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Metaphorical Perceptions of Vocational High School Students on Mathematical Problem Solving*

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ABSTRACT
Since problem solving has an important place in university level mathematics courses, determining university students' perceptions about problem solving can contribute to the field. In this regard, the aim of this study is to determine the mathematical problem solving perceptions of vocational high school students through metaphors. The case survey method was preferred for this study. Participants were 234 students studying in various programs of a vocational high school of a public university in Turkey in the fall semester of the 2019-2020 academic year. "Mathematical Problem Solving Metaphor Survey" were used as data collection tool. This survey included items that asked them to produce a metaphor for mathematical problem solving and its justification. Their written responses were analyzed using content analysis. Analysis of data indicated that participants produced 105 valid metaphors for mathematical problem solving. The most commonly used metaphors were "life", "labyrinth", "playing chess", and "jigsaw puzzles". Also, the metaphors produced by the participants were categorized under ten different conceptual categories. Metaphors were mostly gathered in the categories of "complicated" and "strategically progressive". These categories have a statistically significant relationship with the programs in which the participants study at. Based on the categories, students' perceptions of mathematical problem solving were discussed. Also, the study discusses instructional implications regarding how to change negative perceptions of students.

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Meslek Yüksekokulu Öğrencilerinin Matematiksel Problem Çözmeye İlişkin Metaforik Algıları

ÖZET
Problem çözme üniversite düzeyindeki matematik derslerinde önemli bir yere sahip olduğundan üniversite öğrencilerinin problem çözme ile ilgili algılarını belirlemek alana katkı sağlayabilir. Bu doğrultuda, bu çalışmanın amacı meslek yüksekokulu öğrencilerinin matematiksel problem çözme algılarını metaforlar aracılığıyla belirlemektir. Bunun için bu çalışmada örnek olay tarama modeli tercih edilmiştir. Araştırmanın katılımcıları 2019-2020

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eğitim-öğretim yılı güz döneminde Türkiye'deki bir devlet üniversitesinin meslek yüksekokulunun çeşitli programlarında öğrenim gören 234 öğrencidir. "Matematiksel Problem Çözme Metafor Anketi" veri toplama aracı olarak kullanılmıştır. Bu ankette katılımcılardan matematiksel problem çözme ile ilgili bir metafor üretmeleri ve bunu gerekçelendirmeleri istenmiştir. Katılımcıların yazılı yanıtları içerik analizi ile analiz edilmiştir. Verilerin analizi katılımcıların matematiksel problem çözme ile ilgili 105 geçerli metafor ürettiğini göstermiştir. Katılımcıların en sık "hayat", "labirent", "satranç oynamak" ve "yapboz yapmak" metaforlarını ürettikleri saptanmıştır. Ayrıca katılımcıların ürettikleri metaforlar on farklı kavramsal kategori altında kategorize edilmiştir. Üretilen metaforlar en sık "komplike" ve "stratejik aşamalı" kategorilerinde toplanmıştır. Bu kategorilerin katılımcıların öğrenim gördükleri programlarla istatistiksel olarak anlamlı bir ilişkiye sahip olduğu saptanmıştır. Kategorilere dayalı olarak, öğrencilerin matematiksel problem çözme algıları tartışılmıştır. Ayrıca öğrencilerin olumsuz algılarının nasıl değiştirilebileceğine ilişkin öğretimsel çıkarımlar tartışılmıştır.

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Introduction

One of the areas where problem solving finds its place the most is mathematics. The relationship between real-world problems and mathematics and the increasing interest in such problems have strengthened this position. For this reason, countries give special attention to problem solving in their curricula and set standards for problem solving for various levels from kindergarten to high school (Department for Education, 2014; Ministry of Education [MoE], 2012; National Council of Teachers of Mathematics [NCTM], 2000). In the Turkish context where this study was conducted, problem solving also have a very important place in the curriculum at various levels (Ministry of National Education [MoNE], 2018a). Mathematical problems related to real life have a special place in the secondary mathematics curriculum, and different problem solving strategies are used for the solutions of routine and non-routine problems, and these are addressed in the learning objectives (MoNE, 2018b). Problem solving also plays a crucial role in the evaluation of students' mathematical literacy in international exams such as the Program for International Student Assessment (PISA) and the Trends in International Mathematics and Science Study (TIMSS) (Organization of Economic Co-operation and Development [OECD], 2016; Martin, von Davier, & Mullis, 2020). In Turkey, the questions in the high-stakes exams to enter secondary and high schools are prepared based on the learning objectives in the curriculum as stated in the exam directive (MoNE, 2019). Therefore, candidates are expected to have problem solving competencies in these exams (Measuring, Selection and Placement Center [MSPC], 2019).

