_x = 0, \end{aligned} \tag{2}$$

where $u = u(x,t)$ is magnetization and $v = v(x,t)$ is external magnetic fields related to the ferrite. x represents the displacement and t represents the time. Coefficient s denotes the damping effect [23-28].

Various studies have been carried out by researchers recently to get solutions of the KMM system. For example Younas et al. discussed extended sinh-Gordon equation expansion method and the $\left(\frac{G'}{G^2}\right)$-expansion function method for KMM system [23]. Li and Ma applied generalized $\left(\frac{G'}{G}\right)$-expansion method and truncated Painleve method to KMM system [24,25]. Raza et al. used the new auxiliary equation method and semi inverse technique for KMM system [26]. Ur-Rehman et al. applied new extended direct algebraic method to KMM system [27]. Si and Li constructed the one-soliton and two-soliton for KMM system by the bilinear method [28].

In this study, we will use SGEM, which is one of the widely used methods to find the solutions of NLEEs [29-32]. SGEM has been grown based on traveling wave transformation and the sine-Gordon equation [33]. We perform SGEM to the system to obtain soliton solutions of the Kraenkel-Manna-Merle system.

2. Basic structure of SGEM

We will give the general basic of SGEM. For this, we first consider the sine-Gordon equation

$$v_{xx} - v_{tt} = m^2 \sin(v), \tag{3}$$

where m is a real constant and $v = v(x,t)$ is a function.

Performing the wave transformation $v(x,t) = V(\xi), \xi = \mu(x - kt)$ to Eq. (3), we have following nonlinear ordinary differential equation (ODE),

$$V'' = \frac{m^2}{\mu^2\left(1-k^2\right)} \sin(V), \tag{4}$$

is obtained. Integrating Eq. (4) and setting the integration constant to zero

$$\left[\left(\frac{V}{2}\right)'\right]^2 = \frac{m^2}{\mu^2\left(1-k^2\right)} \sin^2\left(\frac{V}{2}\right). \tag{5}$$

Subsituting $w(\xi) = \frac{V}{2}$ and $b^2 = \frac{m^2}{\mu^2\left(1-k^2\right)}$ in Eq. (5), we get:

$$w' = b\sin(w). \tag{6}$$

If we take $b = 1$, we have:

$$w' = \sin(w). \tag{7}$$

From the Eq. (7), we get:

$$\sin(w) = \sin\left(w(\xi)\right) = \left.\frac{2de^{\xi}}{d^2e^{2\xi}+1}\right|_{d=1} = \operatorname{sech}(\xi), \tag{8}$$

$$\cos(w) = \cos\left(w(\xi)\right) = \left.\frac{d^2e^{2\xi}-1}{d^2e^{2\xi}+1}\right|_{d=1} = \tanh(\xi), \tag{9}$$

To find solution of the nonlinear partial differential equation given below;

$$F(v, v_x, v_t, v_{xx}, v_{tt}, v_{xt}, v_{xxx}, \ldots), \tag{10}$$

we handle the equation given below,

$$V(\xi) = \sum_{i=1}^{n} \tanh^{i-1}(\xi)\left[B_i \operatorname{sech}(\xi) + A_i \tanh(\xi)\right] + A_0. \tag{11}$$

Considering the Eqs. (8) and (9), we can write the Eq. (11) as follows:

$$V(w) = \sum_{i=1}^{n} \cos^{i-1}(w)\left[B_i \sin(w) + A_i \cos(w)\right] + A_0. \tag{12}$$

Here one can specify the value of n in Eq. (12) by means of balance principle, replace Eq. (12) into Eq. (10), and comparison the terms, one can get a system of equations. By solving this obtained system of equations, one can obtain results in travelling wave solutions of the Eq. (10).

