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RESEARCH ARTICLE

Pricing of Contractual Shipments and Slot Allocation in Container Liner Shipping under Stochastic Environment

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ABSTRACT
This study aimed to propose an optimization model for slot allocation and contractual pricing that considers spot and contractual shipments and empty container repositioning under a stochastic environment. In that respect, a two-stage stochastic non-linear programming model was proposed. The model considers contractual pricing that is overlooked by previous studies. Experimentation results revealed that decreasing market demand and spot market prices could cause serious profit loss while creating a high level of idle capacity. With the increasing market demand, capacity utilization reaches saturation at 90% requiring a capacity increase in the service. In the increasing market, slots allocated to empty containers get reduced while taking advantage of other options for empty container supply. Experimentation of symmetric uncertainty revealed that the range of uncertainty should be minimized since it creates a serious loss in profits and capacity utilization. Calculations also demonstrated that the applications of the stochastic modeling solutions would provide higher profit margins than the solutions of their deterministic equivalents. The model can easily be applied to the real-life situations of container liner services for managing and optimization of their service capacities as well as determining optimum contractual prices.

Keywords: Container shipping, contractual pricing, slot allocation, revenue management, capacity management

1. Introduction

Container liner companies must plan for optimal slot allocation for the sake of efficiency and profitability of their operations to survive and prosper in such a competitive market. Optimal slot allocation is not only crucial for the economic sustainability of container liner companies, but also beneficial for environmental sustainability since it increases container ship capacity utilization which in turn reduces environmental emissions per ton-kilometer cargo transported.

Customers of container liner companies, namely the shippers of containers, are mainly segmented as contractual shippers and spot shippers (Wang & Meng, 2021). Contractual shippers can be either freight forwarders or big industry players with a high volume of export/import cargo and regular shipment needs. They draw contracts with container liners to bargain for lower and fixed prices and guarantee a regular shipment of their cargo. Drawing contracts with shippers and freight forwarders are also beneficial for container liners as they guarantee the availability of cargo and stable income. Binding to a contract raises a pricing issue - how should contractual shipments be priced? They are usually priced by considering the expectations regarding future demand and future spot market prices. If the expectation regarding the market is upward, prices negotiated for a contract can be too high, resulting in the loss of the contractual customer. In the future, if the market goes down contrary to past expectations, the container liner company would face a serious loss in its revenue. On the contrary, if the expectation regarding the market is downward, the container liner company would settle for a contract price that is too low. This will again result in a serious loss in revenue if the market goes up contrary to past expectations. Either situation affects the profitability of a container liner company. In addition, in some legs of the service, containers for contractual shipments might not as many as others, and those empty slots can be used for spot shipments so that efficiency and profitability can be maximized. Additionally, it is not possible to predict the future demand and future spot market prices with certainty; therefore, along with the pricing of contractual shipments, stochasticity of demand and spot price expectations must be considered in the modeling of slot allocation in container shipping.

In this regard, considering different segments i.e., contractual spot, and empty container shipments altogether is necessary for efficient slot allocation and profitability. The main research question of this study is how container shipping slot allocation

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with those different segments can be modelled and optimized. To answer the question, this study proposes a slot allocation and contractual pricing model that explicitly considers spot and contractual shipments and empty container repositioning under a stochastic environment. Empty container repositioning must be considered together with contractual and spot shipments in modeling container slot allocation since it is inevitable and uses the capacity of container ships because of the nature of the container shipping industry.

The remainder of this paper is organized as follows. Section 2 reviews the previous studies and reveals the contribution of this study. Section 3 states the slot allocation and container shipping pricing problem that motivated this study. Section 4 formulated the model as a non-linear two-stage stochastic programming model. Section 5 describes the application case and the data for the application instances. Section 6 reveals experimental results. Finally, conclusion was provided in Section 7.

2. Literature Review

The focus of the previous studies has been mostly on spot markets. Feng & Chang (2008) developed a model for slot allocation that maximizes operational profit. To eliminate capacity misutilization because of no-show ups, Wang et al. (2019) proposed a slot allocation model that considers two strategies: overbooking and delivery-postponement. From a different perspective from these two studies, Fu et al. (2016) considered demand uncertainty by putting forward a robust optimization model for slot allocation. Their model accounted for minimum quantity commitment and two types of uncertainty in the demand: bounded and symmetric uncertainty. These studies assumed that a fixed capacity was put aside for empty container repositioning.

Capacity for empty containers on container ships can be arranged more efficiently if empty container repositioning took advantage of the idle capacity resulting from demand fluctuations and demand differences in the different legs of a service route. Considering this fact, optimal slot allocation models proposed in the previous studies accounted for slots allocated for empty containers. Including empty container repositioning, Ting & Tzeng (2004) constructed an optimal slot allocation model that maximizes total freight profit. Feng & Chang (2010) proposed a slot allocation optimization model which considers empty container repositioning while maximizing operational profit. They improved the model developed by Feng & Chang (2008) to include empty container relocation decisions. Zurheide & Fischer (2012) developed a slot allocation optimization model that considers transshipment and prioritization of urgent container shipments. Zurheide & Fischer (2015) modified and improved the model developed by Zurheide & Fischer (2012). Their slot allocation model considered transshipment and proposed a new booking limit strategy called the bid-price strategy. Additionally, they conducted a simulation to compare the newly proposed bid-price strategy with previously presented booking limit strategies. Wong et al. (2015) developed a profit maximization model that incorporates empty and laden container slot allocation. Wang et al. (2015) proposed a non-convex mixed-integer non-linear optimization model to maximize the profits for seasonal container shipping. Their model included shipping speed and realistic non-convex bunker consumption function. Chang et al. (2015) came up with a bi-level optimization model for slot allocation and empty container repositioning. While the upper level maximizes operational profits with optimal slot allocation, the lower level minimizes empty container repositioning costs. Lu & Mu (2016) provided a model for slot reallocation caused by adjustments to shipping schedules after major disruptions. The model put forward by Ting & Tzeng (2016) not only accounted for empty container repositioning, but also considered uncertainties. They proposed a fuzzy multi-objective slot allocation model with uncertain demand and container weight. Contrary to previous studies, their model maximizes both total revenue and agents' degree of satisfaction. In all these studies, capacity for contractual pricing was not considered at best few of them assumed that a certain percentage of the capacity was put aside for contractual containers.

In another study, Wang et al. (2020) constructed an optimal slot allocation and dynamic pricing model considering uncertain demand and port congestion for time-sensitive cargo. Contrary to the other previous studies, their model considers slot allocation for contractual shipments and spot shipments together with pricing of spot shipments, but their model is not applicable for contractual pricing.

2.1. Contribution of the Study

Pricing and slot allocation of contractual shipments can significantly impact the profitability of container shipping lines. Additionally, optimum slot allocation of contractual shipment can increase capacity utilization of container ships thus reduce emissions for per ton kilometers of containerized freight. To the best of the author's knowledge, none of the previous studies related to slot allocation in container shipping considered slot allocation and pricing of contractual shipments. At best some of the previous studies assumed certain percentage of container ships were set aside for contractual shipments and their pricing were determined in an ad hoc manner. In this regard, this study aimed to propose an optimal slot allocation and contractual pricing model that explicitly considers spot and contractual shipments and empty container repositioning under a stochastic environment.

Therefore, a two-stage stochastic non-linear programming model was proposed. The first stage includes the prices for contractual cargo while the second stage includes slot allocation for spot and contractual shipments and empty containers.

