

EFFECT OF THE FINITE ELEMENT MODELING TECHNIQUES ON THE DYNAMIC ANALYSIS OF BEAMS

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ABSTRACT

Analysis of the free and forced vibration responses of beams is one of the most critical problems to be examined in the design step of these structural members. The finite-element method which solves boundary value problems can be applied efficiently to vibration problems. In this study, the natural vibration frequency, damped and undamped transient analyses of the pinned-pinned beams are investigated. The well-known finite-element software packages, ANSYS and SAP2000, are used. The 2-D elastic beam which is based on the Euler-Bernoulli Beam theory, 3-D two-node and 3-D three-node beam elements which are based on Timoshenko beam theory, and four-node shell elements are used in ANSYS, and the frame member is utilized in SAP2000. The effect of these elements on the dynamic behaviors of the isotropic beam is discussed. The results are given in tabular and graphical form for the free and forced vibration, respectively.

Keywords: *Free Vibration, Forced Vibration, Beam, Finite-Element Method, Viscoelastic*

SONLU ELEMAN MODELLEME TEKNİKLERİNİN KİRİŞLERİN DİNAMİK ANALİZİNE ETKİSİ

ÖZET

Kirişlerin serbest ve zorlanmış titreşim davranışlarının analizi, bu yapı elemanlarının tasarım aşamasında incelenmesi gereken en kritik problemlerden biridir. Sınır değer problemlerini çözen sonlu elemanlar yöntemi, titreşim problemlerine de etkin bir şekilde uygulanabilir. Bu çalışmada, iki ucundan sabit mesnetli kirişlerin doğal titreşim frekansları ile sönümlü ve sönümsüz zorlanmış titreşim analizleri incelenmiştir. Analizlerde, iyi bilinen sonlu eleman yazılım paket programları ANSYS ve SAP2000 kullanılmıştır. ANSYS'te Euler-Bernoulli Kiriş teorisine dayanan 2 boyutlu elastik kiriş, Timoshenko kiriş teorisine dayanan 3 boyutlu iki düğümlü ile 3 boyutlu üç düğümlü kiriş elemanları ve dört düğümlü kabuk elemanlar kullanılırken SAP2000'de ise çerçeve elemanı kullanılmıştır. Bu elemanların izotropik kirişin dinamik davranışları üzerindeki etkisi tartışılmıştır. Sonuçlar, serbest ve zorlanmış titreşim için sırasıyla tablo ve grafik şeklinde verilmiştir.

Anahtar Kelimeler: *Serbest Titreşim, Zorlanmış Titreşim, Kiriş, Sonlu Elemanlar Yöntemi, Viskoelastik.*

1. Introduction

Beams are widely used in many engineering applications as structural members therefore understanding their vibration behaviors is an important case. The finite element is one of the most common methods in the analysis of vibration problems of beams., Kapur [1], Thomas and Abbas [2]

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Gupta and Rao [3], Dawe [4], Chen and Yang [5], and Ahmed [6] can be cited as pioneering studies concerning finite-element and beam vibration. Hereafter, the vibration of beam problems solved by the finite element method will be discussed. Yang et al. [7] have studied the free in-plane vibration of generally curved beams with variable curvatures, shear deformation, and rotary inertia using the Galerkin finite element method. Natural vibrations of laminated and delaminated beams have been investigated using a mixed finite-element model by Ramtekkar [8] and Ramtekkar et al. [9].

The vibration of axially or transversely Euler–Bernoulli beams is studied by Alshorbagy et al. [10] using the principle of virtual work. Jafari and Eftekhari [11] have presented a coupled finite element-differential quadrature formulation for the forced vibration of beams subjected to moving dynamic loads. Yang et al. [12] studied the dynamic buckling of the thin walled beams with the aid of spline finite element.

Euler–Bernoulli and Timoshenko cross-ply beams are investigated to determine the natural frequencies by Madenci and Ozutok [13] using the Gâteaux differential approach and finite element. Javid and Hemmatnezhad [14] have studied the large-amplitude oscillations of heterogeneous Euler–Bernoulli beams by employing the von Karman type nonlinear strain-displacement relationship. Vo et al. [15] have examined the vibration and buckling behaviors of functionally-graded sandwich beams. In their study, a refined shear deformation theory with the combination of the finite-element method is applied to determine the natural frequencies and corresponding mode shapes, critical buckling loads, and load–frequency curves. Rakowski and Guminiak [16] have presented the finite-element solution to non-linear free vibration characteristics of isotropic Timoshenko beams.