Problem solving has also an important place in foundation (two-year) degree courses and undergraduate degree courses related to mathematics at the university level. Elective or compulsory mathematics courses in vocational high school programs require solving routine and non-routine problems. Vocational mathematics courses discuss solutions to mathematical problems related to the profession (Hodgen, Wake, & Dalby, 2017). Nonetheless, the scope of problem solving should not only be limited to solving well-structured problems but should also include solving real-world problems (van Merriënboer, 2013) because vocational school graduates are faced with both mathematical and real-world problems in their professional lives and they are expected to solve these problems. However, the findings of Mailisman, Ikhsan, and Hajidin's (2020) study examining the mathematical problem solving skills of vocational high school students showed that their problem solving skills are not sufficient because they do not understand the problems. Furthermore, they were not used to non-routine problems and had difficulties in relating mathematics to real-world situations. Studies in the literature have also revealed that affective aspects such as beliefs, attitudes, or perceptions related to mathematical problem solving affect the academic success and ability to solve mathematical or real-world problems (Callejo & Vila, 2009; Mason, 2003; Mohd & Mahmood, 2011; Or & Bal, 2021; Schoenfeld & Herrmann, 1982; Schommer-Aikins, Duell, & Hutter, 2005; Son & Lee, 2021; Sturm & Bohndick, 2021).

It has long been recognized in mathematics education research that working on students' affective skills is crucial for helping them understand mathematics more deeply (Reyes, 1984). All these imply that it is important for mathematics education research to further investigate the perceptions of vocational school students about mathematical problem solving.

Metaphors as a Tool to Identify Problem Solving Perceptions

Mathematics education research commonly has investigated perceptions using various tools such as scales, interviews, or metaphors. As one of them, metaphors are effective to determine people's perceptions of a concept (Buerk, 1988; Innis, 1982; Latterell & Wilson, 2016, 2017; Wolodko, Willson, & Johnson, 2003). Lakoff (1993) defined metaphor as a poetic expression in which one or more words are used outside their traditional meaning to express a alike concept. We understand most of the concepts from the point of other concepts, therefore "human thought processes are largely metaphorical" (Lakoff & Johnson, 1980a, p. 6). Metaphors enable abstract or less concrete experiences to be understood with more concrete experiences (Lakoff & Johnson, 1980b). Therefore, the concepts we use and our daily activities are metaphorically structured. Since we communicate through the concepts we use in thinking and acting, metaphors are pervasive in thought, language, and action. That is why we use "metaphorical linguistic expressions to study the nature of metaphorical concepts and to gain an understanding of the metaphorical nature of our activities" and thoughts (Lakoff & Johnson, 1980a, p. 8).

According to contemporary metaphor theory, metaphors can be understood as a tightly structured mapping from a source domain to a target domain (Lakoff, 1993, p. 206). This metaphorical mapping is simply done by matching such as "target-domain is source-domain" or "target-domain as source-domain" (Lakoff, 1993, p. 207). A source domain is generally more concrete than a target domain (Mathieu, 2009). According to Lakoff and Núñez (2000), metaphors relate the source domain outside (or inside) mathematics to the target domain within the mathematics that is intended to be grasped or understood. In the metaphor of "problems are puzzles", one of the exemplary metaphors mentioned by Lakoff & Johnson (1980a, p. 145), the problem is the target domain, while the puzzle is the source domain. This kind of metaphorical mapping "preserves the cognitive topology (that is, the image-schema structure) of the source domain, in a way consistent with the inherent structure of the target domain" (Lakoff, 1993, p. 215). This preservation is evident in the metaphor of "problems are puzzles". On the other hand, this mapping might not be necessarily one-to-one. For example, for the "argument as war" metaphor, we highlight a particular aspect of a concept in the source domain (e.g., the battling aspect of arguing) while we hide some other aspects (e.g., cooperative aspect of arguing) which does not belong to the source domain (Lakoff & Johnson, 1980a, p. 10).

