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# Total Completion Time Scheduling Problem with Temperature Considerations

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## Abstract

The focus of this work is to analyze working environment temperature in single-machine scheduling problems. In classical scheduling problems, the processing times are considered usually without the effect of working environment temperature. However, in many real scheduling environments, working environment temperature may affect processing time due to the nature of work. In this paper, we present job processing times based on the working environment temperature for total completion time (square) problem. We derive solutions with polynomial-time for both objectives.

**Keywords:** Scheduling; Ergonomics, Temperature, Total completion time, Physical environment

# Sıcaklık Etkili Toplam Tamamlanma Zamanı Çizelgeleme Problemi

## Öz

Bu çalışmanın odağı tek makineli çizelgeleme problemlerinde iş ortam sıcaklığının analiz edilmesidir. Klasik çizelgeleme problemlerinde çoğunlukla işlem zamanları iş ortamının sıcaklık etkisi olmadan düşünülür. Fakat birçok gerçek çizelgeleme ortamında, iş ortamı sıcaklığı işin doğasından dolayı işlem zamanını etkileyebilir. Bu makalede toplam tamamlanma zamanı (ve tamamlanma zamanlarının karesi toplamı) problemleri için iş ortamı sıcaklığına bağlı işlerin işlem zamanları sunulmuştur. Her iki amaç için de polinom zamanlı çözümler elde edilmiştir.

**Anahtar Kelimeler:** Çizelgeleme; Ergonomi, Sıcaklık, Toplam tamamlanma zamanı, Fiziksel ortam

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## 1. Introduction

In job scheduling problems, the processing times are usually considered without the effect of working environment temperature. However, in many real scheduling environments such as construction industry, steel production, rubber and plastic, production based on metal etc., working environment temperature increases over time due to the nature of work. Some of the jobs are done indoor and while other jobs are performed outdoor. Working environment temperature is so important factor which affects the performance at work. Seppänen et al. [3] show that the worker performance goes up under temperature up to 21-22 $C^0$ when worker performance goes down under temperature above 23-24 $C^0$. Due to the working environment temperature, if a job is assigned to process later, it spends less time than the job when it is assigned to process earlier when temperature is below of 21.75 $C^0$. In this paper, we present that if a job is assigned to process later, it spends more time than the job when it is assigned to process earlier when temperature is above of 21.75 $C^0$. In the literature, some researchers have indicated [1-3] that the most comfortable temperature yields optimal work performance. Seppänen et al. [3] presented the Figure 1, which shows the the peak point of performance curve occurs at temperature of 21.75 $C^0$. Seppänen et al. [3] present that an increase of temperature up to 21 $C^0$ is related with a statistically remarkable increasing productivity and an increase of temperature above 24 $C^0$ is associated with a statistically remarkable dropping in productivity.

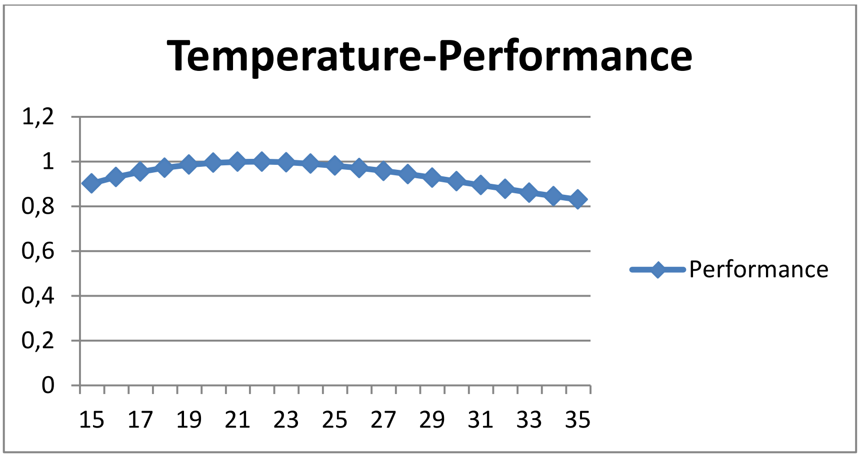

**Figure 1**. Normalized performance under temperature

They proposed the equation for curve is

$$D(Temp) = 0.1647524(Temp) - 0.0058274(Temp)^2 + 0.0000623(Temp)^3 - 0.4685328 \tag{1}$$

where *D(Temp)* is productivity relative to maximum value and *(Temp)* is working environment temperature.

