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ARAŞTIRMA MAKALESİ RESEARCH ARTICLE

Modelling and Forecasting of Usd/Try Exchange Rate Using ARMA-GARCH Approach

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Abstract

Accurately predicting the exchange rate prices helps investors to obtain maximum profit from their exchange rate investments as well as to help firms conducting business with exchange rates to manage their trading based on these predictions. Therefore, the prediction of exchange rate prices is crucial for both investors and companies engaged in exchange rates. In this study, ARMA (Autoregressive Moving Average) models combined with various GARCH (Generalized Autoregressive Conditional Heteroscedasticity) models are applied to model exchange rate prices. For this purpose, ARMA-GARCH (M) models are investigated in order to determine the effect of volatility on exchange rate prices as well as ARMA-GARCH models in which the errors are distributed both symmetrically and skewed. It is concluded that the best fitted model which is determined based on the goodness of fit and estimation accuracy performance criteria, is firstly ARMA-NAGARCH model which can model asymmetric and non-linear structures and secondly ARMA-GJRGARCH model. However, since the performance of ARMA-GARCH (M) models is lower than ARMA-GARCH models, the effect of volatility on exchange rate prices is found to be weak.

Keywords: ARIMA, GARCH Models, Exchange Rates, Forecasting, Model Evaluation

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ARMA-GARCH Yaklaşımı İle Usd/Try Döviz Kurunun Modellenmesi ve Tahmini

Özet

Döviz kuru fiyatlarının doğru bir şekilde tahmin edilmesi, yatırımcıların döviz kuru yatırımlarından maksimum kazanç elde etmelerine ve döviz kurları ile iş yapan firmaların bu tahminlere göre ticaretlerini yönetmelerine yardımcı olur. Bu nedenle döviz kuru fiyatlarının tahmini, hem yatırımcılar hem de döviz kurlarıyla iş yapan şirketler için çok önemlidir. Bu çalışmada, döviz kuru fiyatlarını modellemek için çeşitli GARCH (Genelleştirilmiş Otoregresif Koşullu Değişken Varyans) modelleri ile birleştirilmiş ARMA (Otoregresif Hareketli Ortalama) modelleri uygulanmıştır. Bu amaçla, volatilitenin döviz kuru fiyatları üzerindeki etkisini belirlemek için hataların hem simetrik hem de çarpık olarak dağıldığı ARMA-GARCH (M) modelleri ve ARMA-GARCH modelleri incelenmiştir. Uyum iyiliği ve tahmin doğruluğu performans kriterlerine göre belirlenen en uygun modelin, ilk olarak asimetrik ve doğrusal olmayan yapıları modelleyebilen ARMA-NAGARCH modeli ve ikinci olarak ise ARMA-GJRGARCH modeli olduğu sonucuna varılmıştır. Bununla birlikte ARMA-GARCH (M) modellerinin performansı, ARMA-GARCH modellerine göre daha düşük olması nedeniyle volatilitenin döviz kuru fiyatlarına etkisi zayıf bulunmuştur.

Anahtar Sözcükler: ARIMA, GARCH Modelleri, Döviz Kuru, Öngörü, Model Değerlendirme

