Ladder Operators and Coherent States for Electrons Under Double Parabolic Confinement in a Quantum Wire

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Abstract

In this study, we have chosen the spatial confinement parabolic on semiconductor quantum wire with applied magnetic field. Thus, electrons are confined in zone where two parabola overlapped by using single step between two parabolic potential. The energy eigenvalues and wave functions of electrons under this double parabolic confinement are obtained by solving Schrödinger equation in the framework of asymptotic iteration method. The creation and annihilation operators for the radial wave functions are constructed by using factorization method, it is shown that these ladder operators satisfy the commutation relations for the SU(1,1) group. Closed analytical expressions for the matrix elements of r^2 and $r\ d/dr$ are obtained and the coherent state analysis for the system are carried out.

Keywords — SU(1,1) groups, ladder operators, coherent state, matrix elements, quantum wire.

1 Introduction

The quantum wells, wires and dots can be generated by combining the different types of semiconductor materials [1]. The combination of these semiconductor structures is very important for physical applications due to restriction on the movement of the electrons in the semiconductor materials. Since the confinement effects on electrons in the system increase with decreasing dimensions of these systems, the physical and electronical properties of the system change dramatically. This case is also same for the GaAs-GaAlAs quantum wire. The restriction on movement of the electrons in this quantum wire changes electronic properties of the structure crucially. However, in this study, the dynamical group realization, the ladder operators for radial wave functions, and coherent states for the electrons in this quantum wire have been studied besides these important details. In other words, bound state energies and wave functions of electrons under the double parabolic surrounding created by considering the applied magnetic field and the spatial confinement in the GaAs-GaAlAs quantum wire are examined using the algebraic method. Bound state energies of electrons under influence of such a parabolic confinement can be controlled by changing of applied magnetic field [2]. In order to investigate a given quantum system using algebraic method, analytical expressions for normalization constants of wave functions and the exact solutions of this quantum system need to be obtained.

The relativistic or non-relativistic systems displaying a dynamical symmetry provide the above mentioned conditions can be examined using the algebraic methods [3-6]. However, the obtained ladder operators and their commutation relations provide to establish the dynamical group for corresponding quantum system. The construction of the ladder operators by using factorization method is practical and more functional compare to other methods [7]. It should be pointed out that the algebraic method is very useful for studies in various fields of physics. The Morse potential is important for describing the interaction force of the diatomic molecule. The creation and annihilation operators for wave functions

obtained by solving the Schrödinger equation with the Morse potential have been established by algebraic method [8]. Closed analytical expressions for the matrix elements of relevant functions such as 1/y and d/dy are really practical for important physical calculations as transition probability. By using the solution of the one-dimensional Schrödinger equation in the presence of harmonic oscillator plus an inverse square potential, the wave functions and energy eigenvalues have been found and so the ladder operators have been constructed by factorization method. It has been found that these ladder operators satisfy the commutation relations of the dynamical group SU(1,1). Thus, the some comments have made on Barut-Girardello coherent states [9]. Construction of the ladder operators and realization of dynamical group for some important potentials such as Pöschl-Teller and Morse potential have become the subject of many studies due to their importance in field of molecular physics [10-12]. Unlike these studies, the energy eigenvalues and corresponding wave functions for an electron under only magnetic field without any interaction potential have been obtained by solving Schrödinger equation [13]. Then, ladder operators have been constructed directly from wave functions using factorization method. The commutation relations of these ladder operators have been investigated and it has been concluded that these operators satisfy the commutation relations for the SU(1,1) group.

The first aim of this study is to obtain, using asymptotic iteration method (AIM), the energy eigenvalues and wave functions of the electrons under double parabolic surrounding formed by using magnetic field and parabolic spatial confinement in the quantum wire in Fig.1 [14-16]. The second aim of this paper is to apply algebraic method on obtained wave functions. Then, the creation and annihilation operators are directly found from the generated wave functions by using properties of the confluent hypergeometric functions. Since the goal of an algebraic approach is to establish the dynamical group, investigation of the commutation relations of these ladder operators is crucial. The matrix elements for some different functions are provided in closed analytic form and finally, the calculations of the average values of some observables in the coherent states are performed.