In the scheduling literature, many researchers have been working to optimize scheduling problems under some effects. If a job is assigned to process later, it spends less time than the job when it is assigned to process earlier. In this case, the job is under deterioration effect. Gupta and Gupta [4] and Browne and Yechiali [5] were proposed independently deterioration effect on jobs in scheduling problems. On the other hand, in many real scheduling studies, workstations accelerated continuously due to learning, which by repeating the same or similar activities. So, the job's processing time is shorter if it is assigned to process later, rather than in the sequence [6-7]. In the literature, this phenomenon, which was first entitled by Mosheiov [8], is known as a "learning effect". Recently, researchers [9-12] have been working on job scheduling problems under learning effect. This paper will present the temperature effect in the working environment for two scheduling problems.

We present that the actual processing time of job *j* under temperature effect is such that

$$\overline{p}_j = \frac{p_j}{D_j(T)} \tag{2}$$

where $p_j$ and $\overline{p}_j$ are the basic and actual processing time under temperature effect of job *j* respectively. $D_j(Temp)$ is the effect of temperature to performance.

We organized the remaining part of the paper as follows: The mathematical model of problems is presented in Section 2. In Section 3, we show optimal solutions with polynomial-time for both objectives. In Section 4, computational examples are given. In Section 6, we summarized the results of the study. In Appendix, LINGO codes of the proposed mathematical model are given.

## 2. The Mathematical Model

This section presents the integer programming model of single machine total completion time scheduling problem under working environment temperature as follows:

*Notations:*

| | |
|---|---|
| $n$ | number of jobs |
| $\overline{p}_j$ | actual processing time with respect to job *j* under temperature effect |
| $p_j$ | basic processing time with respect to job *j* |
| $\overline{C}_j$ | actual completion time with respect to job *j* under temperature effect |
| $C_j$ | basic completion time with respect to job *j* |
| $T_j$ | working environment temperature with respect to job *j* |
| $D(T_j)$ | the effect to performance of temperature when it is $T_j$ |
| $X_{jr}$ | 1 if job *j* is processed in position *r*, 0 otherwise |

*Objective function*

$$\min \sum_{i=1}^{n} \overline{C}_i$$

*subject to*

$$\overline{p}_j = \left(\frac{p_j}{D(T_j)}\right).X_{ij} \qquad (i = 1,\dots,n) \tag{3}$$

$$\overline{C}_j = \overline{C}_{j-1} + \overline{p}_j \tag{4}$$

$$\sum_{j=1}^{n} X_{ij} = 1 \qquad (i = 1,\dots,n) \tag{5}$$

$$\sum_{i=1}^{n} X_{ij} = 1 \qquad (j = 1,...,n) \tag{6}$$

The objective function of proposed model is the total completion time minimization. Eq. (3) and Eq. (4) are the actual processing time and the actual completion time of job *j*, respectively. Eq. (5) guarantees that each job can be assigned on just one position and Eq. (6) ensures that each position takes just one job.

We tested the proposed model in follow numerical example. For the sake of simplicity, this is an example with only four jobs.

**Numerical example.** $n = 4$, $p_1 = 3$, $p_2 = 7$, $p_3 = 2$, $p_4 = 4$ and $Temp_1 = 30$, $Temp_2 = 27$, $Temp_3 = 18$, $Temp_4 = 35$.

