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**ARAŞTIRMA MAKALESİ** **RESEARCH ARTICLE**

# Econometric Studies on Wheat Production and Wheat Market of Türkiye (Advanced Application of Verhulst Growth Function and Cobweb Theorems)

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## Abstract

In our study, we make economic analysis of wheat production and of the amounts traded together with the prices in Türkiye in an advanced econometrics and statistical framework. For this purpose, we create a data set of more than one hundred and ten years period using TurkStat data sources and explain the time trend of wheat production with the growth function. In the market research section, the relationships between supply, demand and prices are discussed and the time equilibrium paths of prices and quantities are determined with forty-year data and cobweb theorems. We then investigate the central points of price and quantity. To this end, we use the simple cobweb theorem with a single delay and Goodwin's theory that includes two delays and expectations. After determination of trend functions, cyclical fluctuations around the trend are determined with complementary functions. All functions have been solved simultaneously in computer enviroment. The results obtained with wheat data confirm the economic patterns of production and the market. Then, according to the equations obtained in the Goodwin model, we create the cobweb curve by simulating quantities and prices. We used econometrics and statistics procedures of SAS library for study.

*Keywords: Cobweb Theorem, Wheat Production of Türkiye, Goodwin, Econometry*

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# Türkiye Buğday Üretimi ve Buğday Piyasası Üzerine Ekonometrik Çalışmalar (Verhulst Büyüme Fonksiyonu ve Örümcek Ağı Teoremlerinin İleri Uygulaması)


**Özet**

Çalışmamızda ileri ekonometri ve istatistik yöntemlerle, Türkiye'deki buğday üretiminin ve fiyatlarla birlikte işlem gören miktarların ekonomik analizini yapıyoruz. Bu amaçla TurkStat veri kaynaklarını kullanarak yüz on yılı aşkın bir veri seti oluşturup buğday üretiminin zaman trendini büyüme fonksiyonuyla açıklıyoruz. Piyasa araştırması bölümünde, kırk yıllık veriler ve örümcek ağı teoremleriyle arz, talep ve fiyatlar arasındaki ilişkiler ele alınmakta fiyatlar ve miktarların zaman denge patikaları tespit edilmektedir. Daha sonra fiyat ve miktarın merkez noktalarını araştırmaktayız. Bu amaçla tek gecikmeli basit örümcek ağı teoremini ve Goodwin'in iki gecikmeli ve bekleyişleri de içeren örümcek ağı teorisini kullanıyoruz. Trend tespitinden sonra, trend etrafındaki konjonktürel dalgalanmalar tamamlayıcı fonksiyonlarla tespit edilmektedir. Bütün fonksiyonlar bilgisayar ortamında eşanlı olarak çözülmüştür. Buğday verileriyle elde edilen sonuçlar üretim ve piyasaya ait ekonomik modelleri teyit etmektedir. Daha sonra Goodwin modelinde elde edilen denklemlere göre fiyat ve miktarların simülasyonuyla örümcek ağı eğrilerini oluşturmaktayız. Çalışmamızda SAS kütüphanesinin ekonometri ve istatistik prosedürleri kullanılmıştır.


*Anahtar sözcükler: Örümcek Ağı Teoremi, Türkiye Buğday Üretimi, Goodwin, Ekonometri*

## 1. Introduction and Scope of Research

The aim of this study is to carry out the *production and market analysis of wheat agriculture* which is a long-standing economic activity in the geography of Türkiye, with TurkStat data and according to the principles of economic theory, mathematics, econometric analysis and advanced statistical applications. With the mathematical modeling we made regarding wheat production and the market, the abstractly mathematical representation of Turkish wheat data was aimed and state of affairs was tested with advanced statistical study. After the successful results obtained in the mathematical representation and statistical proof phases, we did not make predictions for the future. We were content to give only brief opinions. With data and modeling, we aimed to stay within the determination of the current situation.

Our econometric study, which consists of two parts as production and market, covers the following details:

a) In the wheat production time series study, the nonlinear univariate Verhulst growth function applied for statistical tests.
b) In market section, first the time-based trends of quantities and prices then cyclical fluctuations around these trends are investigated.
c) Then, simultaneous solutions of quantity and price functions ascertained on the basis of simple cobweb and Goodwin's advanced cobweb theory based on expectations and two-phase delay.
d) Lastly, comments and results are presented.

In accordance with our aims, we have been determining trend and oscillations of the wheat production covering a period of more than a hundred years (1909-2020), ascertaining cobweb theory of price and quantity movements going back to a forty-year history of 1980-2020 and also including the foreign trade statistics with its twenty-year history (2000-2020).

## 2. Source and Explanation of Data Used

One of the oldest and most healthy statistics which goes back to Ottoman period in Türkiye is the amount of land sown and production of cereals and grains. TurkStat, which is at the same age as with Turkish Republic, has kept and published statistics for this group in a regular way ever since its establishment. Statistical publications series that we used in this research are "Statistical Yearbook of Turkey", "Statistical Indicators" and "The Summary of Agricultural Statistics". On the other hand, TurkStat also added agricultural statistics of Ottoman Period to its publications in 1997 with a separate book (Güran 1997). In this way, we have more than a century of data (1909-2020) of Turkish wheat agriculture.

## 3. Statistics and Econometry Application Procedures of SAS/STAT® and SAS/ETS®

With the advances made in statistical science over the last half century, the tests applied in data analysis have increased in addition to traditionel t-test, F-test, Durbin-Watson, correlation, determination coefficients etc. To mention a few, Godfrey's serial correlation, Shapiro-Wilk normality, and the application of generalized Durbin-Watson tests have gained space and become standardized. Theoretical statistical test studies on the structure of the mathematical functions used have also increased. On the other hand, in order for nonlinear mathematical functions to have sound results in statistical application, the conformity tests of these functions to the linearity have been developed. These include skewness, bias and global nonlinearity applications[1]. If the function does not appear to be linear when these tests are applied, then the necessary precautions should be taken to ensure linearity by mathematical transformations, simply by logarithmic and exponential applications, otherwise, the parameters found will be far from full representability. In our wheat study, in addition to the known tests, the volume of our article is expanding because we added the tests listed here to our outputs. However, we wanted a highly respectable and referrable econometric study and included tables in article as much as possible. Although we worked most of the estimation methods mentioned in the explanation of the SAS MODEL procedures below, but only included the "Full Information Maximum Likelihood" (FIML) estimation results here to be short. On the other hand, also collinearity tables, which take up a lot of space according to the volume of the article and the number of parameters, are not included in the article.

Econometric and statistic procedures of the SAS STUDIO library used are:

a) SAS PROC AUTOREG PROCEDURE:

   The AUTOREG procedure estimates and forecasts linear regression models for time series data when the errors are autocorrelated or heteroscedastic. The autoregressive error model is used to correct for autocorrelation, and the generalized autoregressive conditional heteroscedasticity (GARCH) model and its variants are used to model and correct for heteroscedasticity. (SAS Institute Inc. 2014,p. 302)

b) SAS PROC MODEL PROCEDURE:

   The MODEL procedure analyzes models in which the relationships among the variables form a system of one or more nonlinear equations. (SAS Institute Inc. 2018a,pp. 1423–1424)

PROC MODEL uses mainly following statistical calculation methods:

Ordinary least squares (OLS),

Seemingly unrelated regression (SUR) and iterative SUR (ITSUR),

Full information maximum likelihood (FIML),

Two-stage least squares (2SLS),

Three-stage least squares (3SLS) and iterative 3SLS (IT3SLS),

Generalized method of moments (GMM),

Simulated method of moments (SMM),

We extensively used test results of below given headings of SAS Model procedure:

Collinearity Diagnostics,

Nonlinear Summary of Parameter Estimates and Residual Errors,

Heteroscedasticity Test,

Godfrey's Serial Correlation Test,

Durbin-Watson Statistics,

Normality Test,

Structural Change Test: Chow test and

Dynamic Equation Simulation -Theil Statistics.

c) SAS PROC NLIN PROCEDURE: Nonlinear functions

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[1]See (Gebremariam 2014; İskender 2018, pp. 103–105; SAS Institute Inc. 2020,pp. 6956–6964)

> The NLIN procedure fits nonlinear regression models and estimates the parameters by nonlinear least squaresor weighted nonlinear least squares…Nonlinear least-squares estimation involves finding those values in the parameter space that minimize the (weighted) residual sum of squares. (SAS Institute Inc. p. 6956, 2020)

NLIN procedure used in Verhulst groth function.

During our study, following statistical procedures also used when necessary.

d) SAS PROC GLM PROCEDURE
e) SAS PROC CAPABILITY PROCEDURE
f) SAS PROC UNIVARIATE PROCEDURE
g) SAS PROC ROBUSTREG PROCEDURE

## 4. Analysis of Turkish Wheat Production

With the increase in the amount of land sown since the establishment of the Republic, wheat production which is the main grain product, has had a continuous linear increase until 1975, after which the amount of land planted remained constant or decreased until 1995, while production amounts continued to increase with the increase in productivity, and since 1988 to the present day it has been fluctuating around nineteen million metric tons[2] per year. Depending on various factors, it is normal for agricultural products to fluctuate: especially weather and rain are the two most effective factors. The use of up to ten million hectares of arable land (1998) has decreased to 6.8 million hectares since then. The share of wheat in total arable grain land has been in the range of 60-70% in the last three decades. On the other hand, productivity per hectare starting from one tonne has reached to 2.96 tons today.

Since it is the primary source of nutritional needs as a traditional product and production is parallel to domestic consumption, it is worth explaining wheat production together with Turkey's population development. Turkey is not a major wheat exporter country. According to the annual consumption of production, imports were marginally realized in the years when supply was inadequate, and exports were marginal in the case of surplus supply. In other words, it is right to take wheat production as a function of the growing population. On the other hand, with the increase in per capita income, we also see that wheat production and consumption per population decrease with the increase in consumption of alternative and expensive nutrition such as; meat, milk and derivatives, vegetables and fruits, etc. It is possible to see the situation from Figure 2. Per capita production which increased up to 400 kg in the period of1965-1985, decreased steadily after this date to 200 kg.

Direct wheat export figures of Turkey are low: the figures for 2019 and 2020 are 135 and 125 thousand tons respectively. However, Turkey has imported wheat for input purposes in recent years and exported flour, pasta, bulgur, semolina biscuits etc. in significant quantities. Wheat imports increased from 3.8 million tons in 2013 to 9.6 million tons in 2020. As in export figures, also the minor parts of annual imports go to domestic consumption, but we know that most of them are used as inputs and exported as products. I aim to keep import and export information at this level, which is not our main topic.

[2] Throughout the article, the measure of ton means metric ton.