This paper is organized as follows. In Section 2, the Schrödinger equation of the system is established and solved by using AIM. In Section 3, the creation and annihilation operators are obtained and the commutation relations of the ladder operators are examined. Besides, the matrix elements for relevant functions are also found in this section. In Section 4, the coherent states of the system are searched. Section 5 is devoted to conclusion.

2 Hamiltonian and AIM solutions

As shown in Fig.1, the Hamiltonian for an electron in the cylindrical quantum wire with R radius under the applied magnetic field along the z axis is given by

$$H=\frac{(\vec{P}+(e/c)\vec{A})}{2m^*}+V(r) \tag{1}$$

where e is charge of the electron, m^* is effective mass of the electron, $\vec{A}=(1/2)(\vec{B}x\vec{r})$ is the vector potential and V(r) is the spatial confinement potential. The confinement potential V(r) is given as the following form

$$V(r)=\begin{cases}\frac{V_0}{R^2}r^2 & ,r<R\\ 0 & ,r>R\end{cases} \tag{2}$$

where V_0 is height of the potential barrier, $V_0=0.6(1.36x+0.22x^2)$eV [17], x is aluminum concentration for $GaAs-Ga_xAl_{1-x}As$ structure and it changes the height of the potential barrier. If Eq.(1) is rewritten using the $\vec{P}$ and $\vec{A}$, the Hamiltonian turns out

$$H=-\frac{\hbar^2}{2m^*}\left(\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}+\frac{1}{r^2}\frac{\partial^2}{\partial\phi^2}\right)-\frac{\hbar^2}{2m^*}\frac{\partial^2}{\partial z^2}+\frac{eB}{2m^*c}L_z+\frac{e^2B^2r^2}{8m^*c^2}+V(r) \tag{3}$$

where L_z is z component of the angular momentum, B is the magnetic field. Then, the following Schrödinger equation $H\psi(r,\phi,z)=E\psi(r,\phi,z)$ need to be solved. Since the electron is free particle in the z direction, and has 2π period in the ϕ direction, the wave function for this equation is suggested as $\psi(r,\phi,z)=Ne^{ik_zz}e^{im\phi}\psi(r)$ where N is normalization constant. If the Schrödinger equation is rewritten and edited by using this suggested wave function, a second order differential equation is formed such as;

$$\frac{d^2\Psi(r)}{dr^2}+\frac{1}{r}\frac{d\Psi(r)}{dr}-\left(\frac{m^2}{r^2}+\xi_0 r^2+\xi_1\right)\Psi(r)=0 \quad (4)$$

where

$$\xi_0=\frac{e^2B^2}{4\hbar^2c^2}+\frac{2m^*V_0}{\hbar^2R^2},\ \xi_1=k_z^2+\frac{eBm}{\hbar c}-\frac{2m^*E}{\hbar^2}, \quad (9)$$ fdbf

here m is magnetic quantum number.

The AIM is used to solve Eq.(4) in this study. The AIM is very advantageous and practical method to solve second order differential equations in the following form;

$$\psi''=\lambda_0(r)\psi'+s_0(r)\psi \quad (5)$$

$\lambda_0(r)\neq 0, \lambda_0(r)$ and $s_0(r)$ are differentiable functions in $C_\infty(a,b)$. 1st, 2nd, 3rd and others derivatives of Eq.(5) are considered to get general solution of this equation. As a result of these iterations, (k+1)th and (k+2)th derivatives, being k=1,2,3…, are defined as following form:

$$\psi_n^{k+1}(r)=\lambda_{k-1}(r)\psi_n'(r)+s_{k-1}(r)\psi_n(r) \quad (6\text{-a})$$

$$\psi_n^{k+2}(r)=\lambda_k(r)\psi_n'(r)+s_k(r)\psi_n(r) \quad (6\text{-b})$$

where

$$\lambda_k(r)=\lambda_{k-1}'(r)+s_{k-1}(r)+\lambda_0(r)\lambda_{k-1}(r) \quad (7\text{-a})$$

$$s_k(r)=s_{k-1}'(r)+s_0(r)\lambda_{k-1}(r),\ k=1,2,..,n \quad (7\text{-b})$$

These equations are known as recurrence terms. Considering the ratio of (k+1)th and (k+2)th derivatives, for sufficiently large k ($k>0$), it is

