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Research Article

Recalculation of Lost Information in Neuron with Quadratic Spline Interpolation

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Abstract

The main function of neurons in a living creature is to transmit information. Neurons carry out information transmission without loss despite environmental and internal noise sources. However, sometimes there may be losses in the transmission of information. This results in diseases such as Alzheimer's, MS, and Epilepsy. In this study, the information lost in neurons is recalculated with the Quadratic Spline Interpolation method. In cases where it is difficult or impossible to calculate a function, the process of calculating the corresponding value of an unmeasured variable is called interpolation. In this study, first of all, three sample neuron behaviours are created with the Fitzhugh-Nagumo model, and the action potential and recovery parameter variables are obtained. Then, some data in the variables are deleted, resulting in unhealthy neuron behaviour. Then, these deleted data are recalculated using the Quadratic Spline Interpolation method. Various error values are obtained by comparing the actual and calculated data. The data lost in the action potential-recovery variable are detected with a very low error rate of 0.2630-0.0524%, 0.2885-0.0165% and 0.2543-0.0781% for the three sample neuron behaviours, respectively. With this study, it has been demonstrated that information lost or incorrectly coded in neurons for any reason can be corrected. It is also understood that this study can be used to prevent losses in real-time measurement results from biological neurons and to recalculate erroneous values.

Keywords: Quadratic Spline Interpolation, Cubic Spline Interpolation, FitzHugh-Nagumo, Hodgkin-Huxley, Neuron Model

İkinci Dereceden İnterpolasyon ile Nöronda Kayıp Bilginin Yeniden Hesabı

Öz

Canlılarda nöronların temel görevi bilgi iletimidir. Nöronlar çevresel ve içsel gürültü kaynaklarına rağmen bilgi iletimini kayıpsız olarak gerçekleştirirler. Fakat kimi zaman bilgi iletiminde kayıplar meydana gelebilir. Bu durum Alzheimer, MS, Epilepsi gibi hastalıklar ile sonuçlar. Bu çalışmada nöronlarda kaybolan bilgi İkinci Dereceden Şerit İnterpolasyon yöntemi ile yeniden hesaplanması sağlanmıştır. Bir fonksiyonun hesaplanmasının zor veya mümkün olmayan durumlarda, değeri ölçülmemiş bir değişkenine karşılık gelen değerinin hesaplanması işlemine interpolasyon adı verilir. Bu çalışamda öncelikle Fitzhugh-Nagumo model ile üç örnek nöron davranışı oluşturulmuş ve aksiyon potansiyeli ile toparlanma parametresi değişkenleri elde edilmiştir. Ardından değişkenlerdeki bazı veriler silinerek sağlıksız bir nöron davranışı sağlanmıştır. Daha sonra İkinci Dereceden Şerit İnterpolasyon yöntemi kullanılarak silinen bu veriler yeniden hesaplanmıştır. Gerçek ve hesaplanan veriler karşılaştırılarak çeşitli hata değerleri elde edilmiştir. Aksiyon potansiyeli-toparlanma parametresinde kaybolan veriler, üç örnek nöron davranışı için sırasıyla %0.2630-%0.0524, %0.2885-%0.0165 ve %0.254-%0.0781 gibi çok düşük bir hata oranıyla tespit edilir. Bu çalışma ile nöronlarda herhangi bir sebepten dolayı kaybolan veya yanlış kodlanan bilgi düzeltilebilir olduğu ortaya konmuştur. Ayrıca bu çalışmanın biyolojik nöronlardan gerçek zamanlı ölçüm sonuçlarındaki kayıpları önlemek ve hatalı değerleri yeniden hesaplamak için kullanılabileceği anlaşılmaktadır.