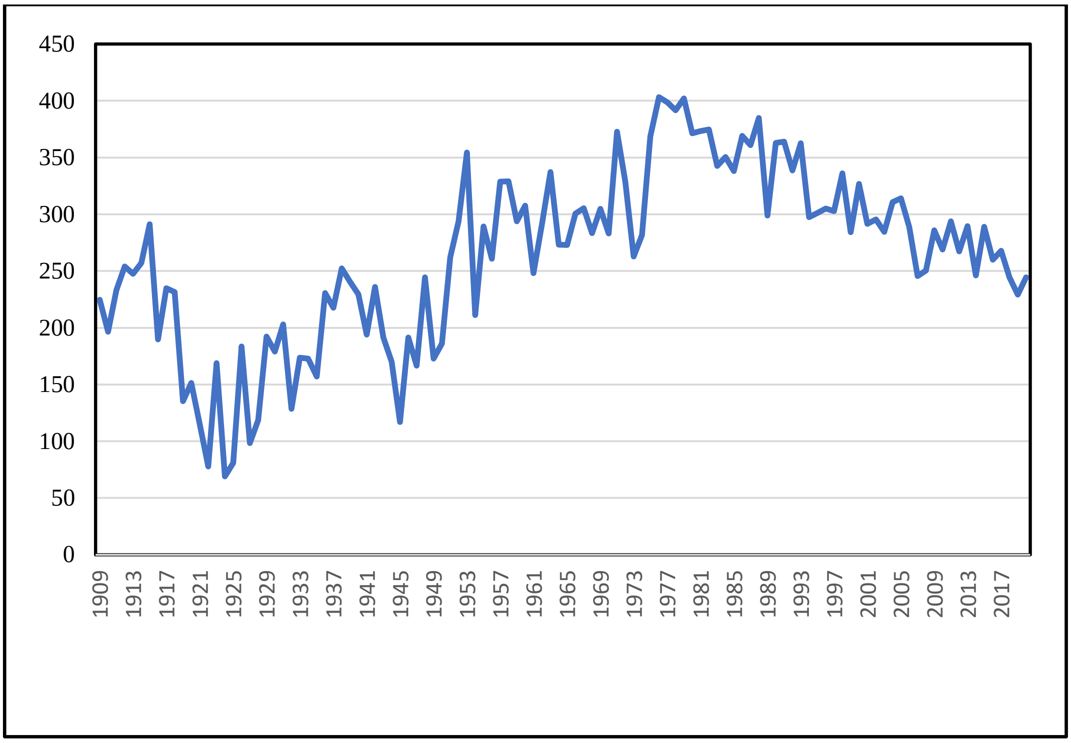


**Figure 1:** Per capita wheat production of Türkiye (Kg)

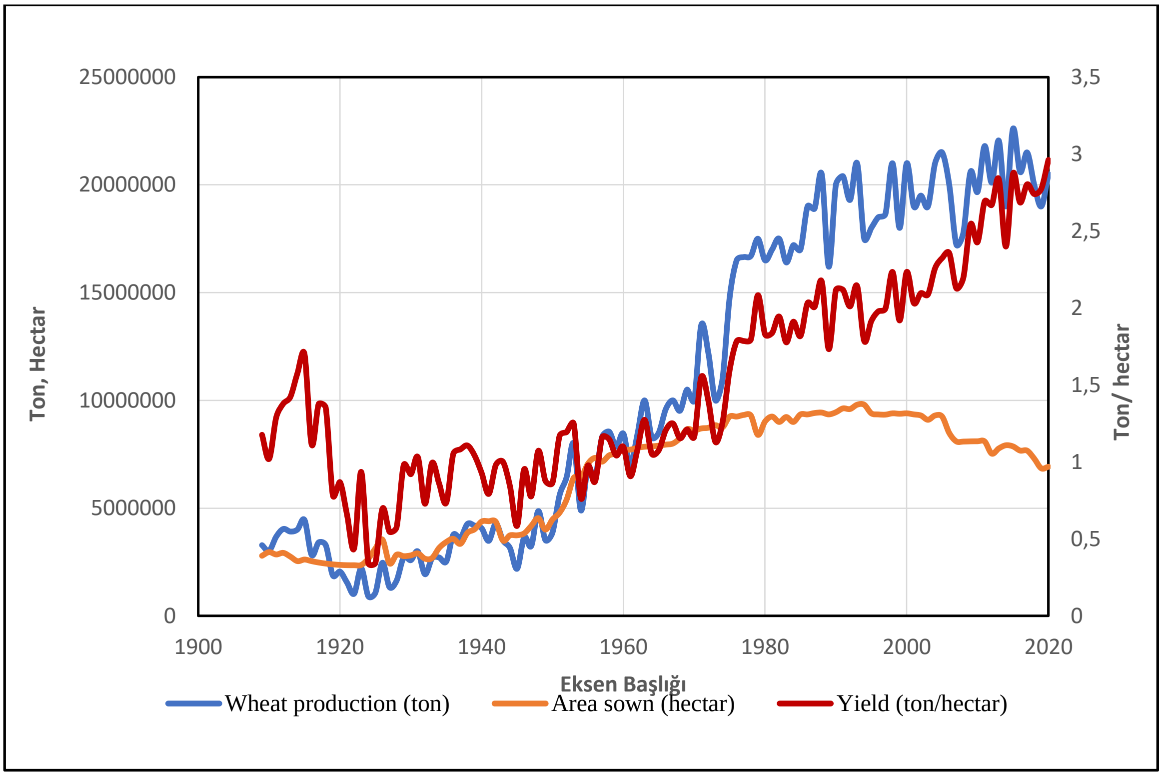


**Figure 2:** Wheat production and area sown of Türkiye (Production and area sown are shown at the lefthand side scale and yield on the right).

In mathematical modeling we applied Verhulst growth function and explained the development of wheat production in the geography of Türkiye. We have a data set of hundred and twelve years for the period 1909-2020 to use in modeling. TurkStat sources have a complete production data for the period 1924-2020. In a publication of TurkStat, we obtain statistics of the Ottoman Period of the years 1909, 1913 and 1914 (Güran

1997). Then we made estimates for the periods 1910-12 and 1915-1923 which cover extraordinary circumstances and have no data: For this purpose, we re-estimated the annual data for two periods applying Microsoft Excel (randbetween function) random numbers function in the range of plus minus seven hundred and fifty thousand tons to the function that is obtained from the fifth-degree polynomial curve fitting of the data of 1909-1950 period.

One of the reasons we included pre-republic period production figures in research is to obtain the data set of the lower asymptote region of the mathematical curve we use and obtain positive results from statistical application. In this way, the statistical application of growth curve has been successful. If we had subjected the wheat production figures for the period 1925-1988 to statistical analysis, there would have been a linear or close to linear production increase which will not reflect reality of the trend. However, adding statistics of Ottoman period and considering that the long-term production conjuncture has reached the upper asymptote region, it would be more accurate to think that wheat production is on a course following the Verhulst growth curve.

During our studies, we saw that the four-parameter Verhulst growth function would be suitable for ascertaining growth in time series analysis.

Symbols used:

*Y*: Wheat Production (dependent variable),

*t*: Time, (explanatory variable),

*Z*: Base year, 1940,

*K*: Upper asymptote,

*L*: Lower asymptote,

*r:* Intrinsic rate of growth,

*Q*: Coefficient of exponential growth base ($e$).

Verhulst function is a symmetric, non-linear structured mathematical growth function with two-variable, four-parameters (*K*, *L*, *Q*, *r*):

Function[3]:

$$Y_{(t-T)} = L + \frac{K - L}{1 + Qe^{-r(t-\mathrm{Z})}} \qquad \text{Eq (1)}$$

According to the statistical study, with 10% growth, the lower level of 2.6 million tons has been reached to the point where it is the upper level of 20.5 million tons. Production is not expected to grow any further. In more than a hundred years of data we used, significant weather-related fluctuations have been observed for every year around the main trend. Since we did not have series on the weather, especially about precipitation and drought, we could not include it as an independent variable in the function. Over the last half century weather-related fluctuations have continued around the trend in the band of plus minus 1.5 million tons. Time series explains 97.11% of the production. Trend growth, which was in a continuous increase until 1969, started to decrease from 1969 and reached zero growth at the level of 20.5 million tons. The price effect, which is important in balancing wheat production at the average trend level of 19.1 million tons, will be discussed in the market section. Wheat production rarely exceeded 4 million tons from 1909 to 1945, depending on weather conditions. The production capacity before the period tractors started to use in agriculture is around four million tons. From 1945 onwards, production has accelerated with the use of tractors and other agricultural equipment, and from 1960 onwards also with the increase in fertilizer and spraying usage. As we will see in the market section below, we expect production to continue at the trend of 19.1 million tons with the current domestic market wheat price, input prices and world wheat prices.

Wheat production function[4]:

[3] See (İskender, 2018, 2021a) for mathematical properties of function.

[4] Logarithmic and exponential equivalents of some figures used to achieve linearity.

$$Y_{(t-T)} = 2645954 + \frac{e^{16.8349} - 2645954}{1 + \mathrm{e}^{e^{1.1284}} e^{(e^{-2.2463}(t-1940))}} \quad \text{Eq (2)}$$

T=1940

Upper asymptote: $e^{16.8349} = 20478784$ tons

Starting level: $\mathrm{e}^{e^{1.1284}} = 21.99$ tons

Intrinsic rate of growth: $e^{-2.4463} = 0.1058$

Lower asymptote: 2654954 tons.

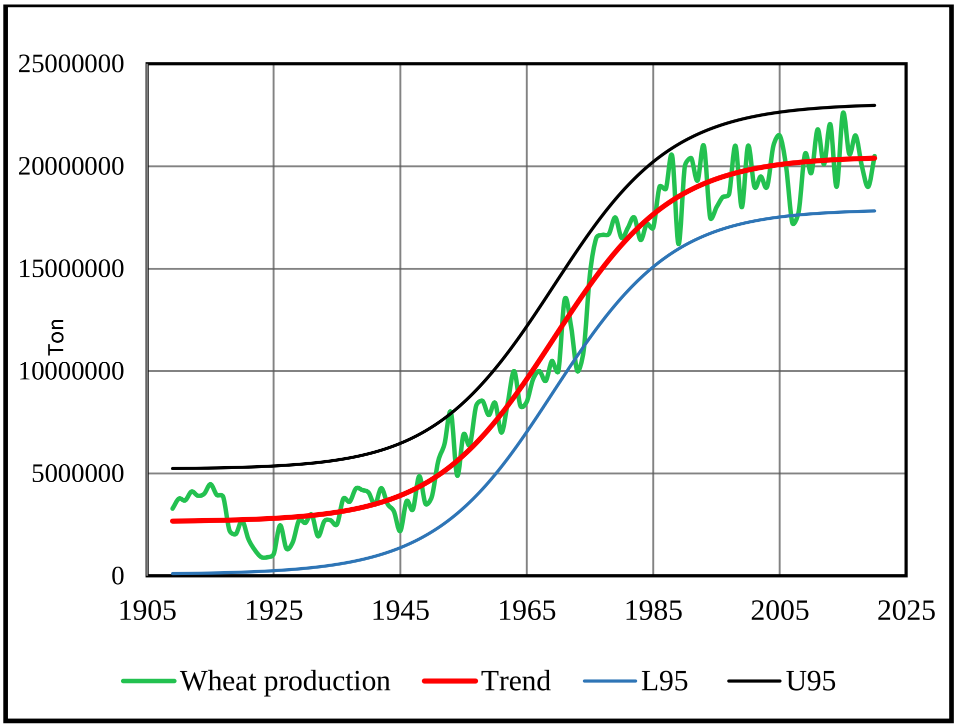


**Figure 3:** More Than a Century Wheat Production Trend of Türkiye

Linearity test results -skewness, bias and global nonlinearity measures- of Verhulst function given in Tables 1-3 below.

The guideline, according to Ratkowsky (1983), is that percentage bias greater than 1% is considered to be significantly nonlinear. Similarly, a value of the standardized Hougaard skewness measure greater than 0.25 in absolute value indicates nonlinear behavior. (Gebremariam 2014, p. 5)

According to Table 2, the skewness numbers of the parameters are below 0.25, bias values are below 1%, and global nonlinearity measures (table 3) are below curvature critical value which is 0.6381. Verhulst growth curve is very close to linear at the evaluation of Turkish wheat productin..

**Table 1:** Statistics of SAS NLIN Procedure

| Source | DF | Sum of Squares Error | Mean Square | F Value | Approx Pr > F |
|---|---|---|---|---|---|
| **Model** | 3 | 5.853E15 | 1.951E15 | 1208.65 | <.0001 |
| **Error** | 108 | 1.743E14 | 1.614E12 | | |
| **Corrected Total** | 111 | 6.028E15 | | | |

**Table 2:** Parameter and Linearity Estimates of NLIN Procedure

| Parameter | Estimate | Approx Std Error | t value | Skewness | Bias | Percent Bias |
|---|---|---|---|---|---|---|
| **L** | 2645954 | 256006 | 10.33552 | -0.0984 | -8021.4 | -0.30 |
| **Log(r)** | -2.2463 | 0.0732 | -30.6872 | 0.0126 | 0.00160 | -0.07 |
| **Log(log(Q))** | 1.1284 | 0.0770 | 14.65455 | 0.00281 | 0.00145 | 0.13 |
| **Log(K)** | 16.8349 | 0.0147 | 1145.231 | 0.1061 | 0.000451 | 0.003 |

**Table 3:** Global Nonlinearity Measures

| | |
|---|---|
| Max Intrinsic Curvature | 0.1307 |
| RMS Intrinsic Curvature | 0.0582 |
| Max Parameter-Effects Curvature | 0.2854 |
| RMS Parameter-Effects Curvature | 0.1241 |
| ***Curvature Critical Value*** | 0.6381 |
| Raw Residual Variance | 1614E9 |
| Projected Residual Variance | 1567E9 |

At the second stage, advanced statistical tests of both trend function and also complementary functions are added in order to explain sinusoidal variations around the trend line. Mathematical model of complementary function used is a two-component sinusoidal function and symbols as follows:

$F_t$ *:* Conjunctural oscillations, Dependent variable,

*t-Z:* Time, independent variable adjusted with base Z,

$\pm A$: Initial amplitude of oscillation or peak and trough coefficient,

*r*: Damping or anti-damping multipliers, growth rates,

$\theta_1$ , $\theta_2$ are angular frequencies,

$\varepsilon_1$, $\varepsilon_2$ are phase lags

And numerical form.

$$F_t = A_1 e^{(r_1(t-Z))} cos(\theta_1(t-Z)+\varepsilon_1) + A_2 e^{(r_2(t-Z))} cos(\theta_2(t-Z)+\varepsilon_2) \quad \text{Eq (3)}$$

With the addition of complementary function, the sum of square error decreased from 1.743E14 to 1.2E14. The function accounts for 98% of production. Although we tried various mathematical models, it has not been possible to further reduce the sum of errors in production which followed a very fluctuating course. Thoroughout the statistical study we added four more variables to eliminate the effects of autogression.

Trend function (particular integral, equilibrium path)[5]:

$$\bar{Y}_t = 2755058 + \frac{e^{17.02301} - 2755058}{1 + e^{e^{1.087176}} e^{(e^{-2.30403}(t-1940))}} \quad \text{Eq (4)}$$

Two-phase complementary function:

$$F_t = -400000e^{(0.0104(t-1940))}cos(0.50795(t-1940) + 2.635506) + 1150000e^{(-0.0187(t-1940))}cos(0.288015(t-1940) + 0.917351) \quad \text{Eq (5)}$$

$A_1 = -400000$ , $A_2 = 1150000, r_1 = 0.0104$ and $r_2 = -0.0187$ taken as given.