1. Introduction

For the investors in the financial area to make maximum profit from their investments and to guide the trade of the firms doing business with the exchange rate, the exchange rate forecast is very important. In terms of that Investors in financial area make maximum profit from their investments and firms doing business with exchange rate govern their trade, the exchange rate forecast is fundamental. Accordingly, it is a critical process to determine the method which can accurately model exchange rate prices. For this purpose, many methods are used in the literature. For this purpose, a large number of methods are used in the literature. One of these methods is artificial neural networks based on artificial intelligence techniques (de Oliveira et al., 2013). Although this method gives very effective results in modelling nonlinear relationships, it does not possess an explicit functional form and thus it is difficult to conduct further analysis by means of this method. Other time series prediction methods commonly used are AR (Autoregressive), MA (Moving Average), ARMA (Autoregressive Moving Average) and ARIMA (Autoregressive Integrated Moving Average) introduced by Box et al. (1994). These methods are modelled with its lagged values, its lagged errors or their combinations. These methods are frequently utilized in time series modelling due to their functional form and high applicability through various computer programs. However, although these models can accurately model exchange rate prices, the issue of heteroscedasticity which is one of time series assumptions and occurs in non-stationary time series cannot be solved by this method and thus the predictions obtained from this model can be misleading. Heteroscedasticity problem encountered in the time series can be solved via ARCH (Autoregressive Conditional Heteroscedasticity) introduced by Engle (1982) and GARCH (Generalized Autoregressive Conditional Heteroscedasticity) models developed by Bollerslev (1986). ARCH models are an effective tool in coping with the heteroscedasticity of time series. The disadvantage of this method is that it requires a high number of parameter estimates to model volatility, causing heavy computational load and loss of time. GARCH models are developed to handle the issue of a large number of parameter estimates in ARCH models and so GARCH models need less parameter estimations compared to ARCH models. On the other hand, ARCH models are modelled with only its lagged errors, while GARCH models are explained based on both lagged errors and lagged conditional variance and indicate stability in conditional variance. Agnolucci (2009) WTI compares the volatility forecast performance of these two models used by estimating the volatility of crude oil futures with GARCH type models and implied models. This study determines that there exist asymmetric effects in volatility of oil prices futures. Lim and Sek (2013) employ GARCH models to model volatilities in stock markets in Malaysia. In the study, pre-crisis and post-crisis periods are considered separately and asymmetric GARCH models are more appropriate for the post-crisis period while symmetrical GARCH models performed better for pre-crisis period. It is also determined that exchange rates and oil prices have a significant impact on the volatility of the Malaysian stock markets. Studies via GARCH models are performed by Teresiene (2009), Efimova and Serletis (2014), Abdalla and Winker (2012) and Liu and Hung (2010). A hybrid method which is called ARMA-GARCH has been developed to model the conditional mean and conditional volatility of financial assets. This method overcomes the issue of heteroscedasticity in residuals obtained from the ARIMA model and thus produces more accurate forecasts. Mohammadi and Su (2010) use ARIMA-GARCH model to model the conditional mean and volatility of weekly crude oil spot prices in eleven international markets. It is concluded that APARCH model gives better results compared to other models and conditional standard deviation is better than conditional variance in catching the volatility of oil returns. Kang and Yoon (2013) investigate the means and volatilities of returns of the three oil types futures through various ARIMA-GARCH models. As a result of the out of sample performance analysis, it is concluded that a single model is not better than the others but that the ARFIMA-GARCH model is better able to capture the long-term memory of the returns and volatilities. Yaziz et al. (2013) employ hybrid ARIMA-GARCH model to forecast gold prices. This study demonstrates that this model is very effective in modeling nonlinear structures in the series and increases the performance of forecasting. Dritsaki (2017) investigates the returns of British pounds and the US dollar exchange rate using three types of ARMA-GARCH models and finds out that ARIMA-EGARCH model that is distributed Student t is the best in modeling the return and volatility of the related exchange rate. Gupta and Kashap (2016) models and forecasts the INR/GBP exchange rate via the hybrid ANN-GARCH approach. Atabani Adi (2019) models RMB/USD exchange rate return volatility using GARCH-type approaches. Epaphra (2016) employs the GARCH and EGARCH methods to model TZS/USD exchange rate volatility. Caporale and Zekokh (2019) investigate the volatility of cryptocurrencies through Markov-Switching GARCH models. Abdullah et al. (2017) models and foracasts the USD/BDT exchange rate using GARCH models where errors are normal and Student t distributed. They also compare the performances of the models used. Pahlavani and Roshan (2015) use ARIMA and hybrid ARIMA-GARCH models to forecast the IRR/USD exchange rate. It then compares the performances of the models used.

This paper aims to investigate the USD/TRY exchange rates in a wide perspective by using ARIMA-GARCH models. It tries to determine the most fitted model by assuming different distributions of errors and also analyzes the effect of volatility on USD / TRY exchange rate prices. For this purpose, the model that forecasts the best exchange rate is estimated via different ARMA-GARCH models. The best fitted model selection is performed using both goodness of fits based on all sample and the measures of out of sample performance and thus more accurate forecast model is chosen in this way. On the other hand, the exchange rate prices are better modelled by approaching with both asymmetric and nonlinear models and the model selection is made with more accurate prediction results.

The rest of the paper is designed as follows. Section 2 describes the ARMA-GARCH methodology and its different types. Section 3 presents model adequacy and the performance measures of forecast accuracy used in the study. Section 4 gives the experimental results obtained from the study. Finally, Section 5 includes the conclusion which summarizes the contribution and important results of the paper.

