Research Article GU J Sci, Part A, 10(1): 1-8 (2023) 10.54287/gujsa.1168439

Gazi University
Journal of Science
PART A: ENGINEERING AND INNOVATION
http://dergipark.org.tr/gujsa

Hilbert I-Statistical Convergence on Neutrosophic Normed Spaces

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Keywords

Neutrosophic Normed Spaces
Hilbert Matrix
Statistical Convergence
Hilbert Sequence Space
Open Sets

Abstract

In this paper, $\lambda\mathfrak{J}$-statistical convergence is defined to generalize statistical convergence on Neutrosophic normed spaces. As it is known, Neutrosophic theory, which brings a new breath to daily life and complex scientific studies which we encounter with many uncertainties, is a rapidly developing field with many new study subjects. Thus, researchers show great interest in this philosophical approach and try to transfer related topics to this field quickly. For this purpose, in this study, besides the definition of $\lambda\mathfrak{J}$-statistical convergence, the important features of Hilbert sequence space and $\lambda\mathfrak{J}$-statistical convergence in Neutrosophic spaces are examined with the help of these defined sequences. By giving the relationship between Hilbert $\lambda\mathfrak{J}$-statistical convergence and Hilbert $\mathfrak{J}$-statistical convergence, it has been evaluated whether the definitions contain a coverage relationship as in fuzzy and intuitionistic fuzzy. As a result, it is thought that the selected convergence type is suitable for the Neutrosophic normed space structure and is a guide for new convergence types.

Cite

Gonul Bilgin, N. (2023). Hilbert I-Statistical Convergence on Neutrosophic Normed Spaces. *GU J Sci, Part A*, *10*(1), 1-8.

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Article Process

Submission Date	30.08.2022
Revision Date	24.11.2022
Accepted Date	06.12.2022
Published Date	21.02.2023

1. INTRODUCTION

Statistical convergence and this concept-based studies play a very important role in areas such as solving some daily life problems, computer science and information theory. Statistical convergence was defined in Fast, 1951. Subsequently, many important modifications of this concept were studied. For example, λ-statistical convergence was introduced in (Mursaleen, 2000) and ideal convergence was given in Kostyrko et al., 2000. Later on, Savas and Das (2011) defined I-λ-statistical convergence. Recently, the types of ideal convergence have been moved to different spaces and important contributions have been made to the theory. Examples of these spaces can be given as fuzzy and intuitionistic fuzzy normed spaces.

Fuzzy theory, allows us to transform imprecise information. This concept was given in (Zadeh, 1965). Fuzzy set theory has been applied to different engineering fields where uncertainty is modeled. e.g. quantum physics (Madore, 1992) computer programming (Giles,1980), control of chaos (Fradkov & Evans, 2005). Moreover, the concept of fuzzy and its modification have been given in different spaces For example, Bilgin & Bozma, 2020; Ali & Ansari, 2022; Guner & Aygun, 2022.

Later, the opinion of the intuitionistic fuzzy set was defined in (Atanassov,1986). This concept and its various generalizations were given in different spaces for example Melliani et al., 2015; Gonul Bilgin & Bozma, 2021. In order to transfer uncertain situations to fuzzy normed spaces, Felbin (1992) defined fuzzy normed space then Saadati and Park (2006) gave basic structures of intuitionistic fuzzy norm. Kumar and Kumar (2008) have been studied ideal convergent in fuzzy number theory. Then, Savas and Gurdal (2015) defined λ-ideal convergence in a intuitionistic fuzzy normed space.

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Neutrosophic set theory; was established by Smarandache (1999) who added the degree of uncertainty of the subject by recognizing the inadequacy and deficiency of the fuzzy logic dealing with the degree of membership and the intuitionistic fuzzy logic dealing with membership-nonmembership. In the scientific world, it is not always possible to definitively interpret a problem in daily life as right or wrong. Therefore, this concept has gained so much popularity. For example uncertainty theory plays an important role in constructing a model in different technological problem. On the other hand, many researchers have been working on this new theory in mathematics (e.g. Kisi, 2021a,b,c; Bilgin, 2022; Gonul Bilgin, 2022).

The transfer of special sequences to normed spaces is one of the popular topics of recent times. Kirisci (2019), examined a generalization of the statistical convergence he has been created with the help of the Fibonacci sequence on the intuitionistic fuzzy normed space. Polat (2016) has defined new type Hilbert sequence spaces and Khan et al. (2020) have been gave some properties of Hilbert I-convergent sequences. After that, Khan et al. (2020) transferred this concept to intuitionistic fuzzy normed spaces.

In the literature review, it was seen that Hilbert sequence space, which is one of the important sequence spaces, was not examined together with this new space and statistical convergence. This study was carried out in order to eliminate this deficiency and to bring a different perspective to the theory. In the study, after the basic definitions and theorems are given, these three concepts and important theoretical information about them have been presented.