The natural frequencies of the composite coated beams with isotropic core have been investigated by Tekili et al. [17]. Enriched finite-element methods have been developed by Hsu [18] to investigate the vibration behavior of beams using hierarchical approximation and partition of unity method where Timoshenko beam theory is considered. The free vibration and buckling analyses of heterogeneous beams are conducted by Kahya and Turan [19]. In their study, the first-order shear deformation and element with five nodes and ten degrees of freedom are used. Natural frequencies of three-dimensional Timoshenko sandwich beams are determined by Hui et al. [20] where a hierarchical one-dimensional unified formulation is adopted to the finite-element method. Karkon [21] has presented an efficient finite-element method to tackle the vibration, bending, and stability analyses of Timoshenko beams. Eroglu and Tufekci [22] have studied in-plane free vibration of planar curved beams with variable curvature, and cross-section. The free vibration of tapered bi-directional heterogeneous beam is studied by Nguyen and Tran [23] using hierarchical approximation and the first-order shear deformation theory. Pegios and Hatzigeorgiou [24] have investigated the free and forced vibration behaviors of the Euler–Bernoulli beam.

To the best of the authors' knowledge only there are few available works that directly dealt with the type of element for the free vibration of structural elements in the open literature. Using the appropriate element type in the dynamic analysis of beam structures via the FEM is an important factor that affects the accuracy of the results. In this regard, this research focuses on the element type for the vibration problems of beams. Several finite element types which are generally used in the FEM are compared for the free vibration and forced vibration analysis. The material of the beam is assumed to be isotropic and homogenous. In the viscoelastic analysis, the Rayleigh damping matrix is applied and the mass proportional Rayleigh damping coefficient is assumed to be zero. To present this study in better means, it is ordered as follows: Section 2 provides information about the details of the analysis. Section 3 shows the results and discussion and finally, Section 4 is dedicated to the most important conclusion of this study.

2. Material and Methods

A beam having a rectangular cross-section with length L , depth h and width b is examined. The simulated boundary condition is pinned-pinned and the beam geometry is uniform for the sake of

simplicity. Steel and Aluminum are used in the analysis as the beam materials. The illustration of beam with its boundary conditions is given in Figure 1.

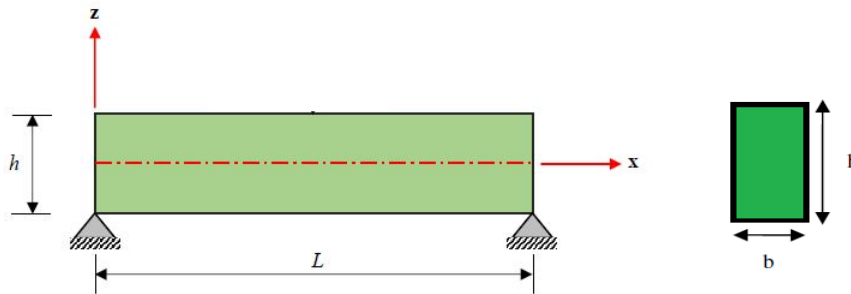


Figure 1. Illustration of homogeneous isotropic beam with pinned ends

The finite element applications on the dynamic analysis of beam are conducted using two different software packages as ANSYS and SAP2000. The element types used in ANSYS are BEAM3, BEAM188, BEAM189, and SHELL181, respectively.

BEAM 3 is a two-node uniaxial beam element in 2-D having tension, compression, and bending capabilities. The element has three degrees of freedom at each node that are translations in the x and y directions and rotation about the z direction. BEAM 3 uses Euler-Bernoulli beam theory where the shear deformations are neglected. BEAM188 uses Timoshenko beam theory where the shear deformation effects are considered. The two-node element in 3-D may be linear, quadratic, or cubic. The degrees of freedom at each node are six or optionally seven having translations in and rotations about the x, y, and z directions. BEAM189 uses the same theory and degree of freedom with BEAM188, however, it is a quadratic three node beam element in 3-D. Both elements are proper for slender to moderately thick beams. SHELL181, based on Mindlin-Reissner shell theory, is a linear four node element. The element has six degrees of freedom at each node as the translations in and rotations about the x, y, and z axes. It is mainly used for the analyzing thin to moderately thick shells. For more detailed information about the restriction and assumptions of these finite element types please see [25].

