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A Soft Set Approach to Relations and Its Application to Decision Making

Kemal Taşköprü* and Elif Karaköse

Abstract
One of the most useful mathematical tools for examining the relationships among objects is the concept of relation. Besides, it can also be necessary to throw light on uncertainties in these relationships. Soft set theory, in which different approaches used in defining the notions bring about different applications in many areas, enables to overcome uncertainties. The purpose of this paper is to define soft relation in a different way and to give a decision making method using the concept of soft relation. For this purpose, firstly, the soft relations are defined on the collection of soft elements, unlike the previous ones. After their basic properties are provided, the correspondence between the soft and classical relations is investigated and some examples are given. Finally, an algorithm is proposed using the soft relation for solving decision making problems, where the decision is related to other circumstances, and given an illustrative example.

Keywords: *soft relation; closure of relations; equivalence; decision making.*

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****Corresponding author***

1. Introduction

The soft sets, introduced by Molodtsov [1], have enabled to be deal with uncertainties such as the fuzzy sets, vague sets, rough sets, intuitionistic fuzzy sets, and neutrosophic sets, which deal with uncertainties in different ways [2]. In substance, a soft set is considered as a parametrized set of alternatives and this parametrization allows the alternatives to be examined according to their properties. The soft set theory, with the integration of other set theories has been the subject of the various scientific fields of study containing vagueness, especially in decision making problems and many different mathematical structures [3–9]. After Maji et al. [10] and Ali et al. [11] laid the foundations of the soft set operations, different interpretations have emerged about extending mathematical structures to the soft set theory (See [12–18], and others in them). The soft elements and elementary (ε-) soft set

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operations were brought forward by Das and Samanta [19], and some mathematical structures have been examined using these concepts by several authors [20–27].

The relations are fundamental concepts that have been used to classify, order or compare objects in many fields of research as a result of the fact that different structures can be related to each other as well. In order to apply the soft set theory to the relations, the relations in the soft sets were introduced to model fuzziness and hesitancy in the relationships between two objects [28–35]. Recently, Alcantud [39] introduced a new concept called softarison defined on a set of alternatives to make parametrized comparisons as the soft sets, which merged the soft sets with the relations and applied it to decision making problems. In addition, for the purpose of handling the decision making process in different ways in vagueness, the relations on the hybrid soft sets have been introduced and applied to decision making problems [40–46]. In the mentioned above studies and others discussing the relations in (hybrid) soft sets, a relation in these sets is actually described as corresponding to a (hybrid) soft set. Furthermore, in all the studies addressing decision making problems, the decision corresponds to an element that is generally determined from among the alternatives, according to the weights and attributes.

The alternatives may consist of certain factors and it may be desired to determine the factors that will form the preference according to the desired criteria. Concordantly, the soft elements correspond to single-element soft sets and a soft element provides a pattern that determines the appropriate alternative for each descriptive attribute from within the soft set. In this study, it is shown that the concepts of soft element and soft relation can be useful and applicable to investigate the factors that constitute the alternatives and the relationships between the alternatives. Unlike the previous studies about the relations in the soft sets, a novel approach is proposed to the soft relation using the concept of soft element and it is demonstrated that using this notion can be operable in decision making.

The paper is organised as follows: In Section 2, the basic information about soft sets and the notion of soft element are given and an overview of previous studies regarding the relations in the soft sets is presented. In Section 3, a soft relation is defined by using a collection of soft elements. After the definitions and properties of the soft relations are given via ε-soft set operations, the interactions of these relations and the classical relations are investigated. It is encountered that soft equivalence relations have different properties from the classical equivalences. In Section 4, a soft relation-based algorithm is proposed for handling decision making problems in which the decision is made as a soft element, that is, by determining the appropriate alternative corresponding to each parameter, and the decision is influenced by other factors. Then, an illustrative example is presented to choose an optimal system and to ensure optimal system integration. Finally, in the concluding section of the paper, various lines for further research on this topic are noted.

2. Preliminaries

Definition 2.1. [1] Let U be a universal set, P be a parameters set and $P(U)$ be the power set of U. A pair (G, P) is called a soft set on U, where $G : P \to P(U)$ is a mapping.

Definition 2.2. Let (G, P) and (H, P) be two soft sets on U. The soft set (G, P) is said to be a null soft set if $G(\alpha) = \emptyset$ and an absolute soft set if $G(\alpha) = U$ for each $\alpha \in P$, denoted by Φ and $\tilde{U}$, respectively. The soft set (G, P) is said to be a soft subset of (H, P) if $G(\alpha) \subset H(\alpha)$ for every $\alpha \in P$ and denoted by $(G, P)\tilde{\subset}(H, P)$. Also, $(G, P) = (H, P)$ if and only if $(G, P)\tilde{\subset}(H, P)$ and vice versa.

Definition 2.3. [33] Let (G, P) be a soft set on U and (H, P') be a soft set on U'. The Cartesian product of (G, P) and (H, P') is defined as $(F, P \times P') = (G, P) \times (H, P')$, where $H : P \times P' \to P(U \times U')$ is given by $H(\alpha, \alpha') = G(\alpha) \times H(\alpha')$ for each $(\alpha, \alpha') \in P \times P'$. Then, a soft set relation from (G, P) to (H, P') is a soft subset of $(G, P) \times (H, P')$ and a soft set relation on (G, P) is a soft subset of $(G, P) \times (G, P)$.

Before the above definition, the definition of Cartesian product and the soft set relation on the same universe U in [28] and it was studied based on this definition in [29–31]. Also, based on these studies, the soft set relations were merged with topology and transferred to the hybrid soft sets (See, [40–42, 45]).

Definition 2.4. [32] Let (ρ, P) be a soft set on $U \times U$, i.e. $\rho : P \to P(U \times U)$. Then, (ρ, P) is called a soft binary relation on U. Here, the soft binary relation is considered as a parametrized collection of binary relations on U.

Based on the above definition, the soft binary relations were merged with algebraic structures and hybrid soft sets (See, [35, 43] and others in them).

Apart from the above studies regarding the relations in the soft sets, by using the partial order relation on the universe U, and by examining the belonging relation of the elements to the set corresponding to the parameters, topological structures were studied on soft sets in [37, 38, 44].

Definition 2.5. [37] A soft point P_α^x of a soft set (G,P) on U is determined by the fact that $x \in G(\alpha)$ for the parameter $\alpha \in P$ and $x \in U$.

In the related studies in Definitions 2.3 and 2.4 and others, an element of a soft set is evaluated via Definition 2.5. Unlike this definition, Das and Samanta [19] introduced the soft element and gave elementary soft set operations. They consider a soft element to be about evaluating not just a single point for a single parameter, but the corresponding points for each parameter.