Metaphors are used not only as rhetorical tools but also as cognitive tools in education (Diaz-Rojas & Soto-Andrade, 2017). Creating metaphors is a cognitive process, and metaphors are cognitive products. Metaphors are used consciously or unconsciously in daily life. In mathematics classrooms, they are used both unconsciously (Font, Bolite, & Acevedo, 2010) and consciously (Yee, 2017) to promote students' understanding of difficult concepts. Since metaphors have pedagogical value in education other than aesthetics and ornaments (Botha, 2009), they should be used deliberately and consistently for certain purposes. Using metaphors in mathematics teaching is a powerful tool for conceptual understanding, and they can also be a source of motivation for students to have a more positive attitude towards mathematics (Hendriana & Rohaeti, 2017). Furthermore, metaphors have the potential to reveal the mathematical identity of students (Latterell & Wilson, 2017). Therefore, studies on metaphors in mathematics education are valuable. There is a growing body of literature that examines the metaphorical perceptions of participants about mathematics (Latterell & Wilson, 2016, 2017; Markovits & Forgasz, 2017; Schinck, Neale, Pugalee, & Cifarelli, 2008; Thibodi, 2017). Although the metaphorical thinking approach has a significant effect on problem solving skills (Purwasi & Fitriyana, 2021), metaphorical studies on mathematical problems and problem solving (Chapman 1997; Saka & Durmuş, 2021; Sezgin-Memnun, 2015; Son & Lee, 2021; Turhan-Türkkan & Yeşilpınar-Uyar, 2016; Uygun, Gökkurt, & Usta, 2016; Yee, 2017) are less common. In these studies, the most popular metaphors used by teachers (Chapman, 1997; Yee, 2017), teacher candidates (Saka & Durmuş, 2021; Son & Lee, 2021; Uygun et al., 2016), high school students (Yee, 2017), postsecondary adults (Chiu, 2001), secondary school students (Sezgin-Memnun, 2015; Turhan-Türkkan & Yeşilpınar-Uyar, 2016), and middle school students (Chiu, 2001) have been investigated.

The studies conducted with teachers investigated how they used metaphors as they teach problem solving. For example, Chapman (1997) found that teachers constructed personal metaphors such as "community", "adventure" and "game" to conceptualize problems and make sense of their teaching. Metaphors were found to be indicators of problem solving performances. In that sense, Son and Lee (2021) found that preservice elementary teachers' metaphors to define problem solving were related to their performances. Participants who mentioned metaphors that imply positive perceptions had a better performance. Studies conducted with students examined their metaphors as indicators of their positive (e.g. sun, game, family) and negative (e.g. trouble, knot, life) perceptions about mathematical problems and problem solving (Saka & Durmuş, 2021; Sezgin-Memnun, 2015; Turhan-Türkkan & Yeşilpınar-Uyar, 2016; Uygun et al., 2016).

As seen in the literature, there is a need for more research investigating students' perceptions of the concept of "mathematical problem solving". In particular, to the authors' knowledge, there is not enough research that determines the perceptions of vocational high school students about problem solving. For these reasons, this study aims to determine the mathematical problem solving perceptions of vocational school students using metaphors. Therefore, we pose the following research questions:

- What are the metaphors that vocational high school students produce for the concept of "mathematical problem solving"?
- What are the conceptual categories that emerged from the metaphors produced by vocational high school students for the concept of "mathematical problem solving"?
- Is there a relationship between the conceptual categories emerged from the metaphors produced by vocational high school students for the concept of "mathematical problem solving" and the programs they study at?

Methodology

Research Design

In line with the purpose of the research, we used the case survey method which is used to study a population by focusing on a sample of that population (Creswell, 2014). The case survey method was preferred for the study because it bridges "the gap between nomothetic surveys and idiographic case studies to combine their respective benefits of generalizable, cross-sectional analysis and in-depth, processual analysis" (Larsson, 1993, p. 1515). In this study, this model was used to investigate how vocational high school students perceive mathematical problem solving in the context of a single higher education institute that offers different programs.

Study Group

This study was carried out with a total of 234 students studying in the first year of various programs of the Vocational School of Technical Sciences of a state university in Turkey in the fall semester of the 2019-2020 academic year and taking the mathematics course as compulsory. This study group was selected by a convenient (easily accessible) sampling method. This sampling technique was preferred because it is possible to reach the study group quickly and easily (Patton, 2001). Another reason for choosing this study group is that the Vocational School of Technical Sciences has a variety of programs. The reason is to explore whether perceptions change according to the programs. Programs that do not have a mathematics course in their curriculum were not included in the study. Table 1 gives the distribution of participants throughout the programs.