3. Problem Statement

Figure 1 illustrates the container liner shipping service provision. Two hypothetic services of a container liner are illustrated in the figure. Service-1 includes four ports, and Service-2 includes five ports, and both of the services are provided in typical cyclic routes that start from P1 and end at P1. A full shipping sequence of a container ship throughout the route i.e., P1-P2-P3-P4-P1 or P1-P5-P6-P7-P8-P1 is called a voyage. And a single shipping activity of a container ship from one port to another such as P1 to P2 is called a leg. The routes and shipping schedules of a container liner shipping service are predetermined and declared to shippers so that they can arrange shipping requirements accordingly. Typically, container liner service is provided according to a weekly schedule, which means that at least once a week a port in a container liner service is called by a container ship. The transportation capacity of a container liner service, particularly how many container ships are to be assigned to that service is determined according to demand predictions. The capacity of a container liner service is fixed unless new container ships are added to the company's fleet, or the company redesigns its container liner services.

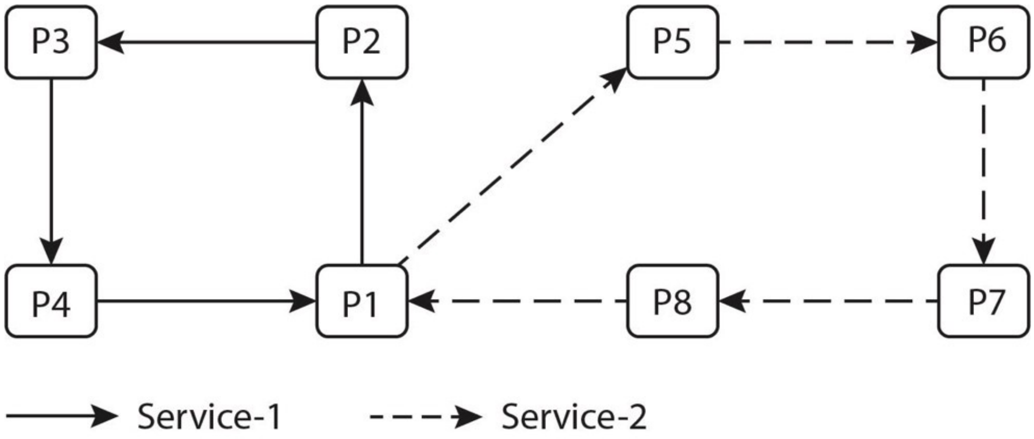

Figure 1. Container Liner Shipping Services

When a port in a container liner service is called by a container ship, certain containers are discharged, certain containers remain on ships, and certain containers are loaded on ships. The discharged containers at a port are the ones that are destined to that port from other ports in the service. The containers that remain on ships are the ones that are shipped from the ports in the previous legs of the ships and destined to upcoming ports in the ships' voyage. The loaded containers at a port are the ones that are shipped from that port and destined to upcoming ports in the ships' voyage. As an example to clarify the container liner service provision, the reader can look at Figure 1 and consider P3 as focal port and consider P1 as starting and ending port of the voyages performed in Service-1. Containers discharged at P3 are the ones sent from P2 to P3 and P1 to P3 in the current voyage and the ones sent from P4 to P3 in the previous voyage. Containers loaded at P3 are the ones sent from P3 to P4 and P3 to P1 and P3 to P2 in the current voyage. The ones sent from P3 to P2 in the current voyage will be discharged at P2 in the next voyage. Containers that remain on ships at P3 are the ones sent from P1 to P4, P2 to P4, and P2 to P1 in the current voyage, and all of them will be discharged in the current voyage. On the other hand, containers that remain on ships at P4 are the ones sent from P2 to P1, P3 to P1, and P3 to P2 in the current voyage, but the ones sent from P3 to P2 will be discharged at P2 in the next voyage.

Considering contractual, spot, and empty container shipments all together is necessary for efficient slot allocation and profitability. As in some legs of a container liner service, demand in the spot market is not as high as others, and similarly, in some legs of the service, containers for contractual shipments are not as many as others, the idle slots resulted in one kind of shipment can be used for the other kinds of shipments so that efficiency and profitability can be maximized.

Pricing of the contractual shipments rises as another issue for slot allocation. The price of the spot shipments is determined by the market. And in turn, spot market prices are determined by demand. Since a container liner company competes with other container liners in destinations where they provide services, the divergence of their spot prices from the spot market prices is usually minuscule. On the other hand, contractual prices are determined by the expectations regarding the future spot market prices as they are usually signed annually, and the contractual prices are fixed in the term of the contract. If the price negotiated for a contract is high, it can cause the loss of a contractual customer. On the other hand, if the price settled for a contract is low, it can result in too many slots being allocated to contractual shipments with a low price. Either situation affects the number of slots allocated to contractual shipments, therefore, the profitability of a container liner company.

4. Model Description

The model proposed in this study is a two-stage stochastic non-linear programming model. There are various types of stochastic programming approaches (Birge & Louveaux, 2011). Two-stage stochastic programming was chosen because it is the most suitable modelling technique for modelling uncertainty for the problem considered in this study. In a two-stage stochastic programming model, uncertainty is revealed in the second stage. The first stage decision variables take values that are best for all occurrences of considered scenarios of the second stage. This is compatible with contractual pricing and slot allocation of contractual shipments. Contractual pricing and slot allocation of contractual shipments are generally decided annually before knowing what the price for spot shipments thorough the year will be. In the first stage of the proposed model, contractual price and slot allocation of contractual container shipments are decided. In the second stage possible occurrence of spot prices are revealed. The solution values of the first stage variables will be the best ones for all considered scenario occurrences of spot prices.

In this section, notations, the objective function, and constraints of the stochastic programming model were also presented. Notations used for sets, parameters, and decision variables of the model are demonstrated in Table 1. While P sands for the set of seaports, V stands for the set of voyages. Ω stands for the set of scenarios. Since it is a stochastic programming model, stochasticity is included in the model through various scenarios.

Table 1. Notations used in the model

Sets	
P	Set of seaports
V	Set of voyages
Ω	Set of scenarios
Parameters	
$r_{ijv}(w)$	The spot price for transportation of 1 container between port i and port j on voyage v
c_{ij}	Cost for transportation of 1 container between port i and port j
$D_{ijv}(w)$	Demand for transportation of container between port i and port j on voyage v
ESD_{iv}	Empty container supply or demand at port i on voyage v
ps	Share of spot shipments
cap	The capacity of the container liner service
h	Inventory holding cost of an empty container per day
l	Leasing cost of an empty container
First-stage Decision Variables	
P_{ij}	Contractual price for transportation of 1 container between port i and port j
Second-stage Decision Variables	
$\beta_{ij}(w)$	The coefficient for contractual price and contractual demand
$XS_{ijv}(w)$	Number of full spot containers transported between port i and port j on voyage v
$XC_{ijv}(w)$	Number of full contractual containers transported between port i and port j on voyage v
$XE_{ijv}(w)$	Number of full empty containers transported between port i and port j on voyage v
$XRS_{iv}(w)$	Number of full spot containers remain on ships at port i on voyage v
$XRC_{iv}(w)$	Number of full contractual containers remain on ships at port i on voyage v
$XRE_{iv}(w)$	Number of full empty containers remain on ships at port i on voyage v
$W_{ijv}(w)$	Auxiliary variable for linearizing $P_{ij} * XC_{ijv}(w)$
$L^{+}_{iv}(w)$	Number of empty containers leased at port i on voyage v
$L^{-}_{iv}(w)$	Number of excess empty containers returned to lessors at port i on voyage v
$EI_{iv}(w)$	Number of empty containers stored at port i on voyage v

Parameters, the data to be included in the model instances, are also described in Table 1. ESD_{iv} denotes empty container demand or supply at port i on the voyage v. When it takes a positive value, it is the supply of empty containers at port i on the voyage v, and when it takes a negative value, it is the demand for empty containers at port i on the voyage v. c_{ij} denotes costs of transporting 1 container between port i and port j. ps denotes the share of the spot shipments in the total demand. While h denotes the storage cost of 1 container at a seaport, l denotes the average costs of leasing 1 container at a seaport. D_{ijv} (w) and $r_i jv$ (w) are stochastic parameters that take a different value in each scenario. D_{ijv} (w) denotes transportation demand from port i to port j on voyage v in scenario w.$r_i jv$ (w) denotes spot market prices for transportation of 1 container from port i to port j on voyage v in scenario w.