Definition 2.6. [19] A function $\varepsilon : P \to U$ is called a soft element of U and ε is said to be member of (G,P) if $\varepsilon(\alpha) \in G(\alpha)$ for each $\alpha \in P$. The class of soft elements of (G,P) are denoted by $SE(G,P)$ and the soft elements by $\tilde{x}$, $\tilde{y}$, $\tilde{z}$, etc.

Throughout the work, the soft sets (G,P) on U such that $G(\alpha) \neq \emptyset$ for every $\alpha \in P$ and the null soft set Φ will be considered. The class of these soft sets is denoted by $S(\tilde{U})$ and $S_P(U)$ represents the set of all soft sets over U with parameters P.

The soft set $SS(\mathfrak{B})$ produced by the class of soft elements $\mathfrak{B}$ is defined by

$$(G,P) = SS(\mathfrak{B}) = \{(\alpha, G(\alpha)) : \forall \alpha \in P,\ G(\alpha) = \bigcup_{\tilde{x}\in\mathfrak{B}} \{\tilde{x}(\alpha)\}\}.$$

The ε-union and ε-intersection of (G,P),$(H,P) \in S(\tilde{U})$ are defined by

$$(G,P) \Cup (H,P) = SS(SE(G,P) \cup SE(H,P))$$

and

$$(G,P) \Cap (H,P) = SS(SE(G,P) \cap SE(H,P)),$$

respectively. The ε-complement of (G,P) is defined $(G,P)^{\mathbb{C}} = SS(SE(G,P)^c)$, where $(G,P)^c = (G^c,P)$ is soft complement of (G,P) and $G^c : P \to P(U)$ is a mapping given by $G^c(\alpha) = U \backslash G(\alpha)$, $\forall \alpha \in P$. (For details, see [25]).

From now on, the notation of a soft set is used as G instead of (G,P) for simplicity and $SE(\tilde{U})$ denotes the set of all soft elements over U with parameters set P.

3. Soft relations

This section proposes a novel approach to the relations in the soft sets. The relations in the soft sets are actually referred to as a soft set in Definitions 2.3 and 2.4, whereas a soft relation based on the concept of soft element is defined to be a subclass of the Cartesian product of any two collections of soft elements.

Definition 3.1. Let U and U' be two universal sets and P be a parameters set. A soft relation $\mathcal{R}$ from $\tilde{U}$ to $\tilde{U}'$ is defined as a subclass of $SE(\tilde{U}) \times SE(\tilde{U'})$ and then a soft relation $\mathcal{R}$ on $\tilde{U}$ is denoted by

$$\mathcal{R} = \{(\tilde{x}, \tilde{y}) : \tilde{x}, \tilde{y} \in \mathfrak{B} \subset SE(\tilde{U})\} \subseteq SE(\tilde{U}) \times SE(\tilde{U}).$$

All the properties of soft relations can be defined similarly to those of classical relations and some situations that make a difference are given as follows.

Definition 3.2. Let $\tilde{U}$ be an absolute soft set with parameter set P having a soft relation $\mathcal{R}$. The soft relation $\mathcal{R}$ is called

- *reflexive* if $\tilde{x}\mathcal{R}\tilde{x}$ for each $\tilde{x} \in SE(\tilde{U})$,
- *irreflexive* if $\neg\tilde{x}\mathcal{R}\tilde{x}$ for each $\tilde{x} \in SE(\tilde{U})$,
- *symmetric* if $\tilde{x}\mathcal{R}\tilde{y} \Rightarrow \tilde{y}\mathcal{R}\tilde{x}$ for each $\tilde{x}, \tilde{y} \in SE(\tilde{U})$,
- *asymmetric* if $\tilde{x}\mathcal{R}\tilde{y} \Rightarrow \neg\tilde{y}\mathcal{R}\tilde{x}$ for each $\tilde{x}, \tilde{y} \in SE(\tilde{U})$,
- *antisymmetric* if $\tilde{x}\mathcal{R}\tilde{y}$ and $\tilde{y}\mathcal{R}\tilde{x} \Rightarrow \tilde{x} = \tilde{y}$ for each $\tilde{x}, \tilde{y} \in SE(\tilde{U})$,
- *transitive* if $\tilde{x}\mathcal{R}\tilde{y}$ and $\tilde{y}\mathcal{R}\tilde{z} \Rightarrow \tilde{x}\mathcal{R}\tilde{z}$ for each $\tilde{x}, \tilde{y}, \tilde{z} \in SE(\tilde{U})$,

- *total (complete, connected, comparable or connex)* if $\tilde{x}\mathcal{R}\tilde{y}$ or $\tilde{y}\mathcal{R}\tilde{x}$ for each $\tilde{x}, \tilde{y} \in SE(\tilde{U})$,

where $\tilde{x}R\tilde{y}$ means that $(\tilde{x}, \tilde{y}) \in \mathcal{R}$. Also, a soft relation $\mathcal{R}$ is called

- **pre-order** if it is reflexive and transitive,
- **total pre-order (weak order)** if it is reflexive, total and transitive,
- **partial order** if it is reflexive, antisymmetric and transitive,
- **strict partial order** if it is irreflexive, asymmetric and transitive,
- **total order (complete order, linear order)** if it is reflexive, antisymmetric, total and transitive,
- **equivalence relation** if it is reflexive, symmetric and transitive.

Proposition 3.1. *Each parametrized family of classical relations can be considered as a soft relation and every soft relation on can be considered as a parametrized family of classical relations.*

Proof. If $\{R_\alpha : \alpha \in P\}$ is a family of classical relations on U with parameter set P then $\mathcal{R}$ is a soft relation on $\tilde{U}$ such that $\mathcal{R}(\alpha) = \mathcal{R}_\alpha = \{(\tilde{x}, \tilde{y})(\alpha) = (\tilde{x}(\alpha), \tilde{y}(\alpha)) \in R_\alpha : \tilde{x}, \tilde{y} \in SE(\tilde{U})\}$ for all $\alpha \in P$. Conversely, if $\mathcal{R}$ is a soft relation on $\tilde{U}$ then for each $\alpha \in P$, $\mathcal{R}_\alpha$ is a classical relation on U. Hence, each parametrized family of classical relations can be considered as a soft relation, and vice versa. □

Suppose that R is a relation on U. Then, $\mathcal{R}$ is a soft relation on $\tilde{U}$ such that $\mathcal{R}_\alpha = R$ for all parameters $\alpha \in P$. So, the soft relation $\mathcal{R}$ determined by using the classical relation R is called a soft relation produced by R.