The SAP2000 offers frame, hinge, cable, shell, plane, asolid, solid, link and tendon elements and the frame element is used in this study. The frame element is based on 3-D beam-column formulation may be prismatic or nonprismatic. The element includes the effects of biaxial bending and shear deformations, torsion and axial deformation and is proper for beams, columns, braces, and trusses in 2-D and 3-D structures [26].

3. Results and Discussions

The material properties used in this paper are: $E_{St} = 210$ GPa, $\rho_{St} = 7850$ kg/m³, $\nu_{St} = 0.3$ and $E_{Al} = 70$ GPa, $\rho_{Al} = 2707$ kg/m³, $\nu_{Al} = 0.3$. The geometric properties are: $h = 0.125$ m, $b = 0.125$ m, and $L = 0.5$ m. Effects of the element type on the free vibration, damped and undamped forced vibration are studied and comprehensive numerical examples are presented hereafter. As a first step, a homogeneous beam with simply-supported ends is considered for the free vibration, and the first 15 natural frequencies of steel and aluminium beam are determined using different elements. The tabulated results are given in Table 1 and 2 and compared with those of Li [27] for the purpose of verifying.

As it can be seen from the Table 2 and 3, each of ANSYS results with all elements considered here is closer to the exact solution of Li [27] than those of the SAP2000. Also, comparing with other ANSYS elements, BEAM188 provides more accurate results. Natural frequency and SAP2000 analyses are concluded here.

Table 1. First 15 natural frequencies (Rad/s) of homogeneous beam with pinned ends (Steel)

Mode	Li [27]	BEAM188	BEAM189	BEAM3	SHELL181	SAP2000
1	6728.89	6717.35	6716.73	6712.33	6714.21	6838.20
2	22279.03	22181.53	22173.99	22136.92	22144.46	22963.49
3	41094.04	40833.16	40806.78	40708.76	40701.85	42184.48
4	60889.98	60424.76	60362.56	60194.80	60135.11	61678.20
5	80895.78	80217.43	80098.05	79865.57	79702.21	64721.27
6	82755.11	81568.31	81574.59	81166.19	81147.34	80592.34
7	90616.10	89478.84	89485.13	89108.13	88523.80	96582.80
8	100855.65	99965.48	99770.70	99487.96	99469.11	98689.99
9	109474.39	108416.36	108422.65	108102.20	106412.03	115887.89
10	120693.16	119619.28	119311.41	119009.81	118299.81	127848.87
11	133540.31	132568.93	132562.64	132317.60	132053.71	132130.93
12	140396.00	139166.27	138720.17	138424.86	138519.10	147365.39
13	159973.97	158625.30	158003.26	157745.65	158380.25	158326.72
14	160222.27	159354.15	159316.45	159178.22	159077.69	161533.53
15	179443.08	178015.21	177179.54	177009.90	176991.05	174575.48

Table 2. First 15 natural frequencies (Rad/s) of homogeneous beam with hinged ends (Al)

Mode	Li [27]	BEAM188	BEAM189	BEAM3	SHELL181	SAP2000
1	6615.66	6604.26	6603.63	6599.23	6601.11	6723.91
2	21904.14	21808.31	21801.40	21764.33	21771.87	22579.70
3	40402.57	40146.41	40120.02	40023.89	40016.98	41479.45
4	59865.40	59408.15	59347.20	59181.95	59122.89	60647.36
5	79534.57	78866.54	78747.16	78520.97	78363.89	63639.56
6	81362.61	80198.58	80198.58	79802.74	81430.08	79245.42
7	89091.33	87970.88	87977.16	87606.45	87034.68	97040.76
8	99158.58	98287.87	98093.09	97816.63	97483.62	97040.78
9	107632.30	106587.96	106600.52	106280.08	104621.32	113951.03
10	118662.29	117602.38	117307.07	117005.48	116308.04	125712.15
11	131293.26	130338.40	130332.11	130093.35	130910.17	129922.64
12	138033.60	136822.64	136389.10	136093.79	136188.04	144902.44
13	157282.14	155954.94	155345.47	155094.15	154440.69	155680.62
14	157526.26	156671.23	156633.53	156495.30	156401.05	158833.84
15	176423.64	175018.13	174195.03	174031.67	174012.82	171656.62

As the elasto-dynamic cases, the maximum vertical displacement and moment are examined. The load type applied to beams for undamped forced vibration is depicted in Figure 2.