$$\frac{s_k}{\lambda_k}=\frac{s_{k-1}}{\lambda_{k-1}}=\alpha. \quad (8)$$

Then, the ratio of (k+1)th and (k+2)th derivatives leads to get general solution of the Eq.(5), which yields to general solution to Eq.(5) as

$$\psi_n(r)=\exp\left(-\int^r \alpha dl\right)\cdot\left[C_2+C_1\int^r \exp\left(\int^l[\lambda_0(\upsilon)+2\alpha(\upsilon)]d\upsilon\right)dl\right] \quad (9)$$

where C_1 and C_2 are integration constants. Using the Eqs.(7) and Eq.(8), the corresponding expression to calculate the energy values is

$$\delta(r)=\lambda_{k-1}(r)s_k(r)-\lambda_k(r)s_{k-1}(r)=0 \quad (10)$$

However, the first part of Eq.(9) have convergent and physical polynomial solutions, whereas the second part of it have nonphysical. So, the coefficient (C_1) of second part of Eq.(9) is taken as zero. Then, the exact eigenfunctions can be derived from the following wave function generator:

$$\psi_n(r)=C_2\exp\left(-\int^r \alpha dl\right) \quad (11)$$

where n is radial quantum number. The detailed information can be found in [12-14].

To solve Eq.(4) using AIM, the asymptotic behavior of equation should be investigated. Therefore, the reasonable physical wave function is proposed as follows:

$$\psi(r)=r^m e^{-\frac{r^2\sqrt{\xi_0}}{2}}f(r) \quad (12)$$

Substituting Eq.(12) into Eq.(4) and defining a new variable$z=r^2\sqrt{\xi_0}$ lead to

$$\frac{d^2f(z)}{dz^2}-\left(1-\frac{1+m}{z}\right)\frac{df(z)}{dz}-\left(\frac{2\sqrt{\xi_0}(1+m)+\xi_1}{4\sqrt{\xi_0}z}\right)f(z)=0 \quad (13)$$

where $\lambda_0(z)=1-\frac{1+m}{z}$, $s_0(z)=\frac{2\sqrt{\xi_0}(1+m)+\xi_1}{4\sqrt{\xi_0}z}$. When Eqs.(7) and Eq.(8) are used together with $\lambda_0(z)$ and $s_0(z)$, the following expressions are generated;

$$s_0\lambda_1-s_1\lambda_0=0\Rightarrow\xi_{10}=-2(1+m)\sqrt{\xi_0} \text{ for k=1,} \quad (14\text{-a})$$

$$s_1\lambda_2 - s_2\lambda_1 = 0 \Rightarrow \xi_{11} = -2(3+m)\sqrt{\xi_0} \quad \text{for k=2,} \tag{14-b}$$

$$s_2\lambda_3 - s_3\lambda_2 = 0 \Rightarrow \xi_{12} = -2(5+m)\sqrt{\xi_0} \quad \text{for k=3,} \tag{14-c}$$

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$$\Rightarrow \xi_{1n} = -2(2n+1+m)\sqrt{\xi_0} \tag{14-d}$$

If the Eq.(14-d) is written by using ξ_0 and ξ_1, the energy values are formed as:

$$E_{n,m} = \frac{\hbar^2 k_z^2}{2m^*} + \frac{\hbar^2}{2m^*}\left(\frac{eBm}{\hbar c} + 2(2n+1+m)\sqrt{\frac{e^2B^2}{4h^2c^2} + \frac{2m^*V_0}{h^2R^2}}\right) \tag{15}$$

As seen in Eq.(15), energy of free particle in the z direction have been shifted a certain amount. The exact eigenfunctions can be derived from the generator in the Eq.(11). Using Eqs.(7-11) leads to

$$f_0(z) = 1 \tag{16-a}$$

$$f_1(z) = z - 1 - m \tag{16-b}$$

$$f_2(z) = (2+m)(1+m) - 2(2+m)z + z^2 \tag{16-c}$$

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$$f_n(z) = (-1)^n (1+m)_n \, {}_1F_1(-n, 1+m; z) \tag{16-d}$$