Remark 3.1. Let $G \in S(\tilde{U})$ be a soft set with parameters set P. If $\mathcal{R}$ is a soft relation on G the family $\{R_\alpha : \alpha \in P\}$ is obtained as a parametrized family of classical relations on U in a similar way to Proposition 3.1. But, if a parametrized family of classical relations $\{R_\alpha : \alpha \in P\}$ on U is given, there can be some $\tilde{x}, \tilde{y} \in SE(G)$ and some $\alpha \in U$ such that $(\tilde{x}, \tilde{y})(\alpha) = (\tilde{x}(\alpha), \tilde{y}(\alpha)) \notin R_\alpha$. Hence, it is not obtained a soft relation on G. In such a case, since each soft element $\tilde{x} \in SE(G)$ is a function from P to U, if the family $\{R_\alpha : \alpha \in P\}$ is given such that

$$R_\alpha \subset \bigcap_{\alpha \in P} G(\alpha) \times \bigcap_{\alpha \in P} G(\alpha)$$

for each $\alpha \in P$, it is obtained a soft relation $\mathcal{R}$ on G in a similar way to Proposition 3.1. An example of such a case is given below in Example 3.1.

Proposition 3.2. *Every parametrized family of classical reflexive, irreflexive, symmetric, antisymmetric, asymmetric, and transitive relations can be considered as a reflexive, irreflexive, symmetric, antisymmetric, asymmetric, and transitive soft relation, respectively.*

Proof. It is easily seen that the proof follows from Proposition 3.1. □

Example 3.1. Let $P = \{\alpha, \beta\}$ and $U = \{u, v, w\}$, then

$$SE(\tilde{U}) = \{\tilde{e}_1, \tilde{e}_2, \tilde{e}_3, \tilde{e}_4, \tilde{e}_5, \tilde{e}_6, \tilde{e}_7, \tilde{e}_8, \tilde{e}_9\},$$

where

$$\begin{aligned} \tilde{e}_1 &= \{(\alpha, u), (\beta, u)\}, & \tilde{e}_4 &= \{(\alpha, v), (\beta, u)\}, & \tilde{e}_7 &= \{(\alpha, w), (\beta, u)\}, \\ \tilde{e}_2 &= \{(\alpha, u), (\beta, v)\}, & \tilde{e}_5 &= \{(\alpha, v), (\beta, v)\}, & \tilde{e}_8 &= \{(\alpha, w), (\beta, v)\}, \\ \tilde{e}_3 &= \{(\alpha, u), (\beta, w)\}, & \tilde{e}_6 &= \{(\alpha, v), (\beta, w)\}, & \tilde{e}_9 &= \{(\alpha, w), (\beta, w)\}. \end{aligned}$$

Suppose that the parametrized classical relations are defined as

$$\begin{aligned} R_1 &= \{(u, u), (v, v), (w, w), (u, v)\}, \\ R_2 &= \{(u, u), (v, v), (w, w), (u, v), (v, u)\}. \end{aligned}$$

The properties of these relations are as follows:

	Reflexive	Irreflexive	Symmetric	Antisymmetric	Asymmetric	Total	Transitive
R_1	✓	✗	✗	✓	✗	✗	✓
R_2	✓	✗	✓	✗	✗	✗	✓

If the parametrized family of classical relations $\{R_1, R_2\}$ is considered, the soft relations are obtained as

$$\mathcal{R}^{(12)} = \{(\tilde{e}_1,\tilde{e}_1),\ldots,(\tilde{e}_9,\tilde{e}_9),(\tilde{e}_1,\tilde{e}_2),(\tilde{e}_1,\tilde{e}_4),(\tilde{e}_1,\tilde{e}_5),(\tilde{e}_2,\tilde{e}_1),(\tilde{e}_2,\tilde{e}_4),(\tilde{e}_2,\tilde{e}_5),(\tilde{e}_3,\tilde{e}_6),(\tilde{e}_4,\tilde{e}_5),(\tilde{e}_5,\tilde{e}_4),(\tilde{e}_7,\tilde{e}_8),(\tilde{e}_8,\tilde{e}_7)\},$$

$$\mathcal{R}^{(21)} = \{(\tilde{e}_1,\tilde{e}_1),\ldots,(\tilde{e}_9,\tilde{e}_9),(\tilde{e}_1,\tilde{e}_2),(\tilde{e}_1,\tilde{e}_4),(\tilde{e}_1,\tilde{e}_5),(\tilde{e}_2,\tilde{e}_5),(\tilde{e}_3,\tilde{e}_6),(\tilde{e}_4,\tilde{e}_1),(\tilde{e}_4,\tilde{e}_2),(\tilde{e}_4,\tilde{e}_5),(\tilde{e}_5,\tilde{e}_2),(\tilde{e}_6,\tilde{e}_3),(\tilde{e}_7,\tilde{e}_8)\}.$$

Also, the soft relations produced by the classical relations R_1 and R_2 are obtained as

$$\mathcal{R}^{(1)} = \{(\tilde{e}_1,\tilde{e}_1),\ldots,(\tilde{e}_9,\tilde{e}_9),(\tilde{e}_1,\tilde{e}_2),(\tilde{e}_1,\tilde{e}_4),(\tilde{e}_1,\tilde{e}_5),(\tilde{e}_2,\tilde{e}_5),(\tilde{e}_3,\tilde{e}_6),(\tilde{e}_4,\tilde{e}_5),(\tilde{e}_7,\tilde{e}_8)\},$$

$$\mathcal{R}^{(2)} = \{(\tilde{e}_1,\tilde{e}_1),\ldots,(\tilde{e}_9,\tilde{e}_9),(\tilde{e}_1,\tilde{e}_2),(\tilde{e}_1,\tilde{e}_4),(\tilde{e}_1,\tilde{e}_5),(\tilde{e}_2,\tilde{e}_1),(\tilde{e}_2,\tilde{e}_4),(\tilde{e}_2,\tilde{e}_5),(\tilde{e}_3,\tilde{e}_6),(\tilde{e}_4,\tilde{e}_1),(\tilde{e}_4,\tilde{e}_2),(\tilde{e}_4,\tilde{e}_5),(\tilde{e}_5,\tilde{e}_1),(\tilde{e}_5,\tilde{e}_2),(\tilde{e}_5,\tilde{e}_4),(\tilde{e}_6,\tilde{e}_3),(\tilde{e}_7,\tilde{e}_8),(\tilde{e}_8,\tilde{e}_7)\}.$$

Here, the notation $\mathcal{R}^{(12)}$ ($\mathcal{R}^{(21)}$) refers to the soft relation generated by R_1 (R_2) and R_2 (R_1) for the α and β parameters, respectively. In addition, the notation $\mathcal{R}^{(1)}$ ($\mathcal{R}^{(2)}$) refers to the soft relation generated by R_1 (R_2) for both the α and β parameters, respectively. Then, the properties of these soft relations are as follows:

	Reflexive	Irreflexive	Symmetric	Antisymmetric	Asymmetric	Total	Transitive
$\mathcal{R}^{(12)}$	✓	✗	✗	✗	✗	✗	✓
$\mathcal{R}^{(21)}$	✓	✗	✗	✗	✗	✗	✓
$\mathcal{R}^{(1)}$	✓	✗	✗	✓	✗	✗	✓
$\mathcal{R}^{(2)}$	✓	✗	✓	✗	✗	✗	✓

In addition, let $G = \{(\alpha, \{v, w\}), (\beta, \{u, w\})\}$ be a soft set on U. Then, $SE(G) = \{\tilde{e}_4, \tilde{e}_6, \tilde{e}_7, \tilde{e}_9\}$. Hence, one can obtain that $(\tilde{e}_4, \tilde{e}_9)(\alpha) = (v, w) \notin R_1, R_2$ and $(\tilde{e}_4, \tilde{e}_9)(\beta) = (u, w) \notin R_1, R_2$. Thus, a soft relation on G having the pair $(\tilde{e}_4, \tilde{e}_9)$ cannot be generated by the classical relations R_1 and R_2.

