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The Investigation of Performances of Different Proportional-Derivative Control Based Fuzzy Membership Functions in Balancing the Inverted Pendulum (IP) on Cart

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Keyword 1, Inverted Pendulum on Cart
Keyword 2, Different Fuzzy Membership Functions
Keyword 3, Proportional-Derivative Control

Abstract: The present study aims to balance an inverted pendulum (IP) system on cart using fuzzy controllers with different membership functions (MFs) based on proportional-derivative control. To this end, a cart was designed to track an intended trajectory in a horizontal (linear) position, while IP was balanced in a vertical (angular) position. This controller system was simulated in Matlab/Simulink, and performance rates were measured for IP's vertical position and cart's horizontal position. Simulation results demonstrated that triangular fuzzy membership function (MF) improved rise time, settling time and overshoot for the IP's vertical position by 8%, 4.35% and 7.7%, respectively, compared to gaussian fuzzy MF. Similarly, for the cart's horizontal position, triangular fuzzy MF improved rise time, settling time and overshoot by 3.8%, 3 and 30%, respectively, compared to gaussian fuzzy MF. When all performance rates are analyzed in terms of IP's vertical position and cart's horizontal position, it was found that triangular fuzzy MF displayed a more satisfactory performance compared to gaussian fuzzy MF.

Arabalı Ters Sarkacın Dengelenmesi için Farklı Oransal-Türevsel Denetim Tabanlı Bulanık Üyelik Fonksiyonlarının Başarımlarının İncelenmesi

Anahtar Kelimeler
Anahtar Kelime 1, Arabalı Ters Sarkaç
Anahtar Kelime 2, Farklı Bulanık üyelik fonksiyonları
Anahtar Kelime 3, Oransal-Türevsel Denetim

Öz: Bu çalışmada, arabalı bir ters sarkaç (TS) sisteminin oransal-türevsel denetim tabanlı farklı üyelik fonksiyonlu (MF) bulanık denetleyiciler kullanılarak dengelenmesi amaçlanmıştır. Bu amaçla araba yatay konumda (çizgisel konum) arzu edilen yörüngeyi takip ederken, sarkacın da dikey konumda (açısal konum) dengede kalması sağlanmıştır. Tasarlanan denetim sistemine ait benzetim çalışmaları Matlab/Simulink ortamında yapılmış olup sarkacın dikey ve arabanın yatay konum denetimi için elde edilen başarım değerleri ayrı ayrı verilmiştir. Benzetim çalışmasından edilen denetim başarım değerleri incelendiğinde sarkacın dikey konumu için yükselme zamanı, yerleşme zamanı ve aşım bakımından üçgen bulanık üyelik fonksiyonunun gauss bulanık üyelik fonksiyonuna göre sırasıyla %8, %4.35 ve %7.7 oranlarında iyileştirme yaptığı görülmüştür. Benzer şekilde arabanın yatay konumu için de yükselme zamanı, yerleşme zamanı ve aşım bakımından üçgen bulanık üyelik fonksiyonunun (ÜF) gauss bulanık ÜF'ye göre sırasıyla %3.8, %3 ve %30 oranlarında daha iyi denetim başarımına sahip olduğu görülmüştür. Sarkacın dikey konum ve arabanın yatay konum denetimi için tüm denetim başarım değerleri analiz edildiğinde üçgen bulanık ÜF'nin gauss bulanık ÜF'ye göre daha tatmin edici sonuçlar verdiği açıkça görülmüştür.

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1. Introduction

Inverted pendulum (IP) on cart is an unstable and non-linear system. It is an important field of study which offers the opportunity to test existing controller systems and develop new controller mechanisms [1]. There are various IP systems in the field of engineering. Among the most popular systems are IP on cart [2-4], double IP [5-6] and rotational single-arm pendulum (Furuta pendulum) [7-8]. Due to the difficulty of control and the need for further improvements on the system, numerous studies have been so far carried out on IP. Many studies on IP in the existing literature focus on the performance of a controller system in balancing IP in a vertical (angular) position [9]. In the current literature, intelligent control systems, linear and non-linear systems and hybrid systems as a combination of different methods have been so far proposed to balance IP [10]. While the most well-known linear control methods can be listed as proportional-integral, proportional-derivative, proportional-integral-derivative [11-12] and linear-quadratic regulator [13-20], non-linear control methods are sliding mode control [21-24] and back-stepping control [25-28]. On the other hand, fuzzy neural networks [29-32], adaptive neuro-fuzzy inference system [33-37], optimization algorithms [38-41] and fuzzy logic (FL) [42-54] are intelligent control systems. It is also possible to benefit from hybrid control systems [55-61] with a combination of different control systems. The literature review above indicates that a high number of control methods have been so far proposed in order to balance an unstable and vertical IP system on cart.

In the present study, a mathematical model was designed for an IP system on cart via Matlab/Simulink. Thanks to a single input-multiple output (SIMO) control approach, different proportional-derivative based fuzzy membership functions (MFs) were used to help the cart track an intended horizontal trajectory and balance the pendulum in a vertical position. Unlike some control methods such as proportional-derivative-integral [57, 59-60], linear quadratic [62], self-tuning fuzzy [61-62] and fuzzy sugeno [53, 57], which were used by the authors in their previous studies on the IP system on cart, the control performances of different proportional-derivative based fuzzy MFs (triangular and gaussian) were analyzed. The main contribution of the present study is that it is the first in the existing literature to have analyzed the control performances of different proportional-derivative based fuzzy MFs in balancing an IP system on cart, which contributes to the originality of the present study.

The rest of the present study is organized as follows: The mathematical model of IP system on cart, basic structure of the FL control system and proportional-derivative FL system are described in Section 2. The comparative results of the detailed simulation studies are presented in Section 3. The obtained numerical data from simulation studies are discussed in Section 4.

2. Material and Method

The present section describes the mathematical modelling of IP system on cart and design of and proportional-derivative FL system.

2.1 The Mathematical Modelling of IP System on Cart

IP on cart system requires a mathematical modelling in order for the cart to track an intended horizontal trajectory and to keep its pendulum in a vertical position. This mathematical model is created using laws of physics. As shown in Figure 1, IP on cart involves the movements of a condensed pendulum pole with a length of L and mass of m along a cart with a mass of M. The angle between the pole and vertical position intersecting its area of movement is represented by Θ, while its distance to a certain reference point in a horizontal position is denoted by x, respectively. While the movement of a pole is limited to x-y axis, the cart is allowed to only move along x axis. At this point, non-linear model of IP is calculated using Newton's laws of motion [63-65].