(Smarandache, 2016) Let's $y_1, y_2, \dots, y_n \in [0,1]$ be n crisp-components. Their sum is next form

1. $0 \leq y_1 + y_2 + \cdots + y_n \leq n$,
   if all of them are 100% independent two by two,

2. $0 \leq y_1 + y_2 + \cdots + y_n \leq 1$,
   if all of them are 100% dependent.

(Neutrosophic Set) (Smarandache, 1999) Let $\mathcal{S} \neq \emptyset$, $\Theta_{\mathfrak{N},M}(z), \Upsilon_{\mathfrak{N},I}(z)$ and $\Psi_{\mathfrak{N},n.M}(z)$ are the degrees of membership, indeterminacy, non-membership of z. A neutrosophic set $\mathfrak{N}$ has the following notation:

$$\mathfrak{N} = \left\{ \left(z, \Theta_{\mathfrak{N},T}(z), \Psi_{\mathfrak{N},n.M}(z), \Upsilon_{\mathfrak{N},I}(z) \right) : z \in \mathcal{S} \right\}$$

where for each z in $\mathcal{S}$, $\Theta_{\mathfrak{N},M}(z), \Upsilon_{\mathfrak{N},I}(z), \Psi_{\mathfrak{N},n.M}(z) \in [0,1]$, $0 \leq \Theta_{\mathfrak{N},M}(z) + \Psi_{\mathfrak{N},n.M}(z) + \Upsilon_{\mathfrak{N},I}(z) \leq 2$.

Here, $\Theta_{\mathfrak{N},M}(z)$, $\Upsilon_{\mathfrak{N},I}(z)$ are dependent components, $\Psi_{\mathfrak{N},n.M}(z)$ is an independent component.

(Neutrosophic Normed Spaces) (Kirisci & Simsek, 2020) Let $\boxtimes$ and $\boxplus$ show the continuous $\mathfrak{t} - norm$ and continuous $\mathfrak{t} - conorm$, $\mathcal{S}$ be a linear spaces on $\mathbb{R}$. A Neutrosophic Normed is a notation of the form $\{ ((z, \mathfrak{s}), \Theta_{\mathfrak{N},M}(z, \mathfrak{s}), \Psi_{\mathfrak{N},n.M}(z, \mathfrak{s}), \Upsilon_{\mathfrak{N},I}(z, \mathfrak{s})) : (z, \mathfrak{s}) \in \mathcal{S} \times \mathbb{R}^+ \}$ where $\Theta_{\mathfrak{N},M}$, $\Upsilon_{\mathfrak{N},I}$ and $\Psi_{\mathfrak{N},n.M}$ are shown the degree of membership, indeterminacy, non-membership of $(z, \mathfrak{s})$ on $\mathcal{S} \times \mathbb{R}^+$ satisfies next:

For every $z, w \in \mathcal{S}$,

1. For all $\mathfrak{s} \in \mathbb{R}^+$ $\Theta_{\mathfrak{N},M}(z, \mathfrak{s}), \Upsilon_{\mathfrak{N},I}(z, \mathfrak{s}), \Psi_{\mathfrak{N},n.M}(z, \mathfrak{s}) \in [0,1]$ then

   $\Theta_{\mathfrak{N},M}(z, \mathfrak{s}) + \Upsilon_{\mathfrak{N},I}(z, \mathfrak{s}) + \Psi_{\mathfrak{N},n.M}(z, \mathfrak{s}) \leq 2$,

2. For all $\mathfrak{s} \in \mathbb{R}^+$,

$$\Theta_{\mathfrak{N},M}(z, \mathfrak{s}) = 1 \Leftrightarrow z = 0, \Upsilon_{\mathfrak{N},I}(z, \mathfrak{s}) = 0 \Leftrightarrow z = 0, \Psi_{\mathfrak{N},n.M}(z, \mathfrak{s}) = 0 \Leftrightarrow z = 0,$$

3. For each $\mathfrak{d} \neq 0$,

$$\Theta_{\mathfrak{N},M}(\mathfrak{d}z, \mathfrak{s}) = \Theta_{\mathfrak{N},M}\left(z, \frac{\mathfrak{s}}{|\mathfrak{d}|}\right), \Upsilon_{\mathfrak{N},I}(\mathfrak{d}z, \mathfrak{s}) = \Upsilon_{\mathfrak{N},I}\left(z, \frac{\mathfrak{s}}{|\mathfrak{d}|}\right), \Psi_{\mathfrak{N},n.M}(\mathfrak{d}z, \mathfrak{s}) = \Psi_{\mathfrak{N},n.M}\left(z, \frac{\mathfrak{s}}{|\mathfrak{d}|}\right)$$

4. For all $\mathfrak{s}_1, \mathfrak{s}_2 \in \mathbb{R}^+$,

$\Theta_{\mathfrak{N},M}(z, \mathfrak{s}_2) \odot \Theta_{\mathfrak{N},M}(\mathfrak{p}, \mathfrak{s}_1) \leq \Theta_{\mathfrak{N},M}(z + \mathfrak{p}, \mathfrak{s}_1 + \mathfrak{s}_2)$,