,Autoregression corrections:

$$AR_t = 0.599928\text{W\_l1} - 0.79948\text{W\_l2} + 0.525013\text{W\_m1} - 0.71312\text{W\_m2} \quad \text{Eq (6)}$$

General solution (Complete primitive) consists of addition of three functions:

$$Y_t = \bar{Y}_t + F_t + AR_t \quad \text{Eq (7)}$$

SAS MODEL procedure FIML application Tables 4-8:

**Table 4:** Nonlinear FIML Summary Table of Residual Errors

| Equation | DF Model | DF Error | SSE | MSE | Root MSE | $R^2$and R | Adj $R^2$ | Durbin Watson |
|---|---|---|---|---|---|---|---|---|
| Wheat | 12 | 100 | 1.2E14 | 1.2E12 | 1095593 | 0.9801<br>0.9900 | 0.9779 | 2.1597 |

[5] Since SAS MODEL procedure applies autoregression corrections, it is clear that the trend function parameters of eq. (4) differ slightly from those in NLIN application eq (2).

**Table 5:** Parameter Estimates

| Parameters | Estimates | Approx Std Err | t Value | Approx Pr > \|t\| | Label |
|---|---|---|---|---|---|
| L | 2755058 | 626481 | 4.40 | <.0001 | Lowest production level of wheat, |
| Log(log(Q)) | 1.087176 | 0.0790 | 13.77 | <.0001 | Exp of exp of e base coefficient |
| Log(r) | -2.30403 | 0.0688 | -33.47 | <.0001 | Log of intrinsic rate of growth |
| Log(K) | 17.02301 | 0.0970 | 175.52 | <.0001 | Highest level of production, |
| $\theta_1$ | 0.50795 | 0.0107 | 47.64 | <.0001 | First angular frequency |
| $\varepsilon_1$ | 2.635506 | 0.5931 | 4.44 | <.0001 | First phase lag |
| $\theta_2$ | 0.288015 | 0.00779 | 37.00 | <.0001 | Second angular frequency |
| $\varepsilon_2$ | 0.917351 | 0.2592 | 3.54 | 0.0006 | Second phase |
| W_l1 | 0.599928 | 0.1573 | 3.81 | 0.0002 | Lag1 parameter of dependent variable wheat |
| W_l2 | -0.79948 | 0.1491 | -5.36 | <.0001 | Lag2 parameter of dependent variable wheat |
| W_m1 | 0.525013 | 0.1906 | 2.76 | 0.0070 | MA lag1 parameter of residuals (error term) |
| W_m2 | -0.71312 | 0.1733 | -4.11 | <.0001 | MA lag2 parameter of residuals (error term) |

*Notes of Table 5: a) Since all the p-values of parameters are lower than confidence level of α=0.05, the null hypothesis that parameters meaningless is rejected. b) Last four parameter applied for autoregressive correction of series. W_l1 and W_l2 are applied to dependant variable wheat instead of structurel residuals of equation. c) For the definitons and details of AR: Autoregressive Errors and MR: Moving-Average Models see (SAS Institute Inc. 2018a, pp. 1565–1580).*

**Table 6:** Heteroscedasticity Test

| **Equation** | **Test** | **Statistic** | **DF** | **Pr > ChiSq** | **Variables** |
|---|---|---|---|---|---|
| **Wheat** | White's Test | 105.3 | 89 | 0.1143 | Cross of all variables |
| | Breusch-Pagan | 0.36 | 1 | 0.5484 | 1, E1 |

*Note to Table 6:*

$$E = 1 + e^{e^{1.087172}}\, e^{e^{-2.30403(t-1940)}}$$

$$E1 = E + F$$

*The White test tests the null hypothesis. $H_0$: $\sigma_i^2 = \sigma^2$ for all i. The p-values>α=0.05 means that there is no heteroskedasticity.*

**Table 7:** Serial Correlation and Autocorrelation Tests

| Godfrey's Serial Correlation Test | | | | Generalized Durbin-Watson Statistics | | | | |
|---|---|---|---|---|---|---|---|---|
| **Equation** | **Alternative** | **LM** | **Pr > LM** | **Equation** | **Order** | **DW** | **Pr < DW** | **Pr > DW** |
| **Wheat** | 1 | 1.81 | 0.1780 | **Wheat** | 1 | 2.16 | 0.4964 | 0.5036 |
| | 2 | 3.30 | 0.1920 | | 2 | 1.76 | 0.0654 | 0.9346 |
| | 3 | 3.34 | 0.3424 | | 3 | 2.18 | 0.8133 | 0.1867 |

*Note of Table 7:a) There is no serial or auto correlation for p values greater than α=0.05 in Table 7. b) Pr < DW is the p-value for testing positive autocorrelation and Pr >DW is the p-value for negative autocorelation, also at Tables 16 and 23.*

**Table 8:** Normality Tets

| **Equation** | **Test Statistic** | **Value** | **Prob** |
|---|---|---|---|
| **Wheat** | Shapiro-Wilk | 0.98 | 0.1400 |
| **System** | Mardia Skewness | 1.92 | 0.1657 |
| | Mardia Kurtosis | 0.04 | 0.9691 |
| | Henze-Zirkler T | 0.59 | 0.1361 |

*Note of Table 8: The null-hypothesis of this test is that the population is normally distributed. Thus, if the p value is less than the chosen alpha level (which is 0.05 in our case), then the null hypothesis is rejected and there is evidence that the data tested are not normally distributed. https://en.wikipedia.org/wiki/Shapiro%E2%80%93Wilk_test*

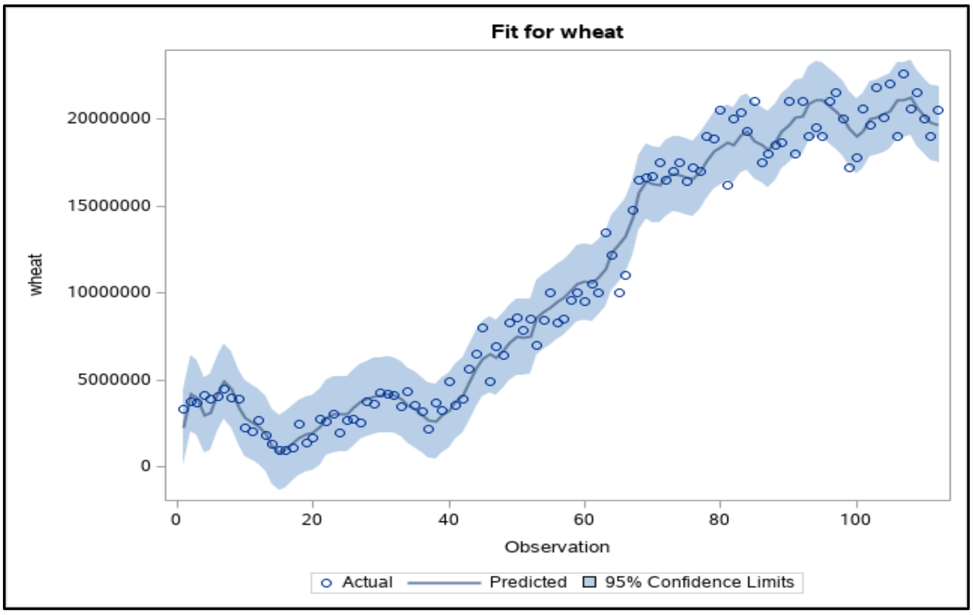


**Figure 4:** Curve Fitting of Wheat Production (SAS Model Procedure output)

# 5. Quantities and Prices: General Wheat Market Equilibrium of Türkiye

## 5.1. Methods

In this part of our study, we investigated the relationship between real wheat prices and traded quantities, with linear and nonlinear functions, simple dynamic cobweb theory, and also advanced cobweb model of Goodwin which includes producer expectations and two period lag of prices. For this purpose, we have compiled wheat producer prices, Producers Price Index (PPI), wholesale price index, production quantities, direct and indirect import and export statistics from the sources of TurkStat and made them ready for our analysis. Our statistical study cover the 1982-2020 period. We also included data from 1980 and 1981 where delayed variables were required. In this way, we have determined the price and supply-demand (traded quantities) trend of the wheat economy with reliable statistical applications for a period of forty years.

Since wheat production in Türkiye is essentially a primary product intended to meet domestic household food consumption and approximately meets the need in quantity, imports and exports figures of Türkiye have remained at marginal levels over decades. Therefore, ignoring the export and import figures and also stocks of which we do not have figures, we have assumed *production = domestic consumption* for the period 1982-1999.

Since 2000, for Türkiye's wheat imports have shown a serious upward trend as a result of growing export-oriented industries -flour, pasta, bulgur, semolina, biscuits and wafers producers- using wheat and flour as intermediate inputs, we have included direct wheat imports and exports figures, and wheat equivalents of exports and imports of the products listed above in the calculations in our work for the period 2000-2020. This approach have aslo been followed by the authors in sector (Polat 2020;Tarım ve Orman Bakanlığı, 2020, 2021). By defining *production +direct import-direct export-indirect export*, we obtained the 2000-2020 series and together with the data of 1982-1999 period, acquired thirty-eight-year quantity series as basis for statistical study.

We studied wheat prices for the period 1982-2020 that compiled from the web pages of Soil Produces Office of Türkiye (TMO), Polatlı Commodity Exchange, Konya Commodity Exchange and TurkStat, and observed that the series were very close to each other and decided to use TurkStat wheat prices together with the PPI (producer price index) index for statistical application. Starting from 1982, we obtained annual real wheat prices as *TurkStat's current price data received by wheat producer /Producer price index (PPI)* and matched this series with quantity figures. Our statistical calculations have been based on these two series.

In the market section of our article, which is already large enough, the linearity studies of the functions are not included. The statistical issues that will extend to the theoretical studies that we have faced in sinusoidal functions have also been effective in this decision.

## 5.2. Determination of Price and Quantity Trends of Wheat Market

The quantities traded in the market and the corresponding prices for these amounts for the period under review are given in Figure 5. The conclusions we obtained from Figure 5 and statistical study results are as follows:

The average of the real wheat price for the period 1982-2020 is 3.1 (*standart deviation: 0.29*) real lira. Over the period of nearly four decades, prices have hovered around this average. The relative share of wheat producers in the economy has not changed from price point of view. There have been years when the actual price has fallen as far below the average as it has been the years when it has risen above average. The real price, which was 3.1 in 1982, is still 3.1 real lira in 2020. Wheat prices, which are in the category of prices monitored by public institutions, have maintained their relative share in the economy. Although it retains its share in the economy, the first half of the 1982-2000 is the period of increase in the average price, while the second half 2001-2020 is the period of decline of the average price: According to our calculations, the actual price is 3.02 real lira in the second half, which was 3.18 real lira in the first period. The price, which fell below 3 real liras in the 2017-19 period, has recovered in 2020 and reached the average. The best year is 1996. This actual price has not been reached in subsequent years. The direction of the trend is still downwards. It is necessary to make a trend detection again with the data of the coming years. The outlook for manufacturers is not good. The state of affairs that is in accordance with economic theory is that prices fluctuate around the average over a long period of nearly forty years. This is the presumption of economic theory. It will be discussed in detail below. It is our personal opinion that the real price will be below average or at most as average in the coming periods.

On the other hand, it is also clear that the price of wheat in Türkiye has been under the influence of the world wheat price of $ 250 / tons. Imports will become attractive when the domestic price rises. The producer's reaction to the lack of real prices was to reduce the amount of land cultivated, in other words, to exclude the land from production that did not cover the cost of 3.1 liras. The amount of land cultivated in 2000 decreased from 9.4 million hectares to 6.9 million hectares in 2020, and it has been possibile to continue production at the level of 19.1 million tons in the same period with the increase in productivity. The hectare yield which was 2.23 tons in 2000 increased to 2.96 tons in 2020 (see Figure 2).

Goodwin wrote:

> It is always assumed that decisions to produce lead to the same output at a later date, but this is not true, especially in agriculture, where the output of most crops is heavily affected by the weather. Thus acreage may consistently be controlled by prices, but yield per acre will have a nonsystematic or random character. (Goodwin 1947,pp. 186–187)

The39-year average of the supply and demand quantities is 19,125 thousand tons and this figure is the functional value of wheat supply and demand trends. Although this is the average, the amount increased from 18.66 million tons in the first half of the period to 19.56 million tons in the second half. There is an upward trend. During the period in question, traded amounts followed a sinusoidal course around the average amount but did not depart from the trend. Although there is an increase in the second half of the period, it is too early to see this as a trend change.

When Figure 5 is examined, it will be seen even with the naked eye that both price and quantity trends fluctuate sinusoidally around a fixed and horizontal axis of their own, each time they turn to trend lines – the invisible hand carefully manages state of affairs – and that cyclical periods are almost equal. This course of the charts has highly forced our econometric studies and has only allowed the *determination coefficient* in price and quantity functions to be obtained as 0.71 and 0.66, respectively in simple cobweb study. It's almost as if there is seemingly no relation between the two variables! Or everyone agree with administrative prices.