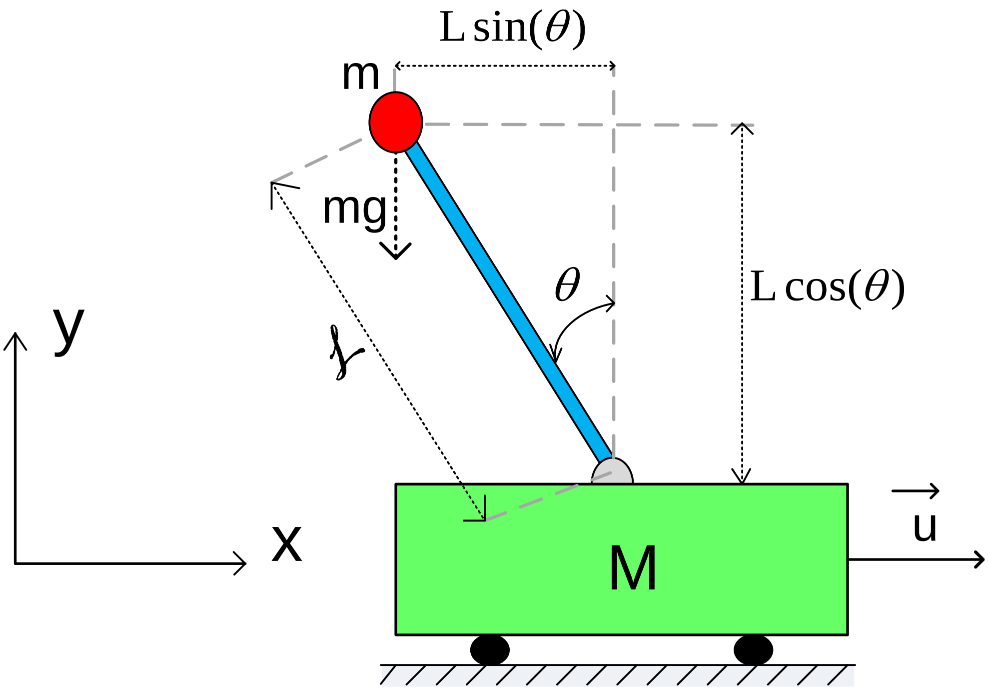

Figure 1. IP on cart

The center of gravity for a mass of m is defined in (x,y) coordinates in Equation (1) and (2).

$$x_g = x + L\sin\theta \tag{1}$$

$$y_g = L\cos\theta \tag{2}$$

If Newton's second law is applied to the movement towards x, it gives a differential equation in Equation (3).

$$M\frac{d^2x}{dt^2} + m\frac{d^2x_g}{dt^2} = u \tag{3}$$

When the expression in Equation (1) is put back in its position in Equation (3), it gives Equation (4).

$$(\mathrm{M+m})\ddot{x} - mL(\sin\theta)\dot{\theta}^2 + mL(\cos\theta)\ddot{\theta} = u \tag{4}$$

Secondly, if Newton's second law is applied to the movement of m around the pole, it gives a differential equation in Equation (5).

$$m\frac{d^2x_g}{dt^2}L\cos\theta - m\frac{d^2y_g}{dt^2}L\sin\theta = mgL\sin\theta \tag{5}$$

When the expressions in Equation (1) and (2) are put back in their respective positions in Equation (5), it gives Equation (6).

$$m\left[\ddot{x} - L(\sin\theta)\dot{\theta}^2 + L(\cos\theta)\ddot{\theta}\right]L\cos\theta - m\left[-L(\cos\theta)\dot{\theta}^2 - L(\sin\theta)\ddot{\theta}\right]L\sin\theta = mgL\sin\theta \tag{6}$$

Element values for IP system on cart are given in Table 1.

Table 1. Element values for IP on cart system

Parameters	Symbol	Value	Unit
Mass of the cart	M	2.4	kg
Mass of the IP	m	0.23	kg
The length of the pole	*L*	0.36	m
Gravitational acceleration	g	9.8	m/sec^2
Track length	*l*	0.5	m

2.2. FL Control System

FL toolbox module was used to control the IP's vertical position and the cart's horizontal position via a FL controller. Matlab FL toolbox window is shown in Figure 2.

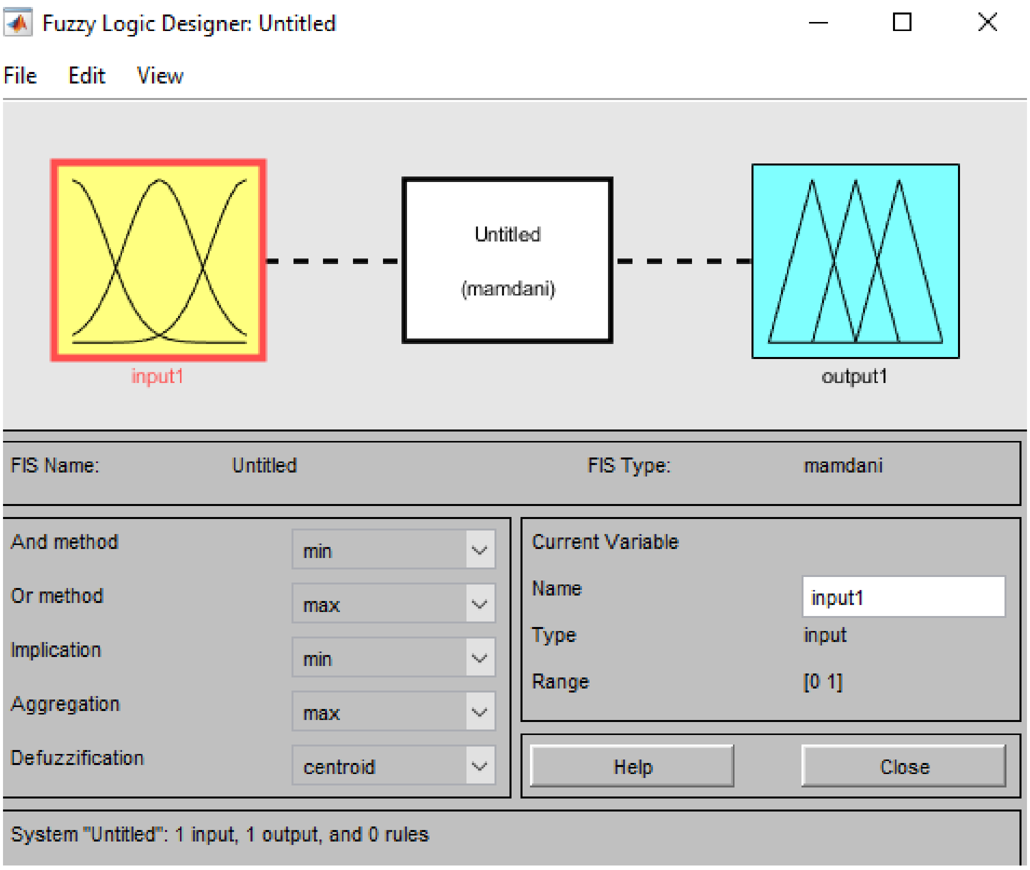

Figure 2. Matlab FL toolbox menu

After the number of optimal input and output variables is selected for the control system on this window, it is possible to select any MFs and types by entering input and output values. In addition, and/or methods, decision, fuzzification, defuzzification, mamdani and sugeno fuzzy inference methods can also be selected from this menu. The membership function (MF) and fuzzy inference windows in Matlab FL toolbox are shown in Figure 3 and 4, respectively.