2. ARIMA-GARCH Models

ARIMA (Autoregressive Combined Moving Average) models Introduced by Box and Jenkins (1976) are time series forecasting methods commonly used in many areas, especially statistics. This method produces forecasts based on the autocorrelation structure in the time series and is defined as follows:

$$y_t = \mu + \sum_{i=1}^{p} \varphi_i y_{t-i} + \sum_{j=1}^{q} \theta_j \varepsilon_{t-j} + \varepsilon_t \qquad (1)$$

Here, μ, φ_i and θ_j are constant term, ith autoregressive coefficient and jth moving average coefficient respectively and ε_t is error term at t time. Moreover, p and q represent autoregressive and moving average term order in turn. Modeling the time series via ARIMA method is performed in four stages. First, the model structure is determined by the autocorrelation (ACF) and partial autocorrelation (PACF) functions which are informative on the characteristic structure of the time series. Second, model parameters can be estimated by the maximum likelihood method. In the third stage, it is checked whether residuals obtained from the estimated model is a white noise process. Finally, future values are forecasted by means of estimated model. It is noted that longer forecast period is, less accurate the forecast is. In traditional ARIMA models error term ε_t should be zero mean and homoscedasticity as well as uncorrelated. When the time series exhibit the conditional heteroscedasticity, the application of ARCH models proposed by Engle (1982) provides more accurate forecast values. ARCH models deal with the serial correlation in time-varying volatility. However, the model does not produce effective results when it is necessary to estimate a large number of parameters in ARCH models. Bollerslev (1986) developed GARCH (Generalized Autoregressive Conditional Variable Variance) models which are generalizations of ARCH models. In this model, conditional variance takes into account not only the lagged errors but also the value of the lagged conditional variance. The general structure of the GARCH model is defined as follows:

$$\varepsilon_t = \sigma_t z_t \tag{2}$$

$$\sigma_t = \alpha + \sum_{i=1}^{m} \beta_i \sigma_{t-i}^2 + \sum_{j=1}^{n} \delta_j \varepsilon_{t-j}^2 \tag{3}$$

Where, α, β_i and δ_j are described as a constant term, ith GARCH parameter and jth ARCH parameter, respectively and these parameters are non-negative values. In the GARCH model given in Eq. (3), the effect of the ε_t shocks is symmetric. That is, positive shocks with the same magnitude negative shocks have the same effect on volatility. The ARIMA-GARCH approach has been developed to handle the serial correlated residuals encountered in ARIMA models. This model allows the modeling of both the conditional means and the volatility of the series. In addition, this approach provides more accurate forecast values and higher forecast performance compared to ARIMA models. There are many types of GARCH models in the literature. Investigation extensively for the most fitted model among these models and determination of the best suitable GARCH model are crucial process in the time series forecasting. Some of the GARCH models can model symmetric time-varying volatility while others model asymmetric volatility and leverage. By investigating comprehensively GARCH models, the features of the time series can be determined more accurately.

Alternative GARCH models are Threshold GARCH (TGARCH) (Zakoian, 1994), Glosten-Jagannathan-Runkle GARCH (GJRGARCH) (Glosten et. Al., 1993), Exponential GARCH (EGARCH) (Nelson, 1991) and Nonlinear Asymmetric GARCH (NAGARCH) (Engle and Ng, 1993). These models are able to capture asymmetric structures and leverage that the standard GARCH model cannot capture. For instance, consider the GJRGARCH model developed by Glosten et al. (1993):

$$\sigma_t = \alpha + \sum_{i=1}^{m} \beta_i \sigma_{t-i}^2 + \sum_{j=1}^{n} \delta_j \varepsilon_{t-j}^2 + \sum_{j=1}^{n} \gamma_j \varepsilon_{t-j}^2 I_{t-j} \tag{4}$$

Here, I_{t-j} is a indicator function and is described as follows:

$$I_{t-j} = \begin{cases} 0, & \varepsilon_{t-j} \geq 0 \\ 1, & \varepsilon_{t-j} < 0 \end{cases} \tag{5}$$

GJRGARCH model can also model asymmetric effects. If $\gamma_j > 0$ there is a leverage effect. That is, it means that the effect of negative shocks $(\delta_j + \gamma_j)$ in modeling volatility it is greater than the effect of positive shocks (δ_j). The NAGARCH model, which can model asymmetric and nonlinear effects, can be defined as follows:

$$\sigma_t = \alpha + \sum_{i=1}^{m} \beta_i \sigma_{t-i}^2 + \sum_{j=1}^{n} \delta_j (\varepsilon_{t-j} - \gamma_j \sigma_{t-j})^2 \tag{6}$$

In this paper, the effect of conditional volatility of exchange rate on the average of exchange rate with GARCH-in-mean (GARCH-M) models is also investigated.

In order to determine the effect of conditional volatility on GARCH (M) models, the term of conditional heteroscedasticity is added to mean equation. For example, the mean equation with the GJRGARCH model defined in Eq. (4) can be written as follows:

$$y_t = \mu + \sum_{i=1}^{p} \varphi_i y_{t-i} + \sum_{j=1}^{q} \theta_j \varepsilon_{t-j} + \eta \sigma_t + \varepsilon_t \tag{7}$$

Here, η is a parameter that measures the effect of volatility on the mean of the time series. η parameter which is statistically significant η means that volatility has a substantial effect in explaining the behavior of exchange rate.