$\Upsilon_{\mathfrak{N},I}(z, \mathfrak{s}_2) \otimes \Upsilon_{\mathfrak{N},I}(z, \mathfrak{s}_1) \geq \Upsilon_{\mathfrak{N},I}(z + \mathfrak{p}, \mathfrak{s}_1 + \mathfrak{s}_2)$,

$\Psi_{\mathfrak{N},n.M}(z, \mathfrak{s}_2) \otimes \Psi_{\mathfrak{N},n.M}(z, \mathfrak{s}_1) \geq \Psi_{\mathfrak{N},n.M}(z + \mathfrak{p}, \mathfrak{s}_1 + \mathfrak{s}_2)$.

5. $\Theta_{\mathfrak{N},M}(z,\cdot)$ is non-decreasing- continuous function;$\Upsilon_{\mathfrak{N},I}(z,\cdot)$, $\Psi_{\mathfrak{N},n.M}(z,\cdot)$ are non-increasing, continuous function,
6. $\lim_{\mathfrak{s}\to\infty} \Theta_{\mathfrak{N},M}(z, \mathfrak{s}) = 1$, $\lim_{\mathfrak{s}\to\infty} \Upsilon_{\mathfrak{N},I}(z, \mathfrak{s}) = 0$, $\lim_{\mathfrak{s}\to\infty} \Psi_{\mathfrak{N},n.M}(z, \mathfrak{s}) = 0$.
7. Let $\mathfrak{s} \leq 0$. thus situated $\Theta_{\mathfrak{N},M}(z, \mathfrak{s}) = 0$, $\Upsilon_{\mathfrak{N},I}(z, \mathfrak{s}) = 1$, $\Psi_{\mathfrak{N},n.M}(z, \mathfrak{s}) = 1$.

So, $(\mathcal{S}, \Theta_{\mathfrak{N},M}, \Upsilon_{\mathfrak{N},I}, \Psi_{\mathfrak{N},n.M}, \odot, \otimes)$ is named Neutrosophic Normed Spaces. In this case, $\Theta_{\mathfrak{N},M}$, $\Psi_{\mathfrak{N},n.M}$ are dependent, $\Upsilon_{\mathfrak{N},I}$ is an independent.

(Ideal) (Kostyrko et al., 2000) Let $\mathcal{S} \neq \emptyset$ and a family of $\mathfrak{I} \subset \mathcal{P}(\mathcal{S})$ is an ideal where,

1. $\emptyset \in \mathfrak{I}$,
2. for every $S_1, S_2 \in \mathfrak{I}$, then $S_1 \cup S_2 \in \mathfrak{I}$,
3. for all $S_1 \in \mathfrak{I}$ and $S_2 \subset S_1$, then $S_2 \in \mathfrak{I}$.

($\mathcal{H}$ − transforms) (Polat, 2016) Let $\mathcal{K}, \mathcal{K}^*$ be a sequence spaces and $x = (x_k) \in \mathcal{K}$, $\mathcal{H} = (\hbar_{rk}) \in \mathcal{K}^*$ be a triangle Hilbert matrix where for $1 \leq k \leq r$, $\hbar_{rk} = \frac{1}{r+k-1}$ and for $k > r$, $\hbar_{rk} = 0$. $\mathcal{H}$ − transform of (x_k) is defined as

$$\mathcal{H}_r(x) = \sum_{k=1}^{r} \frac{1}{r + k - 1} x_k.$$

In the following section, some basic properties of $\lambda\mathfrak{I}$-convergent and Cauchy sequences are given. Throughout this study, the sequence (x_k) is taken as the bounded sequence.

2. MATERIALS AND METHODS

2.1 $\lambda\mathfrak{I}$-Convergence on Neutrosophic Normed Spaces

Definition 2.1 Let $(\mathcal{S}, \Theta_{\mathfrak{N},M}, \Psi_{\mathfrak{N},n.M}, \Upsilon_{\mathfrak{N},I}, \odot, \otimes)$ be a Neutrosophic normed spaces and $\lambda = (\lambda_r)$ be a non-decreasing sequence of positive numbers, $\lim_{r\to\infty} \lambda_r = \infty$ also $\lambda_1 = 1$, $\lambda_{r+1} \leq \lambda_r + 1$. (x_k) is called $\lambda\mathfrak{I}$-convergence to $\mathfrak{L}_{\mathfrak{N}}$ with respect to $(\Theta_{\mathfrak{N},M}, \Psi_{\mathfrak{N},n.M}, \Upsilon_{\mathfrak{N},I})$ in $\mathcal{S}$, if for all $0 < \varepsilon < 1$ and $\mathfrak{s} > 0$, the set

$$\{r \in \mathbb{N}: \Theta_{\mathfrak{N},M}(\mathcal{M}_r(x_k) - \mathfrak{L}_{\mathfrak{N}}, \mathfrak{s}) \leq 1 - \varepsilon \text{ or } \Upsilon_{\mathfrak{N},I}(\mathcal{M}_r(x_k) - \mathfrak{L}_{\mathfrak{N}}, \mathfrak{s}) \geq \varepsilon \text{ and } \Psi_{\mathfrak{N},n.M}(\mathcal{M}_r(x_k) - \mathfrak{L}_{\mathfrak{N}}, \mathfrak{s}) \geq \varepsilon\} \in \mathfrak{I}$$

where $\mathcal{J}_r = [r - \lambda_r + 1, r]$ and $\mathcal{M}_r(x_k) = \frac{1}{\lambda_r} \sum_{k \in \mathcal{J}_r} x_k$.