Suppose that another parametrized classical relations are defined as

$$R_3 = \{(u,u),(v,v),(w,w),(v,u),(v,w),(u,w)\},$$
$$R_4 = \{(u,v),(v,w),(w,u)\}.$$

The properties of these relations are as follows:

	Reflexive	Irreflexive	Symmetric	Antisymmetric	Asymmetric	Total	Transitive
R_3	✓	✗	✗	✓	✗	✓	✓
R_4	✗	✓	✗	✓	✓	✗	✗

If the parametrized family of classical relations $\{R_3, R_4\}$ is considered, the soft relations are obtained as

$$\mathcal{R}^{(34)} = \{(\tilde{e}_1,\tilde{e}_2),(\tilde{e}_1,\tilde{e}_8),(\tilde{e}_2,\tilde{e}_3),(\tilde{e}_2,\tilde{e}_9),(\tilde{e}_3,\tilde{e}_1),(\tilde{e}_3,\tilde{e}_7),(\tilde{e}_4,\tilde{e}_2),(\tilde{e}_4,\tilde{e}_5),(\tilde{e}_4,\tilde{e}_8),(\tilde{e}_5,\tilde{e}_3),(\tilde{e}_5,\tilde{e}_6),(\tilde{e}_5,\tilde{e}_9),(\tilde{e}_6,\tilde{e}_1),(\tilde{e}_6,\tilde{e}_4),(\tilde{e}_6,\tilde{e}_7),(\tilde{e}_7,\tilde{e}_8),(\tilde{e}_8,\tilde{e}_9),(\tilde{e}_9,\tilde{e}_7)\},$$

$$\mathcal{R}^{(43)} = \{(\tilde{e}_1,\tilde{e}_4),(\tilde{e}_1,\tilde{e}_6),(\tilde{e}_2,\tilde{e}_4),(\tilde{e}_2,\tilde{e}_5),(\tilde{e}_2,\tilde{e}_6),(\tilde{e}_3,\tilde{e}_6),(\tilde{e}_4,\tilde{e}_7),(\tilde{e}_4,\tilde{e}_9),(\tilde{e}_5,\tilde{e}_7),(\tilde{e}_5,\tilde{e}_8),(\tilde{e}_5,\tilde{e}_9),(\tilde{e}_6,\tilde{e}_9),(\tilde{e}_7,\tilde{e}_1),(\tilde{e}_7,\tilde{e}_3),(\tilde{e}_8,\tilde{e}_1),(\tilde{e}_8,\tilde{e}_2),(\tilde{e}_8,\tilde{e}_3),(\tilde{e}_9,\tilde{e}_3)\}.$$

Also, the soft relations produced by the classical relations R_3 and R_4 are obtained as

$$\mathcal{R}^{(3)} = \{(\tilde{e}_1,\tilde{e}_1),\ldots,(\tilde{e}_9,\tilde{e}_9),(\tilde{e}_1,\tilde{e}_3),(\tilde{e}_1,\tilde{e}_7),(\tilde{e}_1,\tilde{e}_9),(\tilde{e}_2,\tilde{e}_1),(\tilde{e}_2,\tilde{e}_3),(\tilde{e}_2,\tilde{e}_7),(\tilde{e}_2,\tilde{e}_8),(\tilde{e}_2,\tilde{e}_9),(\tilde{e}_3,\tilde{e}_9),(\tilde{e}_4,\tilde{e}_1),(\tilde{e}_4,\tilde{e}_3),(\tilde{e}_4,\tilde{e}_6),(\tilde{e}_4,\tilde{e}_7),(\tilde{e}_4,\tilde{e}_9),(\tilde{e}_5,\tilde{e}_1),\ldots,(\tilde{e}_5,\tilde{e}_9),(\tilde{e}_6,\tilde{e}_3),(\tilde{e}_6,\tilde{e}_9),(\tilde{e}_7,\tilde{e}_9),(\tilde{e}_8,\tilde{e}_7),(\tilde{e}_8,\tilde{e}_9)\},$$

$$\mathcal{R}^{(4)} = \{(\tilde{e}_1,\tilde{e}_5),(\tilde{e}_2,\tilde{e}_6),(\tilde{e}_3,\tilde{e}_4),(\tilde{e}_4,\tilde{e}_8),(\tilde{e}_5,\tilde{e}_9),(\tilde{e}_6,\tilde{e}_7),(\tilde{e}_7,\tilde{e}_2),(\tilde{e}_8,\tilde{e}_3),(\tilde{e}_9,\tilde{e}_1)\}.$$

Then, the properties of these soft relations are as follows:

	Reflexive	Irreflexive	Symmetric	Antisymmetric	Asymmetric	Total	Transitive
$\mathcal{R}^{(34)}$	✗	✓	✗	✓	✓	✗	✗
$\mathcal{R}^{(43)}$	✗	✓	✗	✓	✓	✗	✗
$\mathcal{R}^{(3)}$	✓	✗	✗	✓	✗	✗	✓
$\mathcal{R}^{(4)}$	✗	✓	✗	✓	✓	✗	✗

Remark 3.2. From Example 3.1, any parametrized family of total classical relations cannot be considered as a total soft relation. Also, the parametrized family of classical relations with various properties cannot be considered as the soft relations with the same properties.

Proposition 3.3. *Every reflexive, symmetric and total soft relation can be considered as a parametrized family of reflexive, symmetric and total classical relations, respectively.*

Proof. Suppose that $\mathcal{R}$ is a reflexive soft relation on $\tilde{U}$ with parameter set P. Then, for all $\tilde{x} \in SE(\tilde{U})$, $(\tilde{x}, \tilde{x}) \in \mathcal{R}$. Hence, for all $\alpha \in P$, $(\tilde{x}(\alpha), \tilde{x}(\alpha)) \in \mathcal{R}_\alpha$ and thus all the classical relations $\mathcal{R}_\alpha$ are reflexive.