3. Model Sufficiency and Accuracy of Prediction

3.1. Evaluation of model sufficiency

Accuracy of model sufficiency is evaluated in two part. In the first part, goodness of fit measures calculated based on all sample are investigated. This measures are AIC (Akaike, 1973) and BIC (Schwarz, 1978) and these are evaluated according to log likelihood. The model that has the smallest AIC and BIC values is determined to be best fitted model to data. In the second stage, assumptions related to residuals from estimated model are tested. To this end, both autocorrelation (ACF) and partial autocorrelation (PACF) functions are investigated and autocorrelation tests such as Ljung-Box are applied to the residuals.

3.2. Evaluation of accuracy of prediction

Evaluating the performance of prediction accuracy also contribute to select the best model for the data along with goodness of fit measures. Prediction accuracies of various model used are analyzed via these measures. In this paper, four performance measures are utilized and these are mean absolute error (MAE), root mean square error (RMSE), mean absolute percentage error (MAPE) and Theil's inequality coefficient (TIC). Prediction performance measures are calculated as follows:

$$\text{Mean absolute error (MAE)} = \frac{\sum_{t=1}^{T} |e_t|}{T}$$

$$\text{Root mean square error (RMSE)} = \sqrt{\frac{e_t^2}{T}}$$

$$\text{Mean absolute percentage error (MAPE)} = \frac{\sum_{t=1}^{T} \frac{e_t}{y_t}}{T}$$

$$\text{Theil's inequality coefficient (TIC)} = \frac{\sqrt{\frac{e_t^2}{T}}}{\sqrt{\frac{1}{T}\sum_{t=1}^{T} y_t^2} + \sqrt{\frac{1}{T}\sum_{t=1}^{T} \hat{y}_t^2}}$$

Where, $e_t = y_t - \hat{y}_t$ and T represents length of test data. mean absolute error (MAE) and root mean square error (RMSE) are sensitive to the scale of variables. That is, they should be used to measure performances of the same variable in different models. On the other hand, mean absolute percentage error and Theil's inequality coefficient don't depend on variable's scale and thus they are commonly utilized for both the same variable and different variables. The models with smallest prediction errors are indicated as the best fitted model.

4. Emprical Findings

4.1. Data description

In this study, daily USD/TRY exchange rate is used. The data set consists of 746 observations ranging from 01/01/2016 to 31/12/2018 and is demonstrated in Fig. 1. To evaluate the accuracy of the estimation, data set is divided into two parts: training data and test data. The training data comprises of 597 observations and this includes approximately 80 percent of all data. The remaining is considered as test data and consists of 149 data.

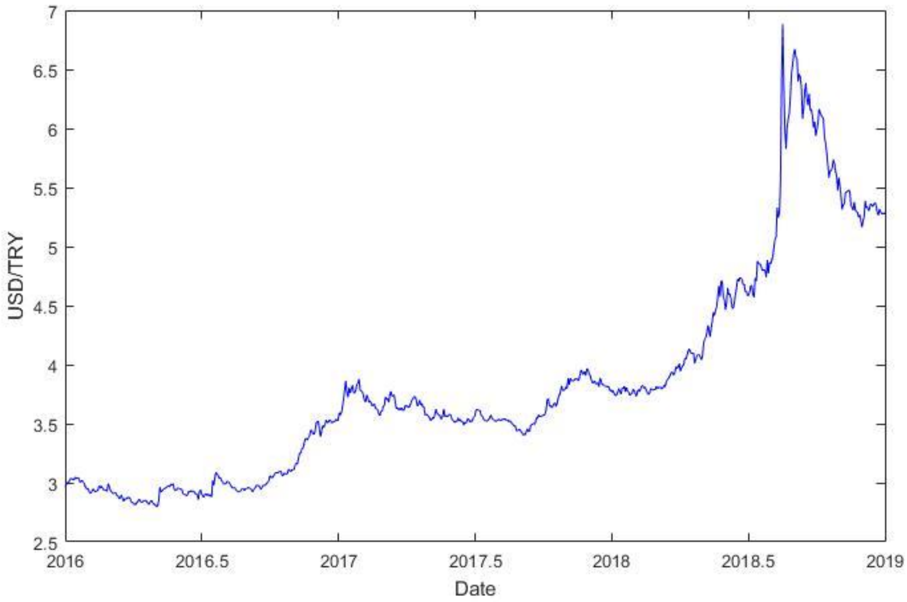

Figure 1: Observed Values of USD/TRY

When viewed Fig. 1, USD / TRY exchange rate has generally an upward trend and this suggests that the series have nonstationary features. However, it exhibits more volatile progress since 2017 while there is an almost constant price movement throughout 2016. The exchange rate reaches the peak point in the second half of 2018 and this period is considered as breaking period for Turkey's economy. After this period, the USD/TRY exchange rate begins quickly to decline and the Turkey's economy gradually moves to normalization period.