Table 1 shows the decision variables of the model. Because the model is a two-stage stochastic programming model, decision variables are distinguished in terms of two stages. P_{ij} denotes the price for contractual shipments from port i and port j. It is the first stage variable since the decision regarding the price of contractual shipments is decided before the uncertainty is revealed. P_{ij} is not scenario dependent - the solution of a model instance will provide a value for the variable that is robust for the realization of all considered scenarios. Other decision variables are the second-stage variables. β_{ij} (w) is the coefficient of the functional relationship between spot market prices and the demand for contractual shipments. XS_{ijv} (w), XC_{ijv} (w), and XE_{ijv} (w) denote the number of full spot containers, the number of full contractual containers, and the number of empty containers that are transported between port i and port j on voyage v, respectively. XRS_{iv} (w), XRC_{iv} (w), and XRE_{iv} (w) denote the number of full spot containers, the number of full contractual containers, and the number of empty containers that are remained on ships at port i on voyage v, respectively. W_{ijv} (w) denotes the auxiliary variable used for linearizing the expression P_{ij}* XC_{ijv} (w) in the objective function. L_{iv}+ (w) and L_{iv}^{-}(w) denote the number of empty containers leased and the number of excess empty containers returned to lessors at port i on voyage v, respectively. At last, EI_{iv} (w) denotes the number of empty containers stored at port i on voyage v.

Model has several assumptions:

1. Containers were segmented under three categories: spot container shipments, contractual container shipments, and empty container shipments.
2. There is an inverse linear relationship between price and demand.
3. Proportion of contractual shipments to spot shipments needs to be decided before solving model instances.
4. The decision variables regarding number of containers included in the model are continuous variables.

The first assumption is straightforward and in line with industry practices were customers of container shipping lines are segmented as shippers of contractual and spot containers (Y. Wang & Meng, 2021). The second assumption is also reasonable because increases in the price of a service or a product reduces its demand. However, the shape of relation might be different for different services or products and may not be linear. The third assumption can be eliminated by solving model insurances for different proportions of contractual and spot shipments. Therefore, the best proportion can be found for different model instances. The fourth assumption also can have a very little impact. Containers are non-dividable entities, but in cases where integer decision variables took high values, they can be treated as continuous variables and rounded to the closest integers with a very minuscule difference compared to integer solutions since it is a lot easier to solve linear programming model instances.

4.1. Objective Function

The expressions from 1.1 to 1.5 present the objective function of the model. As can be seen, the objective function of the model is the profit maximization function. Since the model is a stochastic programming model, the first term in the expression 1.1 is the weighted sum of all the terms in the objective function in terms of the occurrence probability of each scenario. The sum of the occurrence probabilities of all scenarios must be equal to 1. The second term in expression 1.1 represents the profit margin of contractual shipments while expression 1.2 represents the profit margin of spot shipments. The remaining expressions represent the costs; thus, they are subtracted from the profit margins. Expression 1.3, expression 1.4, and expression 1.5 represent the costs of empty container transportation, empty container leasing, and empty container storage, respectively.

$$max\{\sum_{w\epsilon\Omega} pr(w)\{\sum_{i\epsilon P}\sum_{j\epsilon P}\sum_{v\epsilon \vee}(P_{ij}-c_{ij})XC_{ijv}(w)+ \tag{1}$$

$$\sum_{i\epsilon P}\sum_{j\epsilon P}\sum_{v\epsilon \vee}(r_{ijv}(w)-c_{ij})XS_{ijv}(w)- \tag{2}$$

$$\sum_{i\epsilon P}\sum_{j\epsilon P}\sum_{v\epsilon \vee}c_{ij}XE_{ijv}(w)- \tag{3}$$

$$l \sum_{i \epsilon P} \sum_{v \epsilon \vee} L^{+}_{iv}(w) - \tag{4}$$

$$h \sum_{i \epsilon P} \sum_{v \epsilon \vee} El_{iv}(w) \tag{5}$$

4.2. Demand Constraints

Constraints from 2 to 4 represent demand constraints. Constraint 2 is the demand constraint for spot shipments. The number of spot containers that are transported is less than or equal to the spot container transportation demand which is equal to a certain percentage of the total transportation demand. The less than or equal sign indicates that the carrier has an option for accepting spot containers that maximize its profits. Constraints 3.1 and 3.2 represent the demand constraint for contractual shipments. Constraint 3.2 indicates that the contractual price for container transportation from port i to port j equals a ratio of the mean spot rate of all the voyages. Constraint 3.1 indicates that the demand for contractual shipments is inversely proportional to the price of contractual shipments. As can be seen in Constraint 3.1, the relationship is equality, indicating that contractual shipments must be provided by the carrier. Constraint 4 is the flow conservation constraint for empty containers. The terms on the left side of the equation are the incoming empty container flow, and the term on the right side of the equation is the outgoing flow. The first term on the left side is the sum of empty containers that come to port i from each port j while the second and third terms are equal to empty containers stored at port i from the previous voyage and empty containers leased from lessors at port i, respectively. The last term on the left side of the equation is the demand/supply of empty containers at port i. When the parameter is negative, it equals the demand, and when the parameter is positive, it equals to supply of empty containers.

$$XS_{ijv}(W) \leq psD_{ijv}(w) \forall i \in P, \forall j \in P, \forall v \in V, \forall W \in \Omega \tag{6}$$

$$XS_{ijv}(W) = (1 - ps)D_{ijv}(W) - (1 - ps)D_{ijv}(W)\beta_{ij}(W) \forall i \in P, \forall j \in P, \forall v \in V, \forall W \in \Omega \tag{7}$$

$$P_{ij} = meanr_i jv(w) : v \epsilon V \beta_i j(w) \forall i \in P, \forall j \in P, \forall w \in V, \forall W \in \Omega \tag{8}$$

$$\sum_{i \epsilon P} XE_{ijv}(W) + EI_{iv-1}(W) + L^{+}_{iv}(W) + ESD_{iv}(W) = \sum_{i \epsilon P} XE_{ijv}(W) + EI_{iv}(W) + L^{-}_{iv}(W) \tag{9}$$

$$\forall i \in P, \forall v \in P, \forall w \in V, \forall W \in \Omega \tag{10}$$