3. Application of SGEM to KMM

Considering the zero dumping effect $(s = 0)$, we apply the following transformation to Eq. (2)

$$u(x,t) = u(\xi), v(x,t) = v(\xi), \xi = \tau(x - ct). \tag{13}$$

We obtain the following system.

$$\begin{aligned} &-c\tau^2 u'' - \tau uv' = 0, \\ &\tau uu' - c\tau^2 v'' = 0. \end{aligned} \tag{14}$$

From Eq. (14) we have the following equation,

$$v'' = \frac{uu'}{c\tau}. \tag{15}$$

If the integration with respect to ξ is taken in Eq. (15) and by taking the integration constant as zero, we get the following equation,

$$v' = \frac{u^2}{2c\tau} + \theta. \tag{16}$$

If Eq. (16) is written in system (14), we find the following ODE,

$$2c\theta\tau u + u^3 + 2c^2\tau^2 u'' = 0. \tag{17}$$

Balancing the terms u'' and u^3 gives $N = 1$. Using the value of $N = 1$ in Eq. (12), we get:

$$u(w) = B_1 \sin(w) + A_1 \cos(w) + A_0, \tag{18}$$

$$u'(w) = B_1 \cos(w)\sin(w) - A_1 \sin^2(w), \tag{19}$$

$$u''(w) = B_1 \cos^2(w)\sin(w) - B_1 \sin^3(w) \\ -2A_1 \sin^2(w)\cos(w). \tag{20}$$

By placing Eq. (18), (19) and (20) into Eq. (17), we generate trigonometric equations. We obtain an equation system by performing some mathematical operations in these trigonometric equations. By solving the obtained system of equations with the help of Wolfram Mathematica Release 12, the following situations are obtained:

Case1:

$$A_0 = 0, A_1 = -2i\theta, B_1 = -2\theta, \tau = \frac{2\theta}{c}. \tag{21}$$

If we consider the coefficients of (21) in Eq. (11), we get the following solutions:

$$u_1(x,t) = -2\theta \left(\begin{array}{l} \operatorname{sech}\left[\frac{2\theta x}{c} - 2\theta t\right] \\ +i\tanh\left[\frac{2\theta x}{c} - 2\theta t\right] \end{array} \right). \tag{22}$$

$$v_1(x,t) = -2\theta^2 \left(\begin{array}{l} \operatorname{sech}\left[2\theta t - \frac{2\theta x}{c}\right] \\ -i\tanh\left[2\theta t - \frac{2\theta x}{c}\right] \end{array} \right) \\ -\frac{\theta^2}{3}\left(\begin{array}{l} \operatorname{sech}\left[2\theta t - \frac{2\theta x}{c}\right] \\ -i\tanh\left[2\theta t - \frac{2\theta x}{c}\right] \end{array} \right)^3. \tag{23}$$

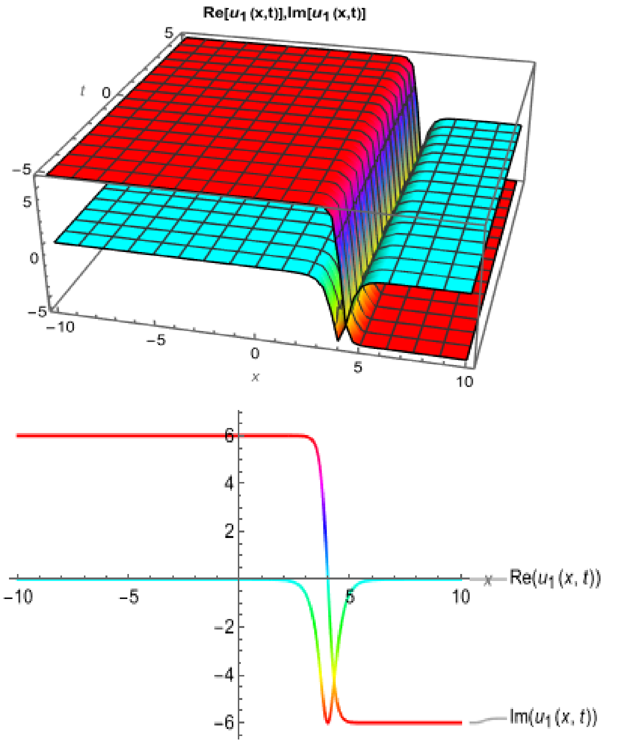


Fig. 1: 3D plot of real and imaginary values of solution (22) for $c = 2, \theta = 3$ values with $-10 < x < 10, -5 < t < 5$ range and 2D plot of solution for $t = 2$ with this values.