Table 1: Descriptive statistics for USD/TRY exchange rate

Statistics	Value
Mean	3.8334
Median	3.6253
Std. Dev.	0.9071
Min.	2.7955
Max.	6.8834
Skewness	1.2580
Kurtosis	0.9362
Observations	746

Descriptive statistics belonging to the USD/TRY exchange rate are presented in Table 1. According to this table, USD/TRY exchange rates show more right skewed and excess kurtosis distribution than normal distribution. This indicates that the exchange rate is not normally distributed. Therefore, Student t and skewed Student t distributions are investigated in time series forecasting models used in the study. A research in broad perspective is carried out for ARMA-GARCH models that can model the exchange rate. While the standard GARCH model can model the symmetrical effects in volatility, alternative GARCH models can model asymmetric effects over time in volatility. For this reason, including alternative GARCH models to the study leads to obtain more accurate results.

Table 2: Evaluation of goodness of fit for ARMA-GARCH models

Model	AIC	BIC
EGARCH-ST	-6.6068	-6.5387
EGARCH-ST-M	-6.6031	-6.5288
EGARCH-T	-6.6128	-6.5509
EGARCH-T-M	-6.5953	-6.5271
GJR-ST	-6.5997	-6.5316
GJR-ST-M	-6.5991	-6.531
GJR-T	-6.6012	-6.5293
GJR-T-M	-6.5916	-6.5334
NAGARCH-ST	-6.6224	-6.5543
NAGARCH-ST-M	-6.6198	-6.5454
NAGARCH-T	-6.609	-6.5471
NAGARCH-T-M	-6.6018	-6.5336
NGARCH-ST	-6.6108	-6.5427
NGARCH-T	-6.6066	-6.5446
SGARCH-ST	-6.5991	-6.5371
SGARCH-ST-M	-6.5964	-6.5283
SGARCH-T	-6.5916	-6.5358
SGARCH-T-M	-6.5913	-6.5293
TGARCH-ST	-6.6153	-6.5471
TGARCH-ST-M	-6.6111	-6.5368
TGARCH-T	-6.6046	-6.5427
TGARCH-T-M	-6.602	-6.5338

However, volatility can also have an impact on USD/TRY exchange rates. ARMA-GARCH (M) models developed for this purpose are analysed in the study and this paper is assessed in very broad manner. Finally, since skewed distributions can model some financial variables better than non-skew distributions, the distribution of errors is assumed to be Student t and skewed Student t distribution. In order to select the model that can best model the exchange rate, goodness of fit measures based on whole data set and performance measures of forecasting accuracy calculated based on the test data are utilized. The model with the smallest value according to the AIC and BIC criteria calculated by regarding the number of parameters used in the model is selected as the best model. Goodness of fit measures for the models is presented in Table 2. When the best fitted model according to AIC values is investigated, the ARMA-NAGARCH model, in which the errors are skewed Student t distributed, is determined as the best model and ARMA NAGARCH (M) model, in which the errors are skewed Student t distributed, is an alternative model to ARMA-NAGARCH model. It is found that GARCH model in which the errors are Student t distributed was the worst model. According to the BIC information criterion, the best fitted model is ARMA-NAGARCH model in which errors are skewed Student t distributed and this model is followed by ARMA-EGARCH model in which errors are Student t distributed. It is selected that the poor model is ARMA-EGARCH (M) where errors are Student t distributed. On the other hand, in this paper the mean absolute error, the square root of the error squared, mean absolute percentage error and Theil's inequality coefficient are used as measures of forecasting performance for the best fitted model selection. The results obtained according to the performance criteria of forecasting accuracy are demonstrated in Table 3.