Table 1

The Distribution of Participants Throughout the Programs

Program	Frequency (f)	Percentage (%)
Computer Programming (CP)	48	20.51
Mechanics (M)	36	15.39
Construction Technology (CT)	65	27.78
Interior Design (ID)	85	36.33
Total	234	100.00

Data Collection

We collected data through the "Mathematical Problem Solving Metaphor Survey". This survey was prepared by making use of the studies in the literature (Sezgin-Memnun, 2015; Son & Lee, 2021; Turhan-Türkkan & Yeşilpınar-Uyar, 2016; Uygun et al., 2016). The survey consists of an open-ended question: "Mathematical problem solving, according to me, is like……, because ……….". The first blank is provided for the metaphor and the second is for its justification. We took expert opinions from three mathematics educators on whether the survey was prepared for its purpose. To make sure that the question is comprehensible in terms of language, we took the opinions of ten students studying at a vocational school of a different university. Based on the feedback, we decided to explain the notion of "metaphor" to the participants. When conducting the survey, we asked participants to fill in the two blanks in the survey by writing only one metaphor, the subject of which was mathematical problem solving, and a justification compatible with this metaphor. The first author collected the data in one session. Before the survey was administered, permission to collect data was obtained and the participants were informed about the research and their rights. An informed consent form was signed by each participant who volunteered to participate in the study.

Analysis of Data

We analyzed data using content analysis which aims to categorize the qualitative textual data into clusters of alike entities to determine coherent patterns and relations between themes (Julien, 2008). We followed the steps below for the content analysis (Creswell, 2014; Julien, 2008; Patton, 2001).

In the first stage, we gave pseudonyms to the participants according to the programs they study: CT1, CT2, CT3,…for the Construction Technology Program, ID1, ID2, ID3,… for the Interior Design Program, CP1, CP2, CP3,…for the Computer Programming, and M1, M2, M3,...for the Mechanical Program. Responses that do not include any metaphors and/or justifications and responses with more than one metaphor were excluded. In qualitative data analysis, researchers may need to sort out data to reach a meaningful data set (Guest, MacQueen, & Namey, 2012). For example, the response from CT25 (mathematical problem solving is like playing football for me, because…) was excluded because it did not include any justification. We also excluded the responses in which the metaphors were not in meaningful harmony with their justification after reaching a consensus among the researchers e.g. "Mathematical problem solving is like black to me because it doesn't exist without white." In addition, responses that do not contribute to the understanding of the concept of mathematical problem solving were also excluded from the research e.g. "Mathematical problem solving is like the scales to me, because it goes down where it weighs." which could only be elaborated using an interviewing technique. In this way, 56 survey forms were excluded and a total of 178 forms, which produced meaningful and valid metaphors that would facilitate understanding the concept of problem solving, were included in the study.

We adopted an inductive analysis approach to establish categories of metaphors. In contrast to deductive analysis, where data are categorized according to existing frameworks, in inductive analysis, categories are discovered through the interaction of the researcher with the research data (Patton, 2001).

The metaphors created by the participants were categorized by the researchers according to their common characteristics, taking into account their justifications. For example, the response from CP13, "Mathematical problem solving is like playing chess to me, because you can't win without the right moves." and CT10 "Mathematical problem solving is like doing a puzzle to me because you have to put the right pieces in the right place." were considered in the category of "strategically progressive". Different justifications for the same metaphors were coded as different categories where necessary. For example, "Mathematical problem solving is like art to me because it requires skill." which implies the "art metaphor" was coded as belonging to the category "skill requiring expertise/mastery" while "Mathematical problem solving is like art for me because it requires care." which also implies the "art metaphor" was coded in another category called "dedication".

To ensure the validity of the research, we took expert opinion as mentioned above. In addition, data collection and data analysis processes are given in detail, also, findings were also supported by direct quotations (Guba & Lincoln, 1982). For the reliability of the research, two researchers coded the data independently. While consensus was reached for 102 metaphors, three metaphors were coded differently. Using the formula of Miles & Huberman (1994, p. 64), the inter-rater reliability was 97%. Since the consistency was higher than 80%, it was accepted that the reliability was ensured (Miles & Huberman, 1994).