The obtained result of the assignment problem is $x_{12} = x_{24} = x_{31} = x_{43} = 1$, and the value of objective function is 34.82. The processing of the first job stars at time zero. Table 1 shows the actual processing times and actual completion times for all jobs. The results show that *SPT* rule (non-decreasing order of jobs' processing time) minimizes the sum of completion time scheduling problem. The obtained result is $(p_3 \le p_1 \le p_4 \le p_2)$.

**Table 1**. Actual processing times and actual completion times for all jobs

| ***Jobs (r)*** | ***Actual processing time*** $(\overline{p})$ | ***Actual completion time*** $(\overline{C})$ |
|---|---|---|
| *Job scheduled in position 1* | 2.19 | 2.19 |
| *Job scheduled in position 2* | 3.13 | 5.32 |
| *Job scheduled in position 3* | 4.11 | 9.44 |
| *Job scheduled in position 4* | 8.43 | 17.87 |

## 3. Total Completion Time Scheduling Problem Under Temperature Effect

The results show that *SPT* rule minimizes the sum of completion time scheduling problem. Let $\sum C_j$ $\left(\text{and} \sum C_j^2\right)$ represents sum of completion times of a given permutation.

**Theorem 1.** *The problem* $1\left| p_j \middle/ D(Temp_j) \right| \sum \overline{C}_j$ *can be optimally solved by SPT rule when* $16C^0 \le Temp \le 35C^0$ *is gap of the most convenient is working temperatures for a human.*

**Proof.** Assume an optimal schedule $\pi$ and $\pi$ has two adjacent jobs, $J_i$ and $J_j$, such that $p_i \le p_j$ and $J_j$ is scheduled directly before $J_i$ in the *r*th position in a sequence. We assume that $16C^0 \le T \le 35C^0$ is gap of the most convenient is working temperatures for a human, and processing times are integer. Let B the completion time of the job scheduled before the $J_i$ and $J_j$ and let $C_{ji}$ be the overall objective function value.

$\overline{C}_j(\pi)=B+\frac{p_j}{D(Temp_r(\pi))}$ and $\overline{C}_i(\pi)=B+\left(\frac{p_j}{D(Temp_r(\pi))}+\frac{p_i}{D(Temp_{r+1}(\pi))}\right)$ then

$$\sum\overline{C}(\pi)=2B+\frac{p_j}{D(Temp_r(\pi))}+\left(\frac{p_j}{D(Temp_r(\pi))}+\frac{p_i}{D(Temp_{r+1}(\pi))}\right)$$

The objective function has includes the sum of the completion times of the jobs scheduled before and after the $J_i$ and $J_j$, actual processing times

If we perform the pairwise interchange on jobs $J_i$ and $J_j$ to obtain schedule $\pi'$ then

$\overline{C}_i(\pi')=B+\frac{p_i}{D(Temp_r(\pi'))}$ and $\overline{C}_j(\pi')=B+\left(\frac{p_i}{D(Temp_r(\pi'))}+\frac{p_j}{D(Temp_{r+1}(\pi'))}\right)$ then

$$\sum\overline{C}(\pi')=2B+\frac{p_i}{D(Temp_r(\pi'))}+\left(\frac{p_i}{D(Temp_r(\pi'))}+\frac{p_j}{D(Temp_{r+1}(\pi'))}\right)$$

If we use Eq.1 that

$$\begin{aligned}D(Temp_r(\pi))&=0.1647524(Temp_r(\pi))-0.0058274(Temp_r(\pi))^2\\&+0.0000623(Temp_r(\pi))^3-0.4685328\end{aligned} \tag{7}$$

On the other hand,

$$\begin{aligned}D(Temp_r(\pi'))&=0.1647524(Temp_r(\pi'))-0.0058274(Temp_r(\pi'))^2\\&+0.0000623(Temp_r(\pi'))^3-0.4685328\end{aligned} \tag{8}$$

We know that $Temp_r(\pi)=Temp_{r+1}(\pi')$ and $Temp_{r+1}(\pi)=Temp_r(\pi')$,

The difference between the obtained sum of completion times is

$$\sum\overline{C}(\pi')-\sum\overline{C}(\pi)=\frac{p_i}{D(Temp_r(\pi'))}-\frac{p_j}{D(Temp_r(\pi))}$$