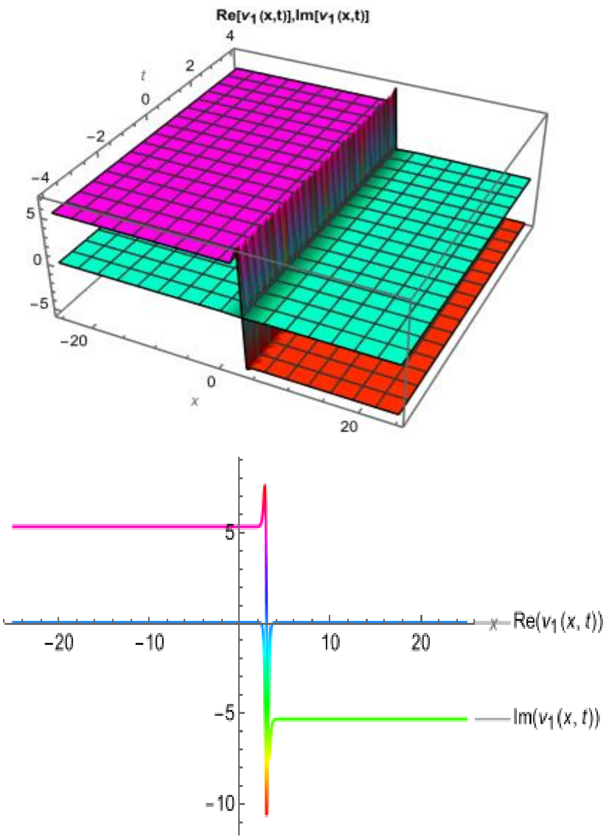


Fig. 2: 3D plot of real and imaginary values of solution (23) for $c = 1, \theta = 2$ values with

$-25 < x < 25, -4 < t < 4$ range and 2D plot of solution for $t = 3$ with this values.

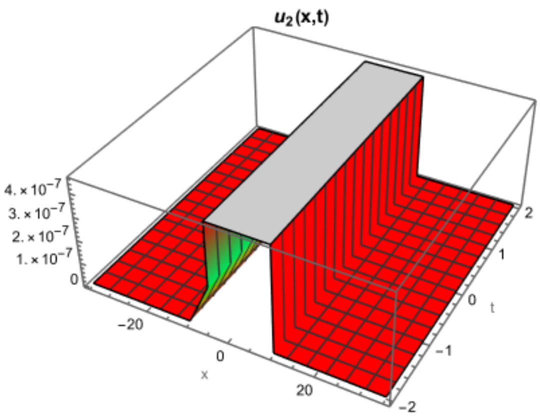
Case2:

$$A_0 = 0, A_1 = 0, B_1 = 2\theta, \tau = -\frac{\theta}{c}. \qquad (24)$$

If we consider the coefficients of (21) in Eq. (11), we get the following solutions:

$$u_2(x,t) = 2\theta \operatorname{sech}\left[\frac{x\theta}{c} - t\theta\right]. \qquad (25)$$

$$v_2(x,t) = 2\theta^2 \operatorname{sech}\left[t\theta - \frac{x\theta}{c}\right] - \frac{4\theta^2}{3}\operatorname{sech}^3\left[t\theta - \frac{x\theta}{c}\right]. \qquad (26)$$