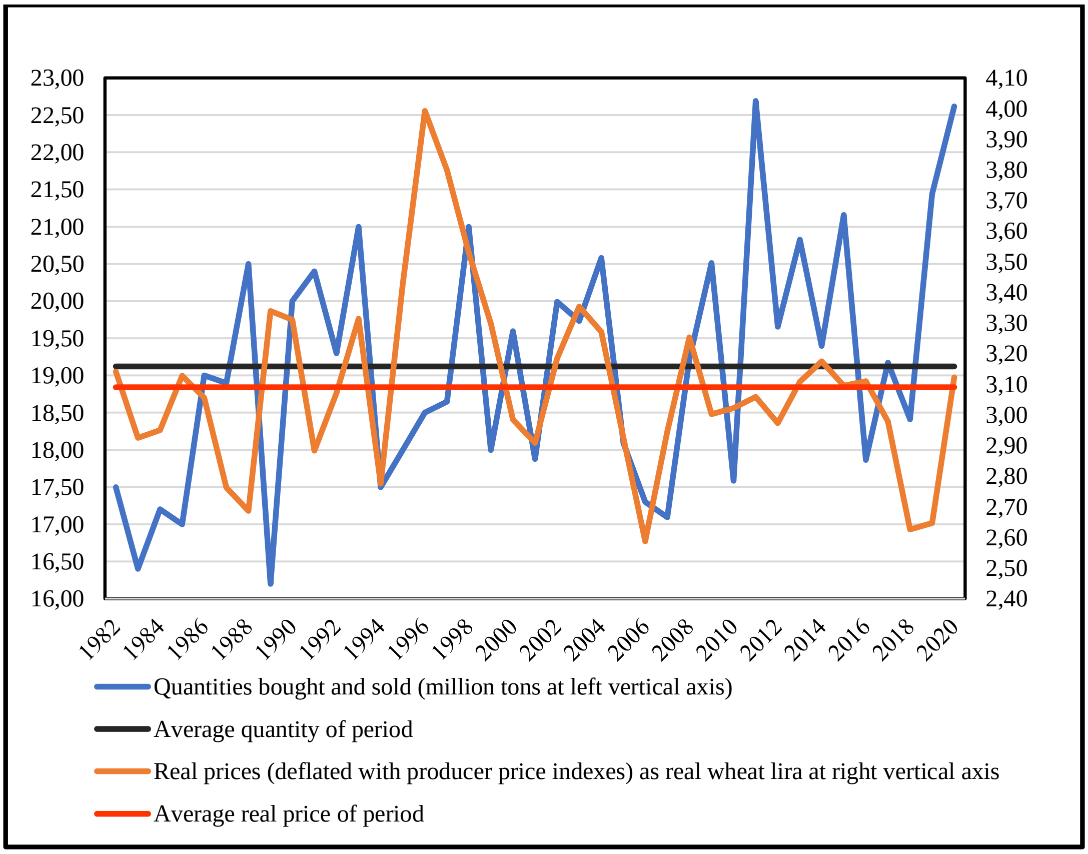


**Figure 5:** Wheat Quantities Bought and Sold and Price Trends of Türkiye

**Table 9:** Price Statistics of Wheat

| | | | |
|---|---|---|---|
| **N** | 39 | **Sum Weights** | 39 |
| **Mean** | 3.09539667 | **Sum Observations** | 120.72047 |
| **Std Deviation** | 0.29176979 | **Variance** | 0.08512961 |
| **Skewness** | 0.84673567 | **Kurtosis** | 1.71768165 |
| **Uncorrected SS** | 376.912666 | **Corrected SS** | 3.23492514 |
| **Coeff Variation** | 9.42592562 | **Std Error Mean** | 0.04672056 |

**Table 10:** Normality Test of Wheat Price

| Test | Statistic | | p Value | |
|---|---|---|---|---|
| **Shapiro-Wilk** | **W** | 0.947530 | **Pr < W** | 0.0679 |

**Table 11:** Quantity Statistics of Wheat

| **N** | 39 | **Sum Weights** | 39 |
|---|---|---|---|
| **Mean** | 19.1247692 | **Sum Observations** | 745.866 |
| **Std Deviation** | 1.63533208 | **Variance** | 2.67431102 |
| **Skewness** | 0.26503539 | **Kurtosis** | -0.5607078 |
| **Uncorrected SS** | 14366.1389 | **Corrected SS** | 101.623819 |
| **Coeff Variation** | 8.55085917 | **Std Error Mean** | 0.26186271 |

**Table 12:** Normality Test of Wheat Quantities

| Test | Statistic | | p Value | |
|---|---|---|---|---|
| **Shapiro-Wilk** | **W** | 0.976315 | **Pr < W** | 0.5712 |

A point that we think will be of interest to the reader about the prices is the following Figure 6 comparing the prices received by the selected countries'producers': The wheat producer prices of Türkiye, United States, Australia, Canada and Argentina included in chart and also producer prices deflated with PPI are given in Figure 6. While Türkiye's producer price deflated by the dollar exchange rate of Turkish lira followed these countries' producer prices, the producer prices deflated with PII remained constant at the level of 3.1. In last fifteen years they caught each other. This means Turkish producers have lost their real lira relative advantage when compared with dollar based prices.

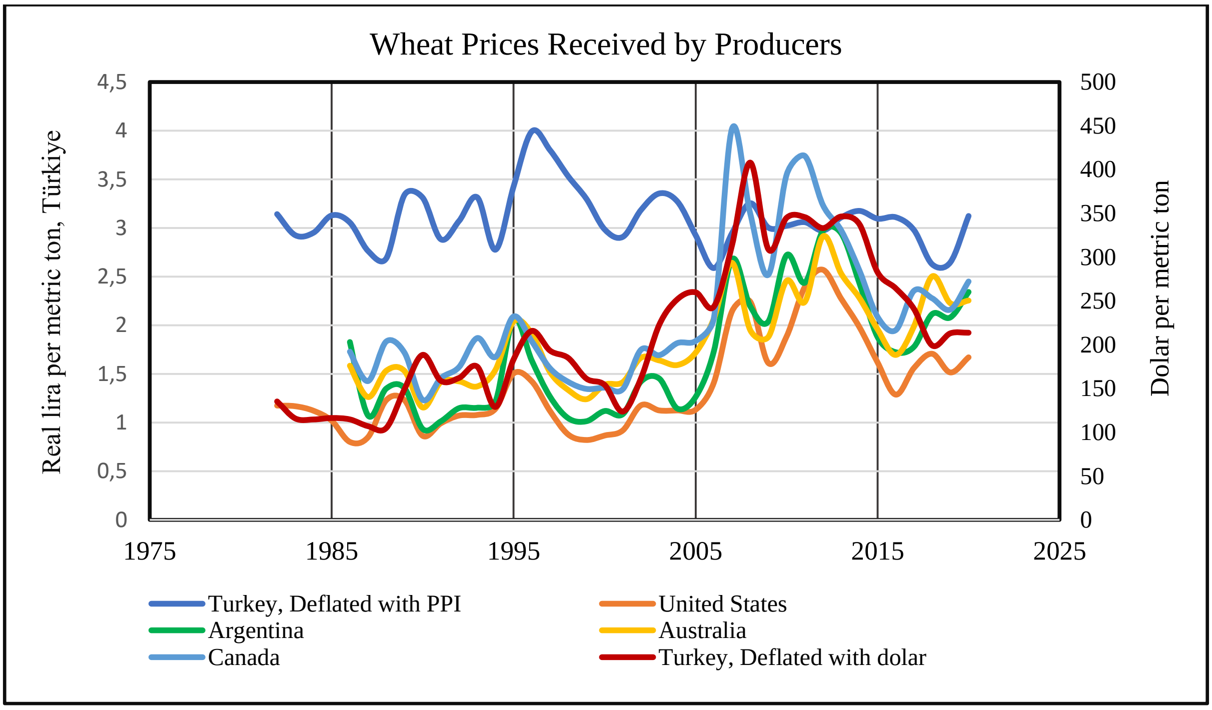


**Figure 6:** Wheat Prices of Selected Countries (Source: https://www.ers.usda.gov and TurkStat websites).

After determining the main trend of the prices and quantities of the wheat, we worked on the fluctuations around the trend with mathematical models that is explained in two sub-headings we have given below; i) Simple, dynamic cobweb theorem application that explains the price with last year's price, and, ii) Analysis with the advanced cobweb theorem (Goodwin 1947) which includes producer expectations in addition to two period delayed supply prices.

### 5.3. Simple Cobweb Theorem

Econometric analysis is the support of the hypotheses put forward by economic theory (wheat prices and quantities which is our subject) with equations established mathematically (differential equations and difference equations) and making them solvable and testing the equations with statistical methods and interpreting the results.

Adhering to approach, we developed mathematical polinomial functions with a lagged explanatory variable in simple cobweb theorem and used these functions with advanced statistical methods.

The annuall quantities of economic variables such as national income, price, production, etc. are never exactly on a linear line. In other words, annual growth rates are not at the same pace. Even if the equation we obtain as a result of our statistical study is linear, there will be variable figures that are not on this line. In this case, in addition to the function that determines the main trend, it is necessary to monitor the course of the data not included in the equation and to investigate the determination of the secondary equation (sinusoidal or cyclical complementary analysis).

Throgouhout the statistical tests, we ascertained that the parabolic fourth-degree function which has previous year's price as independent variable together with the sinusoidal complementary function had very high ability to represent the price trend. For the quantities function, second-degree price multiplied by time series as independent variable gave the best results in polinomial framework.

Symbols:

Price function :

$P_t$ : Curent price, dependent variable,

$P_{t-1}$: Last year's price, independent variable,

*t*: Time, independent variable,

*Z*: Time base,

$a_1, a_2,$ : Parameters,

$a_3$ , $a_4$ Constants of amplitude,

$r_p$: Growth rate,

$a_5$ : Angular frequncy.

Quantities function:

$Q_t$: Current quantity, dependant variable,

$P_t$ : Curent price, independent variable,

*t*: Time, independent variable,

$b_1, b_2$ : Parameters,

$b_3$ : Amplitude,

$r_q$ : Damping or anti-damping multipliers, growth rate,

$b_4$, $b_5$ angular frequency and phase lag of cos.

$b_6$, $b_7$ angular frequency and phase lag of sin.

If *r>1* anti-damped or explosive oscillations occur, when *α* <0 amplitude damped and when *r=1* amplitude has regula*r* oscillations.

Trend and complementary functions used for price and quantities respectively are[6]:

$$P_t = a_1 + a_2 P_{t-1}^5 + a_3^{\left[ r_p^{(t-Z)} [cos(a_4(t-Z) + cos(a_5(t-Z) + a_6))] \right]} \quad \text{Eq (8)}$$

$$Q_t = b_1 + b_2 P^3 (t-Z) + b_3^{\left[ r_q^{(t-Z)} [\cos(b_4(t-Z) + b_5 + \sin(b_6(t-Z) + b_7))] \right]} \quad \text{Eq (9)}$$

Last terms of both functions are complementary functions. Considering the characteristics of the data set used, we saw that complementary functions were more in weight than the trend functions. Although trends can be represented by mean values, it became necessary to make the constants of complementary functions exponential-based both in price and in quantity. In this way, determination coefficients obtained reached to very desirable levels (table 13).

Before work out the parameters of these functions, looking at the actual price and quantity diagram at Figure 7 (blue line) will be more instructive. The presumption of naming diagram as socalled cobweb is obvious. Two variables of diagram have been keeping going around in circles the actual price of 3.1 lira and the amount of 19.1 million tons for forty years. The graph shows the years from 1 to 39 in numbers for ease of tracking (1982=1… 2020=39). Our job as econometrician is to represent the cobweb route on the graph with mathematical functions and statistical applications.

[6] For details of theory and our recent application of complementary functions see (Allen 1956, p. 187-191; İskender 2021b) respectively.