Anahtar Kelimeler: İkinci Dereceden İnterpolasyon, Kübik İnterpolasyon, FitzHugh-Nagumo, Hodgkin-Huxley, Nöron Model

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1. Introduction

In order to understand the behaviour and activities of nerve cells that transmit signals produced in the brain to the organs, many biological neuron models have been developed and expressed with various mathematical equations (FitzHugh 1961; Hindmarsh and Rose 1984; Hodgkin and Huxley 1952; Izhikevich 2003; Morris and Lecar 1981). The common goal of the scientists who presented their studies to the literature until today is to realize the closest and most accurate approach to the action potential diversity produced by biological nerve cells. In this context, in 1952, Alan Lloyd Hodgkin and Andrew Fielding Huxley presented the Hodgkin-Huxley (HH) nerve cell model, which is their joint work, to the literature. In this model, the ionic mechanism of the nerve cell is explained. The electric current created by this ionic flux on the membrane surface was concerned. This model, which describes the behaviour of the biological nerve cell in detail, depends on many parameters because it is comprehensive (4-dimensional) (Hodgkin and Huxley 1952). The simplified version of the Hodgkin-Huxley Model (HH) was expressed as the FitzHugh-Nagumo Model (FHN). The FHN is an example of a relaxation oscillator and has an external exciter input. This 2-dimensional model cannot generate explosive action potentials (FitzHugh 1961; Nagumo, Arimoto, and Yoshizawa 1962). Another two-dimensional neuron model was presented in the literature by Cathy Morris and Harold Lecar. The model mathematically revealed the action potential using calcium and potassium ion channels. The Morris-Lecar Model, as a conductivity-based model, has become popular among scientists working on neurology (Morris and Lecar 1981). Working with Action Potential-Explosive Action Potential (spike-burst), J. L. Hindmarsh and R. M. Rose modelled the nerve cell in 1984 with 3 differential equations. Studies on the Hindmarsh-Rose Model such as bifurcation analysis, synchronization, chaos, and hardware solutions are available in the literature (Hindmarsh and Rose 1984).

The main task of neurons is to transmit information. As a result of electrical and chemical processes, this communication takes place through the Action Potential (Purves et al. 2019). Despite environmental and internal noises, information transmission in a healthy neuron is lossless. However, an unhealthy neuron is affected by noises, and the information from the previous neuron is transferred to another in a lossy way (Kang et al. 2020; Nakamura and Tateno 2019). This situation results in the formation of neurological diseases such as Epilepsy, Alzheimer's and MS (Li et al. 2020). If the lost information is recalculated, it may be possible to reduce the effects of diseases and even cure them. In addition, studies on biological neurons may have losses or erroneous measurements during real-time measurement. In cases where it is difficult or impossible to calculate any function, the process of calculating the value of the function corresponding to an unmeasured variable x is called interpolation (Lunardi and Scuola normale Superiore (Italy) 2009; Werner 1984). Lost, incorrectly coded and incorrectly measured information can be determined by interpolation methods such as Linear, Lagrange, Newton, Chebyshev, etc (Blu, Thévenaz, and Unser 2004; Effenberger and Kressner 2012; Sauer and Xu 1995). Interpolation is used as a solution method in the fields of geographic information systems, digital modems, digital image processing, signal processing, statistics and engineering optimization (Gardner 1993; Keys 1981; Koziel, Bandler, and Madsen 2006; Narang, Gadde, and Ortega 2013; Polytechnica and Eng 1999; Prof et al. 2014; Schafer and Rabiner 1973; Scheuerer 2009). In this study, the FHN neuron model was preferred. Firstly, a numerical solution of a healthy FHN neuron was performed. From the results obtained, the univalent action potential values of the number of steps were extracted. In this way, information losses were created. Then, the missing information was calculated using Quadratic Spline Interpolation and compared with known values. This study is the first in the literature in terms of recalculating the data lost in the action potential using the interpolation method.

In the second part of this study, the numerical solution of the FHN neuron model and the Quadratic Spline interpolation method are explained. In the third chapter, the generation of losses in the action potential and the calculation of the loss values by interpolation are included. In Chapter 4, the results obtained are discussed. In the last section, the results of the study are summarized.

2. Material and Method

2.1. FitzHugh-Nagumo Neuron Model

Although the FHN neuron model has poor biological accuracy, it has a wide place in the literature because of the relaxation oscillator example and its simple structure. The FHN model is expressed by two sets of differential equations (FitzHugh 1961; Nagumo, Arimoto, and Yoshizawa 1962).:

$$\frac{dv}{dt} = d * \left(c \left(v - u + I_s - \frac{v^3}{3} \right) \right)$$
$$\frac{du}{dt} = \frac{v - bu + a}{c} \tag{1}$$

Here v represents the membrane potential of the nerve cell, u represents the recovery parameter, I_s represents the external current applied to the cell membrane. Parameters a, b, c have fixed values. d is the scaling coefficient (FitzHugh 1961; Nagumo, Arimoto, and Yoshizawa 1962). Three FHN neuron dynamics were created by determining various parameters. The parameters are given in Table 1 and the waveforms resulting from the numerical solution are given in Fig 1. In numerical analysis, the 4th Degree Runge-Kutta method was used. The number of steps was $N = 600$, the step size was $h = 0.01$, and the initial values were $(v_0; w_0) = (-2{,}0000; 1{,}1638)$ for the three samples.