By using Eqs.(16), $z = r^2\sqrt{\xi_0}$ and Eq.(12), the radial wave function of the system is derived as

$$\psi_n^m(r) = (-1)^n r^m e^{-\frac{r^2\sqrt{\xi_0}}{2}} (1+m)_n \, {}_1F_1(-n, 1+m; r^2\sqrt{\xi_0}) \tag{17}$$

This wave function can be transformed to another form by considering the following useful relations [18].

$$(1+m)_n = \frac{(m+n)!}{m!} \tag{18}$$

$$L_n^m(r) = \frac{(m+n)!}{n!m!} \, {}_1F_1(-n, 1+m; r) \tag{19}$$

Then, the wave function is

$$\psi_n^m(r) = N_n^m r^m e^{-\frac{r^2\sqrt{\xi_0}}{2}} L_n^m(\sqrt{\xi_0} r^2) \tag{20}$$

where N_n^m is normalization constant. To calculate the normalization constant, when considering the normalization condition $\int_0^\infty (\psi_n^m(r))^2 r dr = 1$ and following another useful relation [16],

$$\int_0^\infty r^\alpha e^{-r} L_n^\alpha(r) L_m^\alpha(r) dr = \frac{\Gamma(n+\alpha+1)}{n!}\delta_{nm} \tag{21}$$

it is

$$N_n^m = \sqrt{\frac{2n!(\sqrt{\xi_0})^{m+1}}{(m+n)!}} \tag{22}$$

3 The construction of the creation and annihilation operators

We adopt the factorization method introduced by Dong [7], based on the Schrödinger factorization method, Infeld-Hull factorization method.

In this section, the finding of the creation and annihilation operators for the wave function of the system is investigated using the basic ideas proposed in [19-20]. The ladder operators should have the following properties

$$\hat{S}_+ \psi_n^m(r) = s_+ \psi_{n+1}^m(r) \tag{23-a}$$

$$\hat{S}_- \psi_n^m(r) = s_- \psi_{n-1}^m(r) \tag{23-b}$$

In Dirac-Braket notation, it is $\hat{S}_\pm |n,m\rangle = s_\pm |n\pm1, m\rangle$ for the operators. In addition, the $\hat{S}_\pm$ ladder operators depend on the physical variable r and these operators should be on the following form

$$\hat{S}_\pm = A_\pm \frac{d}{dr} + B_\pm(r). \tag{24}$$

For this purpose, if the differential operator d/dr is acted on the Eq.(20) and used the following properties

$$r\frac{d}{dr} L_n^m(r) = nL_n^m(r) - (n+m)L_{n-1}^m(r) \tag{25-a}$$

$$r\frac{d}{dr}L_n^m(r)=(n+1)L_{n+1}^m(r)-(n+m+1-r)L_n^m(r) \qquad (25\text{-b})$$

it is obtained that

$$\left(\frac{d}{dr}-\frac{m}{r}-\frac{2n}{r}+\sqrt{\xi_0}r\right)\left|n,m\right\rangle=-\frac{N_n^m}{N_{n-1}^m}\frac{2(n+m)}{r}\left|n-1,m\right\rangle \qquad (26\text{-a})$$

$$\left(\frac{d}{dr}-\frac{m}{r}+\sqrt{\xi_0}r+\frac{2(n+m+1-\sqrt{\xi_0}r^2)}{r}\right)\left|n,m\right\rangle=\frac{N_n^m}{N_{n+1}^m}\frac{2(n+1)}{r}\left|n+1,m\right\rangle \qquad (26\text{-b})$$

Substituting Eq.(22) into the Eq.(26-a) enables us to obtain the following relation for the annihilation operators

$$\frac{1}{2}\left[-r\frac{d}{dr}+m+2n-\sqrt{\xi_0}r^2\right]\left|n,m\right\rangle=\sqrt{n(n+m)}\left|n-1,m\right\rangle \qquad (27)$$

According to Eq.(27), it is clear that

$$\hat{S}_-=\frac{1}{2}\left[-r\frac{d}{dr}+m+2n-\sqrt{\xi_0}r^2\right],\ s_-=\sqrt{n(n+m)}. \qquad (28)$$