At the last stage, the programs that the participants studied, the metaphors they produced, and the conceptual categories of metaphors were entered into the SPSS 16.0 software. We conducted descriptive analysis and found the frequencies (f) and percentages (%) of metaphors and their conceptual categories. We also obtained cross tabulations of the conceptual categories and the program types to explore whether there is a relationship between the categories and programs. Chi-Square Test (Agresti, 1990), one of the quantitative predictive analysis techniques, was applied to determine whether there was a statistically significant relationship between two categorical variables. However, when the expected value of more than 20% of all cells in the cross tabulations is below five, the results of the Chi-Square Test cannot be used (Cochran, 1954). In this case, Fisher's Exact Test (Fisher, 1925) is used in four-cell ($2x2$) cross tabulations, and Fisher-Freeman-Halton test (Freeman & Halton, 1951), which is an extension of this test, is used in multi-cell rxc crosstabs. In cases where the exact p-value can be calculated as a result of these tests, this p-value is used for analysis, while in cases where it cannot be calculated, the Monte Carlo Method is applied and the probability value (p-value) in the output of this test is used (Metha & Patel, 1989). The Conceptual Category*Program cross tabulation in this study is a $10x4$ table. In more than 20% of all cells in this table, the expected value was below five. Since the p-value was not calculated in the Fisher-Freeman-Halton Test (Freeman & Halton, 1951), the Monte Carlo Method was applied and the p-value in the output of this test was used to determine whether there was a statistically significant relationship between the categories of metaphors produced by participants and the programs they studied.

Findings

This section will first give the metaphors produced by the participants and the conceptual categories of these metaphors. Afterward, we will present the findings concerning the relationship between these categories and the programs that the participants study at.

Metaphors and Their Conceptual Categories

178 out of 234 participants (76.07%) produced 105 different metaphors that describe mathematical problem solving. The most frequently used metaphors were "life", "labyrinth", "playing chess", and "jigsaw puzzle". These metaphors were grouped under ten different conceptual categories considering their justifications. These categories are "complicated", "strategically progressive", "skills requiring expertise/mastery", "fun/relaxing", "infinity", "dedication", "spooky/boring", "developing", "reaching

a conclusion/solution", and "indispensable". While some of these categories indicate positive perceptions of problem solving some others indicate negative perceptions.

The metaphors that the participants produced about mathematical problem solving and the conceptual categories that emerged from these metaphors are presented in Table 2. As can be seen from the table, metaphors were mostly gathered in the categories of "complicated" and "strategically progressive". These categories are followed by categories "skills requiring expertise/mastery" and "fun/relaxing".

Table 2

Metaphors Produced by Participants and Their Conceptual Categories

Conceptual categories	Metaphors (Frequency)	Metaphors		Participants	
		f	%	f	%
Complicated	Life (13), labyrinth (7), untying a knot (4), love (2), balls of string (2), driving a car (1), marsh (1), reading a book in a foreign language (1), swirl (1), go adrift (1), obstacles in life (1), holding on to life (1), achieving happiness (1).	13	12.38	36	20.23
Strategically Progressive	Playing chess (7), jigsaw puzzle (7), climbing stairs (5), playing a game (5), to build a building (1), boxing (1), riddle (1), war (1), logic game (1), making a plan (1), playing with a magic cube (1).	11	10.48	31	17.42
Skills Requiring Expertise/Mastery	Coding (3), to build a building (3), interior designing (2), writing a computer program (2), exercising (1), car wiping (1), programming a computer game (1), decorating (1), build a building (1), using a map (1), designing furniture (1), knitting (1), using a compass (1), perspective drawing (1), art (1), plastering (1), cooking (1), knowing a craft (1).	18	17.14	24	13.48
Fun/Relaxing	Drinking coffee (2), game (2), social media (2), drinking water (2), writing a poem (2), holiday (1), journey (1), shopping (1), pouring water on a fire (1), hunting (1), bitter chocolate (1), eat appetizers (1), weekend (1), gift box (1), taking a walk in the forest (1), sugar (1), eating food (1).	17	16.19	22	12.36
Infinity	Looking at the sky (2), road (2), gap (1), trouble (1), loop (1), universe (1), black hole (1), matryoshka (1), Mona Lisa painting (1), troubles (1), ballpoint pen (1), space (1).	12	11.43	14	7.87
Dedication	Growing a tree (5), family (2), love (2), growing a flower (1), to seed (1), weaving a rug (1), art (1).	7	6.67	13	7.30
Spooky/Boring	Going to school (2), herniate (1), people (1), having nightmares (1), winter (1), death (1), spider (1), distress (1), traffic (1).	9	8.57	10	5.62
Developing	Riding a bicycle (4), baby (2), driving a car (1), growing up (1), playing the clarinet (1), riding a motorcycle (1).	6	5.71	10	5.62
Reaching a Conclusion/Solution	Going on the road (5), philosophizing (2), mountain climbing (1), crime scene (1), daydreaming (1).	5	4.76	10	5.62
Indispensable	Oxygen (2), thinking (1), exercise (1), light (1), reading a book (1), breathing (1), doing sport (1).	7	6.67	8	4.49