4.3. Capacity Constraints

Constraints from 5.1 to 5.4 are capacity constraints. Constraints 5.1, 5.2, and 5.3 describe the number of spot containers, contractual containers, and empty containers that remain on ships at port i on voyage v, respectively. As it can be seen, all three constraints are identical except for the type of containers so only one of them will be described in detail. In Constraint 5.1, on the left-hand side of the equation is the decision variable for the number of spot containers that remain on ships at port i on voyage v. The first term on the right-hand side of the equation is the sum of the number of containers that come from preceding ports and are destined to be delivered to upcoming ports on the current voyage. The second term on the right-hand side of the equation is the sum of the number of containers that come from preceding ports and are destined to upcoming ports that precede the origin ports on the cyclic route. And those containers are to be delivered to destination ports on the next voyage. The third and the last terms on the right-hand side of the equation are the sum of the number of containers that come from upcoming ports on the previous voyage and are destined to be delivered on the current voyage to the upcoming ports that precede the origin ports. Constraint 5.4 indicates the ship capacity limitations for the containers that are to be loaded on ships at port i on voyage v. The terms on the left-hand side of the equation represent the sum of the number of spot containers, contractual containers, and empty containers transported from port i to each port j. The first term on the right-hand side of the equation is the total capacity of ships operated on the service. In practice, on a container liner service route, ships' calls to ports are arranged in such a way that each port is visited at least once a week. The slots allocated to the total capacity of ships operated on a container liner service can easily be distributed to each ship operated on that container liner service. The second, third, and last terms on the right-hand side of the equation are, respectively, spot containers, contractual containers, and empty containers that remain on ships at port i on voyage v. As it can be seen in Contraint 5.4, the number of containers to be loaded on ships at port i on voyage v can be at most the remaining empty capacity on the ships at port i on voyage v.

$$XRS_{iv} = (W) = \sum_{\substack{j\epsilon P\\ i>j}} \sum_{\substack{k\epsilon P\\ k>i}} XS_{jkv}(w) + \sum_{\substack{j\epsilon P\\ i>j}} \sum_{\substack{k\epsilon P\\ j>k}} XS_{jkv}(w) + \sum_{\substack{j\epsilon P\\ j>k}} \sum_{\substack{k\epsilon P\\ k>i}} XS_{jkv-1}(w) \tag{11}$$

$$\forall i \in P, \forall v \in V, \forall W \in \Omega \tag{12}$$

$$XRC_{iv} = (W) = \sum_{\substack{j\epsilon P\\ i>j}} \sum_{\substack{k\epsilon P\\ k>i}} XC_{jkv}(w) + \sum_{\substack{j\epsilon P\\ i>j}} \sum_{\substack{k\epsilon P\\ j>k}} XC_{jkv}(w) + \sum_{\substack{j\epsilon P\\ j>k}} \sum_{\substack{k\epsilon P\\ k>i}} XC_{jkv-1}(w) \tag{13}$$

$$\forall i \in P, \forall v \in V, \forall W \in \Omega \tag{14}$$

$$XRE_{iv} = (W) = \sum_{\substack{j\epsilon P\\ i>j}} \sum_{\substack{k\epsilon P\\ k>i}} XE_{jkv}(w) + \sum_{\substack{j\epsilon P\\ i>j}} \sum_{\substack{k\epsilon P\\ j>k}} XE_{jkv}(w) + \sum_{\substack{j\epsilon P\\ j>k}} \sum_{\substack{k\epsilon P\\ k>i}} XE_{jkv-1}(w) \tag{15}$$

$$\forall i \in P, \forall v \in V, \forall W \in \Omega \tag{16}$$

$$\sum_{jeP} XS_{ijv}(w) + \sum_{jeP} XC_{ijv}(w) + \sum_{jeP} XE_{ijv}(w) \leq cap - XRS_{iv}(w) - XRC_{iv} - XRE_{iv}(w) \tag{17}$$

$$\forall i \in P, \forall v \in V, \forall W \in \Omega \tag{18}$$

4.4. Linearization of Non-Linear Objective Term

The objective function includes a bilinear term $P_{ij}\ XC_{ijv}$ (w) since it is a non-linear and non-convex expression that is very difficult to solve, and algorithms that are used for solving linear programming model instances cannot be applied. However, the bilinear term can be linearized using various modeling approaches. One of the modeling approaches used for linearizing bilinear terms in a very efficient way is using McCormick's inequalities (or McCormick's envelopes) (Costa et al., 2017; McCormick, 1976). P $_{ij}$* XC_{ijv} (w) is the bilinear term, and the lower and the upper bounds for the two variables can be defined as $P_{ij} \in [L_{Pij}, U_{(P_ij)}$ and XC_{ijv} (w)$\in [L_{XCijv}(w), U_{XCijv}(w)]$. According to McCormick's inequalities (McCormick, 1976), the convex envelope of the bilinear term is defined by the following inequalities:

$$W_{ijv}(w) \geq L_{XCijv(w)} P_{ij} + Lp_{ij} XC_{ijv}(w) - L_{XCijv}(w) Lp_{ij} \forall i \in P, \forall j \in P, \forall v \in V, \forall w \in \Omega \tag{19}$$

$$W_{ijv}(w) \geq U_{XCijv(w)} P_{ij} + Up_{ij} XC_{ijv}(w) - U_{XCijv}(w) Up_{ij} \forall i \in P, \forall j \in P, \forall v \in V, \forall w \in \Omega \tag{20}$$

$$W_{ijv}(w) \leq L_{XCijv(w)} P_{ij} + Up_{ij} XC_{ijv}(w) - L_{XCijv}(w) Up_{ij} \forall i \in P, \forall j \in P, \forall v \in V, \forall w \in \Omega \tag{21}$$

$$W_{ijv}(w) \leq U_{XCijv(w)} P_{ij} + Lp_{ij} XC_{ijv}(w) - U_{XCijv}(w) Lp_{ij} \forall i \in P, \forall j \in P, \forall v \in V, \forall w \in \Omega \quad (22)$$

P_{ij}* XC_{ijv} (w) on the objective function is replaced by W_{ijv} (w).

4.5. Non-Negativity and Integrality Constraints

All of the decision variables included in the model are non-negative real numbers.

5. Model Application

5.1. Application Case

The model was applied to a case that includes a container liner service of a leading local container liner company that mainly provides services throughout ports of the Mediterranean and the Black Sea. The container liner service is illustrated in Figure 2. The service includes 9 ports, 4 of which are located in Western Turkey (Istanbul, Izmit, Bursa, and Izmir), 3 of which are located in Spain (Valencia, Castellon, and Barcelona), 1 of which is located in France (Fos Sur Mer), and 1 of which located in Greece (Piraeus). The route of the service follows the sequence of ports as Valencia-Castellon-Barcelona-Fos Sur Mer-Piraeus-Istanbul-Izmit-Bursa-Izmir-Valencia. One voyage through the route takes around 3 weeks.

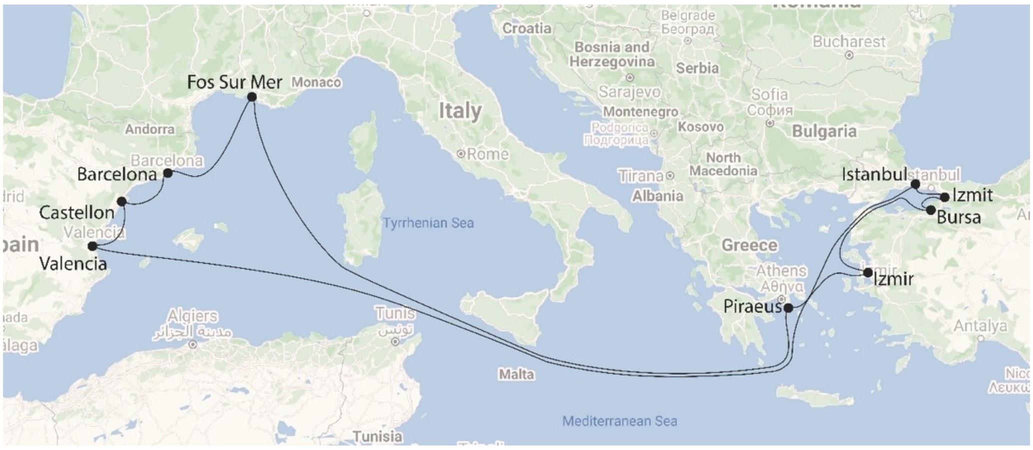

Figure 2. The Container Liner Service that the Model Applied

5.2. Data Description

The instances from the model are constructed by including parameter data regarding the application case of the container liner service. Table 2 shows the spot market price for transportation of 1 TEU full container between each port in the service. The spot market prices shown in the table are gathered from an online shipping platform called Freightos on March 9th, 2021. The prices are not symmetrical, for example, it is shown in the first line of Table 2 that the spot market price for transportation of 1 TEU from Valencia to Fos Sur Mer is \$550, on the other hand, it is shown in the third line that spot market price for transportation of 1 TEU from Fos Sur Mer to Valencia is \$964.