Similarly, when the Eq.(26-b) are used together with Eq.(22) to find the $\hat{S}_+$ creation operator and it is obtained that

$$\frac{1}{2}\left[r\frac{d}{dr}+m+2n+2-\sqrt{\xi_0}r^2\right]\left|n,m\right\rangle=\sqrt{(n+1)(n+m+1)}\left|n+1,m\right\rangle \qquad (29)$$

Then,

$$\hat{S}_+=\frac{1}{2}\left[r\frac{d}{dr}+m+2n+2-\sqrt{\xi_0}r^2\right],s_+=\sqrt{(n+1)(n+m+1)}. \qquad (30)$$

As seen in Eqs.(27)-(29), the ladder operators provide $\hat{S}_\pm\left|n,m\right\rangle=s_\pm\left|n\pm1,m\right\rangle$. The radial wave functions of electrons in the quantum wire can be directly obtained by acting the creation operator $\hat{S}_+$ on the ground state $\left|0,m\right\rangle$, namely

$$\left|n,m\right\rangle=N_n^m\hat{S}_+^n\left|0,m\right\rangle \qquad (31)$$

Now, based on $\hat{S}_\pm\left|n,m\right\rangle=s_\pm\left|n\pm1,m\right\rangle$, dynamic group associated with the system can be examined by considering commutation relations of the ladder operators. The commutator $[\hat{S}_+,\hat{S}_-]$ is calculated:

$$[\hat{S}_+,\hat{S}_-]\left|n,m\right\rangle=2s_0\left|n,m\right\rangle, \qquad (32)$$

where $s_0=n+(m+1)/2$. So, a new operator $\hat{S}_0$ should be defined as $\hat{S}_0=\hat{n}+(m+1)/2$. $\hat{n}$ is number operator with the property of $\hat{n}\left|n,m\right\rangle=n\left|n,m\right\rangle$. This operator $\hat{S}_0$ provides the following eigenvalue equation

$$\hat{S}_0\left|n,m\right\rangle=s_0\left|n,m\right\rangle. \qquad (33)$$

The SU(1,1) generators $\hat{S}_+,\hat{S}_-,\hat{S}_0$ satisfy the following commutation relatios:

$$[\hat{S}_-,\hat{S}_+]=2\hat{S}_0,\quad [\hat{S}_0,\hat{S}_+]=\hat{S}_+,\quad [\hat{S}_0,\hat{S}_-]=-\hat{S}_- \qquad (34)$$

Since the SU(1,1) group is noncompact [21], all its unitary irreducible representations are infinite dimensional [22]. The properties of SU(1,1) group have been reviewed for Morse and Pöschl-Teller potentials [23-24]. However, the Casimir operator is given by

$$\hat{C}\left|n,m\right\rangle=\left(\hat{S}_0(\hat{S}_0-1)-\hat{S}_+\hat{S}_-\right)\left|n,m\right\rangle=\frac{m^2-1}{4}\left|n,m\right\rangle \qquad (35)$$

It is shown that there are four series of irreducible unitary representations for the SU(1,1) algebra except for the identity representation. They are the representation of $D^{\pm}(j)$ with a spectrum bounded above and below, respectively; the supplementary series $D_s(Q,q_0)$ and the principle series $D_p(Q,q_0)$. Since the eigenvalues have the ground state, the representation of dynamical group SU(1,1) belongs to $D^+(j)$:

$$\begin{aligned} &I_0\left|j,v\right\rangle = v\left|j,v\right\rangle \\ &I_-\left|j,v\right\rangle = [(\sqrt{j+v)(v-j-1)}]\left|j,v-1\right\rangle \\ &I_+\left|j,v-1\right\rangle = [\sqrt{(j+v)(v-j-1)}]\left|j,v\right\rangle \\ &v=-j+n, \quad n=0,1,2,...,j<0 \end{aligned} \tag{36}$$