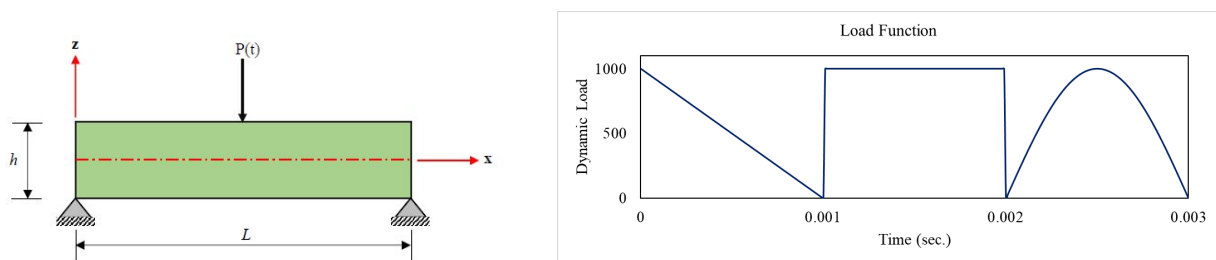


Figure 2. The dynamic load function for the undamped forced vibration

The maximum vertical displacement and moment responses of aluminum and steel beams are illustrated in Figure 3 and 4. It can be observed in the given figures that periods and amplitudes of the vibration are greater when Al is used as the material of the beam.

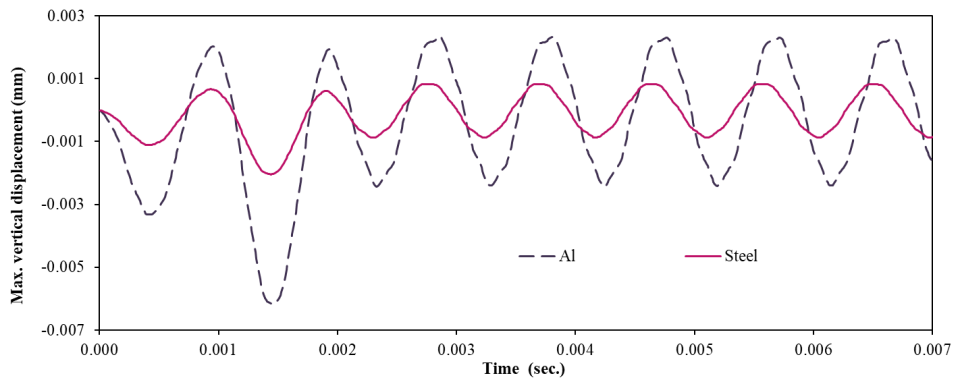


Figure 3. The maximum vertical displacements versus time for Steel and Aluminum beam