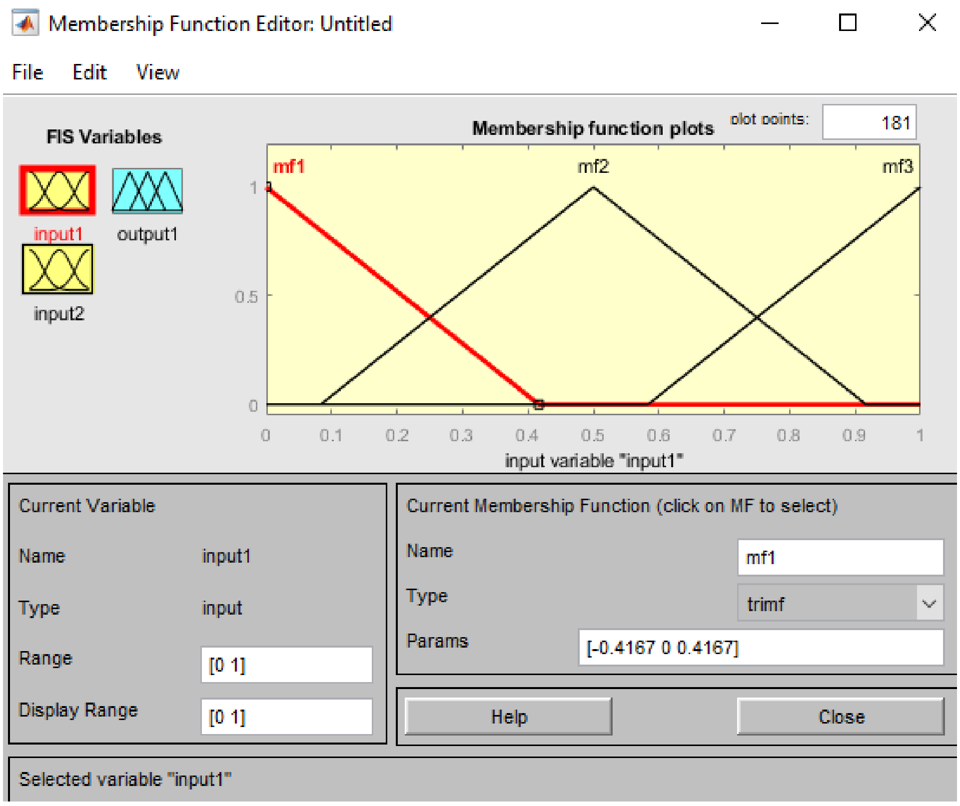

Figure 3. Matlab FL toolbox MF window

Types and ranges of MFs can be selected on Matlab FL toolbox MF window to determine optimal MFs for the designed control system. "Edit" button can be used to add or delete any MFs.

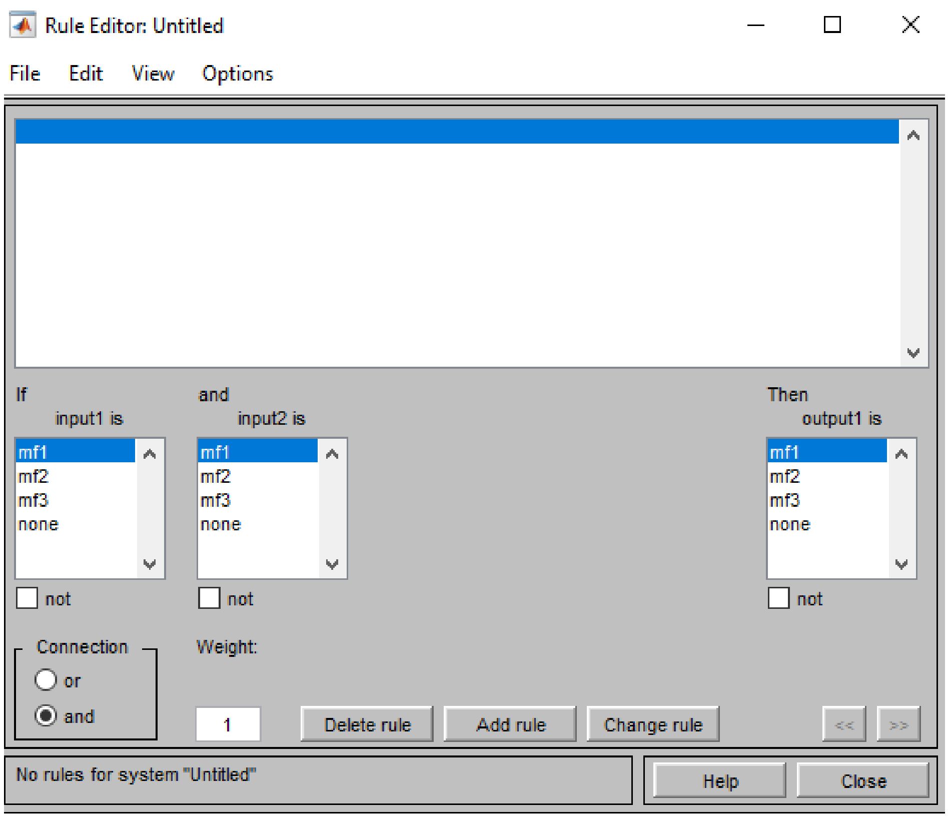

Figure 4. Matlab FL toolbox fuzzy inference window

Using Matlab FL toolbox fuzzy inference window, connection methods are selected to create a rule base suitable to the control system, and MFs are connected to each other via linguistic connectors. It must be noted that a higher number of rules and MFs in a fuzzy controller decreases oscillation and increases stability in the control system. The definition of MFs in a fuzzy controller directly affects the system response. Therefore, input values must be always entered within the range of MFs in the control system to operate rule base effectively.
Maximum error (e) range for cart's horizontal (linear) position is [-1,1]. Error (e), change in error (de) and output value (u) range were selected as [-1,1]. Linguistic variables for triangular and gaussian MFs are negative big (NB), negative medium (NM), negative small (NS), Zero (ZR), positive small (PS), positive medium (PM) and positive big (PB). Meanwhile, 7 different linguistic variables were selected for each input using triangular and gaussian MFs in the design of FL controller. As a result, rule base consisted of 49 rules without excluding any ordered pairs. It was created based on the expert's abilities, observations and system experiments. Mamdani was used as a fuzzy inference method in the designed FL controller. Center of gravity was used as a defuzzification method. In the designed system, regions with a change in (e) value higher than 0 are where the linear position starts to decrease. In particular, impact values in a region with an (e) value higher than 0 must be higher than impact values in other regions. A region with an (e) value higher than 0 and (de) value lower than 0 is the point where linear position attempts to reach a reference value. Settling time and overshoot were taken into account to determine impact values in this region. The region with an (e) value and (de) value equal to 0 is the most critical point for the system, and therefore its impact values must be selected accurately. (e) and (de) were used as inputs in the designed fuzzy control system, and all rules in the rule table were processed to check all probabilities.