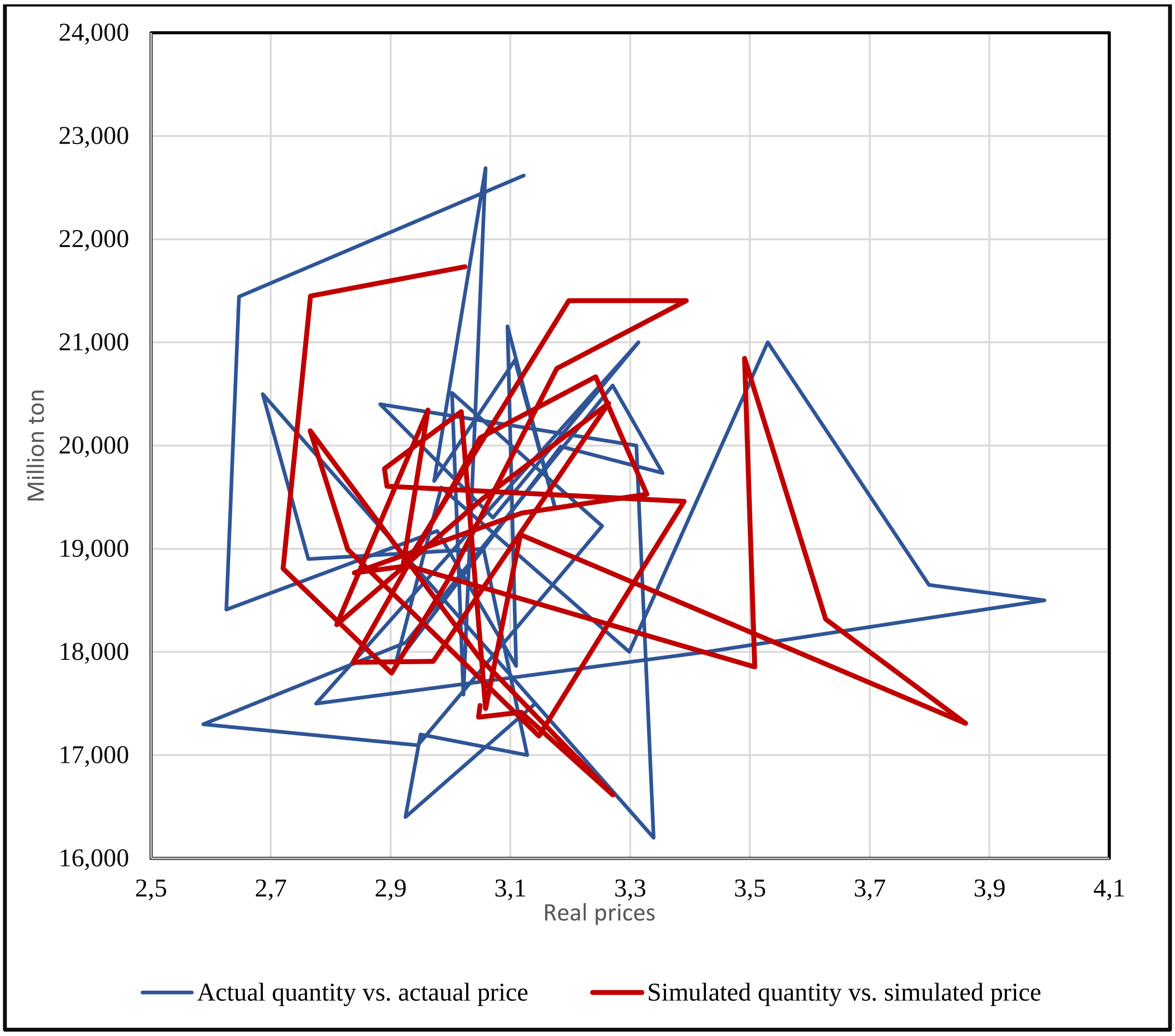


**Figure 7:** Harmony of Actual and Simulated Data : Our trend functions with the complementaries almost catch the actual data. Gravity center of state of affairs is 3.1 real lira on horizontal and 19.1 million tons wheat on vertical axis.

Based on the cobweb theorem, we took the current price as a function of the price of a year ago. In statistical studies, we obtained the best results from the fifth-degree polynomial application. In determining the quantity equation, the current price to the power of three multiplied by time is taken as independant variable, for in the simultaneous solution of only price-based supply equation did not yield the desired results at statistical tables. It seems that there has been no relationship simply between price and supply. This situation can be explained when we consider that the price of wheat is subject to very strict public monitoring, that there is no full competition, that its producers use the powers of institutional organization as well as the market to determine prices, that consumers also strictly control the prices of wheat made final products, etc. In summary, we preferred to work with the function based on price multiplied by time instead of price alone. Both functions are solved simultaneously.

As t: (1982…2020) is time and *Z*=1999 is base year, trend functions of prices and quantities:

$$P_t = 1.72127 + 0.00116P_{t-1}^5 + 0.812015^{[0.998694^{(t-1999)}cos(1.02619(t-1999)+\cos(0.329373(t-1999)+2.152085))]} - 0.43113P\_l2 + 0.454813P\_m3 \quad \text{Eq (10)}$$

$$Q_t = (17.81252 + 0.001902P^3(t-1999) + 0.375931^{\left[1.002928^{(t-1999)}[\cos(1.119806(t-1999)-1.64039+\sin(0.681446(t-1999)+1.106512))]\right]} + 0.704867Q\_l2 - 0.55422Q\_l4$$ Eq (11)

We know from Figure 4 that the price and quantity curves run parallel to the horizontal (time) axis, and both functions almost explain the complete trends. Price and quantities are constant averages throughout the period. In Table 14, we may see those constant terms, $\alpha_1$ and $b_1$. The average price trend of 3.1 (table 9) real lira versus the function constant value is 1.7 real lira. For quantities, it is 17.8 versus 19.1 (table 11). But on the other hand, although the parameters ($a_2$ and $b_2$) of independent variables are close to zero, their standard errors are very low and their significances are high. Price increases made every year on the basis of the previous year's price are effective in determining the current prices. And time is also an effective factor in explaining the course of quantity transections. This leads us to Goodwin's theory of expectations (next chapter). Although the coefficient of independant variable of quantity equation is small, the relationship between quantity and multiplication of time and square of current price as independent variable is high. Hence, the current price follows a course which is based on price of a year ago and the quantity in accordance with price and time variables.

We have obtained positive results from the statistical solutions made with OLS[7], SUR, FIML and ITSUR methods for these two functions, but preferred to give only SAS MODEL procedure[8] FIML (Full Information Maximum Likelihood) simultaneous solution results here with complementary functions of price and quantity functions for the sake of brevity.

Both the equilibrium path and the complementary functions represent the best values reached in statistical studies. We presume that the determination coefficients of the price and quantity equations obtained in our model (0.76 and 0.67) are satisfactory when the particular structure of the Turkish wheat market is taken into consideration.

As we mentioned earlier, it is difficult to measure supply and demand movements with annual data of wheat market which is under serious control by public authorities. If we had daily, weekly or monthly supply, demand and price data of Polatlı, Konya and Ankara Commodity Exchange in addition to the annual data, we could determine the supply and demand functions of these variables based on coefficients as well as time and price trend. Since it is not, what we do in this study consists of determining the path followed by the intersection points of supply and demand functions for a period of forty years.

With the determination of equations, also determined the conjuncture periods of the prices and quantities:

M: Oscillation time,

Price curve: $M_P = 2\pi/a_5$=2π /0.998694= 6.3 years,

Quantity curve: $M_q = 2\pi/b_4$=2π/1.002928= 6.3 years

The fact that cyclical durations of price and quantity functions are very close to each other indicates that the Turkish wheat market has a very stable appearance. Prices and quantities complete their the conjuncturel oscillations in average every six year. The presence of growth factors (*r*) which are close to one both in price and quantity complementary functions also indicate a stable structure, a regular oscillation (see *r* values at Table 14).

Tables 13-17 shows the results of FIML application. After simultaneous solution, the simulation values and graphs of the intersection points of these two equations are given in tables 18 and 19 and graphs 8 and 9, respectively.

In price and quantity equations with nineteen parameters, probabilities are almost at zero levels and the market representation of parameters is very high. At the following tables, the probabilities obtained are well above the 5% alpha level. The obtained test results confirm the validity of the simple cobweb theory of Turkish wheat market.

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[7] For abbreviations see section 3b.

[8] For a very perfect and useful comparison of least squares and likelihood methods see (SAS Institute Inc. 2018b,pp. 37–43).

**Table 13:** Summary of Residuals

| **Equation** | **DF Model** | **DF Error** | **SSE** | **MSE** | **Root MSE** | **$R^2$ and R** | **Adj $R^2$** | **Durbin Watson** |
|---|---|---|---|---|---|---|---|---|
| **P**[9] | 9 | 30 | 0.7914 | 0.0264 | 0.1624 | 0.7553<br>0.8691 | 0.6901 | 1.9599 |
| **Q** | 10 | 29 | 33.9440 | 1.1705 | 1.0819 | 0.6660<br>0.8161 | 0.5623 | 2.4714 |

We think that the determination coefficients 0.7464 and 0.6680 provide a good representation considering seemingly simple nature of the data. It is obvious that the contribution of the application of variable-based amplitude coefficients, which we applied for the first time, has primary role.

[9] P: Price and Q: Quantity throughout the manuscript.

**Table 14:** Nonlinear FIML Parameter Estimates

| Parameter | Estimate | Approx Std Err | t Value | Approx Pr > \|t\| | Label |
|---|---|---|---|---|---|
| ***P*** | | | | | |
| $a_1$ | 1.72127 | 0.0482 | 35.73 | <.0001 | Parameter |
| $a_2$ | 0.00116 | 0.000137 | 8.47 | <.0001 | Parameter |
| $a_3$ | 0.812015 | 0.0216 | 37.66 | <.0001 | First constant of amplitude |
| $a_4$ | 1.02619 | 0.0122 | 83.93 | <.0001 | Second constant of amplitude |
| $r_p$ | 0.998694 | 0.0120 | 83.50 | <.0001 | Growth factor |
| $a_5$ | 0.329373 | 0.0194 | 16.99 | <.0001 | Angular frequency |
| $a_6$ | 2.152085 | 0.2335 | 9.22 | <.0001 | Phase lag |
| ***P_l2*** | -0.43113 | 0.1312 | -3.29 | 0.0026 | AR(P) P lag2 parameter |
| ***P_l3*** | 0.454813 | 0.1297 | 3.51 | 0.0015 | AR(P) P lag3 parameter |
| ***Q*** | | | | | |
| $b_1$ | 17.81252 | 0.1869 | 95.32 | <.0001 | Parameter |
| $b_2$ | 0.001902 | 0.000605 | 3.15 | 0.0038 | Parameter |
| $b_3$ | 0.375931 | 0.0360 | 10.45 | <.0001 | Initial amplitude |
| $r_q$ | 1.002928 | 0.00774 | 129.58 | <.0001 | Growth factor |
| $b_4$ | 1.119806 | 0.0128 | 87.45 | <.0001 | Angular frequency of cos |
| $b_5$ | -1.64039 | 0.1282 | -12.80 | <.0001 | Phase lag of cos |
| $b_6$ | 0.681446 | 0.0295 | 23.10 | <.0001 | Angular frequency of sin |
| $b_7$ | 1.106512 | 0.2143 | 5.16 | <.0001 | Phase lag of sin |
| ***Q_l2*** | 0.704867 | 0.1372 | 5.14 | <.0001 | AR(Q) Q lag2 parameter |
| ***Q_l4*** | -0.55422 | 0.1393 | -3.98 | 0.0004 | AR(Q) Q lag4 parameter |

*Note of Table 14: P_l2, P_l3, Q_l2 and Q_l4 are coefficients of correction for serial autocorrelation See Table 5c note.*

**Table 15:** Heteroscedasticity Test

| Equation | Test | Statistic | DF | Pr>ChiSq | Variables |
|---|---|---|---|---|---|
| **P** | White's Test | 39.00 | 38 | 0.4246 | Cross of all vars |
| | Breusch-Pagan | 5.25 | 5 | 0.3865 | *S1*=(t-1999)(t-2015), *Q*, *P*, *1* |
| **Q** | White's Test | 39.00 | 38 | 0.4246 | Cross of all vars |
| | Breusch-Pagan | 2.71 | 5 | 0.7447 | *S1*=(t-1999)(t-2015), *Q*, *P*, *1* |

**Table 16:** Autoregression Test Results of Prices and Quantities

| Godfrey's Serial Correlation Test | | | | Generalized Durbin-Watson Statistics | | | | |
|---|---|---|---|---|---|---|---|---|
| **Equation** | **Alternative** | **LM** | **Pr> LM** | **Equation** | **Order** | **DW** | **Pr<DW** | **Pr> DW** |
| **P** | 1 | 0.20 | 0.6586 | **P** | 1 | 1.96 | 0.2545 | 0.7455 |
| | 2 | 0.63 | 0.7280 | | 2 | 2.09 | 0.8195 | 0.1805 |
| | 3 | 0.71 | 0.8701 | | 3 | 1.95 | 0.7331 | 0.2669 |
| **Q** | 1 | 3.13 | 0.0767 | **Q** | 1 | 2.47 | 0.8653 | 0.1347 |
| | 2 | 4.44 | 0.1089 | | 2 | 1.68 | 0.3257 | 0.6743 |
| | 3 | 4.44 | 0.2177 | | 3 | 2.13 | 0.7483 | 0.2517 |

**Table 17:** Normality test

| Equation | Test Statistic | Value | Prob |
|---|---|---|---|
| **P** | Shapiro-Wilk W | 0.99 | 0.9225 |
| **Q** | Shapiro-Wilk W | 0.99 | 0.9021 |
| **System** | Mardia Skewness | 2.01 | 0.7348 |
| | Mardia Kurtosis | -0.86 | 0.3871 |
| | Henze-Zirkler T | 0.37 | 0.6875 |

After determining the parameters of the price and quantity functions, we carried out the simulation work required for the comparison and graphing of the function estimates with the real data. Tables 18 and 19 and figures 8, 9 and 10 of this work give us the opportunity of comparison.