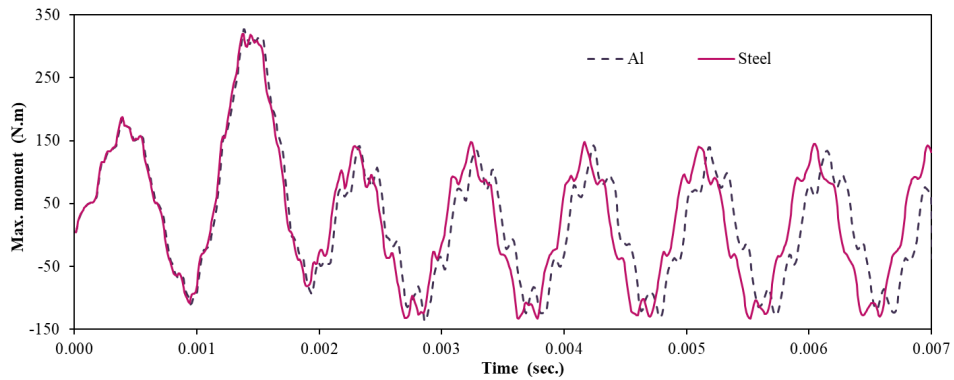
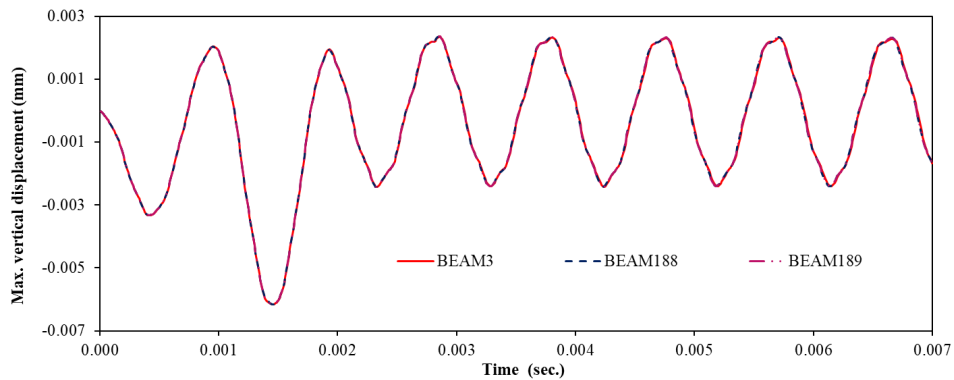
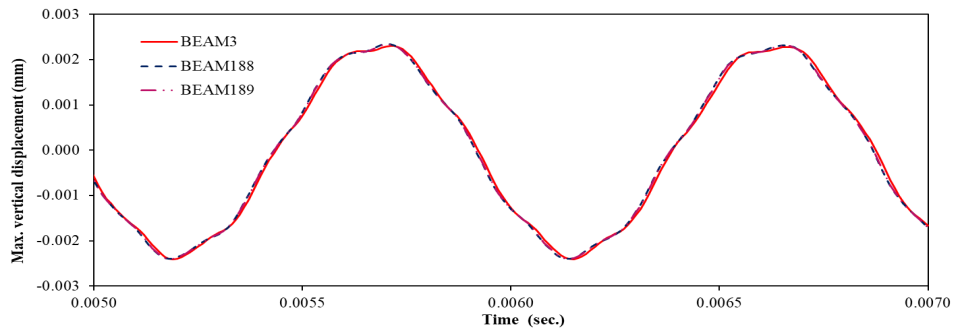


Figure 4. The maximum moments versus time for Steel and Aluminum beam

The influences of element types of ANSYS on the undamped forced vibration of aluminum beam are shown in Figure 5 and 6. The results of BEAM188 and BEAM189 overlap, since both of these elements are based on first-order-shear deformation theory. Note that aluminum is used as the beam material for all the element type analyses hereafter to satisfy a comprehensive reading. The shear deformation is not considered in BEAM3, for this reason the amplituded and periods differ from those of BEAM188 and BEAM189.

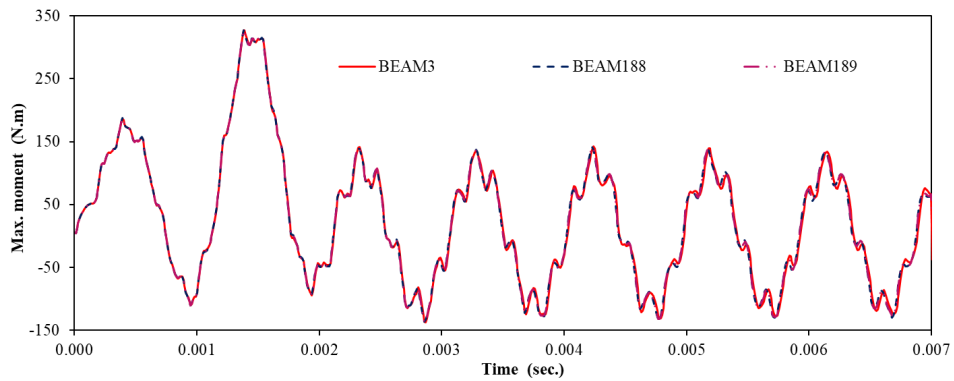


(a)

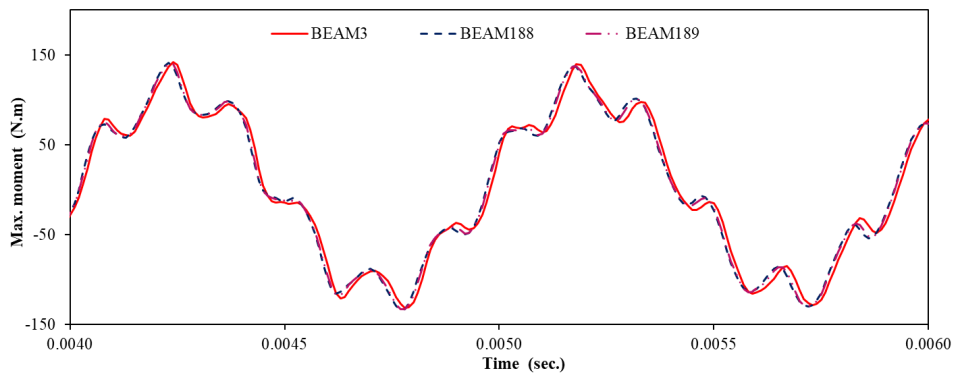


(b)

Figure 5. (a) Maximum vertical displacement versus time (b) its detailed illustration between 0.0050 and 0.0070 seconds for undamped vibration.