The Category of Complicated: This category has the highest frequency. 36 (20.23%) participants produced 13 different metaphors in this category. The common features of the metaphors and their justifications in this category point out the complex structure and difficult processes of problem solving. The most frequently used metaphors in this category are “life”, “labyrinth”, and “untying a knot”. Examples of metaphors in this category are given below:

"Mathematical problem solving, according to me, is like life because it's complex with ups and downs." (ID8)

“Mathematical problem solving, according to me, is like a labyrinth because there is always a way out, but you can't find it easily." (ID34)

"Mathematical problem solving, according to me, is like untying a knot because it's so complicated." (ID5)

The Category of Strategically Progressive: This category has the second-highest frequency among the conceptual categories. 11 metaphors produced by 31 (17.42%) participants were coded in this category. The common features of the metaphors and their justifications in this category point out to strategic stages of problem solving. Within this category, the most frequently used metaphors were determined as "playing chess", "jigsaw puzzle", "climbing stairs", and "playing a game". Examples of metaphors in this category are given below:

"Mathematical problem solving, according to me, is like playing chess because if you make the right moves, you win." (CP17)

“Mathematical problem solving, according to me, is like climbing stairs because you progress step by step." (CP46)

"Mathematical problem solving, according to me, is like playing a game because you should always have a strategy." (CP1)

The Category of Skills Requiring Expertise/Mastery: This category is the third-highest frequency. 24 (13.48%) participants produced 18 different metaphors in this category. The metaphors produced by the participants differed according to their interests and programs. The common feature of metaphors and their justifications is that problem solving is not an ordinary job and it is a skill that requires expertise or mastery. The most frequently used metaphors were "coding", "to build a building", "interior designing", and "writing a computer program". Examples of metaphors in this category are given below:

"Mathematical problem solving, according to me, is like coding because only experts can." (CP22)

"Mathematical problem solving, according to me, is like knowing a craft because it requires mastery." (ID1)

The Category of Fun/Relaxing Category: 22 (12.36%) participants emphasized the fun and relaxing aspects of problem solving. These participants produced 17 different metaphors. Apart from the metaphors used once, “drinking coffee”, “game”, “social media”, “drinking water”, and “writing a poem” were reported twice. Examples of metaphors in this category are given below:

"Mathematical problem solving, according to me, is like writing a poem because it's fun." (CT2)

"Mathematical problem solving, according to me, is like hunting because it causes addiction." (ID40)

"Mathematical problem solving, according to me, is like pouring water on a fire because the sound of a dying fire is comforting." (ID11)

The Category of Infinity: The metaphors produced by 14 (7.87%) participants are in this category. Participants reported 12 different metaphors and the common feature of these metaphors is the sense of infinity. “Looking at the sky”, “road”, “gap”, “universe”, and “space” were remarkable metaphors. Examples are given below:

"Mathematical problem solving, according to me, is like a road because it has no end." (M13)

"Mathematical problem solving, according to me, is like a gap because you never see the end." (ID28)

"Mathematical problem solving, according to me, is like a loop because when one ends, another begins." (CP24)

The Category of Dedication: 13 (7.30%) participants produced seven different metaphors in this category. The common feature of the metaphors in this category is that problem solving requires dedication, such as patience, attention, and hard work. “Growing a tree”, “family”, and “love” are more common metaphors than others. Examples of this category are given below:

"Mathematical problem solving, according to me, is like growing a tree because the more you care to look after it, the more beautiful fruits you will get." (ID4)

"Mathematical problem solving, according to me, is like love because it requires dedication." (CT14)

The Category of Spooky/Boring: In this category, 10 (5.62%) participants produced nine different metaphors. Metaphors that were coded in this category imply the scary or boring nature of problem solving. Examples of metaphors are given below:

"Mathematical problem solving, according to me, is like having nightmares because it's scary." (M15)

"Mathematical problem solving, according to me, is like distress because it causes cancer." (CT24)

The Category of Developing: 10 (5.62%) participants produced six different metaphors in this category. Their main characteristic is that the development of problem solving requires a process. Examples of metaphors in this category are given below:

"Mathematical problem solving, according to me, is like a baby because it takes time to grow." (M21)

"Mathematical problem solving, according to me, is like playing the clarinet because you can learn both in time." (CP29)

The Category of Reaching a Conclusion/Solution: In this category, 10 (5.62%) participants produced five different metaphors. The common feature of these metaphors is that problem solving should lead someone to a result or a solution. Examples are presented below:

"Mathematical problem solving, according to me, is like philosophizing, because the more you think, the more you get results." (ID29)

"Mathematical problem solving, according to me, is like a crime scene because if you can't solve it you can't find the truth." (ID15)

The Category of Indispensable: In this category, eight (4.49%) participants produced seven different metaphors. The metaphors in this category are related to the indispensable nature of problem solving. In this category, the metaphor of "oxygen" was used the most frequently:

"Mathematical problem solving, according to me, is like oxygen because without it there is no life." (CT15)

"Mathematical problem solving, according to me, is like breathing because it is indispensable." (ID13)

Findings regarding the relationship between the conceptual categories of metaphors and programs types

In this section, we will examine whether the conceptual categories that emerged from the metaphors produced by the participants and the programs they study at are related. The frequencies and percentages of valid and meaningful metaphors produced by the participants according to the programs are presented in Table 3.

Table 3

Frequencies and Percentages of Valid Metaphors Produced by the Participants according to the Program Types

Program	Number of participants (N)	Number of participants who created valid metaphors	Percentages (%)
Computer Programming (CP)	48	38	79.17
Mechanics (M)	36	29	80.56
Construction Technology (CT)	65	45	69.23
Interior Design (ID)	85	66	77.65
Total	234	178	76.07

As seen in Table 3, participants in all programs were successful in generating valid and meaningful metaphors. The most successful program in generating valid metaphors was the Mechanics program (80.56%), while the students in Construction Technology program (69.23%) were less successful than in other programs.

In this section, metaphors and conceptual categories were analyzed according to the program variable. To determine whether there is a significant relationship between them, the crosstabulation technique and the Monte Carlo method were applied. The findings are presented in Table 4.

Table 4

*Conceptual Category*Program Crosstabulations and Monte Carlo Method Output*

Conceptual category*Program	CP	M	CT	ID	Total	p
	f	f	f	f		
Complicated	5	3	12	16	36	0.025^{a}
Strategically Progressive	13	7	6	5	31	
Skills Requiring Expertise/Mastery	7	2	5	10	24	
Fun/Relaxing	5	3	5	9	22	
Infinity	2	2	3	7	14	
Dedication	0	4	5	4	13	
Spooky/Boring	0	1	3	6	10	
Developing	2	5	1	2	10	
Reaching a Conclusion/Solution	4	1	1	4	10	
Indispensable	0	1	4	3	8	
Total	38	29	45	66	178	

a Monte Carlo Method: $p < 0.05$

As can be seen from Table 4, the percentages of the conceptual categories that emerged from the metaphors produced by the participants differ across the programs. Since the Monte Carlo Method's p-value is p =0.025<0.05, there is a statistically significant relationship between the conceptual categories and the programs. In other words, the conceptual categories differ in a statistically significant way according to the programs. While the metaphors produced by the students studying in Computer Programming and Mechanics programs clustered in the "strategically progressive" category, the metaphors produced by the students of Construction Technology and Interior Design programs clustered around the "complicated" category.

Discussion and Conclusion

The findings of the study showed that the majority of vocational high school students were able to produce a meaningful and valid metaphor for mathematical problem solving. The metaphors of "life", "labyrinth", "playing chess", and "jigsaw puzzle" were the most frequent ones. Conceptual categories that emerged from these metaphors indicated both positive and negative perceptions about problem solving. The frequent metaphors and their conceptual categories show similarities with the results of other studies in the literature (Buerk, 1988; Latterell & Wilson, 2016; Saka & Durmuş, 2021; Sezgin-Memnun, 2015; Son & Lee, 2021; Thibodi, 2017; Turhan-Türkkan & Yeşilpınar-Uyar, 2016; Uygun et al., 2016; Yee, 2017).

The most frequent metaphors, "life" and "labyrinth", which belong to the category of "complicated" might be an indication of participants' difficulties with problem solving, because according to Chapman (1997), problem solving is a complicated process all by itself. The source of this perception might also be related to participants' self-efficacy beliefs related to problem solving because, as Pajares and Miller (1994) pointed out, the self-efficacy beliefs of the students affect their problem solving performances.