Also, for every $\tilde{x}, \tilde{y} \in SE(\tilde{X})$, $(\tilde{x}, \tilde{y}) \in \mathcal{R}$ implies $(\tilde{y}, \tilde{x}) \in \mathcal{R}$ if $\mathcal{R}$ is symmetric soft relation. Hence, for all $\alpha \in P$, $(\tilde{x}(\alpha), \tilde{y}(\alpha)) \in \mathcal{R}_\alpha$ implies $(\tilde{y}(\alpha), \tilde{x}(\alpha)) \in \mathcal{R}_\alpha$ and thus all the classical relations $\mathcal{R}_\alpha$ are symmetric.

In the case of the total soft relation, the proof obtains similarly to above. □

Example 3.2. From Example 3.1, suppose that a soft relation $\mathcal{R}_1$ is defined as

$$\mathcal{R}_1 = \{(\tilde{e}_1, \tilde{e}_1), \ldots, (\tilde{e}_9, \tilde{e}_9), (\tilde{e}_1, \tilde{e}_5), (\tilde{e}_5, \tilde{e}_1), (\tilde{e}_4, \tilde{e}_6), (\tilde{e}_6, \tilde{e}_4), (\tilde{e}_7, \tilde{e}_8), (\tilde{e}_8, \tilde{e}_7)\}.$$

So, $\mathcal{R}_1$ is reflexive, symmetric, and transitive and hence it is equivalence relation. Then, $\mathcal{R}_{1_\alpha}$ is reflexive, symmetric, and transitive and $\mathcal{R}_{1_\beta}$ is reflexive, symmetric, and non-transitive such that

$$\mathcal{R}_{1_\alpha} = \{(u, u), (v, v), (w, w), (u, v), (v, u)\},$$
$$\mathcal{R}_{1_\beta} = \{(u, u), (v, v), (w, w), (u, v), (v, u), (u, w), (w, u)\}.$$

Thus, although $\mathcal{R}_{1_\alpha}$ is an equivalence relation, $\mathcal{R}_{1_\beta}$ is not.

Suppose that a soft relation $\mathcal{R}_2$ is defined as

$$\mathcal{R}_2 = \{(\tilde{e}_1, \tilde{e}_1), \ldots, (\tilde{e}_9, \tilde{e}_9), (\tilde{e}_1, \tilde{e}_2), (\tilde{e}_1, \tilde{e}_5), (\tilde{e}_2, \tilde{e}_5), (\tilde{e}_8, \tilde{e}_7)\}.$$

So, $\mathcal{R}_2$ is reflexive, antisymmetric, and transitive and hence it is partial order. Then, $\mathcal{R}_{2_\alpha}$ is reflexive, antisymmetric, and transitive and $\mathcal{R}_{2_\beta}$ is reflexive, symmetric, and transitive such that

$$\mathcal{R}_{2_\alpha} = \{(u, u), (v, v), (w, w), (u, v)\},$$
$$\mathcal{R}_{2_\beta} = \{(u, u), (v, v), (w, w), (u, v), (v, u)\}.$$

Thus, although $\mathcal{R}_{2_\alpha}$ is a partial order, $\mathcal{R}_{2_\beta}$ is an equivalence relation.

Suppose that soft relation $\mathcal{R}_3$ is defined as

$$\mathcal{R}_3 = \{(\tilde{e}_1, \tilde{e}_7), (\tilde{e}_2, \tilde{e}_1), (\tilde{e}_2, \tilde{e}_8), (\tilde{e}_5, \tilde{e}_4), (\tilde{e}_7, \tilde{e}_8)\}.$$

So, $\mathcal{R}_3$ is irreflexive, asymmetric, and non-transitive. Then, $\mathcal{R}_{3_\alpha}$ is reflexive, antisymmetric, and transitive and $\mathcal{R}_{3_\beta}$ is non-reflexive, symmetric, and transitive such that

$$\mathcal{R}_{3_\alpha} = \{(u, u), (v, v), (w, w), (u, w)\},$$
$$\mathcal{R}_{3_\beta} = \{(u, u), (v, v), (u, v), (v, u)\}.$$

Remark 3.3. From Example 3.2, the irreflexive, transitive, asymmetric, and antisymmetric soft relation cannot be considered as a parametrized family of irreflexive, transitive, antisymmetric, and asymmetric classical relations, respectively.

Definition 3.3. Let $\tilde{U}$ be an absolute soft set with parameter set P having soft relation $\mathcal{R}$ and s be a property of $\mathcal{R}$. The closure of soft relation $\mathcal{R}$, denoted by $cl\mathcal{R}$ according to the property s is a soft relation on $\tilde{U}$ with property s which contains $\mathcal{R}$ such that $cl\mathcal{R}$ is a subset of each soft relation containing $\mathcal{R}$ with property s.

- The reflexive closure of $\mathcal{R}$ is $cl\mathcal{R}^r = \mathcal{R} \cup \Delta$, where Δ denotes the diagonal or identity soft relation on $\tilde{U}$ such that
$$\Delta = \{(\tilde{x}, \tilde{x}) : \tilde{x} \in SE(\tilde{U})\}.$$
- The symmetric closure of $\mathcal{R}$ is $cl\mathcal{R}^s = \mathcal{R} \cup \mathcal{R}^{-1}$, where $\mathcal{R}^{-1}$ denotes the inverse soft relation of $\mathcal{R}$ on $\tilde{U}$ such that
$$\mathcal{R}^{-1} = \{(\tilde{y}, \tilde{x}) : (\tilde{x}, \tilde{y}) \in \mathcal{R}\}.$$
- The transitive closure of $\mathcal{R}$ is $cl\mathcal{R}^t = \bigcup_{n=1}^{\infty} \mathcal{R}^n$, where $\mathcal{R}^n$ denotes the nth power of soft relation of $\mathcal{R}$ on $\tilde{U}$ such that $\mathcal{R}^1 = \mathcal{R}$ and $\mathcal{R}^n = \mathcal{R}^{n-1} \circ \mathcal{R}$ with
$$\mathcal{R} \circ \mathcal{R} = \{(\tilde{x}, \tilde{z}) : \exists \tilde{y} \in SE(\tilde{U}) \ni (\tilde{x}, \tilde{y}), (\tilde{y}, \tilde{z}) \in \mathcal{R}\}.$$

Remark 3.4. From Proposition 3.3, the reflexive and symmetric closures of the soft relation $\mathcal{R}$ can be considered as a parametrized family of reflexive and symmetric closures of the classical relations $\mathcal{R}_\alpha$ for $\alpha \in P$, respectively. But, from Remark 3.3, the transitive closures of the soft relation $\mathcal{R}$ cannot be considered as a parametrized family of transitive closures of the classical relations $\mathcal{R}_\alpha$ for $\alpha \in P$.