(a)



(b)

Figure 6. (a) Maximum moment versus time (b) its detailed illustration between 0.0040 and 0.0060 seconds for undamped vibration.

The step load function shown in Figure 7 is used for the element type investigation of damped forced vibration. Note that ANSYS offers many forms of damping and in this study, the Rayleigh damping matrix is applied and the mass proportional Rayleigh damping coefficient (α) is ignored. The coefficient of damping, g , is calculated from values of ζ (the ratio of actual damping to critical damping), and ω_1 by $\beta=2 \zeta / \omega_1$, and ω_1 is the first fundamental frequency of the structure here.

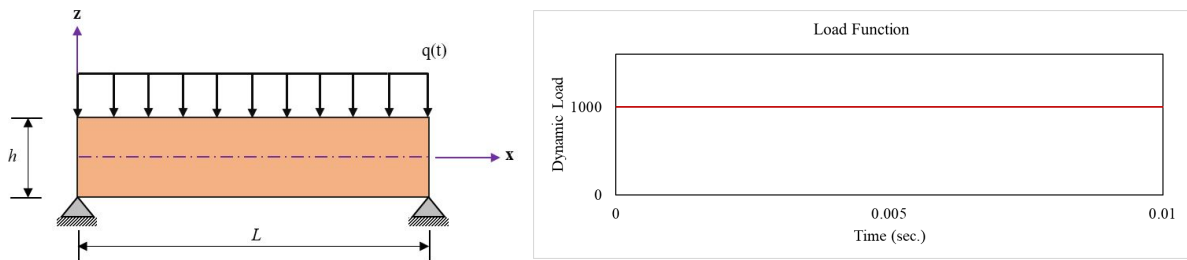
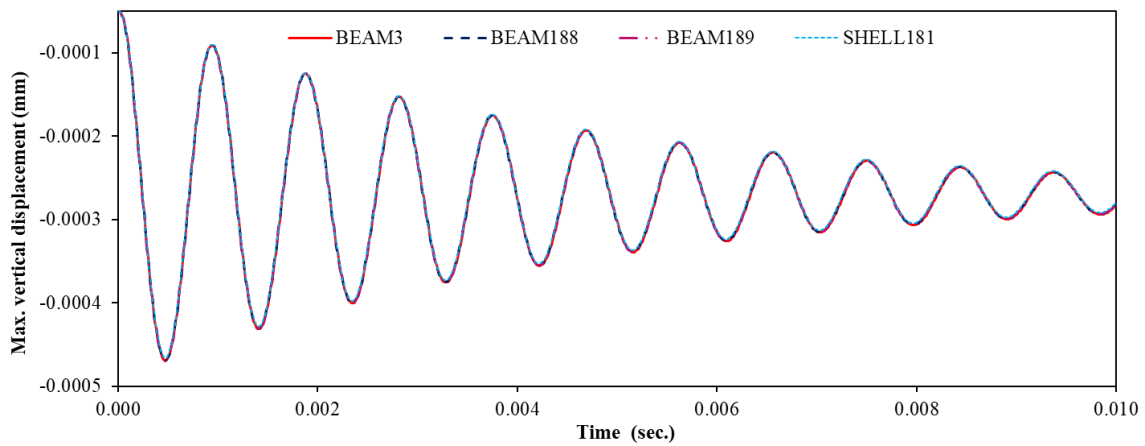
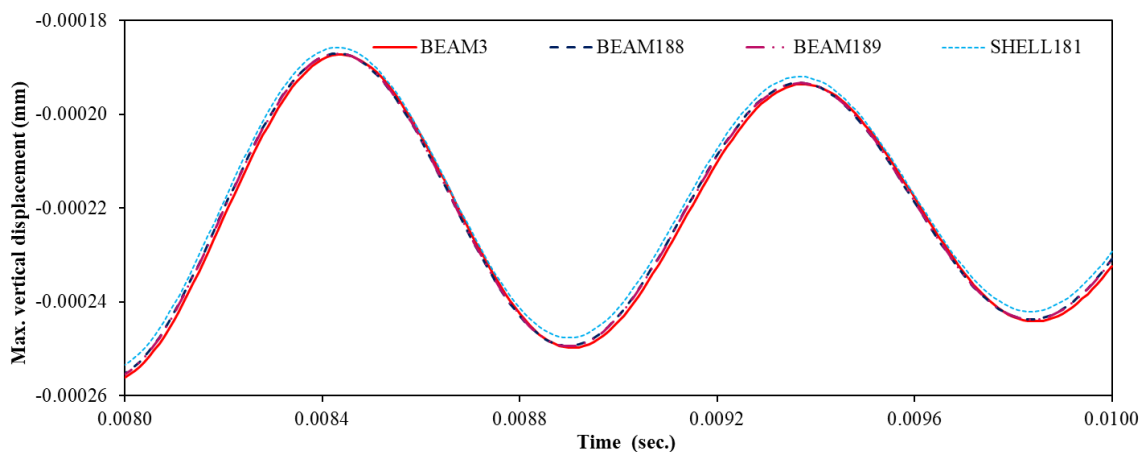