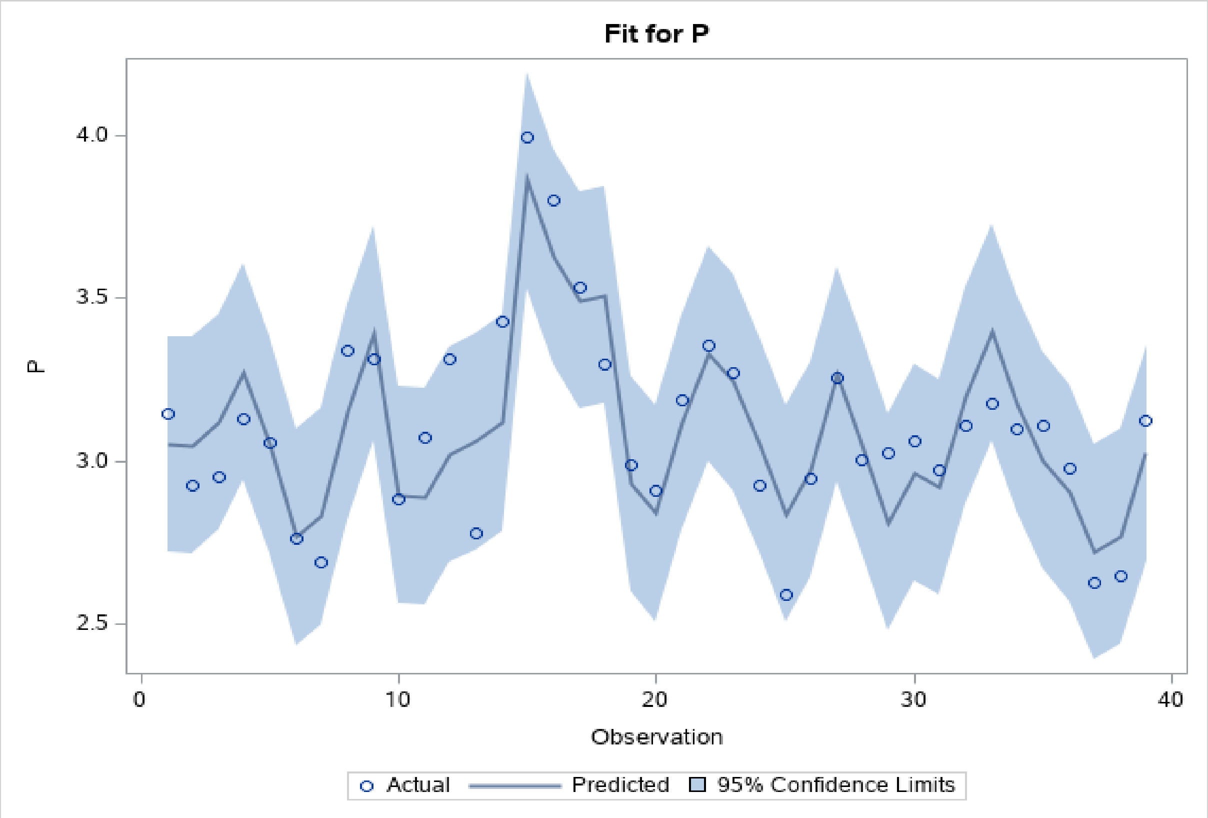


**Figure 8:** Fit for Price

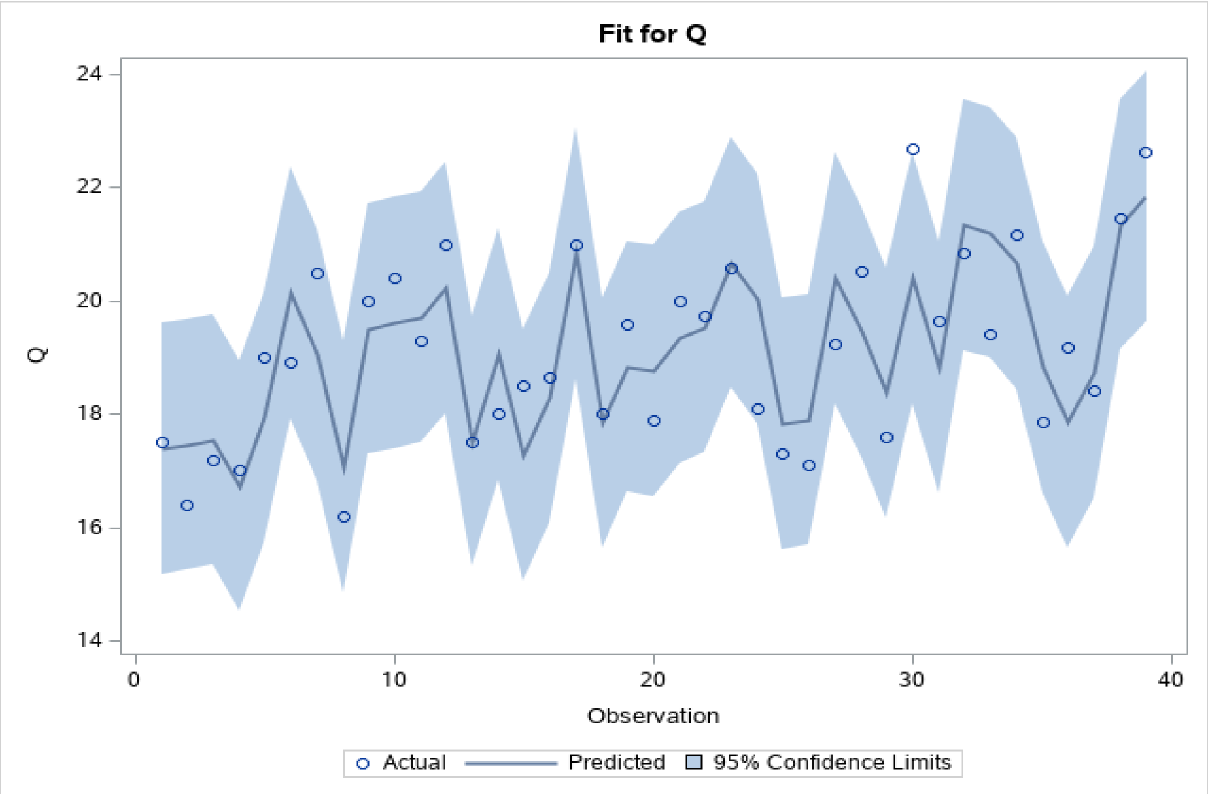


**Figure 9:** Fit for Q

**Table 18:** Simultaneous Simulation Resluts

| | | | Actual | | Predicted | | |
|---|---|---|---|---|---|---|---|
| **Variable** | **N Obs** | **N** | **Mean** | **Std Dev** | **Mean** | **Std Dev** | **Label** |
| **P** | 39 | 39 | 3.0954 | 0.2918 | 3.0933 | 0.2520 | Price |
| **Q** | 39 | 39 | 19.1248 | 1.6353 | 19.1183 | 1.3842 | Quantity |

**Table 19:** Simultaneous Simulation Statistics of Fit

| Variable | N | Mean Error | Mean % Error | Mean Abs Error | Mean Abs % Error | RMS Error | RMS % Error | $R^2$ |
|---|---|---|---|---|---|---|---|---|
| **P** | 39 | -0.00213 | 0.1465 | 0.1165 | 3.7820 | 0.1425 | 4.6365 | 0.7553 |
| **Q** | 39 | -0.00650 | 0.1871 | 0.7813 | 4.0856 | 0.9549 | 4.9477 | 0.6501 |

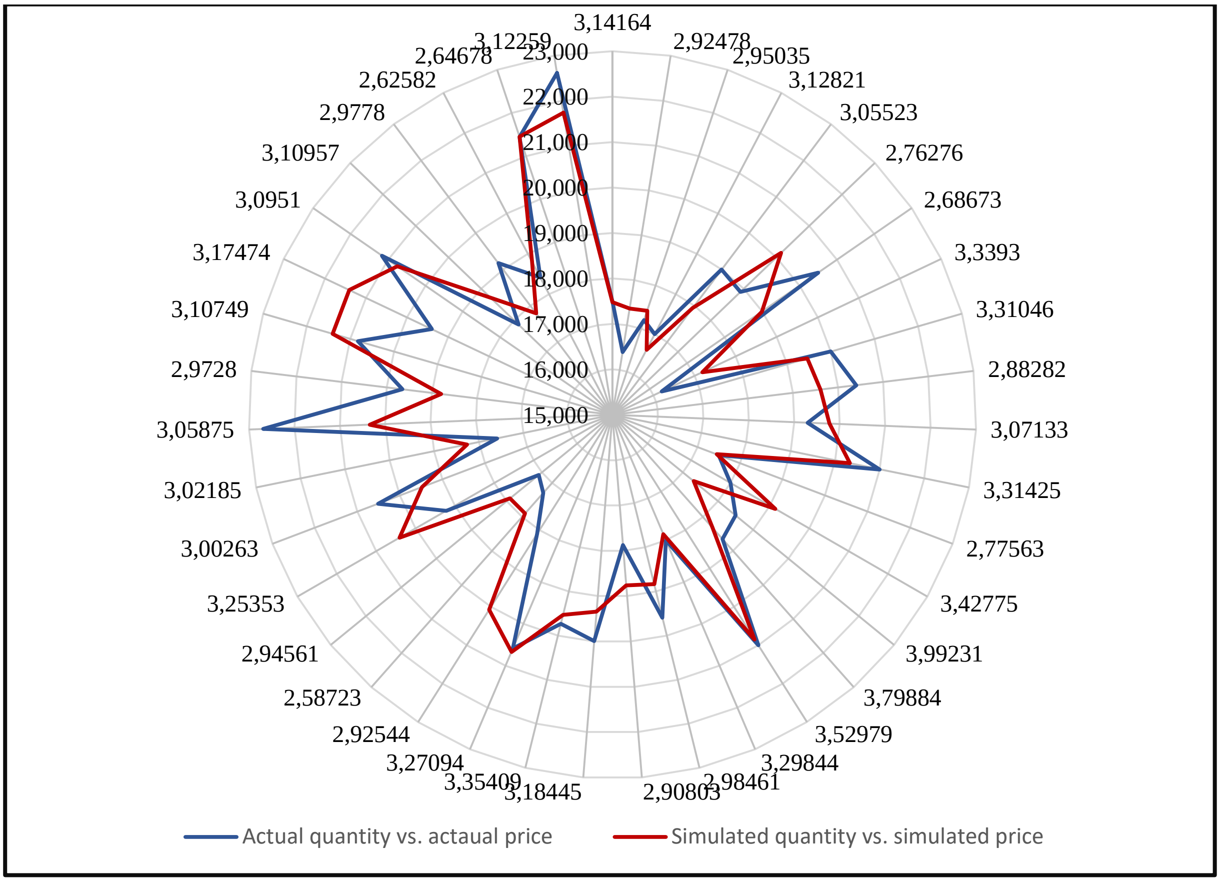


**Figure 10:** Satellite View of Turkish Wheat Market:Actual vs. Simulated Data (Prices are around the most outside circle, quantities (mil.tons) center to upwards, starting point is north pole 1982=3.14164), clockwise)

## 5.4. Producers' Expectations and Goodwin's Cobweb Model

In macro and microeconomic theories, expectations have an important place. While short-term expectations for producers cover price and sales amounts, long-term expectations include investments, production arrangements and employment according to price and quantity expectations (Keynes 1936, pp. 46–47). When the situation is looked at from the point of view of agricultural production and when there are effective public institutions in determining the price, it becomes even more important that price expectations direct production and investments. As we explained above, while the real prices remained stable, the area of cultivation in wheat in the period 1993-2020 decreased from 9.8 million hectares to 6.9 million hectares, the number of tractors increased from 746000 to 1370000. The sacrifice made from the land was closed by capital-intensive investment. Real prices, which have remained constant for forty years, have led to the abandonment of marginal land for wheat cultivation (which may have been used for other crops or purposes) and to increased investment in mechanization.

As the price expectations for annual production in agriculture gained importance, economists developed supply and demand models which include previous period price and price increases (Goodwin 1947) for commodity markets. Altough simple dynamic cobweb model sees supply as a function of the price of previous period $P_{t-1}$, Goodwin's theory explains the cobweb theorem with the price of the past two years and the expectations of price increase attributed and based on last year's price. Being $Q_t$:Quantity and $P_t$ : Price,

Supply equation:

$$Q_t = \psi_{t-1} + \beta P_t \qquad \text{Eq (12)}$$

If we add a constant number ($\rho$) for the expected price of the next production period by the producers, then expected price equation may be written as:

$$Ex\ P_t = (1+\rho)P_{t-1} - \rho P_{t-2} \qquad \text{Eq (13)}$$

Substituting in eq (15):

$$Q_t = \psi_{t-1} + \beta((1+\rho)P_{t-1} - \rho P_{t-2}) \qquad \text{Eq (14)}$$

If $\rho = 0$ , the simple cobweb theorem is obtained. If$\rho > 0$, it is expected that prices will increase in the same direction, and if $\rho < 0$, they will move in the opposite direction. Economically reasonable range is -1< $\rho$ <1. This is a summary of Goodwin model.

In statistical application phase of our work, we took Goodwin's expectetions coefficient, added $(P_{t-1}^2 + (P_{t-1} - P_{t-2})^2)$, $\bar{P}$ and also complementary equation which was most suitable with wheat data for price function .