Definition 3.4. Let $\mathcal{R}$ be a soft equivalence relation on $\tilde{U}$ and $\tilde{x} \in SE(\tilde{U})$. The soft equivalence class of $\tilde{x}$ determined by $\mathcal{R}$ is the subset of $\tilde{U}$ defined by
$$[\tilde{x}]_{\mathcal{R}} = SS(\{\tilde{y} \in SE(\tilde{U}) : (\tilde{x}, \tilde{y}) \in \mathcal{R}\}).$$
The family of all soft equivalence classes of $\tilde{U}$ is called as soft quotient set of $\tilde{U}$ reduced by $\mathcal{R}$ and denoted by $\tilde{U}/\mathcal{R}$.

Remark 3.5. From Example 3.2, if the soft equivalence relation $\mathcal{R}_1$ is considered, then the soft equivalence classes are obtained as follows:
$$\begin{aligned}
&[\tilde{e}_1] = [\tilde{e}_5] = SS(\{\tilde{e}_1, \tilde{e}_5\}) = \{(\alpha, \{u, v\}), (\beta, \{u, v\})\},\\
&[\tilde{e}_2] = SS(\{\tilde{e}_2\}) = \{(\alpha, \{u\}), (\beta, \{v\})\},\\
&[\tilde{e}_3] = SS(\{\tilde{e}_3\}) = \{(\alpha, \{u\}), (\beta, \{w\})\},\\
&[\tilde{e}_4] = [\tilde{e}_6] = SS(\{\tilde{e}_4, \tilde{e}_6\}) = \{(\alpha, \{v\}), (\beta, \{u, w\})\},\\
&[\tilde{e}_7] = [\tilde{e}_8] = SS(\{\tilde{e}_7, \tilde{e}_8\}) = \{(\alpha, \{w\}), (\beta, \{u, v\})\},\\
&[\tilde{e}_9] = SS(\{\tilde{e}_9\}) = \{(\alpha, \{w\}), (\beta, \{w\})\}.
\end{aligned}$$
Here, $[\tilde{e}_1]$ or $[\tilde{e}_5]$ is generated by the class of soft elements $\{\tilde{e}_1, \tilde{e}_5\}$ but $\tilde{e}_2$ and $\tilde{e}_4$ are also members of these soft equivalence classes. Hence, $[\tilde{e}_1] \Cap [\tilde{e}_2] \neq \Phi$ and $[\tilde{e}_1] \Cap [\tilde{e}_4] \neq \Phi$. Thus, unlike the classical case, it is encountered that the soft equivalence classes are not disjoint.

Theorem 3.1. *Let $\mathcal{R}$ be a soft equivalence relation on $\tilde{U}$.*

1. *There exists a soft equivalence class of all soft elements of $\tilde{U}$ that is different from the null soft set.*
2. *The ϵ-union of all soft equivalence classes is equal to the soft set $\tilde{U}$, i.e.*
$$\Cup_{\tilde{x} \tilde{\in} \tilde{U}} [\tilde{x}]_{\mathcal{R}} = \tilde{U}.$$
3. *For a pair of soft equivalence classes, they are equal or one is a subset of the other or disjoint if and only if the classical relations $\mathcal{R}_\alpha$ are equivalence relations for all $\alpha \in P$.*

Proof. Since the first and second items can be proven similarly to the classical cases, only the third is proven.

Suppose that for any $\tilde{x}, \tilde{y} \in SE(\tilde{U})$, $[\tilde{x}] \Cap [\tilde{y}] \neq \Phi$. There exists $\tilde{z} \in SE(\tilde{U})$ such that $\tilde{z} \in SE([\tilde{x}])$ and $\tilde{z} \in SE([\tilde{y}])$. If $\tilde{z}$ is not a member of one of the classes of soft elements generating $[\tilde{x}]$ and $[\tilde{y}]$, then $[\tilde{x}] \tilde{\subset} [\tilde{y}]$ or $[\tilde{y}] \tilde{\subset} [\tilde{x}]$. Hence, for all $\alpha \in P$, $[\tilde{x}](\alpha) \subset [\tilde{y}](\alpha)$ or $[\tilde{y}](\alpha) \subset [\tilde{x}](\alpha)$. If $\tilde{z}$ is a member of the classes of soft elements generating $[\tilde{x}]$ and $[\tilde{y}]$, then it is clear that $[\tilde{x}] = [\tilde{y}]$. Hence, for all $\alpha \in P$, $[\tilde{x}](\alpha) = [\tilde{y}](\alpha)$. Suppose that for any $\tilde{x}, \tilde{y} \in SE(\tilde{U})$, $[\tilde{x}] \Cap [\tilde{y}] = \Phi$. Then, for at least one $\alpha \in P$, $[\tilde{x}](\alpha) \cap [\tilde{y}](\alpha) = \emptyset$. In case of $[\tilde{x}](\alpha) \cap [\tilde{y}](\alpha) \neq \emptyset$, $[\tilde{x}](\alpha) = [\tilde{y}](\alpha)$ or $[\tilde{x}](\alpha) \subset [\tilde{y}](\alpha)$ or

$[\tilde{y}](\alpha) \subset [\tilde{x}](\alpha)$. Thus, $[\tilde{x}](\alpha)$ is a partition on U for all $\tilde{x} \in SE(\tilde{U})$ and $\alpha \in P$. Since every partition of U determines an equivalence relation on U, the classical relations $\mathcal{R}_\alpha$ coincide with these relations for all $\alpha \in P$. Thus, the classical relations $\mathcal{R}_\alpha$ are equivalence relations for all $\alpha \in P$.

Conversely, suppose that the classical relations $\mathcal{R}_\alpha$ are equivalence relations for all $\alpha \in P$. From Proposition 3.2, the classical relations $\mathcal{R}_\alpha$ produce the $\mathcal{R}$ that is a soft equivalence relation. Then, the equivalence classes of $\mathcal{R}_\alpha$, which are equal or disjoint, correspond to the sets $[\tilde{x}](\alpha)$ for all $\tilde{x} \in SE(\tilde{U})$ and $\alpha \in P$. Hence, the soft equivalence classes of any $\tilde{x}, \tilde{y} \in SE(\tilde{U})$ are equal or disjoint. □

4. Soft relations applied to decision making

In this section, an application of how the soft relations can be used in decision making is presented and an algorithm for dealing with decision making problems is provided based on the weighted method in [3].

In decision making applications, where the concept of relation is used in the previously mentioned (hybrid) soft sets, the decision is made as a single element among the alternatives by determining the attributes and their weights. However, the decision may consist of certain factors and the decision-makers may want to determine each factor that will form the decision in accordance with their current criteria. While making this decision, the possible relations with other situations also occur as an issue. Here, it is proposed that using the soft elements and soft relations to deal with the situations mentioned in the decision making process.