Table 3 illustrates the distance between each port in the service. Table 3 is created by using the data from an online shipping platform called Searates. Similar to the spot market prices, the distances shown in the table are also not symmetrical because they are not direct distances, but distances through the route. For example, it is shown in the first line of Table 3 that the distance from Valencia to Castellon through the route is 39 miles, but the distance from Castellon to Valencia through the route is 3386 miles since a ship must complete one voyage to arrive from Castellon to Valencia through the route of the service.

Table 4 shows the container transportation demand between each pair of ports in the service. The demand data is hypothetically created since it is considered sensitive information and not to be shared by container shipping lines. The main flows of container transportation through the service are between West Mediterranean (Valencia, Castellon, Barcelona, and Fos Sur Mer) and East

Table 2. Spot Price for Transportation of 1 TEU

	Valencia	Castellon	Barcelona	Fos	Piraeus	Istanbul	Izmit	Bursa	Izmir
Valencia	-	$793	$793	$550	$793	$848	$848	$848	$848
Castellon	$793	-	$793	$550	$793	$848	$848	$848	$848
Barcelona	$793	$793	-	$550	$793	$848	$848	$848	$848
Fos	$964	$964	$964	-	$964	$903	$903	$903	$903
Piraeus	$793	$793	$793	$550	-	$848	$848	$848	$848
Istanbul	$400	$400	$400	$500	$400	-	$500	$500	$500
Izmit	$400	$400	$400	$400	$400	$500	-	$500	$500
Bursa	$400	$400	$400	$400	$400	$500	$500	-	$500
Izmir	$400	$400	$400	$400	$400	$500	$500	$500	-

Source. www.freightos.com

Mediterranean (Piraeus) and Marmara Sea (Istanbul, Izmit, Bursa, and Izmir). The hypothetical demand data is created considering this fact to reflect the current sector practice. Table 4 illustrates that the container transportation demand between ports of West Mediterranean and East Mediterranean and the Marmara Sea is created according to uniform distribution between 200 and 500 containers. Since there is a minuscule number of containers transported among ports of the Marmara Sea and ports of Spain and France, the container transportation demands between those ports are created according to uniform distribution between 0 and 50 containers. It can be seen in Table 4 that there is a container transportation demand from Valencia to Castellon, but there is not from Castellon to Valencia. Since the transportation of containers from Castellon to Valencia requires one complete voyage, it is not economically and practically viable to transport containers from Castellon to Valencia. This is also applicable for other legs that require one complete voyage to transport containers between them.

Table 3. Distance Between Ports in the Service (Nautical Miles)

	Valencia	Castellon	Barcelona	Fos	Piraeus	Istanbul	Izmit	Bursa	Izmir
Valencia	-	39	168	345	1407	1738	1773	1816	2052
Castellon	3386	-	129	306	1368	1699	1734	1777	2013
Barcelona	3257	3296	-	177	1239	1570	1605	1648	1884
Fos	3080	1746	3248	-	1062	1393	1428	1471	1707
Piraeus	2018	2057	2186	2363	-	331	366	409	645
Istanbul	1687	1726	1855	2032	3094	-	35	78	314
Izmit	1652	1691	1820	1997	3059	3390	-	43	279
Bursa	1609	1648	1777	1954	3016	3347	3382	-	236
Izmir	1373	1412	1541	1718	2780	3111	3146	3189	-

Source. www.searates.com

Data related to other parameters are also included in the instances. The time period included in the instances is 1 year. Since 1 voyage takes 21 days, the number of voyages equals 17. Inventory holding cost of 1 empty TEU (h) for a duration of 1 voyage at a port in the service equals $105 ($5 per day). The average cost of leasing 1 empty TEU (l) is $300. The share of spot container transportation demand in the market (ps) equals 0.6 meaning that 60% of the container transportation demand in the market is for spot container transportation. The empty container demand/supply at each port (ESD_{iv}) is hypothetically created according to uniform distribution between -100 and 100. When it is below zero, it means the number of empty containers demanded at port i on voyage v. When it is above zero, it means the number of empty containers supplied at port i on voyage v. Container transportation cost between each pair of ports is determined according to transportation distance (L_{ij}), which is shown in Table 3. The transportation cost equals $0.05 per TEU per mile ($c_{ij} = 0.05 * L_{ij}$). Additionally, upper, and lower bounds for the two decision variables, i.e.,

XC_{ijv}(w) and P_{ij}, must be determined as described in equations (6), (7), (8), and (9). Lower bound for contractual price P_{ij} equals $0.1 * L_{ij}$, which is higher than the transportation cost of 1 container between port i and port j ($L_{(P_i j)} = 0.1 * L_{ij}$) since it is not reasonable that a container shipping company would provide contractual transportation prices lower than or equal to transportation cost. At least reasonable profits should be made; thus, it is twice as much higher than the transportation costs. The upper bound of the contractual price is the average spot market price throughout all 17 periods $U_{(P_i j)} = meanr_i jv(w) : v$. It is reasonable that the contractual price should be less than the average spot market price, otherwise there is no reason for customers to sign a contract, they can get the transportation service from the spot market. The lower bound for XC_{ijv} (w) equals 0 ($L_{XC_{ijv}}(w) = 0$) since the company can choose not to provide service for contractual shipments. The upper bound for XC_{ijv}(w) equals the total demand for contractual shipments ($U_{XCijv}(w) = (1 - ps)\ D_{ijv}(w)$. The transportation capacity of the service (cap) equals 8200 TEUs. The capacity is determined by turning the capacity parameter (cap) into a decision variable and solving the deterministic equivalent of the model instance. The solution showed that to maximize the profits, the container liner company should provide at least 8201.89 TEUs of transportation capacity in the service. Assuming that the container liner company determined the service capacity that maximizes its profits as consistent with industry practice, the capacity parameter *(cap)* was set to 8200 TEUs.

Table 4. Distance Between Ports in the Service Container Transportation Demand Between Ports in the Service (Number of TEUs)

	Valencia	Castellon	Barcelona	Fos	Piraeus	Istanbul	Izmit	Bursa	Izmir
Valencia	-	$U[0,50]$	$U[0,50]$	$U[0,50]$	$U[200,500$	$U[200,500$	$U[200,500$	$U[200,500$	$U[200,500$
Castellon	0	-	$U[0,50]$	$U[0,50]$	$U[200,500$	$U[200,500$	$U[200,500$	$U[200,500$	$U[200,500$
Barcelona	0	0	-	$U[0,50]$	$U[200,500$	$U[200,500$	$U[200,500$	$U[200,500$	$U[200,500$
Fos	0	0	0	-	$U[200,500$	$U[200,500$	$U[200,500$	$U[200,500$	$U[200,500$
Piraeus	$U[200,500$	$U[200,500$	$U[200,500$	$U[200,500$	-	$U[200,500$	$U[200,500$	$U[200,500$	$U[200,500$
Istanbul	$U[200,500$	$U[200,500$	$U[200,500$	$U[200,500$	$U[200,500$	-	$U[0,50]$	$U[0,50]$	$U[0,50]$
Izmit	$U[200,500$	$U[200,500$	$U[200,500$	$U[200,500$	$U[200,500$	0	-	$U[0,50]$	$U[0,50]$
Bursa	$U[200,500$	$U[200,500$	$U[200,500$	$U[200,500$	$U[200,500$	0	0	-	$U[0,50]$
Izmir	$U[200,500$	$U[200,500$	$U[200,500$	$U[200,500$	$U[200,500$	0	0	0	-