(e), (de) and linguistic variables for output (u) in triangular MF are as follows:

NB= [-1.333 -1 -0.6667],

NM= [-1 -0.6667 -0.3333],

NS= [-0.6667 -0.3333 -5.551e-17],

ZR= [-0.3333 0 0.3333],

PS= [-5.551e-17 0.3333 0.6667],

PM= [0.3333 0.6667 1]

PB= [0.672 1.005 1.338],

Triangular MF for (e), (de) and (u) is shown in Matlab FL toolbox in Figure 5.

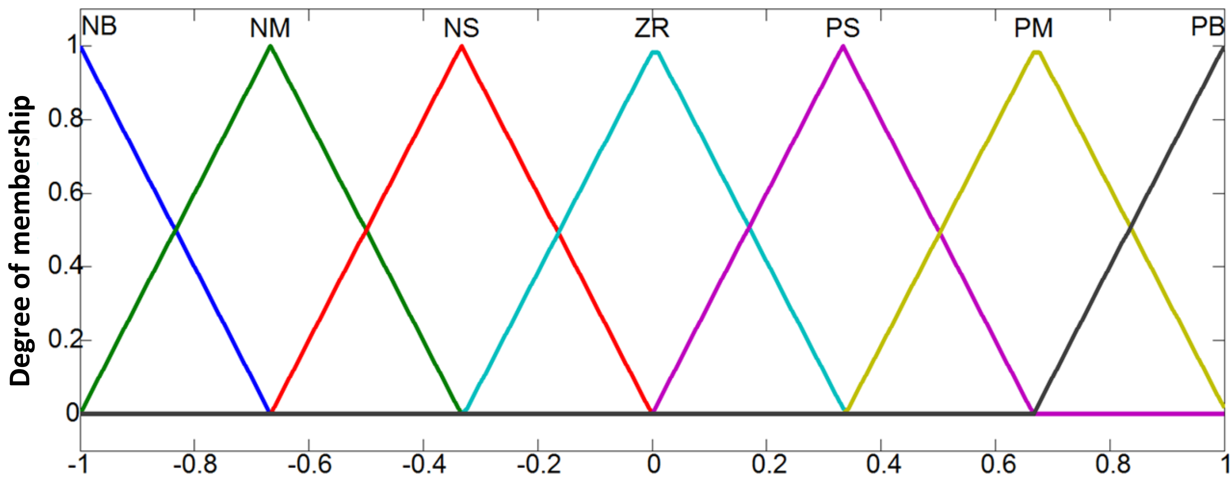

Figure 5. Triangular MF for (e), (de) and (u)

(e), (de) and linguistic variables for (u) in gaussian MF are as follows:

NB= [0.1416 -1],

NM= [0.142 -0.6613],

NS= [0.1416 -0.3334],

ZR= [0.1416 0],

PS= [0.1416 0.3334],

PM= [0.1416 0.6666],

PB= [0.1416 -1],

Gaussian MF for (e), (de) and (u) is shown in Matlab FL toolbox in Figure 6.

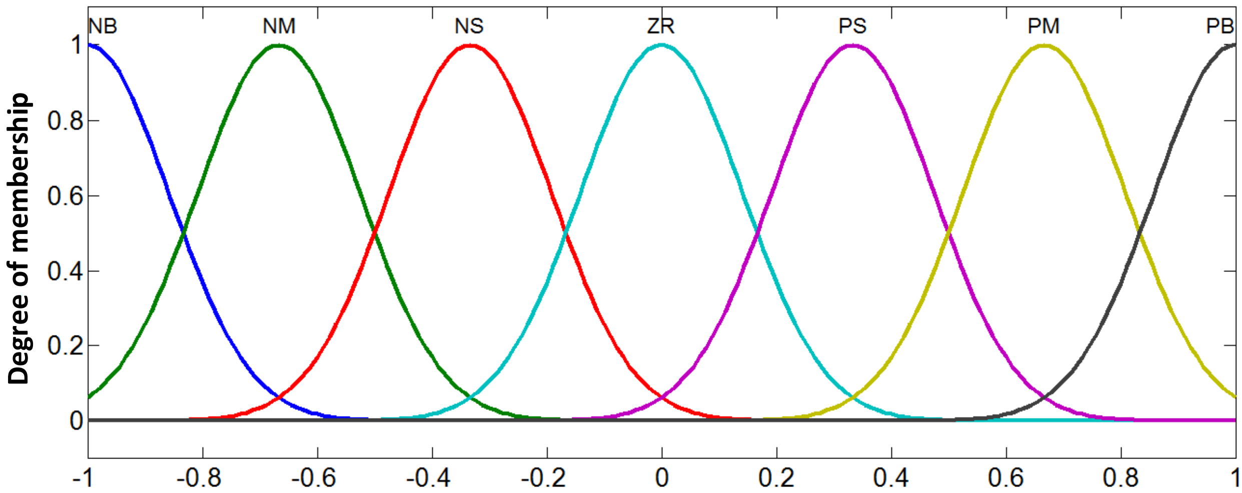

Figure 6. Gaussian MF for (e), (de) and (u)

Maximum (e) range for IP's angular position is [-0.3, 0.3]. (e) value range in triangular and gaussian MF was selected as [-0.3, 0.3]. (de) and (u) value range was selected as [-1,1]. Linguistic variables for triangular and gaussian MFs are (NB), (NM), (NS), (ZR), (PS), (PM) and (PB). Meanwhile, 7 different linguistic variables were selected for each input using triangular and gaussian MFs for IP's vertical (angular) position in the design of FL controller. As a result, rule base consisted of 49 rules without excluding any ordered pairs. Mamdani was used as a fuzzy inference structure in the designed FL controller. Center of gravity was used as a defuzzification method. In the designed system, regions with a change in (e) value higher than 0 are those where the angular position starts to decrease. In particular, impact values in a region with an (e) value higher than 0 must be higher than impact values in other regions. A region with an (e) value higher than 0 and (de) value lower than 0 is the point where the angular position attempts to reach a reference value. Settling time and overshoot were taken into account to determine impact values in this region. The region with an (e) value and (de) value equal to 0 is the most critical point for the system, and therefore its impact values must be selected accurately. (e) and (de) were used as inputs in the designed fuzzy control system, and all rules in the rule table were processed to check all probabilities.