For further calculations, the matrix elements of relevant physical functions as r^2 and rd/dr are considered in terms of the ladder operators by using Eq.(28) and (30). The matrix elements for these physical functions are obtained in closed analytic form as

$$\begin{aligned} \left\langle m\left|r^2\right|n\right\rangle = &\frac{2n+m+1}{\sqrt{\xi_0}}\delta_{m,n} \\ &-\frac{1}{\sqrt{\xi_0}}\sqrt{(n+1)(m+n+1)}\delta_{m,n+1} \\ &-\frac{1}{\sqrt{\xi_0}}\sqrt{n(m+n)}\delta_{m,n-1} \end{aligned} \tag{37}$$

$$\begin{aligned} \left\langle m\left|r\frac{d}{dr}\right|n\right\rangle = &\sqrt{(n+1)(m+n+1)}\delta_{m,n+1} \\ &-\sqrt{n(m+n)}\delta_{m,n-1}-\delta_{m,n} \end{aligned} \tag{38}$$

These relations are practical for finding the matrix elements from the ladder operators.

4 Coherent States

The coherent states were suggested by Schrödinger in the initial years of quantum mechanics. In 1963, Glauber approved giving name of the Coherent States to these states previously proposed by Schrödinger, to make mathematical description of coherent laser beam in quantum optic. The most important property of the coherent state system is to be consist of an overcomplate set and not orthogonal. However, the coherent states can be defined from the point of minimum uncertainty states. The coherent state system was applied not only quantum optic but also to many fields of physics such as superfluidity theory in solid state physics [25]. The Landau diamagnetizma was investigated by constructing coherent state of an electron in a uniform magnetic field [26]. The spin coherent state was developed to investigate semiclassic properties of spin functions, which is similar to coherent state of harmonic oscillator [27]. In addition, the coherent state of the quantum mechanical asymmetric gyroscope was obtained by examining of semiclassic properties of a rotating core ensured the possibility of investigation mono nucleus as well as pair core. The coherent states were used in the description of the condensation of nuclear material in nuclear physics [28]. The corresponding coherent states for particle trapped in an infinite square-well and Pöschl-Teller potentials were constructed [29]. To establish a coherent state, different three ways in the consideration of displacement operator acting on the ground state, annihilation operator and minimum uncertainty states can be used. While coherent states for the square-well potential were investigated by using ladder operator formalism in [5], displacement operator method was considered to discuss same quantum system in [30]. In this section of paper, the coherent states are established by using below defined the unitary operators as also called displacement operator. This formalism is also known as Perelomov's coherent states [4], which is different from considered ladder operator formalism to obtain Klauder-Glauber coherent states examined in [29] in detail.

$$D(\alpha)=\exp[\alpha\hat{S}_+ - \alpha^*\hat{S}_-] \tag{39}$$

where $\alpha \in C$ and $\hat{S}_\pm^\dagger = \hat{S}_m$. In order to construct the coherent states, it should be based on that

$$\left|\alpha\right\rangle = D(\alpha)\left|0\right\rangle \tag{40}$$

The Baker-Campbell-Hausdorff formula is used to calculate the average of some observables, which is

$$e^{-\hat{A}}\hat{B}e^{\hat{A}} = \hat{B}+\frac{1}{1!}[\hat{B},\hat{A}]+\frac{1}{2!}[[\hat{B},\hat{A}],\hat{A}]+\frac{1}{3!}[[[\hat{B},\hat{A}],\hat{A}],\hat{A}]+... \tag{41}$$

Eqs.(39)-(40) should be used together with below expression to calculate the average value of annihilation operator $\hat{S}_-$ in the coherent states.

$$\left\langle\alpha\left|\hat{S}_-\right|\alpha\right\rangle = \left\langle 0\left|D^+(\alpha)\hat{S}_- D(\alpha)\right|0\right\rangle = (m+1)(\alpha-\alpha^2\alpha^*) \tag{42}$$