Using Eq. 7 and Eq. 8,

$$|D(Temp_{r+1}(\pi))-D(Temp_{r+1}(\pi'))|\le 0.2,\ 0\le(D(Temp_{r+1}(\pi)),D(Temp_{r+1}(\pi')))\le 1$$

when the working temperatures for a human is $16C^0\le T\le 35C^0$. So, these equations show that the proposed limits are obtained using extreme points for temperature $(Temp_r(\pi)$ or $Temp_r(\pi'))$.

Finally, we obtain $\sum\overline{C}(\pi')<\sum\overline{C}(\pi)$ if all processing times are integer and $p_i\le p_j$.

$\pi$ dominates $\pi'$. This contradicts the optimally of $\pi'$.

Townsed [13] proposed quadratic objectives in single machine scheduling. We present Theorem 2 quadratic objectives in the single machine scheduling.

**Theorem 2.** *The problem* $1\left|{p_r}/{D(Temp_r)}\right|\sum \overline{C}_j^2$ *can be optimally solved by SPT rule when* $16C^0 \le Temp \le 35C^0$ *is gap of the most convenient is working temperatures for a human.*

**Proof.** This proof is the similar with Theorem 1, expect that $\overline{C}_i(\pi') > \overline{C}_j(\pi)$ and $\overline{C}_j(\pi') > \overline{C}_i(\pi)$, thus $\overline{C}_i^2(\pi') > \overline{C}_j^2(\pi)$ and $\overline{C}_j^2(\pi') > \overline{C}_i^2(\pi)$.

## 4. Conclusions

This paper considers minimizing the total completion time (square) with working environment temperature in single machine environment. It should develop a structure to better reflect real life systems where the worker performance is decreasing if the temperate is below of 21.75 $C^0$ and where worker performance is increasing if it is bigger than 21.75 $C^0$. This paper shows that the problems under study can be polynomially solved. The both problems (sum of completion times and square) are minimized how jobs are sequenced according to the SPT rule.

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## Appendix: LINGO codes for the proposed model

```
MODEL:
            n=4;   ! Problem size;

SETS:
      J / 1.. 4/;   ! Job number;
  POS / 1.. 4/;   ! Position number;

     LINK(J,POS):
         Z;   ! Z( I, R) = 1 if job i is done in position R, 0 otherwise;


     LINK1(POS):
       PRO_TIME,
       COMP_TIME;

     LINK2(J):
     W,      !Weight;
     Temp,      !Temperature;
     PRO_TIME1;    ! Basic processing time;

ENDSETS

DATA:
      PRO_TIME1= 3 7 2 4;
      Temp= 30 27 18 35;
            W= 0.1 0.2 0.3 0.4;
 ENDDATA

!Objective function for total completion time;
MIN = @SUM(   J(I):
 @SUM(   POS(R):(COMP_TIME(R)*Z(I,R))));

!Objective function for total weighted completion time;
MIN = @SUM(   J(I):
 @SUM(   POS(R):(COMP_TIME(R)*W(I)*Z(I,R))));

@FOR( POS( R):
           PRO_TIME(R)=@SUM( J( I):PRO_TIME1(I)/(0.1647524*Temp(R)-
0.0058274*Temp(R)*Temp(R)+0.0000623*Temp(R)*Temp(R)*Temp(R)-0.4685328)*Z(I,R)) ;);

COMP_TIME(1)=(PRO_TIME(1));

   @FOR( POS( R)|R#GT#1:
COMP_TIME(R)=COMP_TIME(R-1)+(PRO_TIME(R));
);

@FOR( J( I): @SUM( POS( R): Z( I,R))=1;
);

@FOR( POS( R): @SUM( J( I): Z( I,R))=1;
);
@FOR( LINK: @BIN( Z));

END
```