Table 3: Evaluation of model forecasting performance for ARMA-GARCH models

Model	MAE	RMSE	MAPE	TIC
EGARCH-ST	0.07651	0.129	1.36327	0.02373
EGARCH-ST-M	0.07525	0.12821	1.33816	0.02358
EGARCH-T	0.07727	0.13003	1.37276	0.02391
EGARCH-T-M	0.07715	0.13	1.37083	0.02391
GJR-ST	0.0751	0.12776	1.33461	0.0235
GJR-ST-M	0.07605	0.12869	1.35364	0.02367
GJR-T	0.07637	0.12748	1.35813	0.02345
GJR-T-M	0.07528	0.12811	1.33873	0.02357
NAGARCH-ST	0.07623	0.12876	1.35482	0.02368
NAGARCH-ST-M	0.08236	0.13483	1.46122	0.02478
NAGARCH-T	0.07645	0.12879	1.35876	0.02368
NAGARCH-T-M	0.07687	0.12983	1.36631	0.02388
NGARCH-ST	0.07741	0.13065	1.37794	0.02403
NGARCH-T	0.07638	0.12971	1.35694	0.02385
SGARCH-ST	0.07634	0.12941	1.35871	0.0238
SGARCH-ST-M	0.0754	0.12863	1.34151	0.02366
SGARCH-T	0.07521	0.1283	1.33572	0.0236
SGARCH-T-M	0.07575	0.12883	1.34736	0.0237
TGARCH-ST	0.07589	0.12909	1.34928	0.02374
TGARCH-ST-M	0.07502	0.12784	1.3356	0.02351
TGARCH-T	0.07542	0.12813	1.33927	0.02357
TGARCH-T-M	0.07525	0.12781	1.33802	0.02351

The ARMA-TGARCH (M) model is the most appropriate model in which the errors are skewed Student t distributed in terms of the mean absolute error value according to the value of mean absolute error value from performance measure of forecasting accuracy used and then the ARMA-TGARCH model in which the errors are skewed Student t distributed follows this model. The worst model is ARMA-NAGARCH (M) in which errors are skewed Student t distributed. According to other performance evaluation criteria, the most compatible models are RMSE, MAPE and TIC ARMA-GJRGARCH model where errors are Student's distributed, ARMA-GJRGARCH model where errors are skewed Student, and ARMA-GJRGARCH models where errors are distributed. When similar evaluations are performed according to other performance evaluation criteria such as RMSE, MAPE and TIC, the best fitted models are ARMA-GJRGARCH with Student t errors, and ARMA-GJRGARCH with skewed Student t errors, and ARMA-GJRGARCH with Student t errors, respectively. The model that possesses the worst performance is ARMA-NAGARCH (M) in which errors are skewed Student t distributed in terms of RMSE, MAPE and TIC measures. There is no single result for the best model when both the goodness of model fit criteria and the performance criteria of forecasting accuracy are taken into account. However, ARMA-NAGARCH model in which errors are skewed Student t distributed can be regarded as the most fitted model according to AIC and BIC models. GJRGARCH model with Student t errors can be taken into account as alternative to the best fitted model in terms of performance criteria of estimation accuracy and the most compatible model is Student's GJRGARCH model. The values of estimates obtained from two candidate models are shown in Fig. 2 and Fig. 3.

Table 4: Parameter estimates of mean equations for the best fitted models

	μ	φ_1	φ_1	θ_1	θ_2
NAGARCH	0.0007	-0.7677	-0.9874	0.7692	1.0040
	(0.0001)	(0.0059)	(0.0020)	(0.0013)	(0.0003)
GJRGARCH	0.0001	-0.4387	0.4637	0.3943	-0.4797
	(0.0002)	(0.3213)	(0.2976)	(0.3162)	(0.2857)

In the stage of the best fitted model selection, it is found out that conventional ARMA-GARCH model gives poor results compared to alternative GARCH models in modelling the USD/TRY exchange rate. This may be due to the fact that the GARCH model can only capture symmetrical effects. However, considering the most suitable models, it is discovered that USD/TRY exchange rate has the leverage effect and an asymmetric effect that is thought as common feature of financial assets. In addition, it is found that non-linear models have better results in modelling exchange rates. However, it is determined that the proposed ARMA-GARCH models increased the forecasting accuracy performance by approximately 5 % to 8 %.