Following to same method, the average values of

$\langle\alpha|\hat{S}_-^2|\alpha\rangle$, $\langle\alpha|\hat{S}_+|\alpha\rangle$, $\langle\alpha|\hat{S}_+^2|\alpha\rangle$, $\langle\alpha|\hat{S}_0|\alpha\rangle$ and $\langle\alpha|\hat{S}_0^2|\alpha\rangle$ are calculated, which are

$$\langle\alpha|\hat{S}_-^2|\alpha\rangle=\frac{(2n+m)(m+1)}{3}(3\alpha^2-2\alpha^3) \quad (43)$$

$$\langle\alpha|\hat{S}_+|\alpha\rangle=-\alpha^*(m+1) \quad (44)$$

$$\langle\alpha|\hat{S}_+^2|\alpha\rangle=(\alpha^*)^2(2n+m+2)(m+1) \quad (45)$$

$$\langle\alpha|\hat{S}_0|\alpha\rangle=\frac{1+2\alpha\alpha^*}{2}(m+1) \quad (46)$$

$$\langle\alpha|\hat{S}_0^2|\alpha\rangle=\frac{(m+1)^2}{2} \quad (47)$$

It should be pointed out that displacement operator method leads to get simple forms for expectation values in the coherent states of S_0, $S_\pm$ operators.

5 Conclusion

The physical implication of this study has been discussed as follows:
In the presence of magnetic field, the investigation of behavior of electron gas in low-dimensional systems is very important for possible applications. Hamiltonian of this quantum system is

$$H=-\frac{\hbar}{2m^*}\nabla^2+\frac{eB}{2m^*c}L_z+\frac{e^2B^2}{8m^*c^2}r^2+V(r,\phi) \quad (48)$$

As seen in the Hamiltonian, the magnetic field is applied on the system along the z axis. This applied magnetic field, together with parabolic spatial confinement, leads to create total effective potential in quantum wire. In other words, electrons in quantum wire are confined under the influence of magnetic field and spatial (parabolic) confinement. The aim is here to confine of the electrons in zone overlapped two parabola using single step between two parabolic potential

$$V_{eff}(r)=\begin{cases}\frac{e^2B^2}{8m^*c^2}r^2 & ,r>R\\ \frac{e^2B^2}{8m^*c^2}r^2+\frac{V_o}{R^2}r^2 & ,r<R\end{cases} \quad (49)$$

It is clear in Fig. 2, 3 and 4 that when B magnetic field increases from 0.5 to 3, the potential profile, and thus the localizations of bound states, changes significantly. Since parameters (V_0 and R) in parabolic spatial confinement relative to structure of semiconductor, parabolic spatial confinement cannot be changed. But, changing of strength of magnetic field means changing of parabolic confinement as can be seen in Fig. 1, 2 and 3.

Realization of the dynamical group for the Morse, Pöschl-Teller, Pseudoharmonic potentials important potentials in atomic, molecular and condensed matter physics was examined using algebraic method. Because, constructing the ladder operators for these potentials allows to find the explicit bound state energies and the eigenfunctions directly in a simple and unique way, which is advantage of algebraic approach. Considered double parabolic potential to model semiconductor heterostructures in this study is important a potential from the point of view its applicable in semiconductor technology such as optoelectronic and sensors that sensitive to magnetic field. Due to these reasons, investigation of this quantum system by using algebraic method, as well as AIM, is also very necessary. I sure that construction of ladder operator for semiconductor heterostructures will be interest to some researchers in this field.

As a result of applying magnetic field on GaAs-GaAlAs quantum wire and changing a parabolic spatial surrounding, electrons in the system have localized in overlapped zone of two parabolic potential. The employed Schrödinger equation for these localized electrons has been solved by using AIM, the energy eigenvalues and corresponding wave functions including confluent hypergeometric functions have been obtained. The annihilation and creation operators for the normalized wave functions have been constructed using the factorization method. By examining commutation relations of the obtained operators $\hat{S}_{\pm,0}$, it has been shown that SU(1,1) group is the dynamical group for the bound states of electrons in overlapped zone of two parabolic potential. It has been shown that corresponding wave functions can be obtained by acting the creation operator $\hat{S}_+$ on the ground state. The matrix elements of functions r^2 and rd/dr have been obtained by using ladder operators in closed analytic form. Finally, the average values of the ladder operators $\hat{S}_\pm$, operator $\hat{S}_0$ and squares of these operators in the coherent states have been calculated. But, since the Hamiltonian of the system can not be exactly expressed in terms of operators $\hat{S}_{\pm,0}$, the time depending of operators $\hat{S}_{\pm,0}$ could not be

obtained by using the evolution operator $U(t)=\exp[(-i/h)\hat{H}t]$.