5.3. Sampling approach

The uncertain input parameters of the model, namely spot container transportation price r_{ijv} (w) and container transportation demand D_{ijv} (w), are included in the instances as a finite number of scenarios that are constructed from a random sample with equal occurrence probabilities as consistent with the usual stochastic modeling practice (Birge & Louveaux, 2011). The random samples in the application case are taken from uniform distribution, and the lower and upper bounds of the uniform distribution were determined according to probable market expectations that were investigated. For example, when the expectation of a down to 10% decrease in the market demand is tested in the experiments, it is assumed that the demand will gradually decrease down to 10% at the last voyage of 17 voyages. In that situation, the created scenarios include random demand realizations between the current level and 10% lower than the current level at the last voyage. As an example, in one of the scenarios, if the random demand realization is -8.4%, it is assumed that the demand on the last voyage will be 8.4% lower than on the first voyage with a gradual decrease of 8.4/17% in each voyage. When drawing random samples for spot price and transportation demand, the correlation between those two random parameters was also considered according to the 0.8 Pearson correlation. For instance, if a 10% decrease in the market demand is assumed, it is also assumed that it will create down to a 10% decrease in the spot market price, and random realization of those demands and spot market prices are 0.8 correlated in terms of Pearson correlation.

Because stochastic modeling instances are required to include a finite number of scenarios, the number of scenarios to be included needs to be decided. If the number of scenarios is too many, the instance would not be solved in a reasonable time. However, if the number of scenarios is too few, some portion of uncertainty would not be captured in the instances. Table 5 shows the solution performance with regard to the number of scenarios. Model instances with 80, 90, and 100 scenarios as shown in Table 5 were solved by Gurobi Solver. The differences in the objective values in each set of scenarios are less than 1%. This indicates that increasing the number of scenarios to more than 100 would bring very little improvement. Therefore, it is decided that 100 scenarios can provide adequate representation for uncertainty performing experimentations that will be explained in the next section.

Table 5. Solution Performance with Different Number of Scenarios

Number of Scenarios	Solution Time (Seconds)	Objective Value ($)
80	184	126,301,214
90	211	127,162,364
100	221	128,736,500

6. Experimental Results

6.1. Sensitivity Evaluations for Market Expectations

Experimentations have been performed considering various market situations in the application context. Particularly, three market situations were evaluated concerning the expectation toward the future market. They include downward market expectations down to 50%, upward market expectations up to 50%, and expectations in a range between down to 50% and up to 50%. In the experimentation, it is evaluated how these market situations impact various performance criteria i.e., average contractual price, gross profit (objective value), and capacity utilization.

6.1.1. Downward Market Expectations

The impacts of downward market expectations are evaluated by assuming that the future demand and spot market price will be lower than the current level. 5 different levels of downward demand are evaluated. Each time, it is considered that the demand and spot market prices would take values in between the current values and the downward level. For example, in the first level, it is considered that the demand and spot market prices would take random values in between their current values and 10% downward of the current market level. Down to 50% decreases in the demands and spot market prices were considered and illustrated in Figure 3 and Figure 4.

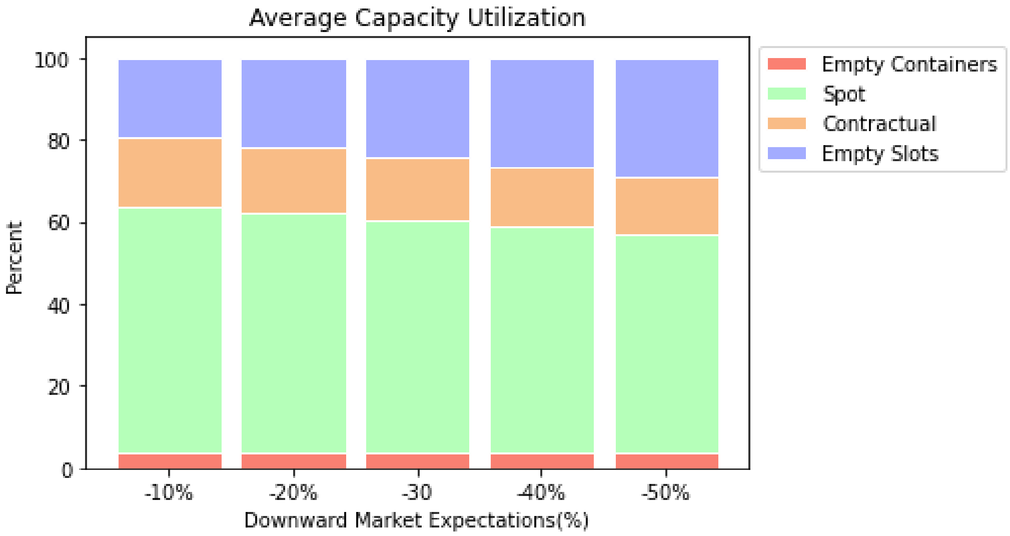

Figure 3. Sensitivity of Capacity Utilization

Figure 3 shows that when a down to 10% decrease is expected in the market, the average capacity utilization would be around 80%. However, if the expected decrease is steeper i.e., 50%, the capacity utilization decreases a little less than 10% and becomes 71%. Increasing the range of downward uncertainty reduced the number of both contractual and spot containers while the number of empty containers stayed about the same. A similar decrease is also observed for contractual freight as shown in Figure 4. When the expected market decrease goes down to a range of 50% from a range of 10%, the contractual price decreases from 359*to*324.8, which is around 10%. Objective value, however, is more sensitive as it decreases 23% by going down from $123,060,930 to

$94,485,490. These results indicate that a possible decrease in demand might cause serious financial difficulties and a high level of idle capacity. Therefore, the liner company should prepare for the downward market in advance. One solution can be redeploying its fleet to shift the capacity to the markets where the market outlook is more promising. Forming long-term relations with their customers can be another option. The experiment instances assumed that 60% of the total demand is spot market demand while 40% of it is contractual demand from the contractual market. By forming long-term relations with customers, this share of the contractual market can be increased.

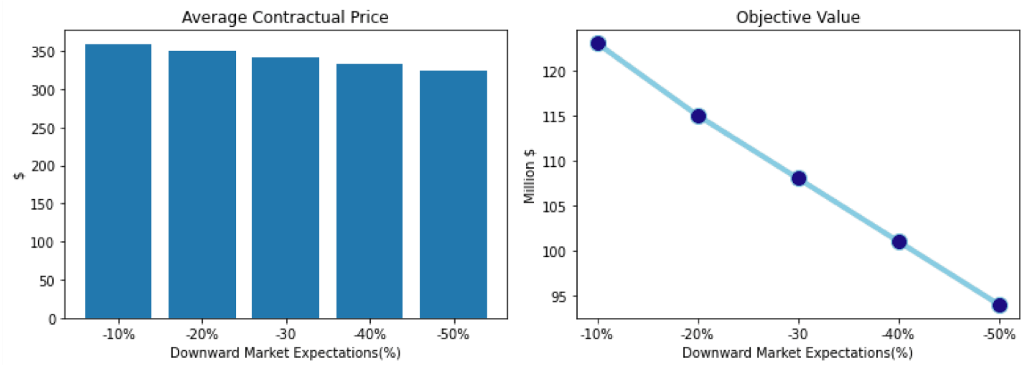

Figure 4. Sensitivity Contractual Price and Objective Value

6.1.2. Upward Market Expectations

Upward market expectations were evaluated in the same way as the downward expectations, but the five-level demand changes were upward as illustrated in Figure 5 and Figure 6. As shown in Figure 5, the capacity utilization resulted from the increasing number of spot containers. When the range of upward market expectations goes to 50% from 10%, the increase in the average capacity utilization would be around 5% increasing from 84.5% to 88.6%. Compared to the impact of downward expectations, the change is almost half of the change resulting from the same amount of change in the downward expectations range. This indicates that capacity utilization is more sensitive to demand decreases than demand increases. Additionally, at around 90%, the increase in capacity utilization slows down and reaches saturation point even though there is enough demand for further increase. At this point, the liner company should take measures to increase the capacity of the service to take advantage of the high demand. Various options can be considered depending on the situation in the market. The capacity from other services that are less profitable can be redeployed to the current service or extra container ships can be bought from the second-hand market or chartered or capacity can be hired from other alliance members' services.