Figure 7. The dynamic load function for the undamped forced vibration

The viscoelastic response of beam are illustrated in Figures 8 and 9 for the maximum vertical displacement and maximum moment, respectively. From the figures, the results obtained by using Beam elements are in exact agreement with each other, however, results obtained by using SHELL181 do not overlap those of BEAM3, BEAM188 and BEAM189.



(a)



(b)

Figure 8. (a) Maximum vertical displacement versus time (b) its detailed illustration between 0.0080 and 0.0100 seconds for damped vibration.

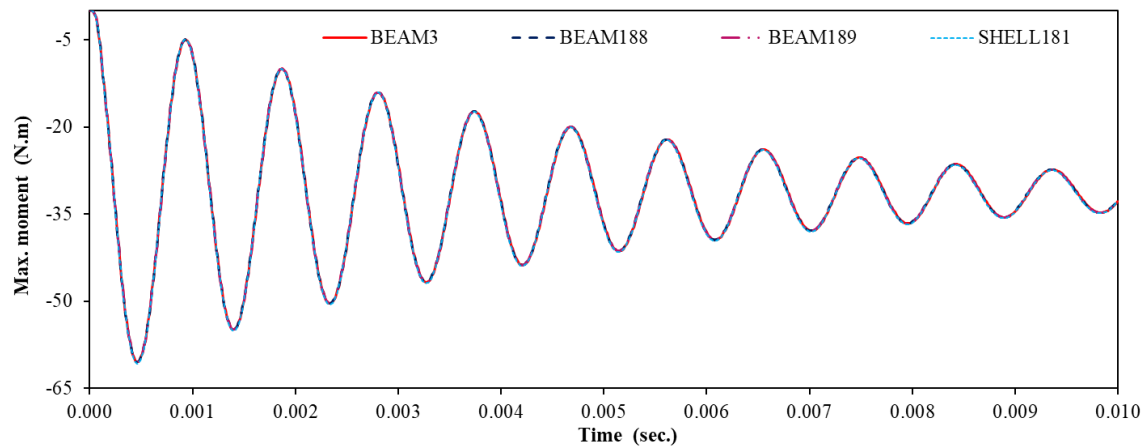


Figure 9. Maximum moment versus time for damped vibration

4. Conclusions

In this study, the finite-element method is used for the vibration problems of homogeneous isotropic beams. BEAM3, BEAM188, BEAM189 and SHELL181 are compared for free vibration and forced vibration analysis of the beams. BEAM 3 is based on the Euler-Bernoulli beam theory or classical beam theory. BEAM188 and BEAM189 are based on the first order shear deformation theory. SHELL181 is based on the Mindlin-Reissner shell theory. The influence of the type of element on the free vibration response of the beams is obvious. Results of ANSYS are closer to the exact solution than those of the SAP2000. It is suggested to use BEAM188 or BEAM 189. In the elastic case results of BEAM188 and BEAM189 overlap, because both of these elements are based on FSDT. In the viscoelastic case, it is concluded that results obtained by using SHELL181 do not overlap those of BEAM3, BEAM188 and BEAM189.

Declaration of Conflicting Interests

The author declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

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