Table 1. Parameter values for three samples created

FHN Model	FHN-Sp1	FHN-Sp2	FHN-Sp3
a	0,07	0,2	0,35
b	0,08	0,1	0,4
c	0,5	2	2,5
I_s	0,15	0,05	0,75
d	10	10	10

2.2. Quadratic Spline Interpolation

In cases where it is difficult or impossible to calculate the $f(x)$ function, the process of calculating the value of the $f(x)$ function corresponding to an unmeasured variable x is called interpolation (Lunardi and Scuola normale Superiore (Italy) 2009; Werner 1984). In this method, the points where the function is defined are called "interpolation points". Determining the intermediate value by using higher-order interpolation functions in cases where the function does not change linearly is called nonlinear interpolation. Quadratic Spline Interpolation is a nonlinear method. As shown in Fig 2, interpolation is done by finding quadratic functions between the interpolation points. For all data points, a quadratic polynomial is derived between these data points.

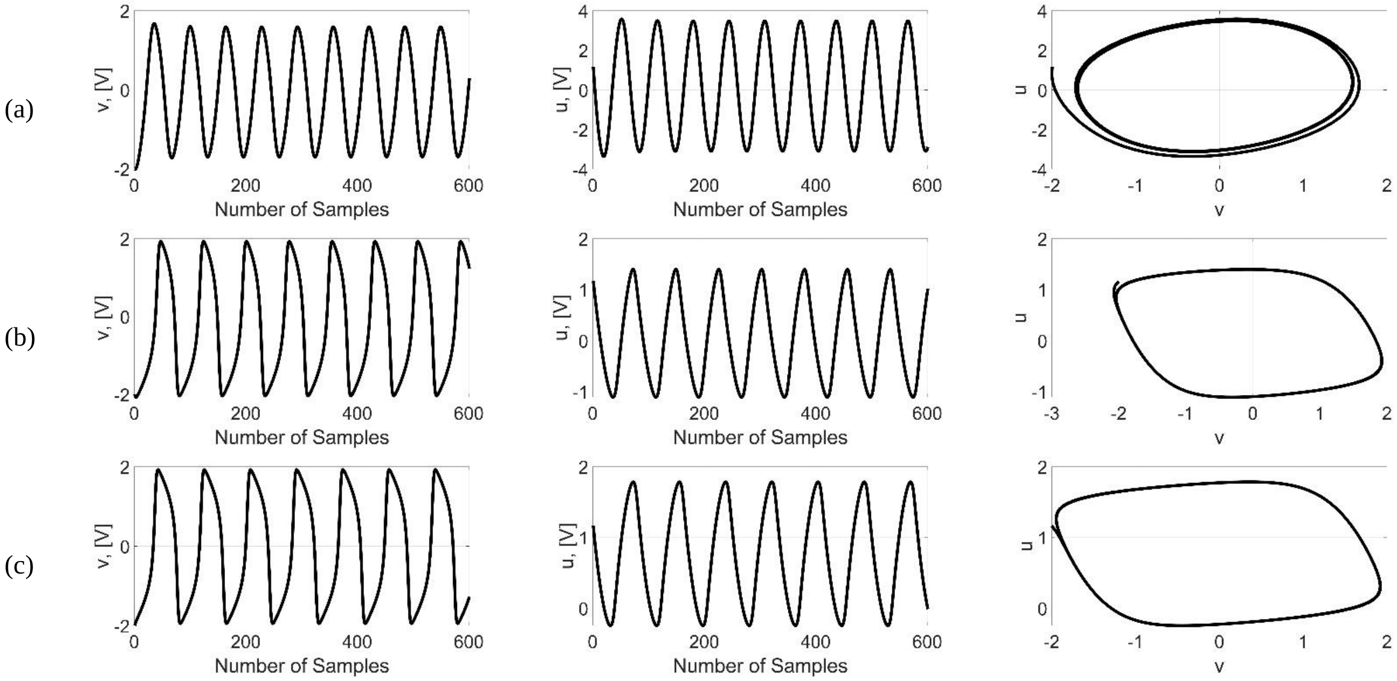

Figure 1. Using the 4th Degree Runge-Kutta method of the FHN neuron model, numerical solution waveforms, first column membrane potential, second column recovery parameter, third column v-u phase portraits a) example 1, b) example 2, c) example 3

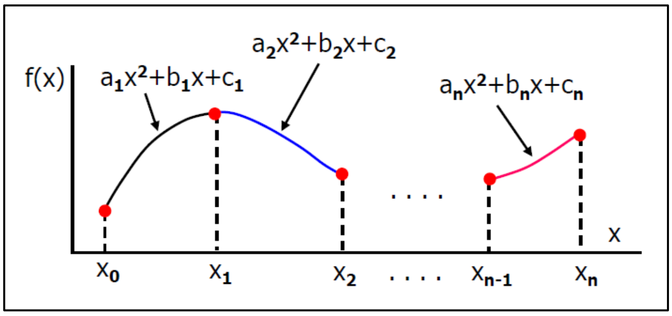

Figure 2. Quadratic Spline interpolation functions

Here, each point is connected by a quadratic function. For $(N+1)$ data points, there are N strips and $3N$ unknown constants. In order to determine the constants, $3N$ equations must be solved. In $3N$ equations, representing the start and end points,

$$a_1x_0^2 + b_1x_0 + c_1 = f(x_0)$$
$$a_Nx_N^2 + b_Nx_N + c_N = f(x_N) \tag{2}$$

Two equations, representing points in the interior region,

$$i = 2,3,4,\dots,N$$
$$a_{i-1}x_{i-1}^2 + b_{i-1}x_{i-1} + c_{i-1} = f(x_{i-1}) \tag{3}$$