Linguistic variables for (e) in triangular MF are as follows:

NB= [-0.4 -0.3 -0.2],

NM= [-0.3 -0.2 -0.1],

NS= [-0.2 -0.1 -1.388e-17],

ZR= [-0.1 8.674e-19 0.1],

PS= [-1.388e-17 0.1 0.2],

PM= [0.1 0.2 0.3],

PB= [0.2 0.3 0.4],

Membership functions for (e) in triangular MF are shown in Matlab FL toolbox in Figure 7.

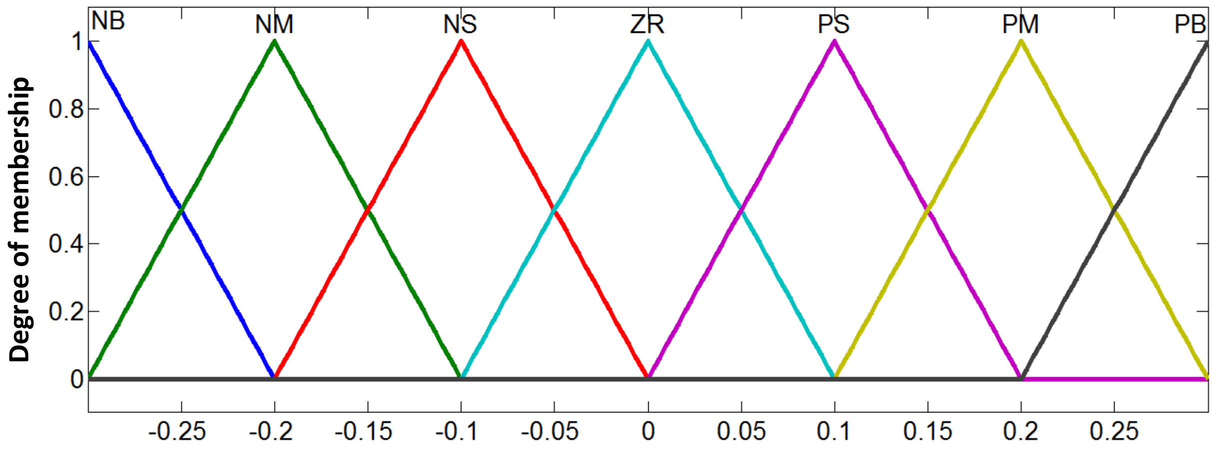

Figure 7. Triangular membership function for (e)

Membership functions for (de) and (u) in triangular MF are shown in Matlab FL toolbox in Figure 8.

(de) and linguistic variables for (u) in triangular MF are as follows:

NB= [-1.333 -1 -0.6667],

NM= [-1 -0.6667 -0.3333],

NS= [-0.6667 -0.3333 -5.551e-17],

ZR= [-0.3333 0 0.3333],

PS= [-5.551e-17 0.3333 0.6667],

PM= [0.3386 0.672 1.005],

PB= [0.6667 1 1.333],

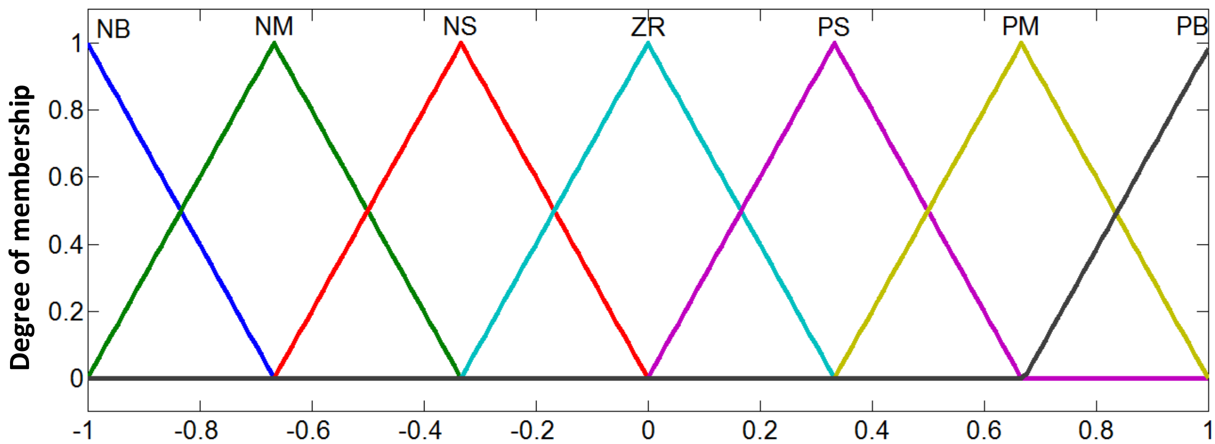

Figure 8. Triangular MF for (de) and (u)

Linguistic variables for the (e) in gaussian MF are as follows:

NB= [0.04247 -0.3],

NM= [0.0425 -0.203],

NS= [0.04247 -0.09842],

ZR= [0.04247 0],

PS= [0.0425 0.1],

PM= [0.04247 0.2],

PB= [0.0425 0.3],

MFs for (e) in gaussian MF are shown in Matlab FL toolbox in Figure 9.

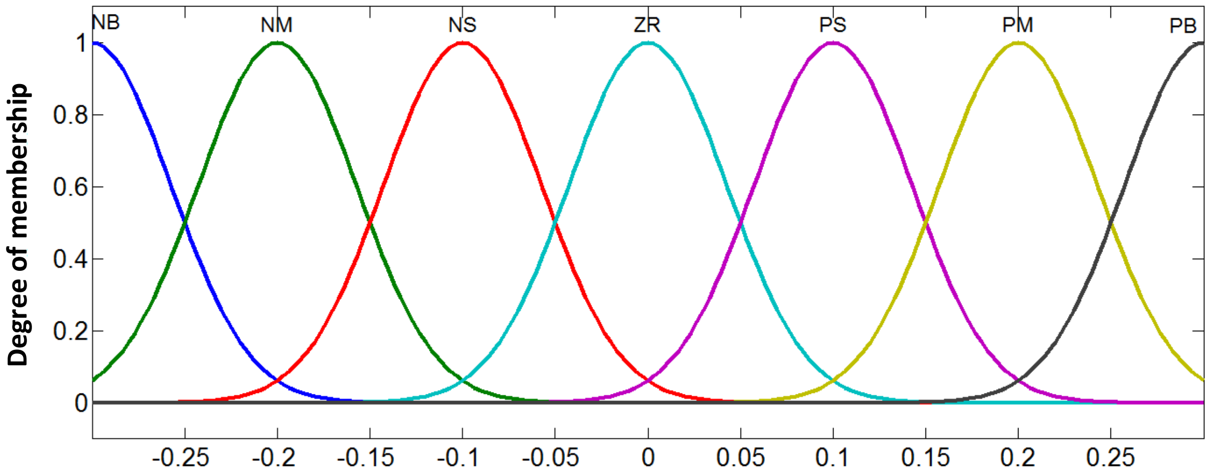

Figure 9. Gaussian MF for (e)