Table 1. Comparison of the decision according to the decision making applications, where the relations are used in the soft sets

Soft set relation [28, 33]	$H : P \times P' \to P(U \times U')$	The decision is an element of U or U'.
Soft binary relation [32]	$\rho : P \to P(U \times U)$	The decision is an element of U.
Softarison [39]	$\mathcal{S} : U \to S_P(U)$	The decision is an element of U.
Soft relation	$\mathcal{R} \subset SE(\tilde{U}) \times SE(\tilde{U})$	The decision is a soft element of $\tilde{U}$.

The following notions are provided to obtain a mathematical framework for the proposed decision making method.

Definition 4.1. Let $\tilde{U}$ be an absolute soft set with parameter set P, $G \in S(\tilde{U})$ and $\mathcal{R}$ be a soft relation on G.

- The number of soft elements other than itself related to a soft element $\tilde{e}_m$ in $\mathcal{R}$ is called the degree of $\tilde{e}_m$, denoted by $deg(\tilde{e}_m) = d_m$. If there exists $\tilde{e}_m$ such that related to itself i.e. $(\tilde{e}_m, \tilde{e}_m) \in \mathcal{R}$, then two degrees are added to $deg(\tilde{e}_m)$.
- The tabular form of the parametrized classical relations $\mathcal{R}_{\alpha_i}$ reduced from $\mathcal{R}$ is defined by entries p_{ij} for each $\alpha_i \in P$, where $p_j \in U \times U$ such that if $p_j \in \mathcal{R}_{\alpha_i}$ then $p_{ij} = 1$, otherwise $p_{ij} = 0$.
- The weighted value of a pair p_j is defined by

$$s_j = \sum_i \omega_i p_{ij},$$

  where $\omega_i \in (0, 1]$ are imposed on the parameters in P.

Now, a decision making method using the soft elements and soft relations can be created with the algorithm below.

Algorithm Decision making by using the soft elements and the soft relations

Step 1. Construct a feasible soft set G over U with the parameter set P based on the decision-maker,
Step 2. Construct a soft relation $\mathcal{R}$ on G as requested,
Step 3. Find $cl\mathcal{R}^t$ and find $\mathcal{L} = \{l : d_l = \max d_m\}$ in $cl\mathcal{R}^t$,
Step 4. If there is only one $l \in \mathcal{L}$, then $\tilde{e}_l$ may be chosen,
Step 5. Else find the pairs $(\tilde{e}_l, \tilde{e}_{l'}) \in cl\mathcal{R}^t$, where $l, l' \in \mathcal{L}$,
Step 6. Present $cl\mathcal{R}^t_{\alpha_i}$ in tabular form by computing the s_j for all $\alpha_i \in P$ and find k, for which $s_k = \max s_j$,
Step 7. If there is no pairs such that $(\tilde{e}_l, \tilde{e}_{l'})(\alpha_i) = p_k$ for all $l, l' \in \mathcal{L}$ and $\alpha_i \in P$, then any $\tilde{e}_l$ may be chosen for all $l \in \mathcal{L}$,
Step 8. Else $\tilde{e}_l$ or $\tilde{e}_{l'}$ may be chosen as the most related in $(\tilde{e}_l, \tilde{e}_{l'})$ pairs and having the most $(\tilde{e}_l, \tilde{e}_{l'})(\alpha_i) = p_k$.

4.1 Illustrative example

A company wants to create the most optimal system that can be integrated with other existing systems and choose the components required for the system with the specified parameters. The vendors offer various brands to the company for each system component according to the desired system and ensure the integration of systems that can be obtained with preferred brands.

Let $U = \{u, v, w, x, y\}$ be a set of the brands offered by the vendors for the components and $P = \{\alpha_1 =$ Adaptable, $\alpha_2 =$ Customizable, $\alpha_3 =$ Cheap, $\alpha_4 =$ Durable$\}$ be a set of the parameters determined by the company, where each parameter also corresponds to a component required for the system. Assume that there is a vendor and the soft set G corresponding to this vendor describes the brands of components provided by the vendor according to the parameters as follows.

$$G = \{(\alpha_1, \{u, w, y\}), (\alpha_2, \{v, x, y\}), (\alpha_3, \{u, x\}), (\alpha_4, \{y\})\}.$$

Table 2. The soft elements of G

$\tilde{e}_1 = \{(\alpha_1, u), (\alpha_2, v), (\alpha_3, u), (\alpha_4, y)\}$,	$\tilde{e}_{10} = \{(\alpha_1, w), (\alpha_2, x), (\alpha_3, x), (\alpha_4, y)\}$,
$\tilde{e}_2 = \{(\alpha_1, u), (\alpha_2, v), (\alpha_3, x), (\alpha_4, y)\}$,	$\tilde{e}_{11} = \{(\alpha_1, w), (\alpha_2, y), (\alpha_3, u), (\alpha_4, y)\}$,
$\tilde{e}_3 = \{(\alpha_1, u), (\alpha_2, x), (\alpha_3, u), (\alpha_4, y)\}$,	$\tilde{e}_{12} = \{(\alpha_1, w), (\alpha_2, y), (\alpha_3, x), (\alpha_4, y)\}$,
$\tilde{e}_4 = \{(\alpha_1, u), (\alpha_2, x), (\alpha_3, x), (\alpha_4, y)\}$,	$\tilde{e}_{13} = \{(\alpha_1, y), (\alpha_2, v), (\alpha_3, u), (\alpha_4, y)\}$,
$\tilde{e}_5 = \{(\alpha_1, u), (\alpha_2, y), (\alpha_3, u), (\alpha_4, y)\}$,	$\tilde{e}_{14} = \{(\alpha_1, y), (\alpha_2, v), (\alpha_3, x), (\alpha_4, y)\}$,
$\tilde{e}_6 = \{(\alpha_1, u), (\alpha_2, y), (\alpha_3, x), (\alpha_4, y)\}$,	$\tilde{e}_{15} = \{(\alpha_1, y), (\alpha_2, x), (\alpha_3, u), (\alpha_4, y)\}$,
$\tilde{e}_7 = \{(\alpha_1, w), (\alpha_2, v), (\alpha_3, u), (\alpha_4, y)\}$,	$\tilde{e}_{16} = \{(\alpha_1, y), (\alpha_2, x), (\alpha_3, x), (\alpha_4, y)\}$,
$\tilde{e}_8 = \{(\alpha_1, w), (\alpha_2, v), (\alpha_3, x), (\alpha_4, y)\}$,	$\tilde{e}_{17} = \{(\alpha_1, y), (\alpha_2, y), (\alpha_3, u), (\alpha_4, y)\}$,
$\tilde{e}_9 = \{(\alpha_1, w), (\alpha_2, x), (\alpha_3, u), (\alpha_4, y)\}$,	$\tilde{e}_{18} = \{(\alpha_1, y), (\alpha_2, y), (\alpha_3, x), (\alpha_4, y)\}$.