This sequences is shown with $(x_k) \to \mathfrak{L}_{\mathfrak{N}_{\lambda\mathfrak{I}}}$ or $(\lambda\mathfrak{I})_{\mathfrak{N}} - \lim x_k = \mathfrak{L}_{\mathfrak{N}}$.

Lemma 2.1 Let $(\mathcal{S}, \Theta_{\mathfrak{N},M}, \Upsilon_{\mathfrak{N},I}, \Psi_{\mathfrak{N},n.M}, \odot, \otimes)$ be a Neutrosophic normed spaces and (x_k) be a $\lambda\mathfrak{I}$-convergent sequences. Then, for all $0 < \varepsilon < 1$, $\mathfrak{s} > 0$, the next situations are equivalent,

1. $(\lambda\mathfrak{I})_{\mathfrak{N}} - \lim x_k = \mathfrak{L}_{\mathfrak{N}}$,

2. $\{r \in \mathbb{N}: \Theta_{\mathfrak{N},M}(\mathcal{M}_r(x_k) - \mathfrak{L}_{\mathfrak{N}}, \mathfrak{s}) \leq 1 - \varepsilon\} \in \mathfrak{I}$, $\{r \in \mathbb{N}: \Psi_{\mathfrak{N},n.M}(\mathcal{M}_r(x_k) - \mathfrak{L}_{\mathfrak{N}}, \mathfrak{s}) \geq \varepsilon\} \in \mathfrak{I}$,

$\{r \in \mathbb{N} : \Upsilon_{\mathfrak{N},I}(\mathcal{M}_r(x_k) - \mathfrak{L}_{\mathfrak{N}}, \mathfrak{s}) \geq \varepsilon\} \in \mathfrak{I}$.

3. $(\lambda\mathfrak{I})_{\mathfrak{N}} - \lim \Theta_{\mathfrak{N},M}(x_k - \mathfrak{L}_{\mathfrak{N}}, \mathfrak{s}) = 1$,

$(\lambda\mathfrak{I})_{\mathfrak{N}} - \lim \Psi_{\mathfrak{N},n.M}(x_k - \mathfrak{L}_{\mathfrak{N}}, \mathfrak{s}) = 0$,

$(\lambda\mathfrak{I})_{\mathfrak{N}} - \lim \Upsilon_{\mathfrak{N},I}(x_k - \mathfrak{L}_{\mathfrak{N}}, \mathfrak{s}) = 0$.

Proof Using Definition 2.1, the equivalence of properties are easily obtained.

Lemma 2.2 Let $(\mathcal{S}, \Theta_{\mathfrak{N},M}, \Upsilon_{\mathfrak{N},I}, \Psi_{\mathfrak{N},n.M}, \odot, \otimes)$ be a Neutrosophic normed spaces, (x_k) be a $\lambda\mathfrak{I}$-convergent to $\mathfrak{L}_{\mathfrak{N}}$. Then, $\mathfrak{L}_{\mathfrak{N}}$ is unique.

Proof It can be easily proved by assuming that there are two $\mathfrak{L}_{\mathfrak{N}}$ and making a contradiction.

Now, let's recall the definition of the filter (Mursaleen, 2000). Let $\mathcal{S} \neq \emptyset$ and a family of $\mathcal{F} \subset \mathcal{P}(\mathcal{S})$ is a filter where, $\emptyset \notin \mathcal{F}$; for all $S_1, S_2 \in \mathcal{F}$ then $S_1 \cap S_2 \in \mathcal{F}$; for all $S_1 \in \mathcal{F}$ and $S_1 \subset S_2$ then $S_2 \in \mathcal{F}$.

Let $\mathfrak{I} \neq \emptyset$, $\mathcal{S} \notin \mathfrak{I}$. Then, $\mathcal{F} = \mathcal{F}(\mathfrak{I}) = \{(\mathcal{S} \setminus I) : I \in \mathfrak{I}\}$ is a Filter.