Price expectation's function:

$$Ex\ P_t = (1+\rho)P_{t-1} + \rho(P_{t-1}^2 + (P_{t-1} - P_{t-2})^2)) \qquad \text{Eq (15)}$$

Price function:

$$P_t = a_1\big((1+\rho)P_{t-1} + \rho(P_{t-1}^2 + (P_{t-1} - P_{t-2})^2)\big) + \bar{P} + [a_2 P_{t-1}]^{r_p^{(t-Z)}[\cos(a_3(t-Z)+cos(a_4*(t-Z)+a_5))]} \qquad \text{Eq (16)}$$

Quantities function:

$$Q_t = b_1 + b_2\, P^4(t-Z) + b_3^{r_q^{(t-Z)}[cos(b_4(t-Z)+\boldsymbol{b_5})+\sin(b_6 P)]} \qquad \text{Eq (17)}$$

Since the supply and demand series data are not separately available in our hand, we have to work with price and quantity series. Although Goodwin established the equations according to the differences from the averages in order to explain the price and quantity fluctuations and to find the roots of auxiliery equation in his theoretical explanations as so-called homogenous form, on the contrary we performed statistical applications by leaving the averages in the equation for best statistical test results.

$$P_t = -1.59938\big((1-0.19108)P_{t-1}\big) - 0.19108(P_{t-1}^2 + (P_{t-1} - P_{t-2})^2) + 3.1 + 0.267075^{1.002198^{(t-1999)}[cos(1.024524(t-1999)+\cos(0.327539(t-1999)+1.826656))]} - 0.56031P_{l2} + 0.307509P\_m3 \quad \text{Eq (18)}$$

Average price of period 3.1 lira also added to price equation as given.

$$Q_t = 16.91016 + 0.001138P_t^4(\mathrm{t}-1999) + 0.559911^{((0.99479)^{(t-1999)}[cos(1.128678(\mathrm{t}-1999)-1.4864)+\sin(-0.70981P)]} - 0.86449Q_{l2} - 2.47242Q_{m2} - 1.7301Q\_m4 \quad \text{Eq (19)}$$

In the statistical application of the Goodwin model, we preferred to analyze trend and complementary functions together and solved simultaneously. We found it appropriate to use exponential forms both in price and quantities complementary functions. Also, variables of auotcorrelation correction added both for price and quantitities equations. Detailed statistical tables are given below in Table 20-24.

Goodwin developed a two-dimensional chart, $\boldsymbol{\rho}$ (expectations coefficient) on the horizontal axis and $\boldsymbol{a}$ (the ratio of slopes of supply and demand curves) on the vertical axis to determine the effects of producer expectations on price and quantity conjunctures. The economically significant values of $\rho$ are -1< ρ<+1 Value ($\rho = -0.19108$) founded in this statistical study meet this condition and near to zero point. Most stable place is zero point on horizontal axis. $\rho$ -values moving away from zero will increase instability.

Although Goodwin did not give an explanation for the negative value of $a_1$, if we take it as an absolute value, the value we find $a_1 = 1.6$ (the growth of the slope of thesupply and demand curves) will increase the likelihood of instability. The stable pitch around the vertical axis is like a conical tent that sits on the horizontal axis, and large $a_1$ values will push the system to outside stabel area. The fact that the expectations coefficient is very close to zero prevents this situation. As we surely expected, the wheat market is in a stable place near the zero point on the horizontal axis of Goodwin[10]. We confirm a stable market outlook for Turkish wheat market with the Goodwin model. The determination coefficients we obtained indicate a stable market. Especially in the quantities equation, the appearance that the price has no effectiveness is obtained. In this case, the independent variable time is effective and explains the market of the product whose production period is one year. We presume that the determination coefficients obtained as 0.78 and 0.81 respectively for the price and quantities equations have an adequate explanation for the wheat data set we have. Notice that we have been evaluating a market which has public control, uncontrollable prices and supply quantities by producers, import factors etc.

When the statistical tables are examined, with sufficiently higher than expected t-values of seventeen parameters of Table 21 is also an indicator of the market representation capability of study. Autocorrelation arrangements are sufficient and contributive, heteroskedasticty, normality and the multicollinearity[11] results are very satisfactory, and the probabilities in every table exceed the 5% alpha level in all tests.

Cojuncturel cycles of prices and quantities which are very close to the case of simple cobweb theorem are given below. Price expectations and quantities bought and sold are completing their cycyle in every 6.3 and 6.3 years.

M: Oscillation time:

Price: $M_p = 2\pi/a_3$=2π /1.002198=6.3 years

years.

Quantities: $M_q = 2\pi/b_4$=2π/0.99479= 6.3 years.

[10] See (Allen 1956,pp. 13-14, 196-200) for a very detailed and advanced explanation of Goodwin model.

[11] Not included in manuscript because of very vast volume which has a matrix of 19 rows and 21 columns. Maximum "condition number" is at an acceptable level of 27.

**Table 20:** Nonlinear FIML Summary of Residual Errors

| Equation | DF Model | DF Error | SSE | MSE | Root MSE | $R^2$ and R | Adj $R^2$ | Durbin Watson | Label |
|---|---|---|---|---|---|---|---|---|---|
| **P** | 9 | 30 | 0.7241 | 0.0241 | 0.1554 | 0.7762 0.8810 | 0.7165 | 1.7257 | P |
| **Q** | 10 | 29 | 18.8375 | 0.6496 | 0.8060 | 0.8146 0.9025 | 0.7571 | 2.0756 | Q |

**Table 21:** Nonlinear FIML Parameter Estimates

| Parameter | Estimate | Approx Std | t Value | Approx | Label |
|---|---|---|---|---|---|
| **P** | | | | | |
| $a_1$ | -1.59938 | 0.0920 | -17.38 | <.0001 | Coefficient of price equation |
| $\boldsymbol{\rho}$ | -0.19108 | 0.00282 | -67.87 | <.0001 | Coefficient of expectations |
| $a_2$ | 0.267075 | 0.00750 | 35.60 | <.0001 | Amplitude of cosine |
| $\boldsymbol{r_p}$ | 1.002198 | 0.0118 | 85.18 | <.0001 | Growth factor |
| $a_3$ | 1.024524 | 0.0119 | 86.19 | <.0001 | Angular frequency of |
| $a_4$ | 0.327539 | 0.0164 | 19.93 | <.0001 | Angular frequency |
| $a_5$ | 1.826656 | 0.2283 | 8.00 | <.0001 | Phase lag |
| ***P_l2*** | -0.56031 | 0.1220 | -4.59 | <.0001 | AR(P) P lag2 parameter |
| ***P_l3*** | 0.307509 | 0.1247 | 2.47 | 0.0196 | AR(P) P lag3 parameter |
| **Q** | | | | | |
| $\boldsymbol{b_1}$ | 16.91016 | 0.3940 | 42.92 | <.0001 | First constant of quantities |
| $\boldsymbol{b_2}$ | 0.001138 | 0.000256 | 4.45 | 0.0001 | Second constant of quantities |
| $\boldsymbol{b_3}$ | 0.559911 | 0.0581 | 9.63 | <.0001 | Amplitude of cosine |
| $\boldsymbol{r_q}$ | 0.99479 | 0.00792 | 125.65 | <.0001 | Growth factor |
| $\boldsymbol{b_4}$ | 1.128678 | 0.0128 | 88.00 | <.0001 | Angular frequency of cos |
| $\boldsymbol{b_5}$ | -1.4864 | 0.1664 | -8.93 | <.0001 | Phase lag |
| $\boldsymbol{b_6}$ | -0.70981 | 0.2055 | -3.45 | 0.0017 | Angular frequency of sinus |
| ***Q_l2*** | -0.86449 | 0.1060 | -8.16 | <.0001 | AR(Q) Q lag2 parameter |
| ***Q_m2*** | -2.47242 | 0.1140 | -21.69 | <.0001 | MA(Q) Q lag2 parameter |
| ***Q_m4*** | -1.7301 | 0.0511 | -33.84 | <.0001 | MA(Q) Q lag4 parameter |

*Note of Table 21: P_l2, P_l3, Q_l2, Q_m2 and Q_m4 are coefficients of correction for serial autocorrelation. See Table 5c note.*

**Table 22:** Heteroscedasticity Test

| Equation | Test | Statistic | DF | Pr > ChiSq | Variables |
|---|---|---|---|---|---|
| **P** | White's Test | 39.00 | 38 | 0.4246 | Cross of all vars |
| | Breusch-Pagan | 7.20 | 5 | 0.2061 | s1=(t-2015)(t-1999),Q, P, $P_{t-1}$, $P_{t-2}$, 1 |
| **Q** | White's Test | 39.00 | 38 | 0.4246 | Cross of all vars |
| | Breusch-Pagan | 1.65 | 5 | 0.8956 | S1=(t-2015)(t-1999),Q, P, $P_{t-1}$, $P_{t-2}$, 1 |

**Table 23:** Goodwin Model: Autoregression Test Results of Prices and Quantities

| Godfrey's Serial Correlation Test | | | | Generalized Durbin-Watson Statistics | | | | |
|---|---|---|---|---|---|---|---|---|
| **Equation** | **Alternative** | **LM** | **Pr > LM** | **Equation** | **Order** | **DW** | **Pr < DW** | **Pr > DW** |
| **P** | 1 | 1.03 | 0.3098 | **P** | 1 | 1.73 | 0.0858 | 0.9142 |
| | 2 | 3.84 | 0.1469 | | 2 | 2.31 | 0.9172 | 0.0828 |
| | 3 | 3.97 | 0.2644 | | 3 | 1.88 | 0.6108 | 0.3892 |
| **Q** | 1 | 0.39 | 0.5329 | **Q** | 1 | 2.08 | 0.4726 | 0.5274 |
| | 2 | 0.48 | 0.7861 | | 2 | 1.96 | 0.8759 | 0.1241 |
| | 3 | 2.40 | 0.4940 | | 3 | 1.66 | 0.2769 | 0.7231 |

**Table 24:** Normality Tests

| Equation | Test Statistic | Value | Prob |
|---|---|---|---|
| **P** | Shapiro-Wilk | 0.96 | 0.2296 |
| **Q** | Shapiro-Wilk | 0.98 | 0.8366 |
| **System** | Mardia Skewness | 4.22 | 0.3771 |
| | Mardia Kurtosis | -1.01 | 0.3138 |
| | Henze-Zirkler | 0.52 | 0.3441 |