Findings concerning the conceptual category of "complicated" for problem solving are in line with the findings of the studies that investigated students' perceptions of the concept of mathematics. Because, mathematics and problem solving are closely related concepts and phenomena (Ergen, 2020).

In the study of Latterell and Wilson (2016, 2017), the metaphors produced by the participants about the concept of mathematics were gathered within the 'difficult and impossible' category the most frequently referring to the relationship between problem solving and mathematics.

"Playing chess" and "jigsaw puzzle" were also common and were considered in the conceptual category of "strategically progressive". These metaphors reflect participants' perceptions that problem solving requires strategy and proceeds progressively. This finding shows that the participants were aware of the necessity of the strategies in the process of problem solving because acting on the strategies is a well-accepted phase that has been described by all models on problem solving processes (Polya, 1945).

The perceptions of some participants are in the direction of considering mathematical problem solving as a skill requiring expertise or mastery. While the participants expressed their perceptions in this direction, they used metaphorical expressions related to their areas of profession or interests. This is not surprising since metaphors provide for explaining unknown or less known concepts using well-known concepts (Lakoff, 1993).

While problem solving is a fun activity for some participants, it is a boring activity for some others. It can be considered that emotional reactions are significant factors in problem solving (McLeod, 1988), so they can affect problem solving performances of the participants. Teachers must consider the significance of emotions in the teaching of problem solving and they should be able to help students regulate their emotions (Hannula, 2015). In other words, they should help them in developing the emotions such as 'I can do it, I can solve it' when they face mathematical problems (Wilson, Fernandez, & Hadaway, 1993).

The fact that problem solving is an action that should be able to reach a conclusion or a solution is among the perceptions of some participants in this study. They think of problem solving as a result, not a process. We question whether the product-oriented approach while solving problems might cause this kind of situation (Lachner & Nückles, 2016).

Some of the participants' perceptions point out the role of dedication and effort required for problem solving. Both learning and teaching problem solving require effort. However, referring to the previous paragraph, teachers should teach students how to think instead of presenting the solution (Wilson et al., 1993).

Another finding was the statistically significant relationship between the conceptual categories of metaphors produced by the participants and the programs they were studying at. Participants' perceptions of problem solving varied according to the programs. This finding points out the effects of departmental practices on students' perceptions of problem solving. While the metaphors generated by the students studying in Computer Programming and Mechanics programs clustered in the "strategically progressive" category, the metaphors generated by the students of Interior Design and Construction Technology programs clustered around the "complicated" category. Different ways of using problem solving in the vocational courses across programs might have caused this finding. This finding is parallel to the findings of some other studies in the literature (Pajares & Miller, 1994; Sağlam & Dost, 2014; Uygun et al., 2016).

The problem solving perception of more than one-fifth of the participants who produced valid metaphors is that problem solving is difficult and complex. Some participants also find problem solving spooky and boring. Considering this finding, we suggest intervention studies to change the negative perceptions of the participants that problem solving is a difficult and boring activity. By setting out the question "*How can productive beliefs toward mathematical problem solving be nurtured?*", intervention studies should focus on ways of strengthening the positive attitudes of the students (Lester & Cai, 2016, p. 117). Because affective skills such as perceptions, attitudes, and beliefs are improvable (Hannula, 2002; Wilson et al., 1993).

One of the limitations of this study is that the data were gathered in a single vocational high school. The other limitation is that this study serves only for detecting the perceptions of the students in a particular country. There is research evidence that the perceptions of mathematics might change according to the culture (Tsao, 2004). Therefore, it can be suggested that problem solving perceptions of vocational high school students in different countries could also be explored and compared. Another limitation of this study is that the data were collected solely by employing a questionnaire. It is suggested that the perceptions must be examined more thoroughly by using various data collecting tools such as interviews or observation. In this research, only the relationship between the perceptions of the participants and the programs they are studying were examined. It was determined that the perceptions differed according to the programs. Because different perceptions may require different teaching approaches, the relationship between the perceptions of the participants and various factors can also be examined in the following studies.

As a final remark, we suggest future studies which could focus on problem solving perceptions of lecturers who work at vocational high schools because their perceptions might affect their teaching methods in their classes and students' perceptions and performances (Cai, 2004; Son & Lee, 2021). Future studies could also investigate metaphorical perceptions formed by in-class interaction in a qualitative manner.

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