Definition 2. 2 Let $(\mathcal{S}, \Theta_{\mathfrak{N},M}, \Psi_{\mathfrak{N},n.M}, \Upsilon_{\mathfrak{N},I}, \odot, \otimes)$ be a Neutrosophic normed spaces and $\mathcal{J}_r = [r - \lambda_r + 1, r]$. (x_k) is named $\lambda\mathfrak{I}$-Cauchy sequences in $\mathcal{S}$, where for each $\varepsilon \in (0,1)$, $\mathfrak{s} > 0$, there exist $\tilde{n} \in \mathbb{N}$:

$$\{r \in \mathbb{N} : \Theta_{\mathfrak{N},M}(\mathcal{M}_r(x_k) - \mathcal{M}_r(x_{\tilde{n}}), \mathfrak{s}) > 1 - \varepsilon \ or \ \Upsilon_{\mathfrak{N},I}(\mathcal{M}_r(x_k) - \mathcal{M}_r(x_{\tilde{n}}), \mathfrak{s}) < \varepsilon \ and \ \Psi_{\mathfrak{N},n.M}(\mathcal{M}_r(x_k) -$$

$$- \mathcal{M}_r(x_{\tilde{n}}), \mathfrak{s}) < \varepsilon\} \in \mathcal{F}(\mathfrak{I}),$$

where $\mathcal{M}_r(x_k) = \dfrac{1}{\lambda_r} \sum_{k \in \mathcal{J}_r} x_k$.

2.2 $\mathcal{H}$- Sequences Spaces Using Neutrosophic Norm

Definition 2.3 Let $(\mathcal{S}, \Theta_{\mathfrak{N},M}, \Psi_{\mathfrak{N},n.M}, \Upsilon_{\mathfrak{N},I}, \odot, \otimes)$ be a Neutrosophic normed spaces and $\lambda = (\lambda_r)$ be a sequence whose properties are given above. The sequences space of the $\mathcal{H} -$ transforms of (x_k) $\lambda\mathfrak{I}$-converging to $\mathfrak{L}_{\mathfrak{N}}$ with respect to Neutrosophic norm $(\Theta_{\mathfrak{N},M}, \Psi_{\mathfrak{N},n.M}, \Upsilon_{\mathfrak{N},I})$ in $\mathcal{S}$ is defined as follows.

$$\mathfrak{C}_{\mathfrak{N}} = \{x = (x_k) : \{r \in \mathbb{N} : \Theta_{\mathfrak{N},M}(\mathcal{M}_r(\mathcal{H}_k x) - \mathfrak{L}_{\mathfrak{N}}, \mathfrak{s}) \leq 1 - \varepsilon \ or \ \Upsilon_{\mathfrak{N},I}(\mathcal{M}_r(\mathcal{H}_k x) - \mathfrak{L}_{\mathfrak{N}}, \mathfrak{s}) \geq \varepsilon \ and$$

$$\Psi_{\mathfrak{N},n.M}(\mathcal{M}_r(\mathcal{H}_k x) - \mathfrak{L}_{\mathfrak{N}}, \mathfrak{s}) \geq \varepsilon\} \in \mathfrak{I}\}$$

where $\mathcal{J}_r = [r - \lambda_r + 1, r]$, $0 < \varepsilon < 1$,

$$\mathcal{M}_r(x_k) = \frac{1}{\lambda_r} \sum_{k \in \mathcal{J}_r} x_k.$$

Lemma 2.3 Let $(\mathbb{R}, \Theta_{\mathfrak{N},M}, \Psi_{\mathfrak{N},n.M}, \Upsilon_{\mathfrak{N},I}, \odot, \otimes)$ be a Neutrosophic normed spaces. The sequences spaces $\mathfrak{C}_{\mathfrak{N}}$ is a linear spaces on $\mathbb{R}$.

Now, using techniques similar to those in intuitinistic fuzzy normed spaces an open ball with center x, radius a will be given as:

Definition 2.4 Let $(\mathcal{S}, \Theta_{\mathfrak{N},M}, \Upsilon_{\mathfrak{N},I}, \Psi_{\mathfrak{N},n.M}, \odot, \otimes)$ be a Neutrosophic normed spaces. An open ball is given next form using $\mathcal{H} -$ transforms:

$\mathcal{B}_{\mathcal{H}}\big((x,a),\mathfrak{s}\big) = \{(\mathfrak{z}_k):\{\, r \in \mathbb{N}: \Theta_{\mathfrak{N},M}(\mathcal{M}_r(\mathcal{H}_k x) - \mathcal{M}_r(\mathcal{H}_k \mathfrak{z}),\mathfrak{s}) > 1 - a \ \ and \ \Upsilon_{\mathfrak{N},I}(\, \mathcal{M}_r\ (\mathcal{H}_k x) - \mathcal{M}_r(\mathcal{H}_k \mathfrak{z}),\mathfrak{s}) < a$

$and\ \Psi_{\mathfrak{N},n.M}(\, \mathcal{M}_r(\mathcal{H}_k x) - \mathcal{M}_r(\mathcal{H}_k \mathfrak{z}),\mathfrak{s}) < a\}\}$

Lemma 2.4 Let $\mathfrak{C}_{\mathfrak{N}}$ is a sequences spaces given in Definition 2.3. In this case, every open ball is an open set in $\mathfrak{C}_{\mathfrak{N}}$.