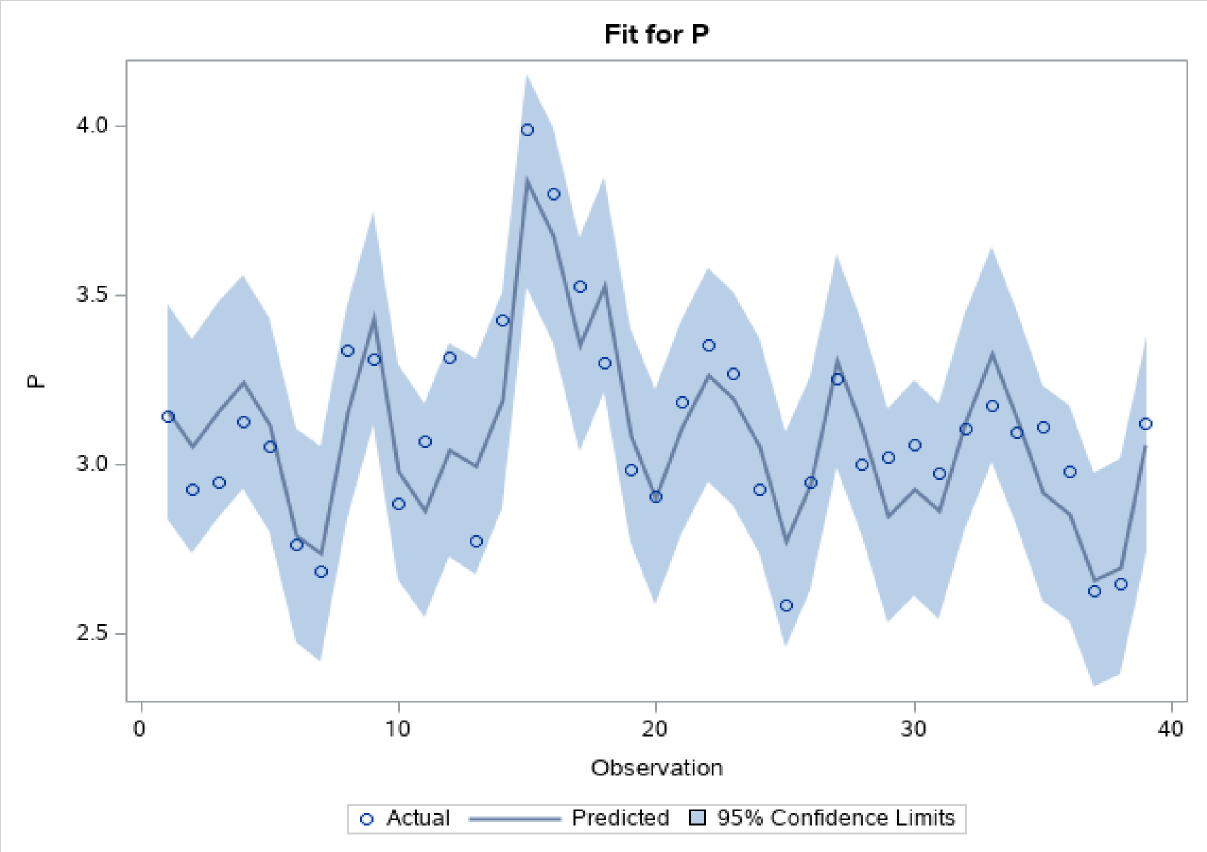


**Figure 11:** Wheat Prices

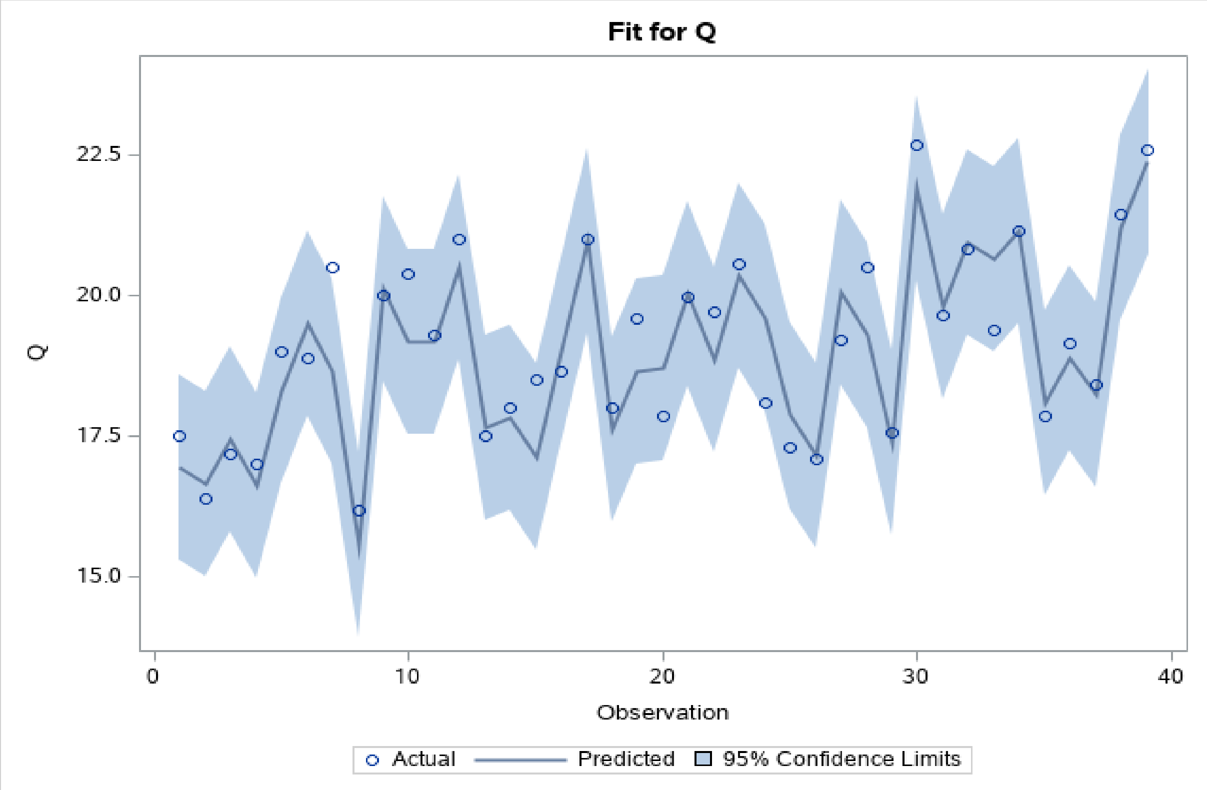


**Figure 12:** Fit for Quantities

Just as we find the estimated values of the dependent variable after the parameters are ascertained in case of a single function, we can also find the estimated values of the independent and dependent variables through these functions in cases where we use multiple functions. It is possible to solve equations simultaneously in SAS MODEL Procedure which takes lots of time to do manually. In the two-variable (price and quantities equations) model in our example, the statistics of simultanouous solution and graph of the estimated results are given Tables 25-26 and Figures 11-12. According to the Goodwin model, the center of the cobweb is the price of 3.1 real lira and the amount of 19.1 mil. tons of wheat. The entire cobweb is located around these coordinates. Results are almost same as in simple model. State of affairs confirms that producers' expectations theory is in conformity with cobweb model.

Below given statistical tables of price and quantity estimates based on equations and a graph of actual figures and forecast figures. We leave it to the readers to interpret the representation capability of tables and graphs in accordance with their purposes.

**Table 25:** Descriptive Statistics

| | | | Actual | | Predicted | | |
|---|---|---|---|---|---|---|---|
| **Variable** | **N Obs** | **N** | **Mean** | **Std Dev** | **Mean** | **Std Dev** | **Label** |
| **P** | 39 | 39 | 3.0954 | 0.2918 | 3.0892 | 0.2558 | P |
| **Q** | 39 | 39 | 19.1248 | 1.6353 | 18.9703 | 1.6365 | Q |

**Table 26:** Statistics of Fit

| Variable | N | Mean Error | Mean % Error | Mean Abs Error | Mean Abs % Error | RMS Error | RMS % Error | $R^2$ | Label |
|---|---|---|---|---|---|---|---|---|---|
| **P** | 39 | -0.00619 | -0.0102 | 0.1166 | 3.7406 | 0.1363 | 4.3612 | 0.7762 | P |
| **Q** | 39 | -0.1545 | -0.7497 | 0.5325 | 2.7767 | 0.7250 | 3.7523 | 0.7983 | Q |

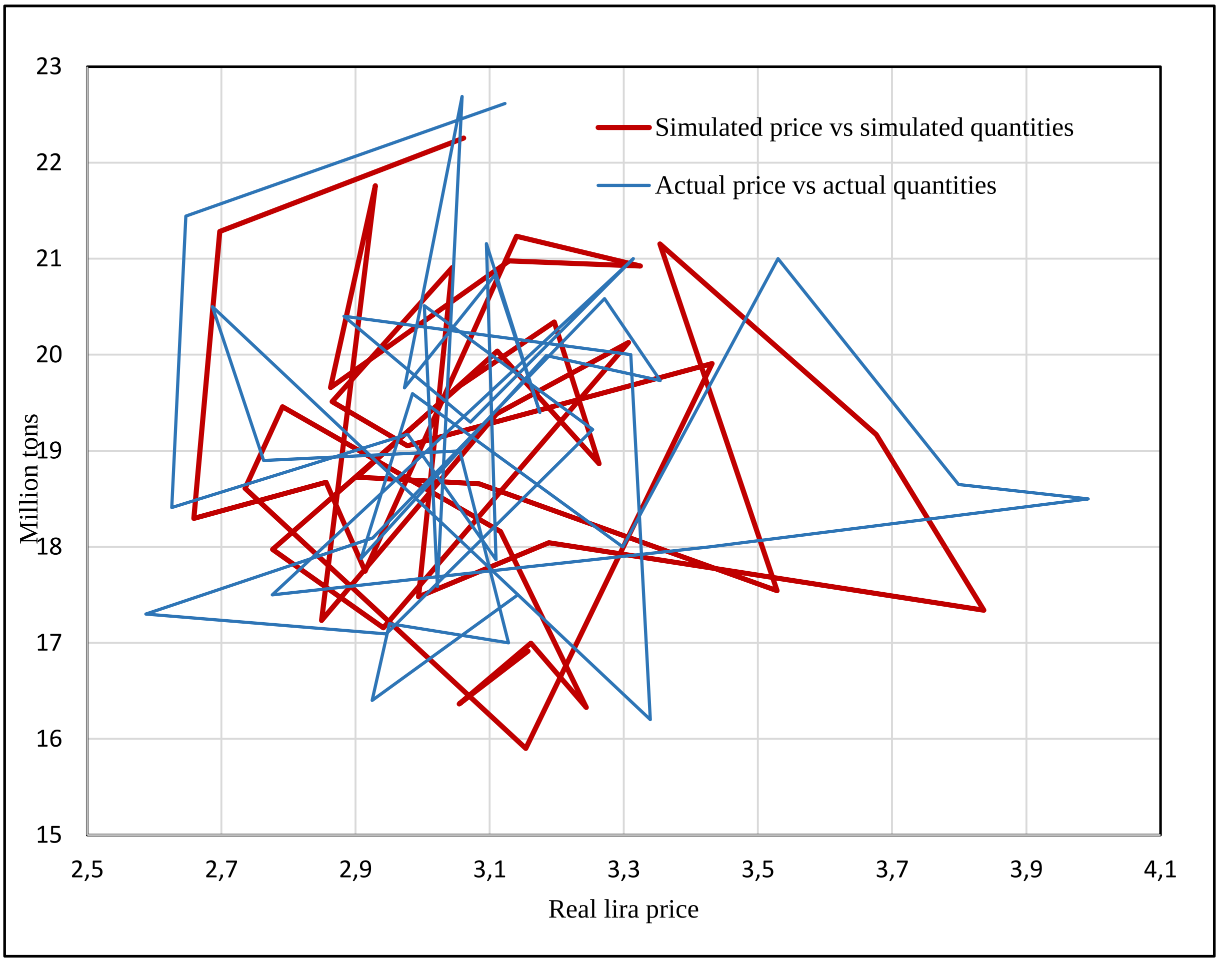


**Figure 13:** Actual and Simulated Figures of Goodwin Cobweb Functions

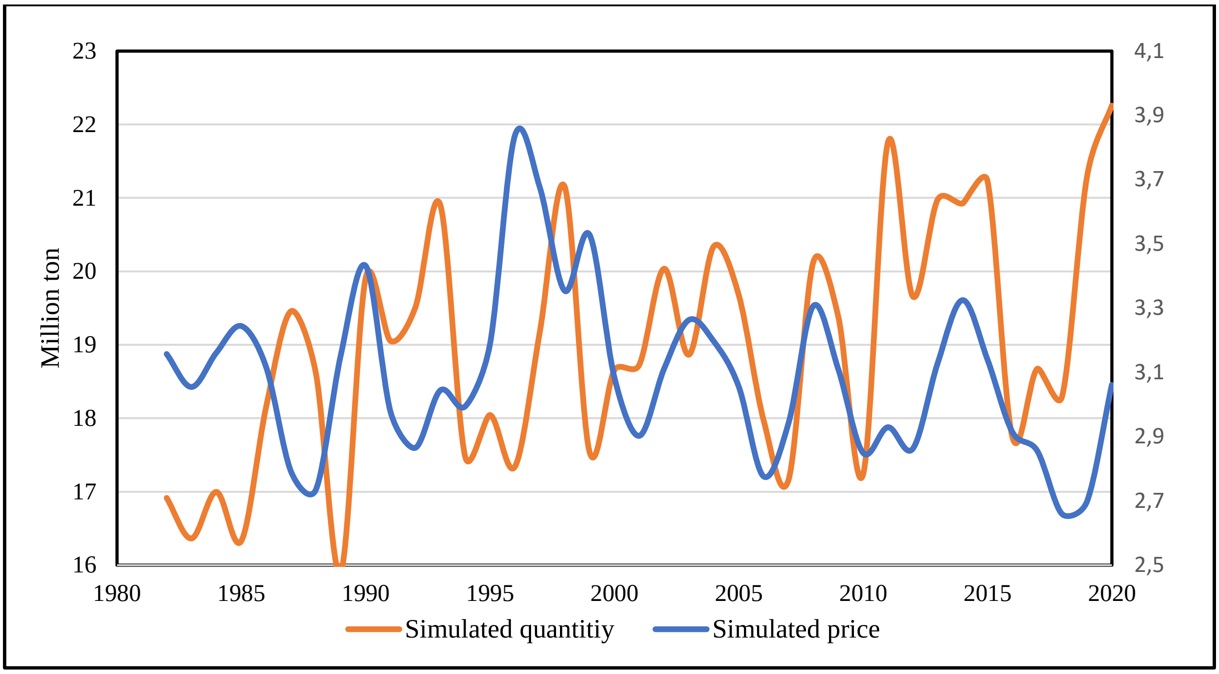


**Figure 14:** Simulated Quantity and Price Trends of Goodwin Model (Left axis: Quantities as million tons, right axis: Prices as real lira)

According to the Figure 14 simulation, production is on the rising trend, while prices are in stable and/or downward trend.

## 6. Comments and Results

Our wheat production and market study which we have chosen as a subject matter of referable and respectable econometric analysis of a product, has achieved its purpose in terms of the results obtained from economic theory, modeling and application. Successful and detailed results have benn only possible with advanced econometrics, statistical softwares. Although the developing and increasing number of theoretical econometrics and statistical studies increase the work volume of the research, the expectations are met more and the degree of health increases as a result of the expansion of the results in terms of the scope obtained.

Data we collected and the methods we applied confirm the validity of simple and advanced cobweb theorems. It is clear that prices and quantities do not form rectangular cobwebs as described in ecomomics textbooks. Supply and demand curves appear to form angular and complex cobwebs according to their slopes. In our study, supply and demand curves were not determined, but trend (equilibrium path) that the intersection points of supply and demand curves followed over the years ascertained. The axis of the price path and central point are 3.1 real lira. For quantity, the time axis and the center point of the cobweb is 19.1 million tons of wheat.

According to the production function, the wheat production capacity is 20.5 million tons. Changes below and above the trend value of 19.1 million tons occur as a result of weather conditions. With current input, output and world prices of wheat, trend value of production is expected to remain at 19.1 million tons and the price at the level of 3.1 real lira. Despite the fact that the amount of cultivated land decreased from 10 million hectares to 6.9 million hectares, it has been possible with mechanization (capital-intensive production) that production remained at the level of 19.1 million tons. No change is expected in terms of both supply and demand namely in production and consumption levels, which are under the supervision of the relevant public institutions.

Our simulation study reveals that the price trend is downward, while the quantity trend has yet set a new direction upwards for forty years of period. Interim periods might be quite different than that of long period.

Our study does not carry comprehensive objectives for the sectoral problems and solutions of wheat production, market and distribution. On the contrary, econometric study will contribute to the economy of the sector to the extent that it reveals valuable points related to the sectorial policies in accordance with its purpose and framework.

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