An interesting result regarding empty containers was observed as illustrated in Figure 5. When the upward range increases, the number of empty containers decreases. This result partially sheds light on the current empty container shortage in the market. Among other reasons, when the market is expected to increase, it is more profitable for the liner company to allocate its slots to spot containers instead of empty containers and looking for other options for empty container supply such as leasing since the option of empty container leasing is included in the model. Even if the other options are more expensive than relocating empty containers, the profit generated from increased spot rates and demand would compensate for it.

Figure 6 illustrates the sensitivity of contractual prices and gross profit (objective value). Contrary to capacity utilization, the contractual price is more sensitive to the upward market expectations because the changes in the average contractual price resulting from upward market expectations are higher compared to the changes resulting from downward market expectations. When the upward expectations range goes up to 50% from 10%, the average contractual price increases 12% from $423 to $377.25. And the objective value increases around 22% from $138,675,370 to $168,865,890.

6.1.3. Expectations towards Both Sides

In addition to downward and upward demand expectations, the symmetrical expectations towards both sides were evaluated by assuming the market will be in a particular range between a certain percentage lower and upper than the current market. Experiments were conducted the same way as the previous experiments, by evaluating 5-level range changes as illustrated in Figure 7 and Figure 8.

Figure 7 illustrates the impacts on average capacity utilization. For example, the first level range in the graph shows the market

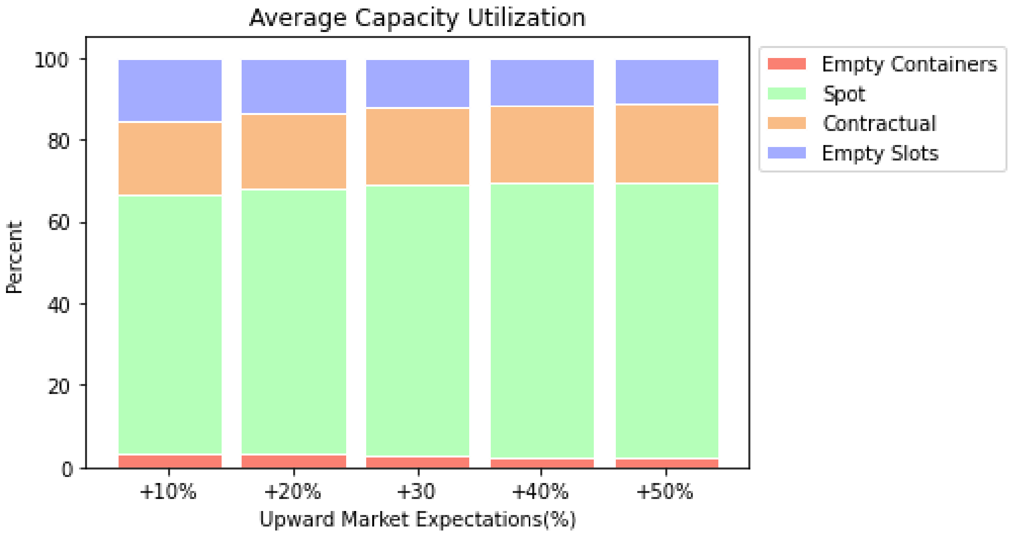

Figure 5. Sensitivity of Capacity Utilization

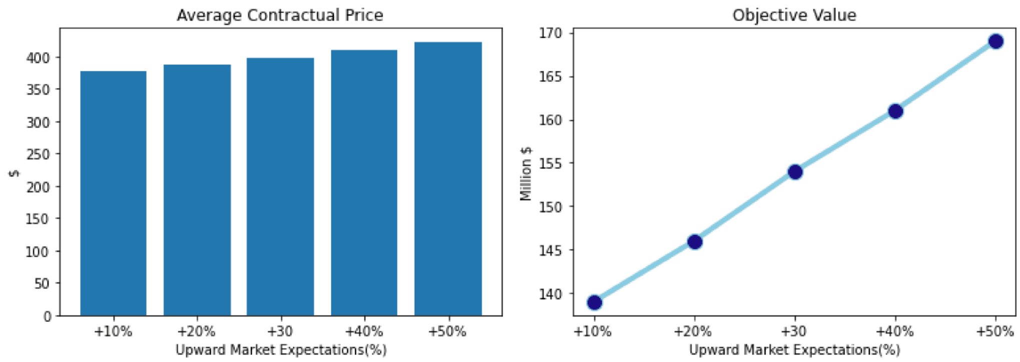

Figure 6. Sensitivity of Contractual Price and Objective Value

expectations between 10% up and 10% lower than the current market, and the demand and spot market prices take random values in between this range. The graphs in Figure 5 show that even if the market expectations are symmetrical on both sides (downward and upward), the increasing uncertainty ranges negatively affect both capacity utilization and objective value while increasing contractual price. When the uncertainty exceeds the range between -20% and +20%, the number of contractual and spot containers starts to shrink. While the number of empty containers stays about the same unit the uncertainty range hits -40% and +40% then the number of empty containers starts to shrink as well. The increase in the uncertainty ranges from between +10% and -10% to between +50% and -50% decreases average capacity utilization around 5% from 82.4% to 78.5% while decreasing objective value around 1.5% from $130,600,900 to 128,736,500$. On the other hand, this change in the uncertainty range increases the average contractual price by around 5% from \$369.47 to \$390.43.

These results indicate that the increasing range of uncertainty negatively impacts the business of the liner company. Therefore, the level of uncertainty should be reduced by taking various measures. To minimize uncertainty, a company can increase its integration and information sharing with its customers. This can be accomplished by long-term relationships as stated earlier as a measure to deal with downward market expectations. Additionally, taking advantage of state-of-the-art forecasting techniques can help to reduce the level of uncertainty.