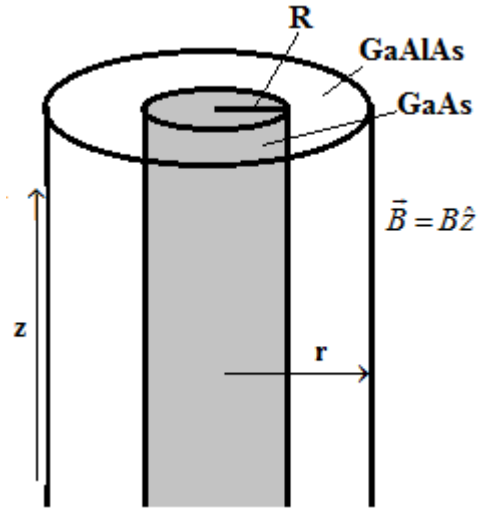

Figure 1. Schematic representation of cylindrical quantum wire [2].

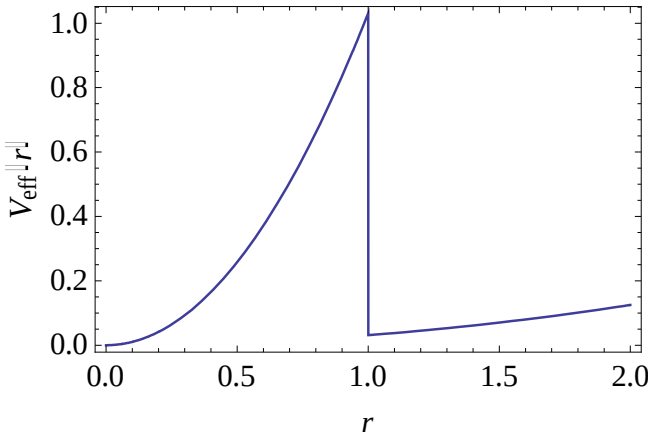

Figure 2. Plot of effective potential for B=0.5, e= $m^*=V_0$ =R=c=1.

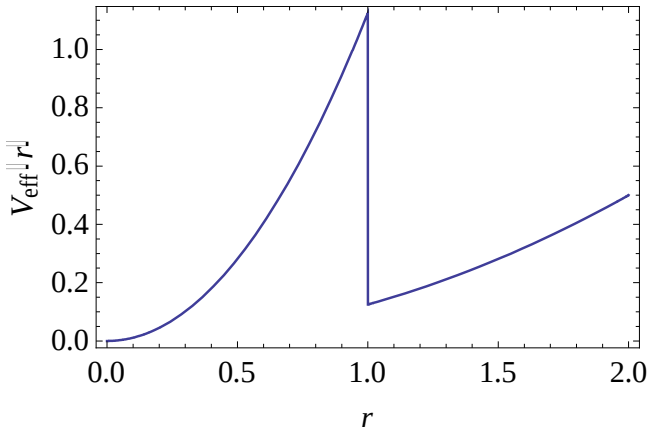

Figure 3. Plot of effective potential for B=1, e= $m^*=V_0$ =R=c=1.

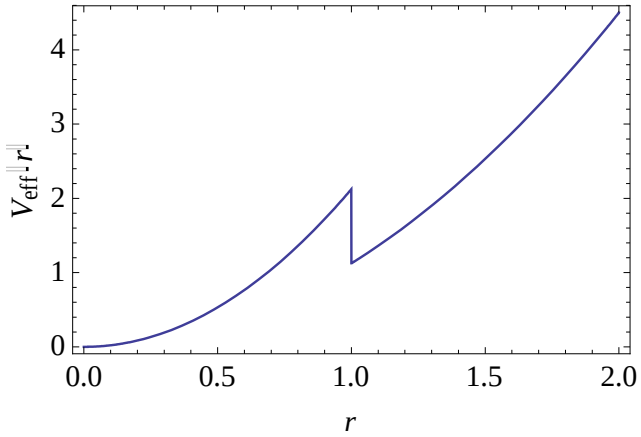

Figure 4. Plot of effective potential for B=3, e= $m^*=V_0$ =R=c=1.

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