(de) and linguistic variables for (u) in Gaussian MF are as follows;

NB= [0.1416 -1],

NM= [0.1416 -0.6667],

NS= [0.1416 -0.3333],

ZR= [0.1416 0],

PS= [0.142 0.339],

PM= [0.1416 0.6667],

PB= [0.1416 1],

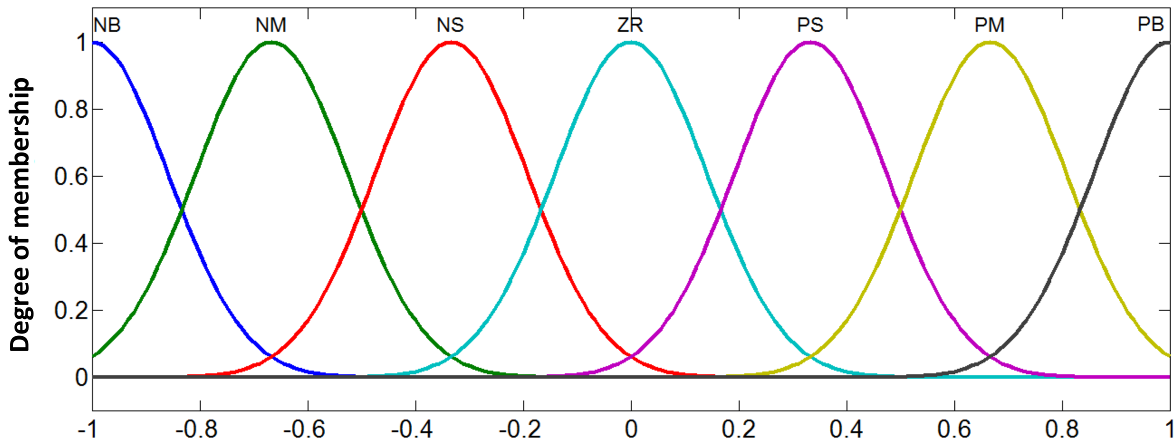

Figure 10. Gaussian MF for the (de) and (u)

Rule table used for IP system on cart is given in Table 2.

Table 2. The rule table used for IP system on cart

u \ *e* / *de*	NB	NM	NS	ZR	PS	PM	PB
NB	NB	NB	NB	NB	NM	NS	ZR
NM	NB	NB	NB	NM	NS	ZR	PS
NS	NB	NB	NM	NS	ZR	PS	PM
ZR	NB	NM	NS	ZR	PS	PM	PB
PS	NM	NS	ZR	PS	PM	PB	PB
PM	NS	ZR	PS	PM	PB	PB	PB
PB	ZR	PS	PM	PB	PB	PB	PB

2.3. Proportional-Derivate Control Based FL System

Proportional-derivative control based FL system is a two-input single-output control system which was developed based on conventional proportional-derivative. A proportional-derivative controller consists of proportional and derivative gain factors. It is used to minimize rise time, overshoot and instability in a control system. The output of proportional-derivative controller is given in Equation (7). Matlab/Simulink diagram of a proportional-derivative control based FL system is shown in Figure 11.

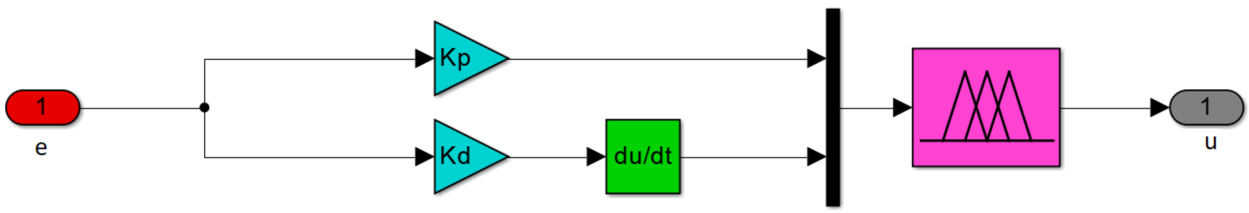

Figure 11. Matlab/Simulink diagram of a proportional-derivative control based FL system

$$u(t) = K_p.e(t) + K_d.\frac{d}{dt}e(t) \tag{7}$$

K_p and K_d gains are particularly important for the design of a proportional-derivative control based fuzzy controller. When changing gains, to determine the most optimal values for the designed control system, K_p value was increased if the system response was slow, while K_d value was increased if overshoot and oscillation rates were high (65). Proportional and derivative gains for the control of IP's vertical (angular) position were adjusted as K_p=8 and K_d=5, respectively. On the other hand, for the control of cart's horizontal (linear) position, the proportional and derivative gains were adjusted as K_p=1.6 and K_d=3, respectively. The initial and reference vertical (angular) positions were set to 0.035 radians (2 degrees) and 0 radians, respectively, for IP. On the other hand, initial and reference horizontal (linear) positions were set to 0 m and 0.1 m, respectively, for cart [4]. As shown in Figure 12, disturbance in the designed system was a pulse signal with an amplitude of 0.25N. The model designed for the control system simulated on Matlab/Simulink is shown in Figure 13.