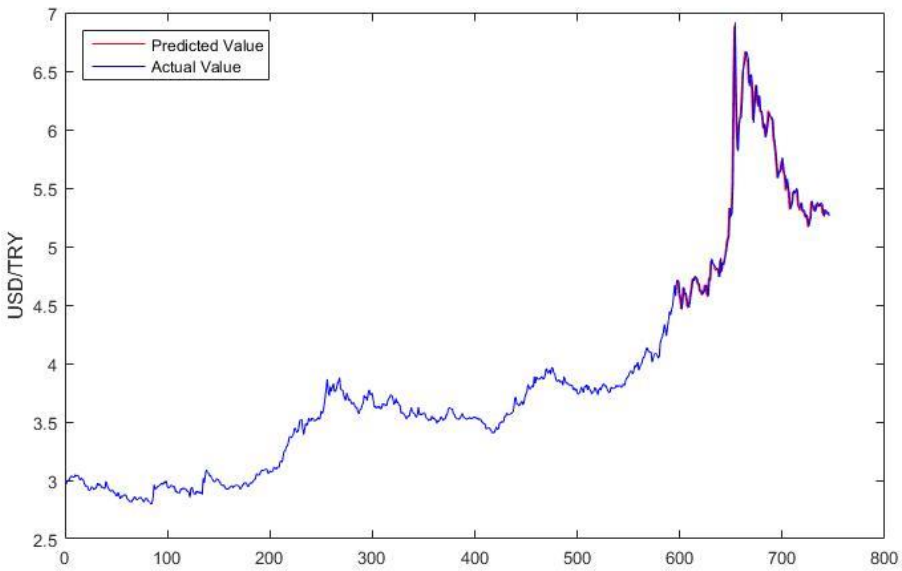

Figure 2: Actual and predicted values for ARMA-NAGARCH

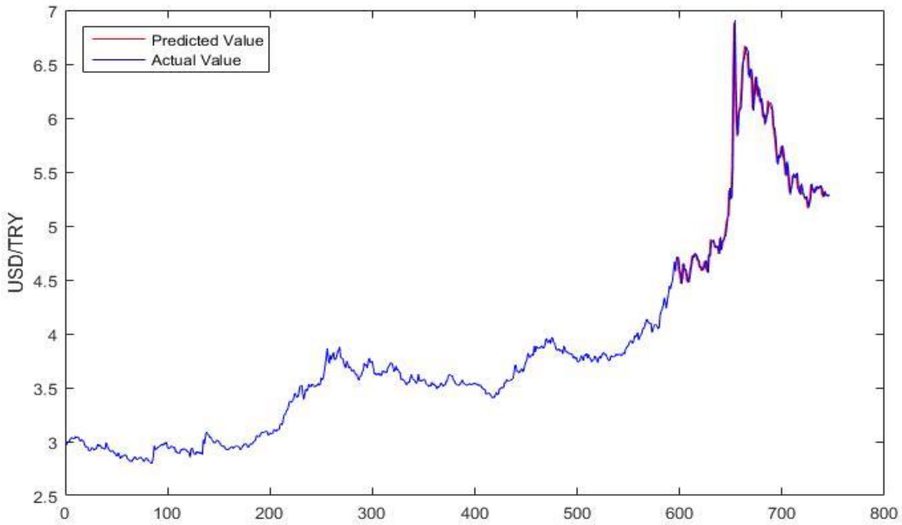

Figure 3: Actual and predicted values for ARMA-GJRGARCH

Tablo 5: Parameter estimates of variance equations for the best fitted models

	α	δ	β	η	ν	λ
NAGARCH	0.0000	0.0076	0.5691	-7.3366	4.9695	1.2021
	(0.0000)	(0.0000)	(0.0024)	(0.0252)	(0.6547)	(0.0589)
GJRGARCH	0.0000	0.1547	0.8427	-0.0688	4.8944	
	(0.0000)	(0.0405)	(0.0356)	(0.0536)	(0.9304)	

* Values in parenthesis show the standard error of the estimated parameters

Parameter estimates of the best fitted ARMA-GARCH models are presented in Table 4 and Table 5. All parameter results are tested 5% significance level and it is found that almost all parameters are statistically significant. This indicates that the price of the exchange rate has a changing volatility changing over time. However, NAGARCH model, which is determined as the best fitted model, can model asymmetric and nonlinear structure in volatility and it implies that volatility of the exchange rate has a time varying asymmetric and nonlinear effect. The fact that skewed Student t distribution for errors is statistically significant indicates that skewed distributions are better than non-skewed distributions in modelling the exchange rates. Both ARMA-NAGARCH and ARMA-GJRGARCH model have persistence in volatility over time. In both models, the effects of lagged conditional variance are rather higher than the effect of lagged errors.

Table 6: Results of autocorrelation test for residuals from best fitted ARMA-GARCH models

	$Q(10)$ stat	$Q^2(10)$ stat	$Q(20)$ stat	$Q^2(20)$ stat
ARMA-NAGARCH	6.2789	14.193	16.034	29.346
	(0.7913)	(0.1644)	(0.7145)	(0.08116)
ARMA-GJRGARCH	4.5341	7.9025	14.479	20.244
	(0.9201)	(0.6384)	(0.8054)	(0.4427)

It is investigated whether residuals from the best fitted ARMA-GARCH model exhibit autocorrelation by means of Ljung-Box test and results are presented in Table 6. It is found that residuals from the best appropriate models are identically and independently distributed such as white noise. This means that estimated models are suitable for modelling USD/TRY exchange rate. ACF and PACF of these residuals are demonstrated between Fig. 4 and Fig 5. and these also indicates that the residuals are white noise. On the whole, it is found out that the exchange rate has an asymmetric and non-linear effect and alternative ARMA-GARCH models such as NAGARCH and GJRGARCH are effective tool in modelling these feature.