$$a_1x_{i-1}^2 + b_1x_{i-1} + c_1 = f(x_{i-1})$$

$2N-2$ equations, the 1st derivatives of these functions must be equal, and in this case,

$$i = 1,2,3,\dots,N$$
$$2a_{i-1}x_{i-1} + b_{i-1} = 2a_ix_{i-1} + b_i \tag{4}$$

There are $N-1$ equations. In addition, one constant must be determined beforehand in order to find all the constants. It is set to $a_1 = 0$. Then, by solving all these equations, the values of a_i, b_i, c_i being $i = 1,2,3,\dots,N$ are found and quadratic equations between both points are obtained. The function $f(x)$ corresponding to any value of x can be found.

2.3. Error Calculation

Various error values were calculated to compare the results obtained as a result of the FHN numerical solution with the values calculated by the Quadratic Spline interpolation method. These errors are Mean Absolute Error (MAE), Mean Square Error (MSE), Root Mean Square Error (RMSE), and Normalized Root Mean Square Error (NRMSE).

$$MAE = \frac{1}{N}\sum_{i=1}^{N}\left|v_i^{original} - v_i^{interpolation}\right| \tag{5}$$

$$MSE = \frac{1}{N}\sum_{i=1}^{N}(v_i^{original} - v_i^{interpolation})^2 \tag{6}$$

$$RMSE = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(v_i^{original} - v_i^{interpolation})^2} \quad (7)$$

$$NRMSE = \frac{\sqrt{\frac{1}{N}\sum_{i=1}^{N}(v_i^{original} - v_i^{interpolation})^2}}{(v_{i,max}^{original} - v_{i,min}^{original})} \quad (8)$$

3. Results and Discussion

The $v - u$ variables obtained in the second chapter will be used to examine the Quadratic Spline Interpolation method. Let there be a situation where $v - u$ values corresponding to $t = 1,3,5,\dots,N-1$ value is known but $v - u$ variables corresponding to $t = 2,4,6,\dots,N$ value is not known. That is, there are $N/2 = 300$ values and the corresponding is known $v - u$ values. These values are given in Table 2.