Proof Let $(\mathcal{H}_k \mathfrak{z}) \in \mathcal{B}_{\mathcal{H}}\big((x,a),\mathfrak{s}\big)$ and then $\Theta_{\mathfrak{N},M}\ (\, \mathcal{M}_r(\mathcal{H}_k x) - \mathcal{M}_r(\mathcal{H}_k \mathfrak{z}),\mathfrak{s}) > 1 - a$, $\Upsilon_{\mathfrak{N},I}(\, \mathcal{M}_r(\mathcal{H}_k x) - \mathcal{M}_r(\mathcal{H}_k \mathfrak{z}),\mathfrak{s}\big) < a$ and $\Psi_{\mathfrak{N},n.M}(\, \mathcal{M}_r(\mathcal{H}_k x) - \mathcal{M}_r(\mathcal{H}_k \mathfrak{z}),\mathfrak{s}) < a$. So there exists a $\mathfrak{s}^* \in (0,\mathfrak{s})$: $\Theta_{\mathfrak{N},M}\ (\, \mathcal{M}_r(\mathcal{H}_k x) - \mathcal{M}_r(\mathcal{H}_k \mathfrak{z}),\mathfrak{s}^*) > 1 - a$, $\Upsilon_{\mathfrak{N},I}(\, \mathcal{M}_r(\mathcal{H}_k x) - \mathcal{M}_r(\mathcal{H}_k \mathfrak{z}),\mathfrak{s}^*\big) < a$ and $\Psi_{\mathfrak{N},n.M}(\mathcal{M}_r$

$(\mathcal{H}_k x) - \mathcal{M}_r(\mathcal{H}_k \mathfrak{z}),\mathfrak{s}^*) < a$.

On the other hand, let $u \coloneqq \Theta_{\mathfrak{N},M}\ (\, \mathcal{M}_r(\mathcal{H}_k x) - \mathcal{M}_r(\mathcal{H}_k \mathfrak{z}),\mathfrak{s}^*)$. So, $u > 1 - a$ then, there exists $0 < h < 1$: $u > 1 - h\ > 1 - a$. Using the properties of $\mathfrak{t} - norm$ and $\mathfrak{t} - conorm$, given $u > 1 - h$, $u^*, u^{**} \in (0,1)$ is getting. Here, $u \odot u^* > 1 - h$ and $(1-u) \otimes (1 - u^{**}) < h$. Then $u^{***} := max\{u^*, u^{**}\}$. To show that $\mathcal{B}_{\mathcal{H}}\big((\mathcal{H}_k \mathfrak{z}, 1 - u^{***}),\mathfrak{s} - h\big) \subset \mathcal{B}_{\mathcal{H}}\big((x,a),\mathfrak{s}\big)$, let $v \in \mathcal{B}_{\mathcal{H}}\big((\mathcal{H}_k \mathfrak{z}, 1 - u^{***}),\mathfrak{s} - h\big)$.

So, $\Theta_{\mathfrak{N},M}(\, \mathcal{M}_r(\mathcal{H}_k x) - \mathcal{M}_r(\mathcal{H}_k v),\mathfrak{s}^*) \geq \Theta_{\mathfrak{N},M}(\, \mathcal{M}_r(\mathcal{H}_k x) - \mathcal{M}_r(\mathcal{H}_k \mathfrak{z}), h) \odot \Theta_{\mathfrak{N},M}(\, \mathcal{M}_r(\mathcal{H}_k \mathfrak{z}) - \mathcal{M}_r(\mathcal{H}_k v)$

$, u - h) > u \odot u^{***} \geq u \odot u^* > 1 - h > 1 - a$.

Moreover,

$\Upsilon_{\mathfrak{N},I}(\, \mathcal{M}_r(\mathcal{H}_k x) - \mathcal{M}_r(\mathcal{H}_k v),\mathfrak{s}\big) \leq \Upsilon_{\mathfrak{N},I}(\, \mathcal{M}_r(\mathcal{H}_k x) - \mathcal{M}_r(\mathcal{H}_k \mathfrak{z}), h) \otimes \Upsilon_{\mathfrak{N},I}(\, \mathcal{M}_r(\mathcal{H}_k \mathfrak{z}) - \mathcal{M}_r(\mathcal{H}_k v),\mathfrak{s} - h\big)$

$\leq (1-u) \otimes (1 - u^{***}) \leq (1-u) \otimes (1 - u^{**}) < a$.

$\Psi_{\mathfrak{N},n.M}(\, \mathcal{M}_r(\mathcal{H}_k x) - \mathcal{M}_r(\mathcal{H}_k v),\mathfrak{s}^*)$ can be obtained in a similar way.