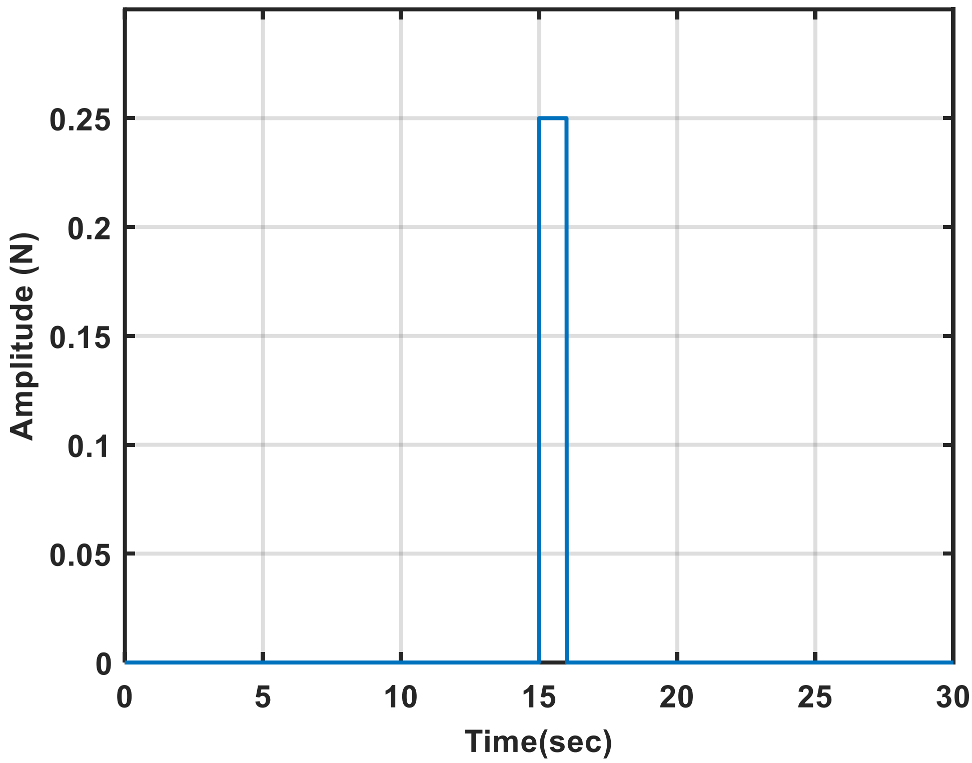

Figure 12. Disturbance Input Signal

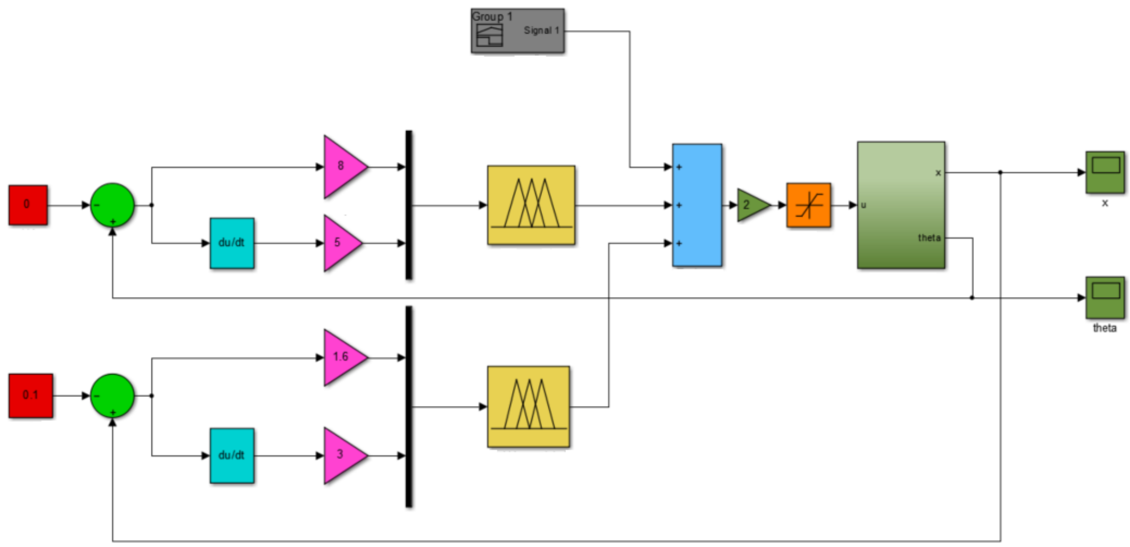

Figure 13. Matlab/Simulink model designed for the proportional-derivative control-based FL system

3. Results

3.1 Simulation Studies for the IP System on Cart

IP system on cart is a single-input multiple-output (SIMO) one. In the current literature, there are several studies which focused on independent control of cart's horizontal (linear) position against the reference input or IP's vertical (angular) position against the reference input. However, the present study proposes a proportional-derivative control based controller with different fuzzy (MFs) for the simultaneous control of cart's horizontal (linear) position and IP's vertical (angular) position. The disturbances for IP on cart's horizontal (linear) and vertical (angular) position were added to control signals and given to the system as an input. The disturbance pulse signal was applied to the system at the 15th second and lasted 1 second. The control signal was limited within a range of u=[-1,1]. The control of IP on cart's vertical (angular) position using triangular and Gaussian MFs are shown in Figure 14.

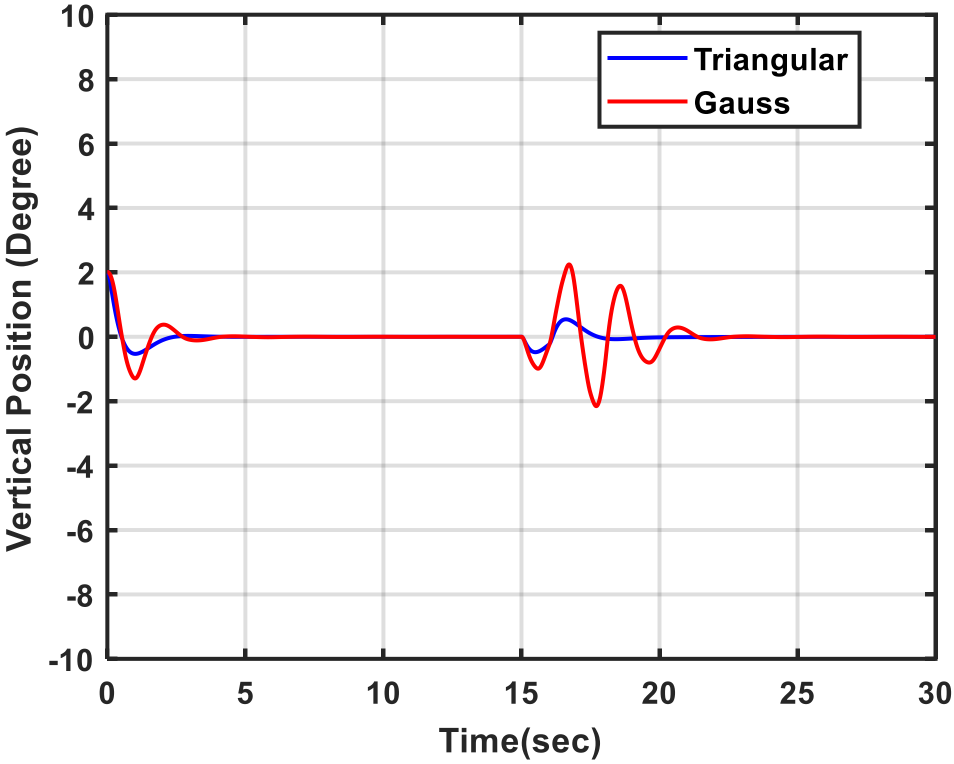

Figure 14. The control of IP on cart's vertical (angular) position using triangular and gaussian MFs

The control of IP on cart's horizontal (linear) position using triangular and gaussian MFs is shown in Figure 15.