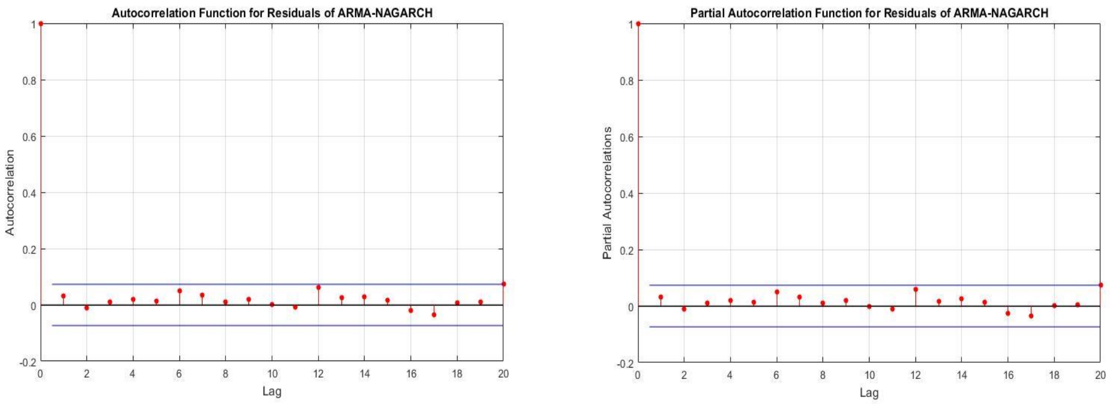

Figure 4: Autocorrelation and partial autocorrelation functions for residuals from ARMA-NAGARCH models

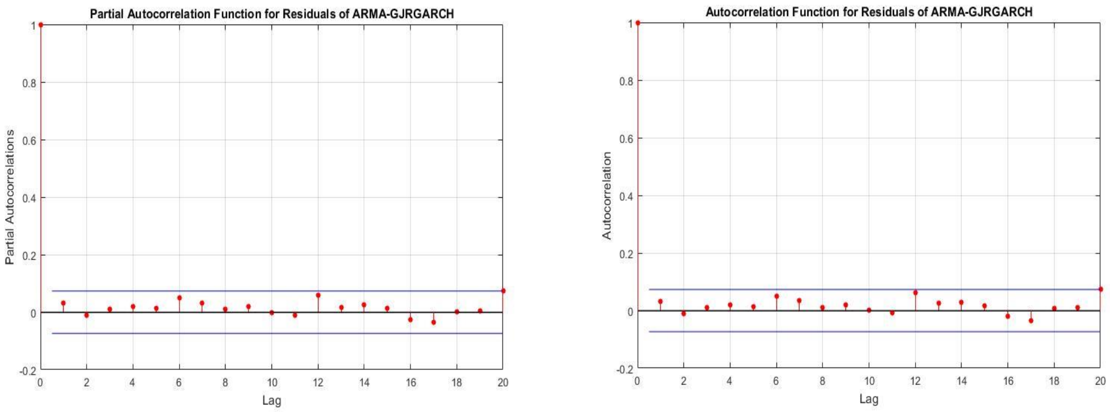

Figure 5: Autocorrelation and partial autocorrelation functions for residuals from ARMA-GJRGARCH models

5. Conclusion

Depending on the exchange rate forecasting values, the investors make a plan to make the maximum profit from their exchange rate investments, and the firms that do business with the exchange rate direct their trade based on these forecasts. Therefore, the forecasts of exchange rate prices are crucial for both investors and companies engaged in exchange rates. It is a critical process to determine the method that can accurately model behaviours of exchange rates. The most important contribution of this study is that it includes a comprehensive model determination process by using a wide range of methods to model exchange rate prices. ARMA-GARCH and ARIMA-GARCH (M) models are used to model exchange rate prices and this includes many models. Model selection is performed based on both goodness of fit measures and performance measures of forecasting accuracy. Empirical results suggest that ARMA-GARCH models are more effective than ARMA-GARCH (M) in modelling USD/TRY exchange rate. Moreover, it is found that alternative ARMA-GARCH models which can model asymmetric and nonlinear effects give better results than conventional ARMA-GARCH model which can model only symmetric effects. It is determined that ARMA-NAGARCH in which errors are skewed Student t distributed can model the UST/TRY exchange rate and it is followed by ARMA-GJRGARCH with Student t errors. It implies that fitting only symmetric distribution in modelling exchange rate can give misleading results. İn this paper, the prices of exchange rate are modelled based on its lagged values. However, the exchange rate depends on other variables such as inflation rate and terms of trade. In future studies, exchange rate is investigated by adding variables stated above as exogenous variables to the model.

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