Thus, $(\mathcal{H}_k v) \in \mathcal{B}_{\mathcal{H}}\big((x,a),\mathfrak{s}\big)$, $\mathcal{B}_{\mathcal{H}}\big((\mathcal{H}_k \mathfrak{z}, 1 - u^{***}),\mathfrak{s} - h\big) \subset \mathcal{B}_{\mathcal{H}}\big((x,a),\mathfrak{s}\big)$. So, every open ball is an open set in $\mathfrak{C}_{\mathfrak{N}}$.

Definition 2.5 Let $\big(\mathcal{S}, \Theta_{\mathfrak{N},M}, \Upsilon_{\mathfrak{N},I}, \Psi_{\mathfrak{N},n.M}, \odot, \otimes\big)$ be a Neutrosophic normed spaces, (x_k) is called Hilbert $\lambda\mathfrak{I}$-statistical convergence to $\mathfrak{L}_{\mathfrak{N}}$ if, for all $\varepsilon > 0$, $s > 0$, there exist a $\mathfrak{L}_{\mathfrak{N}}$:

$$\Big\{ r \in \mathbb{N}: \frac{1}{\lambda_r} |\, k \in \mathcal{I}_r: \Theta_{\mathfrak{N},M}(\, \mathcal{M}_r(\mathcal{H}_k x) - \mathfrak{L}_{\mathfrak{N}},\mathfrak{s}) \leq 1 - \varepsilon \ or\ \Upsilon_{\mathfrak{N},I}(\, \mathcal{M}_r(\mathcal{H}_k x) - \mathfrak{L}_{\mathfrak{N}},\mathfrak{s}\big) \geq \varepsilon \ and$$

$$\Psi_{\mathfrak{N},n.M}(\, \mathcal{M}_r(\mathcal{H}_k x) - \mathfrak{L}_{\mathfrak{N}},\mathfrak{s}\big) \geq \varepsilon \big| \Big\} \in \mathfrak{I}.$$

This situation is denoted with $(x_k) \to \mathfrak{L}_{\mathfrak{N}}\big(st^{\mathcal{H}}_{\lambda\mathfrak{I}}\big)$.

The set of Hilbert $\lambda\mathfrak{I}$-statistical convergence according to neutrosophic normed, is denoted by $St^{\mathcal{H}}_{\lambda\mathfrak{I}}$. For $\lambda_r = r$, it is called Hilbert $\mathfrak{I}$-statistical convergent to $\mathfrak{L}_{\mathfrak{N}}$.

Theorem 2.1 Let (x_k) be Hilbert $\mathfrak{I}$-statistical convergent to $\mathfrak{L}_{\mathfrak{N}}$ on $\big(\mathcal{S}, \Theta_{\mathfrak{N},M}, \Upsilon_{\mathfrak{N},I}, \Psi_{\mathfrak{N},n.M}, \odot, \otimes\big)$. If $\liminf_{r\to\infty} \frac{\lambda_r}{r} > 0$, then (x_k) is Hilbert $\lambda\mathfrak{I}$-statistical convergent to $\mathfrak{L}_{\mathfrak{N}}$.

Proof For given $0 < \varepsilon < 1$,

$\frac{1}{r}|\, k \leq n: \Theta_{\mathfrak{N},M}(\mathcal{M}_r(\mathcal{H}_k x) - \mathfrak{L}_{\mathfrak{N}}, \mathfrak{s}) \leq 1-\varepsilon \text{ or } \Upsilon_{\mathfrak{N},I}(\mathcal{M}_r(\mathcal{H}_k x) - \mathfrak{L}_{\mathfrak{N}}, \mathfrak{s}) \geq \varepsilon \text{ and } \Psi_{\mathfrak{N},n.M}(\mathcal{M}_r(\mathcal{H}_k x) - \mathfrak{L}_{\mathfrak{N}}$

$, \mathfrak{s}) \geq \varepsilon| \geq \frac{1}{r}|\, k \in \mathcal{I}_r: \Theta_{\mathfrak{N},M}(\mathcal{M}_r(\mathcal{H}_k x) - \mathfrak{L}_{\mathfrak{N}}, \mathfrak{s}) \leq 1-\varepsilon \text{ or } \Upsilon_{\mathfrak{N},I}\big((\mathcal{M}_r \mathcal{H}_k x) - \mathfrak{L}_{\mathfrak{N}}, \mathfrak{s}\big) \geq \varepsilon, \Psi_{\mathfrak{N},n.M}(\mathcal{M}_r(\mathcal{H}_k x$

$) - \mathfrak{L}_{\mathfrak{N}}, \mathfrak{s}) \geq \varepsilon\,| \geq \frac{\lambda_r}{r}\frac{1}{\lambda_r}|\, k \in \mathcal{I}_r: \Theta_{\mathfrak{N},M}(\mathcal{M}_r(\mathcal{H}_k x) - \mathfrak{L}_{\mathfrak{N}}, \mathfrak{s}) \leq 1-\varepsilon \text{ or } \Upsilon_{\mathfrak{N},I}(\mathcal{M}_r(\mathcal{H}_k x) - \mathfrak{L}_{\mathfrak{N}}, \mathfrak{s}) \geq \varepsilon\,,$