900 equations are obtained by using two equations, equation 2, which defines the start and end points, equation 3 for intermediate values, and equation 4, which defines the case where the first derivatives are equal to each other. Using MATLAB, all equations are solved and quadratic equations between both points are obtained. Some of the coefficients determined for $v - u$ variables are given in Table 3.

Table 2. Some of the known $v - u$ values corresponding to $t = 1,3,5,\dots,N-1$

	N	**1**	**3**	**5**	**…**	**595**	**597**	**599**
FHN-Sp1	v [V]	-1,9936	-1,9213	-1,8021	…	-0,6193	-0,3414	-0,0372
	u [V]	0,3662	-0,3901	-1,0844	…	-3,0278	-3,0946	-3,0446
	N	**1**	**3**	**5**	**…**	**595**	**597**	**599**
FHN-Sp2	v [V]	-2,0750	-2,0518	-2,0018	…	1,5836	1,4796	1,3634
	u [V]	0,9680	0,7726	0,5830	…	0,5568	0,7236	0,8779
	N	**1**	**3**	**5**	**…**	**595**	**597**	**599**
FHN-Sp3	v [V]	-1,9059	-1,8372	-1,7734	…	-1,5136	-1,4393	-1,3599
	u [V]	1,0014	0,8501	0,7088	…	0,2478	0,1513	0,0639

Table 3. Some of the coefficients a, b and c were obtained as a result of the Quadratic Spline Interpolation method

FHN-Sp1	v	a_2	a_{150}	a_{250}	b_{25}	b_{185}	b_{250}	c_{40}	c_{260}	c_{299}
		0,0165	0,0053	0,0057	1,2403	10,0994	-5,8044	74,2632	4068,9564	0,4642
	u	a_{80}	a_{150}	a_{173}	b_{133}	b_{260}	b_{282}	c_{175}	c_{274}	c_{297}
		0,0094	-,0067	0,0115	-1,31636	-11,2890	12,9067	754,5496	786,5849	3418,4125
FHN-Sp2	v	a_2	a_{150}	a_{250}	b_{25}	b_{185}	b_{250}	c_{40}	c_{260}	c_{299}
		0,0246	-0,0029	0,0230	1,5628	14,8896	-22,7138	134,9728	-315,8449	1,2104
	u	a_{80}	a_{150}	a_{173}	b_{133}	b_{260}	b_{282}	c_{175}	c_{274}	c_{297}
		0,000628	-0,0039	0,0063	-1,7969	2,0815	-1,6732	881,2019	334,3197	-540,3704
FHN-Sp3	v	a_2	a_{150}	a_{250}	b_{25}	b_{185}	b_{250}	c_{40}	c_{260}	c_{299}
		-0,0022	0,0025	0,0256	-0,1871	-51,7809	-25,5614	-14,3591	6912,2	-14775
	u	a_{80}	a_{150}	a_{173}	b_{133}	b_{260}	b_{282}	c_{175}	c_{274}	c_{297}
		0,000143	-0,00015	0,000055	-0,0130	-0,1513	-0,2246	6,4370	44,1310	-8,6863

By using the constants a_i, b_i, c_i, $i = 1,2,3,\dots,N$ obtained by using the Quadratic Spline Interpolation method, quadratic equations are obtained between both points. By using the equations, the normally unknown $v - u$ variables corresponding to $t = 2,4,6,\dots,N$ are calculated. Some of these values are expressed in Table 4. In addition, the calculated 300 unknown action potential values and the waveforms of the known values are given in Figure 3. The actual values for $t = 2,4,6,\dots,N$ of the FHN neuron model are shown with a red asterisk marker. In addition, the values calculated by the Quadratic Spline Interpolation method for $t = 2,4,6,\dots,N$ are expressed with a black round marker. When Figure 3 is examined, it is understood that the values are similar to each other.

Here, MAE, MSE, RMSE and NRMSE are used as error calculations. Obtained error values are given in Table 5. All values are expressed as percentages. Considering the NRMSE, it is seen that the error values for all samples are quite low. With the Quadratic Spline Interpolation method, the unknown action potential and recovery parameter values in the neuron were calculated with high accuracy. When Table 4 is examined, it has been observed that as N increases, unknown values are calculated with lower error values and results close to the original value are obtained. As stated in the Quadratic Spline Interpolation method equations 3 and 4, the previous values are also taken into account when calculating the a, b, c coefficients. Therefore, as N increases, the solution of the function is closer to the correct value as the coefficients are calculated with low error.