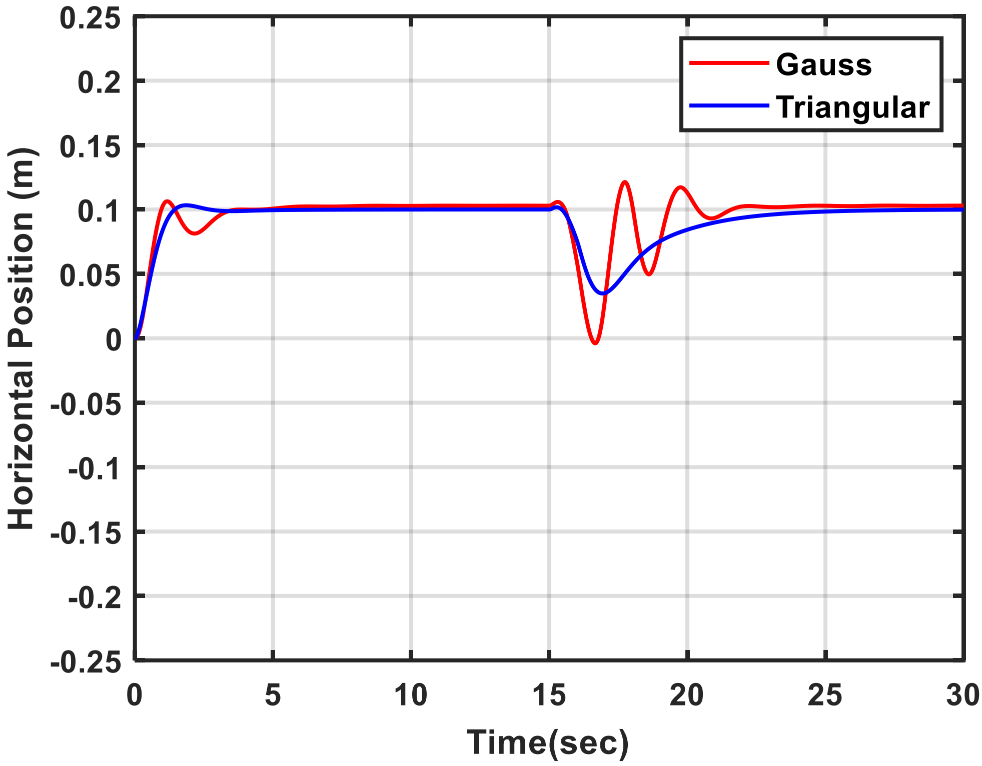

Figure 15. The control of IP on cart's horizontal (linear) position using triangular and gaussian MFs

The impact of triangular and gaussian MFs on the proposed controller's performance in terms of IP on cart's vertical (angular) position was analyzed. As shown in Figure 14, three important performance parameters for the proposed controller, namely rise time (t_r), settling time (t_y) and overshoot (M%), were assessed in terms of IP on cart's vertical (angular) position. The results are summarized in Table 3.

Table 3. The control performance for IP on cart's vertical (angular) position

Performance values	t_r (sec)	t_y(sec)	M%
Triangular	0.23	4.4	12
Gaussian	0.25	4.6	13

The impact of triangular and gaussian MFs on the proposed controller's performance in terms of IP on cart's horizontal (linear) position was analyzed. As shown in Figure 15, three important performance parameters for the proposed controller, namely rise time (t_r), settling time (t_y) and overshoot (M%), were assessed in terms of IP on cart's vertical (angular) position. The results are summarized in Table 4.

Table 4. The control performance for IP on cart's horizontal (linear) position

Performance Values	t_r (sec)	t_y(sec)	M%
Triangular	0.625	3.3	7
Gaussian	0.650	3.4	10

According to Tables 3 and 4, improved performance values of triangular membership function for controlling IP's vertical and the cart's horizontal position are shown in Figure 16.

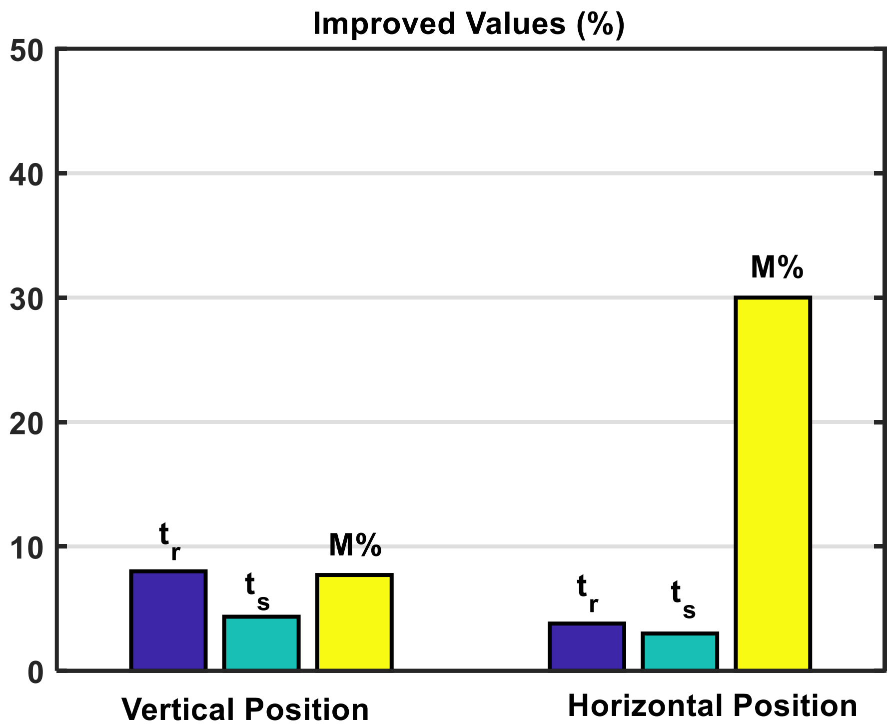

Figure 16. Improved performance values of triangular membership function for controlling IP's vertical and cart's horizontal position

As for the control of IP's vertical position, a disturbance signal was applied at the 15th second for 1 second, and the system with triangular membership function was balanced and came to a steady-state (without oscillation) position after 4 seconds. On the other hand, the system with gaussian membership function underwent oscillation and was balanced after 6 seconds. Similarly, as for the control of cart's horizontal position, the system with a gaussian membership function underwent oscillation and was balanced in an unstable position. However, the system with a triangular membership function was balanced in a steady-state position. Settling time performance values of both membership functions for vertical and horizontal positions following disturbance signals are given in Tables 5 and 6, respectively.

Table. 5 IP's vertical position control values following a disturbance signal

Performance Values	t_y(sec)
Triangular	4
Gaussian	6

Table. 6 Cart's horizontal position control values following a disturbance signal

Performance Values	t_y(sec)
Triangular	10
Gaussian	10

4. Discussion and Conclusion

The present study designed a non-linear model for an IP system on a cart to control the IP's vertical (angular) position and the cart's horizontal (linear) position simultaneously using different proportional-derivative control-based fuzzy MFs against disturbance signals. The results demonstrated that fuzzy triangular MF displayed the highest performance in the control of the IP on cart's vertical (angular) and horizontal (linear) position thanks to its narrower peak, higher slope, and quicker response to change in error. It was also observed that steady-state error in the system response was caused by the system input value range and that a narrower range would contribute to a more accurate system. In addition to the methods used in the present study, future studies are recommended to focus on obtaining better system responses through the optimization of the most optimal rules and membership functions for FL control systems.

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