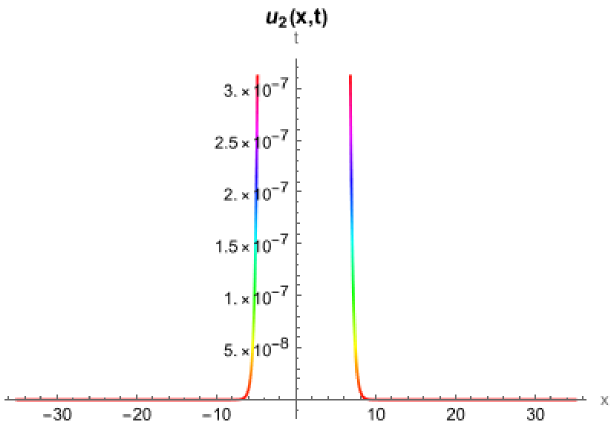
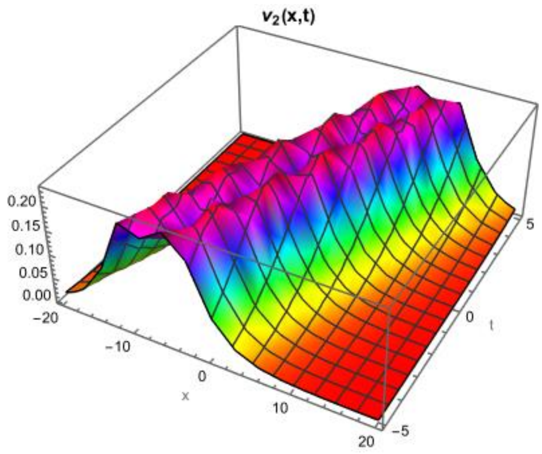


Fig. 3: 3D plot of real values of solution (25) for $c = 1, \theta = 3$ values with $-35 < x < 35, -2 < t < 2$ range and 2D plot of solution for $t = 1$ with this values.



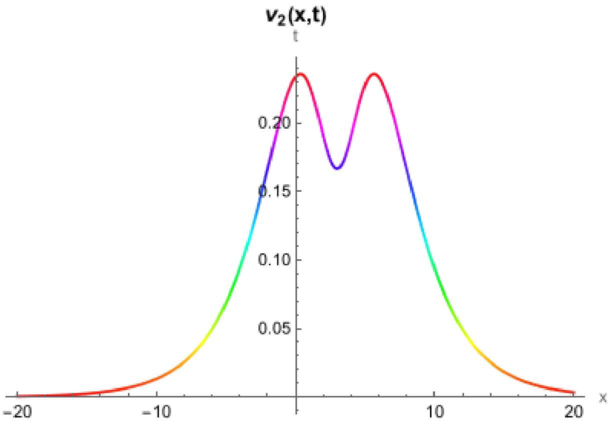


Fig. 4: 3D plot of real values of solution (26) for $c = 1.5, \theta = 0.5$ values with $-20 < x < 20, -5 < t < 5$ range and 2D plot of solution for $t = 2$ with this values.

Case3:

$$A_0 = 0, A_1 = -i, B_1 = 0, c = \frac{\theta}{2\tau}. \qquad (27)$$

If we consider the coefficients of (27) in Eq. (11), we get the following solutions:

$$u_3(x,t) = i\theta \tanh\left[\frac{\theta t}{2} - \tau x\right]. \qquad (28)$$

$$v_3(x,t) = i\theta^2 \tanh\left[\frac{\theta t}{2} - \tau x\right] - \frac{i\theta^2}{3}\tanh^3\left[\frac{\theta t}{2} - \tau x\right]. \qquad (29)$$