$\Psi_{\mathfrak{N},n.M}(\mathcal{M}_r(\mathcal{H}_k x) - \mathfrak{L}_{\mathfrak{N}}, \mathfrak{s}) \geq \varepsilon|.$

If $\liminf_{r\to\infty}\frac{\lambda_r}{r} = c$, then from definition $\left\{r \in \mathbb{N}: \frac{\lambda_r}{r} < \frac{c}{2}\right\}$ is finite. For $\delta > 0$,

$\Big\{n \in \mathbb{N}: \frac{1}{\lambda_r}|\, k \in \mathcal{I}_r: \Theta_{\mathfrak{N},M}(\mathcal{M}_r(\mathcal{H}_k x) - \mathfrak{L}_{\mathfrak{N}}, \mathfrak{s}) \leq 1-\varepsilon \text{ or } \Upsilon_{\mathfrak{N},I}(\mathcal{M}_r(\mathcal{H}_k x) - \mathfrak{L}_{\mathfrak{N}}, \mathfrak{s}\big) \geq \varepsilon, \Psi_{\mathfrak{N},n.M}(\mathcal{M}_r(\mathcal{H}_k x)$

$-\mathfrak{L}_{\mathfrak{N}}, \mathfrak{s}) \geq \varepsilon| \geq \delta\} \subset \Big\{n \in \mathbb{N}: \frac{1}{r}|\, k \in \mathcal{I}_r:\ \Theta_{\mathfrak{N},M}(\mathcal{M}_r(\mathcal{H}_k x) - \mathfrak{L}_{\mathfrak{N}}, \mathfrak{s}) \leq 1-\varepsilon \text{ or } \Upsilon_{\mathfrak{N},I}(\mathcal{M}_r(\mathcal{H}_k x) - \mathfrak{L}_{\mathfrak{N}}, \mathfrak{s}\big) \geq \varepsilon,$

$\Psi_{\mathfrak{N},n.M}(\mathcal{M}_r(\mathcal{H}_k x) - \mathfrak{L}_{\mathfrak{N}}, \mathfrak{s}) \geq \varepsilon| \geq \frac{c}{2}\delta\Big\} \cup \Big\{n \in \mathbb{N}: \frac{\lambda_r}{r} < \frac{c}{2}\Big\}.$

Here, since the set on the right belongs to $\mathfrak{I}$ then the desired result is getting.

3. RESULTS AND DISCUSSION

In this study, the definition of $\lambda\mathfrak{I}$ −convergence for Neutrosophic normed spaces and important properties for, $\lambda\mathfrak{I}$ −convergent sequences are given. In addition, the definition of $\lambda\mathfrak{I}$ −Cauchy sequence is established. The $\mathcal{H}$ −transforms of, $\lambda\mathfrak{I}$ −sequences and the open ball definition have been studied. The definition of Hilbert statistical convergence is given and an important property of these sequences is proved.

Khan et al. (2020) defined Hilbert I-convergent sequence spaces in the classical sense and Khan et al. (2022) introduced the concept of Hilbert ideal convergent series for intuitionistic fuzzy normed spaces. On the other hand, Savas and Gurdal (2015) defined the concept of λ −convergent sequence for intuitionistic fuzzy normed spaces using ideals. Then, for Neutrosophic normed spaces, Khan et al. (2019) gave the definition of statistical convergence using Fibonacci matrices. In addition, the definition of Fibonacci I-convergent in intuitionistic fuzzy normed spaces has been given by Kisi & Guler (2019).

In this study, an important convergence definition has been made on Neutrosophic normed spaces, where convergence types in fuzzy and intuitionistic fuzzy normed spaces have been transferred quickly in the last few years. Moreover, due to its connection with statistical convergence and its use in the $\mathcal{H}$ −transform, many important concepts in the mentioned space were brought together for the first time in this study.

4. CONCLUSION

In this paper, firstly, the definition of $\lambda\mathfrak{I}$ −convergence is created for Neutrosophic Normed Spaces, which forms the basis of the research. Important properties are given for $\lambda\mathfrak{I}$ −convergent sequences also the unique of such convergent sequences is proven. Then, the definition of the Cauchy sequence for the $\lambda\mathfrak{I}$ −sequence structure has been given. An $\mathcal{H}$ −transform of the sequences set up in this space is created and an open ball definition has been given with the help of this transformation. By showing that the open ball-open set relationship is preserved in this defined space, the Hilbert statistical convergence definition is given, and in this sense, an important property for convergent sequences has been proven.

In this study, a concept is defined that will allow the evaluation of important studies in Neutrosophic normed spaces from different perspectives. In future studies, it is aimed to obtain different properties of the defined concept and to create the equivalent of the concept in different sequence spaces.

ACKNOWLEDGEMENT

The author thanks the editors and referees.

CONFLICT OF INTEREST

The author declares no conflict of interest.

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