Each soft element of G given in Table 2 is considered to indicate the systems that the vendor provides. Also, each soft relation on this soft set is considered to correspond to the integrated version of the systems created with the components provided by the vendor. Assume that the following soft relation $\mathcal{R}$ on G is the system integrations that the vendor can provide

$$\mathcal{R} = \{(\tilde{e}_3, \tilde{e}_{13}), (\tilde{e}_6, \tilde{e}_3), (\tilde{e}_6, \tilde{e}_{16}), (\tilde{e}_{11}, \tilde{e}_8), (\tilde{e}_{13}, \tilde{e}_{11}), (\tilde{e}_{13}, \tilde{e}_{16})\}.$$

Then, the transitive closure of $\mathcal{R}$, obtained in below, is considered possible system integrations that can be created.

$$\begin{aligned} cl\mathcal{R}^t = \{&(\tilde{e}_3, \tilde{e}_8), (\tilde{e}_3, \tilde{e}_{11}), (\tilde{e}_3, \tilde{e}_{13}), (\tilde{e}_3, \tilde{e}_{16}), (\tilde{e}_6, \tilde{e}_3), (\tilde{e}_6, \tilde{e}_8), (\tilde{e}_6, \tilde{e}_{11}), \\ &(\tilde{e}_6, \tilde{e}_{13}), (\tilde{e}_6, \tilde{e}_{16}), (\tilde{e}_{11}, \tilde{e}_8), (\tilde{e}_{13}, \tilde{e}_8), (\tilde{e}_{13}, \tilde{e}_{11}), (\tilde{e}_{13}, \tilde{e}_{16})\}. \end{aligned}$$

Since the degree of soft elements are found as $d_3 = d_6 = d_{13} = 5$, $d_8 = d_{11} = 4$ and $d_{16} = 3$, there exist three soft elements which are $\tilde{e}_3$, $\tilde{e}_6$ and $\tilde{e}_{13}$ having the maximum degree. Hence, the members of $cl\mathcal{R}^t$, where these soft elements are related, are $(\tilde{e}_3, \tilde{e}_{13})$, $(\tilde{e}_6, \tilde{e}_3)$ and $(\tilde{e}_6, \tilde{e}_{13})$.

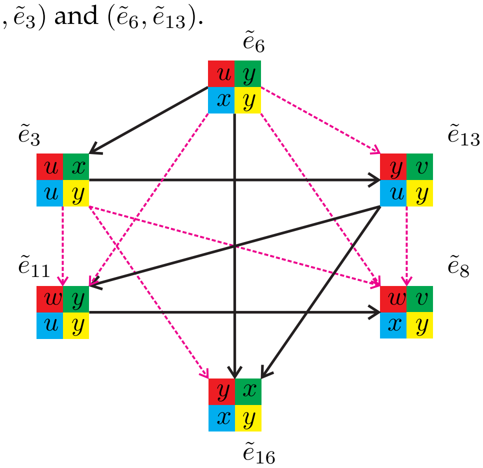

Figure 1. Visualisation of the $cl\mathcal{R}^t$ as a directed graph. Red, green, blue and yellow labels correspond to the parameters α_1, α_2, α_3 and α_4, respectively.

Table 3. The tabular form of parametrized classical relations

p_i	α_1 ($\omega_1 = 0.4$)	α_2 ($\omega_2 = 0.3$)	α_3 ($\omega_3 = 0.8$)	α_4 ($\omega_4 = 0.3$)	s_j
(u, u)	1	0	1	0	1.2
(u, v)	0	0	0	0	0
(u, w)	1	0	0	0	0.4
(u, x)	0	0	1	0	0.8
(u, y)	1	0	0	0	0.4
(v, u)	0	0	0	0	0
(v, v)	0	1	0	0	0.3
(v, w)	0	0	0	0	0
(v, x)	0	1	0	0	0.3
(v, y)	0	1	0	0	0.3
(w, u)	0	0	0	0	0
(w, v)	0	0	0	0	0
(w, w)	1	0	0	0	0.4
(w, x)	0	0	0	0	0
(w, y)	0	0	0	0	0
(x, u)	0	0	1	0	0.8
(x, v)	0	1	0	0	0.3
(x, w)	0	0	0	0	0
(x, x)	0	1	1	0	1.1
(x, y)	0	1	0	0	0.3
(y, u)	0	0	0	0	0
(y, v)	0	1	0	0	0.3
(y, w)	1	0	0	0	0.4
(y, x)	0	1	0	0	0.3
(y, y)	1	1	0	1	1.0

It can be expected that many subsystems, i.e. the components of the systems, will be integrated with each other to increase the functionality of the systems. Assume that the company assigns the weight of the parameters as $\omega_1 = 0.4$, $\omega_2 = 0.3$, $\omega_3 = 0.8$ and $\omega_4 = 0.3$ to assess the relevance between the components of the systems i.e. the pairs p_j.

From the tabular form of parametrized classical relations reduced from $cl\mathcal{R}^t$ in Table 3, it is seen that the company will choose the system $\tilde{e}_3$ according to the parameters and the system integrations since $(\tilde{e}_3, \tilde{e}_{13})(\alpha_3) = (\tilde{e}_6, \tilde{e}_3)(\alpha_1) = (u, u)$ such that $p_k = (u, u)$ and the most related in the pairs $(\tilde{e}_3, \tilde{e}_{13})$ and $(\tilde{e}_6, \tilde{e}_3)$ is $\tilde{e}_3$.

5. Conclusion

In this study, a basis for researches is presented that will use soft relations via soft elements and ε-soft set operations. By using this basis, one can concentrate on the theoretical foundations of the concepts extended to soft set theory. In addition, while someone makes a decision, it should be noted that the decision can consist of certain factors, and it can be desirable to determine these factors according to their attributes. In such cases, which are not considered in any decision making application using the (hybrid) soft sets, it is shown that the concepts of soft element and soft relation are useful. These concepts and the mentioned decision making method can be integrated into the fuzzy sets, vague sets, rough sets, intuitionistic fuzzy sets, and neutrosophic sets and more confirmative solutions can be obtained in decision making problems.

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Affiliations

Kemal Taşköprü
Address: Bilecik Şeyh Edebali University, Department of Mathematics, Faculty of Science, Bilecik 11230, Türkiye.
e-mail: kemal.taskopru@bilecik.edu.tr
ORCID ID: 0000-0002-0760-3782

Elif Karaköse
Address: Bilecik Şeyh Edebali University, Department of Mathematics, Faculty of Science, Bilecik 11230, Türkiye.
e-mail: elf.ozdamar.1.9.9.3@gmail.com
ORCID ID: 0000-0001-6204-0890