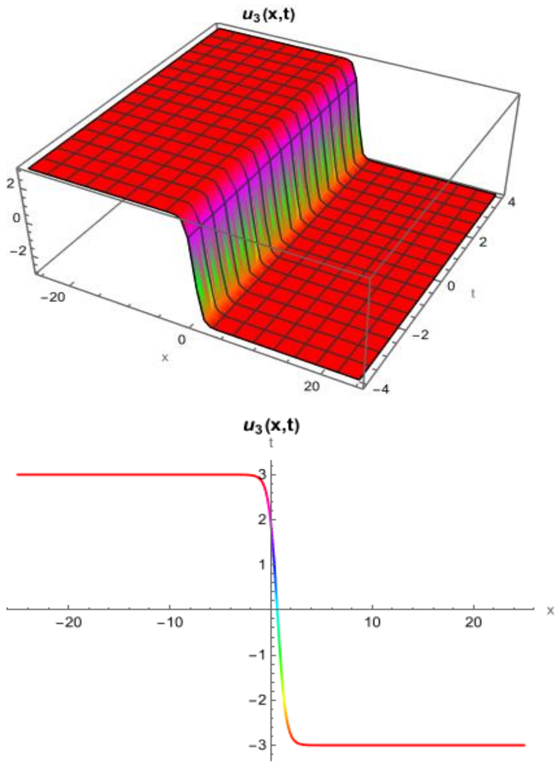


Fig. 5: 3D plot of imaginary values of solution (28) for $\tau = 1.2, \theta = 3$ values with $-25 < x < 25, -4 < t < 4$ range and 2D plot of solution for $t = 0.5$ with this values.

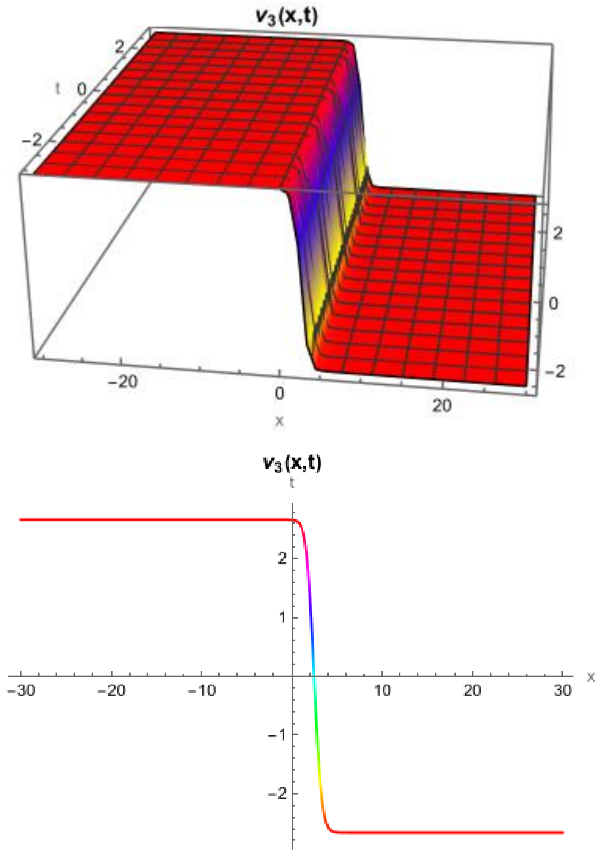


Fig. 6: 3D plot of imaginary values solution (29) for $\tau = 0.8, \theta = 2$ values with $-30 < x < 30, -3 < t < 3$ range and 2D plot of solution for $t = 2$ with this values.

4. Result and discussions

In this study, a mathematical approach that has not been applied to the KMM system before was used. Thus, some soliton solutions were obtained by using the sine-Gordon expansion method for the KMM system. Looking at the obtained solutions, $u_1(x,t)$ and $v_1(x,t)$ dark-bright soliton, $u_2(x,t)$ and $v_2(x,t)$ bright soliton, and $u_3(x,t)$ and $v_3(x,t)$ dark soliton solutions were obtained. In addition, 2D and 3D graphics of the obtained solutions have been drawn. Graphics of dark-bright soliton solutions are plotted for both imaginary and real situations. Real graphics of bright soliton solutions and imaginary graphics of dark soliton solutions have been drawn. The SGEM contains trigonometric functions that will be used to obtain new solutions of the equation considered in equation 12. Thanks to the properties of these trigonometric functions, various new solutions can be obtained. This is one of the main features of SGEM. For this reason, it gives many coefficients such as complex, exponential, and trigonometric to the model under consideration. Thus, SGEM is an easy-to-use method applied to obtain various solutions of nonlinear partial differential equations.

5. Conclusion

In this study, we had some soliton solutions for the KMM system by applying SGEM. Thus, we obtained new soliton solutions of the KMM system. We drew the 2D and 3D graphical representations of these solitons with the help of a Wolfram Mathematica 12. As far as we know, SGEM has not been applied to the KMM system before. The solutions we obtained have not been presented in previous studies and are new. In the light of this results, we consider the SGEM as an effective method in NLEEs calculation.

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