The calculated error values expressed in Table 5 are quite small. This showed that the Quadratic Spline Interpolation method can be used in the calculation of missing data in the neuron. However, it is understood that the recovery parameter u is calculated with a much lower error value than the membrane potential v. When NRMSE values are compared, v for SHN-Sp1 was calculated with an error of 0.2630%, while u was calculated with an error of 0.0524%. This difference is directly related to the waveform. v has a unique waveform, while u is more like a sinusoidal signal. Since the interpolation method is created with 2nd order equations, signals with sinusoidal-like waveforms are expected to be calculated with less error. Although there is some difference between the error values in the calculation of v and u, both variables were calculated with high accuracy.

Table 5. Calculated error values for the three samples generated

		MAE	MSE	RMSE	NRMSE
FHN-Sp1	v	0,0097	0,00009346	0,0097	0,2630
	u	0,0036	0,00001318	0,0036	0,0524
		MAE	**MSE**	**RMSE**	**NRMSE**
FHN-Sp2	v	0,009000	0,000132800	0,011500	0,2885
	u	0,000342	0,000000172	0,000413	0,0165
		MAE	**MSE**	**RMSE**	**NRMSE**
FHN-Sp3	v	0,0081	0,00009685	0,0098	0,2543
	u	0,0016	0,00000251	0,0016	0,0781