6.2. Value of Stochastic Solution

The value gained from including stochasticity in the modeling is evaluated in this section. The value of the stochastic solution (VSS) is defined as the benefit gained from including uncertainty in a model (Birge, 1982; Birge & Louveaux, 2011; Maggioni & Wallace, 2012). Equation 11.1 shows that in a maximization problem, it equals the recourse problem (RP) solution minus

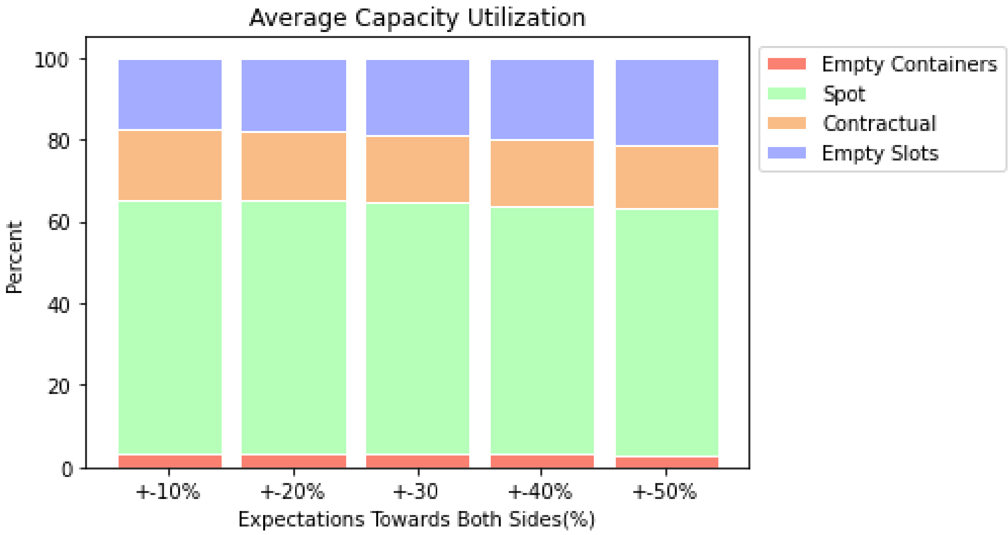

Figure 7. Sensitivity of Capacity Utilization

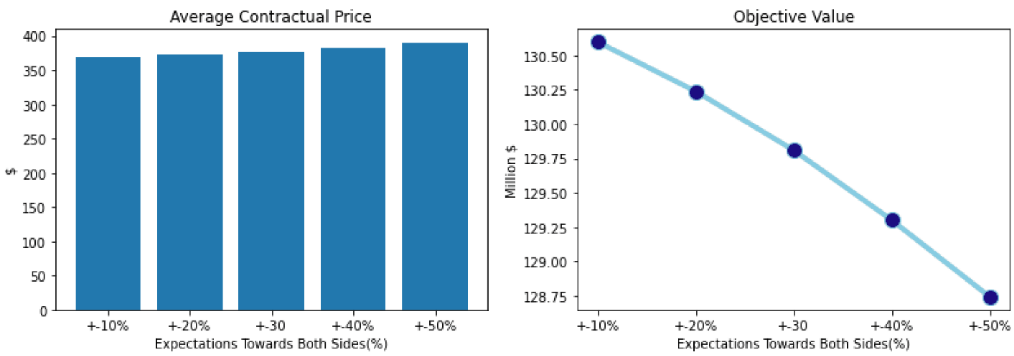

Figure 8. Sensitivity of Contractual Price and Objective Value

the expectation of the expected value solution (EEV). Recourse problem solutions equal the solution of the stochastic instance. Expectations of the expected value solutions equal the mean of deterministic instance solutions of every scenario by fixing the first stage variable values to the deterministic instance solution of the mean value scenario (Birge (1982), Birge & Louveaux (2011), and Maggioni & Wallace (2012).

$$VSS = RP - EEV \tag{23}$$

$$RP = \min_{x} E_{\omega} z(x, \omega) \tag{24}$$

$$EEV = E_{\omega}(\min_{x} z(x(\omega), \omega)) \tag{25}$$

Table 6 shows values of stochastic solution calculated for three instances; the first instance includes downward expectations in a range from the current level to down to 50%, and the second instance includes upward expectations in a range from the current level to up to 50%, and the third instance includes the expectations in a range between down to 50% and up to 50%. Table 6 shows that when the market is expected to decrease by 50%, applying stochastic modeling would bring up to $1,815,490 more profit instead of applying a deterministic equivalent. The benefit of stochastic solution would be higher, particularly $4,390,640 when the market is expected to increase up to 50%. On the other hand, when the market is expected to be in a range between up to 50% and down to 50%, the value of the stochastic solution would be $386,980. Results for all three instances confirm that including stochasticity in the modeling brings benefits. Since life is full of uncertainties, it can be assumed that it is certain, but it is not possible to avoid the impacts of uncertainties. Therefore, accounting for uncertainty in the modeling of slot allocation and contractual pricing of container liner shipping would be beneficial.

Table 6. Value of Stochastic Solutions ($)

Instance	EEV	RP	VSS
Down to 50%	92,670,000	94,485,490	1,815,490
Up to 50%	164,475,250	168,865,890	4,390,640
Between down to 50% and up to 50%	128,349,520	128,736,500	386,980

7. Conclusions

The product of container liner companies is their transportation capacity, and by selling their transportation capacities in particular routes to their customers, they generate revenue. Therefore, managing their transportation capacity is a topmost priority for container liner companies. The capacity usage can be segmented into three as the capacity for contractual, spot, and empty containers. Additionally, the number of slots allocated to contractual and spot containers depends on the prices provided to each type of customer. The transportation price for spot containers is determined in accordance with the spot market prices; however, contractual prices are determined according to future expectations regarding demand and spot market prices. Container shipping lines usually have room for deviating from the contractual prices of their competitors. Therefore, optimum contractual prices and slot allocation that maximize profitability and capacity utilization of container shipping lines can be determined by taking advantage of optimization modelling. In this regard, this study aimed to propose an optimization model for slot allocation and contractual pricing that considers spot and contractual shipments and empty container repositioning under a stochastic environment. The model was applied in the case of container liner shipping service in between ports of western Mediterranean and ports of Marmara Sea.

Experimentation for evaluating the downward market expectations showed that a possible decrease in the demand and spot prices could cause a tremendous loss in revenue and create a high level of idle capacity. The liner company should take measures to soften the blow by redeploying its ships to services that have promising outlooks or focusing on developing long-term relationships with its customers to increase the share of contractual shipment demand in terms of total demand. Experiments regarding the upward demand expectation showed that the capacity utilization reach a saturation point at around 90%. After this point, the company needs to increase its service capacity to take advantage of the increasing market. Increasing range of upward demand results in a reduction of empty container transportation since it becomes more profitable to allocate the slots to spot market demand rather than empty containers and to take advantage of other means of empty container supply. Experimentation of symmetric uncertainty revealed that the increasing range of uncertainty creates a serious loss in profits and capacity utilization. It can be minimized by higher integration and information sharing with customers and taking advantage of state-of-the-art demand forecasting techniques.

The results of this study can have various implications. Decreases in demand for container transportation can have a tremendous impact on profitability of container lines. Container shipping lines can soften the impact by developing long-term relationships to increase the share of contractual shipments. Increases in the uncertainty can also cause big losses in profits. This shows container shipping lines the importance of integration with supply chain partners and taking advantage of new technologies such as AI and big data analytics for state-of-the-art demand forecasting. Besides, calculations of the value of stochastic solutions showed that the application of the stochastic modeling solutions would provide a higher profit margin than the solution of its deterministic equivalents. Therefore, container shipping lines should include uncertainty in their optimization modelling to increase their profitability.

This study has several limitations. First, it is apparent that the experimental results highly depend on the functional relationship between contractual price and the number of contractual slots. Second, the model includes only two stochastic parameters, namely sport prices and demand for container transportation. Third, the container transportation costs do not explicitly include speed and bunker consumption function. The application instances include container transportation cost that was determined according to transportation distance multiplied with a constant that equals to $0.05.

Future studies can overcome those limitations. First, the application of the model can be improved by providing a data-driven approach to determine the functional dependence between contractual price and the number of slots allocated to contractual shipments. Second, the model can include other stochastic parameters such as inventory costs, container leasing costs and transportation costs. Third, the model can be improved by including bunker consumption function and speed of container ships as decision variables. However, inclusion of more stochastic parameters and additional decision variables might increase the difficulty of model solution. Therefore, future studies can also develop specialized solution algorithms for their models.

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