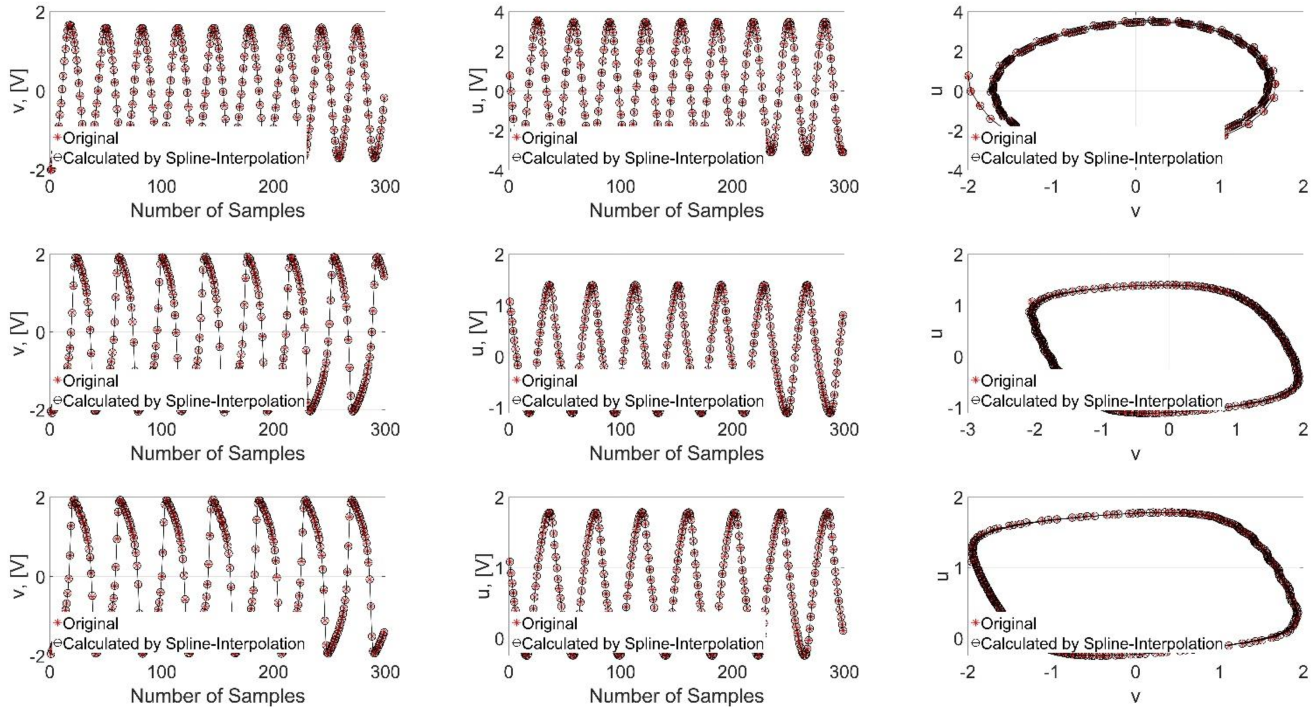

Figure 3. In the FHN neuron model, the red star shows the known value for the marker $t = 2,4,6,\ldots,N$ and the black round marker shows the values calculated by Quadratic Spline Interpolation method for $t = 2,4,6,\ldots,N$. Column 1 shows the action potential, column 2 shows the recovery parameter and Column 3 shows the $v - u$ phase portrait. a) FHN example 1, b) FHN example 2, and c) FHN example 3.

Table 4. Some of the known $v - u$ values calculated with Quadratic Spline Interpolation corresponding to the $t = 2,4,6,\ldots,N$ value

	N		**2**	**4**	**6**	**…**	**594**	**596**	**598**
FHN-Sp1	**Original**	v [V]	-2.0065	-1.9643	-1.8666	…	-0.7476	-0.4838	-0.1923
	Interpolation		-1.9968	-1.9739	-1.8569	…	-0.7379	-0.4935	-0.1758
	Original	u [V]	0.7614	-0.0186	-0.7459	…	-2.9528	-3.0753	-3.0846
	Interpolation		0.7650	-0.0223	-0.7424	…	-2.9492	-3.0790	-3.0924
	N		**2**	**4**	**6**	**…**	**594**	**596**	**598**
FHN-Sp2	**Original**	v [V]	-2,0586	-2,0692	-2,0284	…	1.6317	1.5329	1.4232
	Interpolation		-2,0375	-2,0880	-2,0089	…	1.6339	1.5307	1.4292
	Original	u [V]	1.0665	0.8697	0.6769	…	0.4689	0.6417	0.8023
	Interpolation		1.0659	0.8702	0.6764	…	0.4689	0.6417	0.8041
	N		**2**	**4**	**6**	**…**	**594**	**596**	**598**
FHN-Sp3	**Original**	v [V]	-1.9469	-1.8703	-1.8051	…	-1.5492	-1.4769	-1.4002
	Interpolation		-1.9529	-1.8652	-1.8104	…	-1.5549	-1.4712	-1.4123
	Original	u [V]	1.0810	0.9244	0.7782	…	0.2994	0.1983	0.1064
	Interpolation		1.0826	0.9229	0.7797	…	0.3009	0.1968	0.1082

4. Conclusions and Recommendations

The neuron undertakes the task of transmitting information in living things (Purves et al. 2019). There may be losses due to any environmental and internal noise in the transmission of information (Casado 2003; Faisal, Selen, and Wolpert 2008). As a result, neurological diseases may occur (Li et al. 2020). Preventing the loss of information or recalculating the lost data can have a corrective effect on the irregular operation of the unhealthy neuron. This study, it was aimed to prevent information loss that may occur in the neuron, data losses were created in the FHN neuron model and these data were recalculated with the Quadratic Spline Interpolation method. Quadratic Spline Interpolation is a method used to calculate nonlinear, 2nd order, unknown or unsolvable function values (Lunardi and Scuola normale Superiore (Italy) 2009; Werner 1984). In this study, three samples were created in the FHN neuron, the double sample values of the v and u variables were deleted and then recalculated with the Quadratic Spline Interpolation method. The waveforms for the $\boldsymbol{v}-\boldsymbol{u}$ variable for the 3 samples created are given in figure 3. In addition, NRMSE error was calculated as 0.2630-0.0524%, 0.2885-0.0165% and 0.2543-0.0781%, respectively, and other error values are given in Table 5. Since the $\boldsymbol{u}$ variable has a sinusoidal waveform and the interpolation method is of the 2nd order, it is calculated with a lower error value compared to the $\boldsymbol{v}$ variable. Despite this, the error values are quite small and the Quadratic Spline Interpolation method has become a new tool for recalculating the information lost in the neuron.

The development of the study is aimed to compare the results by using other interpolation methods such as Lagrange, Cubic, and Chebyshev. In addition, it is aimed to increase the neuron behaviour examples created and to apply these interpolation methods to other neuron models in the literature.

5. Acknowledge

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