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Stability Of a Quadratic Functional Equation in Intuitionistic Fuzzy Banach Spaces

Pratap Mondal[a,1] (pratapmondal111@gmail.com)
Nabin Chandra Kayal[b] (kayalnabin82@gmail.com)
T. K. Samanta[c] (mumpu_tapas5@yahoo.co.in)

[a]*Department of Mathematics, Bijoy Krishna Girls' College, Howrah-711101, West Bengal, India*
[b]*Department of Mathematics, Moula Netaji Vidyalaya, Moula, Howrah - 711312, West Bengal, India*
[c]*Department of Mathematics, Uluberia College, Uluberia, Howrah-711315, West Bengal, India*

Abstract - Hyers-Ulam-Rassias stability theorem has been applied to several functional equations for studying stability in case of approximation of a given functional equation in Banach spaces, fuzzy Banach spaces etc. In this paper, we wish to study generalized Hyers-Ulam-Rassias stability regarding the approximation of the following quadratic functional equation

$$f(2x+y) - f(x+2y) = 3f(x) - 3f(y) \tag{1}$$

in intuitionistic fuzzy Banach spaces.

Keywords - *Intuitionistic fuzzy norm, Hyers-Ulam stability, quadratic functional equation, Intuitionistic fuzzy Banach spaces.*

1 Introduction

The study of stability regarding approximation of a functional equation is related to a question of Ulam [16], while delivering his speech at the University of Wisconsin in 1940, concerning the stability of group homomorphisms. In Banach spaces, first positive answer to this equation of Ulam was provided by Hyers [6] in 1941. His exposition is textured with equation that if $\delta > 0$ and $f : E \longrightarrow E_1$ with E and E_1 Banach spaces , such that $\|f(x+y) - f(x) - f(y)\| \leqslant \delta$ for all $x, y \in E$, then there exists a unique mapping $g : E \longrightarrow E_1$ such that $g(x+y) = g(x) + g(y)$ and $\| f(x) - g(x) \| \leqslant \delta$ for all $x, y \in E$.
The quadratic function $f(x) = cx^2$ fulfils the functional equation

$$f(x+y) + f(x-y) = 2f(x) + 2f(y) \tag{2}$$

[1]Corresponding Author

which is why the equation is redefined as the quadratic functional equation. It is by F. Skof [15] who justified the Hyers -Ulam stability theorem for (2) in favour of the function $f : E \rightarrow E_1$ where E is a normed space at the same time E_1 is a Banach space. Replacing E_1 by an abelian group, the Hyers -Ulam stability theorem for (2) had been evidently established by P. W. Cholewa [3] and Czerwik [4]. Remarkable, the outcome of it got more widespread perspective at the handle of Th. M. Rassias [10], C. Borelli and G. L. Forti [2]. Subsequently the authors succeeding effort in the paper of Jun [7] cast a rather bright radiance in term of the result for the new quadratic functional equation (1). To define the situation marked with data uncertain, vague or imprecise the fuzzy set theory can be utilized comfortable as an instrument. In a certain status in which precisely objects are attached to a set, the intuitionistic fuzzy set theory makes themselves easily operative by bestowing a degree of membership and the non-membership to the object. Atanassov [1] is ascribed to be the introducer of the concept of the intuitionistic fuzzy set, as a generalized perspective of fuzzy set [17].

A new vista was opened for further progress following the initiation of intuitionistic fuzzy set and consequently several authors [13, 8] engaged themselves to make headway upon it for last four decades. Afterwards, a new version by Shakeri [14] on the notion of the intuitionistic fuzzy set come out for studying Hyers-Ulam-Rassias stability of a Jenson type mapping in intuitionistic fuzzy Banach spaces. Considering the notion of intuitionistic fuzzy Banach spaces due to Shakeri [14], here we apply generalized Hyers-Ulam-Rassias stability theorem to approximate the quadratic functional (1).

2 Preliminary

A new concept of intuitionistic fuzzy normed linear space was marked out by the author S. Shakeri [14] by adopting the idea of the intuitionistic fuzzy metric space initiated by Park [9] and Saadati-Park [11, 12]. In this section, first this definition of intuitionistic fuzzy norm [14] and subsequently a few results have been enfolded, that would be applied in the sequel.

Definition 2.1. : ([5]) *Consider the set* L^* *and the order relation* $\leq_{L^*}$ *define by*

$$L^* = \{ (x_1, x_2) : (x_1, x_2) \in [0,1]^2 \text{ and } x_1 + x_2 \leq 1 \},$$

$$(x_1, x_2) \leq_{L^*} (y_1, y_2) \Leftrightarrow x_1 \leq y_1, x_2 \geq y_2, \forall (x_1, x_2), (y_1, y_2) \in L^*.$$

Then $(L^*, \leq_{L^*})$ *is a complete lattice.*

Definition 2.2. : ([1]) *Let E be any set. An intuitionistic fuzzy set A of E is an object of the form* $A = \{ (x, \mu_A(x), \nu_A(x)) : x \in E \}$, *where the functions* $\mu_A : E \rightarrow [0, 1]$ *and* $\nu_A : E \rightarrow [0, 1]$ *denotes the degree of membership and the degree of non-membership of the element* $x \in E$ *respectively and for every* $x \in E$,
$0 \leq \mu_A(x) + \nu_A(x) \leq 1$.
We denote its units by $0_{L^*} = (0, 1)$ *and* $1_{L^*} = (1, 0)$

Definition 2.3. : ([5]) *A triangular norm (t-norm) on* L^* *is a mapping* $\Gamma : (L^*)^2 \rightarrow L^*$ *satisfying the following conditions :*

(a) $(\forall x \in L^*)(\Gamma(x, 1_{L^*}) = x)$ *(boundary condition),*

(b) $(\forall (x, y) \in (L^*)^2)(\Gamma(x, y) = \Gamma(y, x))$ *(commutativity),*

(c) $(\forall (x, y z) \in (L^*)^3)(\Gamma(x, \Gamma(y, z)) = \Gamma(\Gamma(x, y), z))$ *(associativity),*

(d) $(\forall (x, x', y, y') \in (L^*)^4)(x \leq_{L^*} x'$ *and* $y \leq_{L^*} y' \Rightarrow \Gamma(x, y) \leq_{L^*} \Gamma(x', y'))$ *(monotonicity).*
If $(L^*, \leq_{L^*}, \Gamma)$ *is an Abelian topological monoid with unit* 1_{L^*}*, then* Γ *is said to be a continuous t-norm.*

Definition 2.4. : ([5]) *A continuous t-norm* Γ *on* L^* *is said to be continuous t-representable if there exists a continuous t-conorm* $\diamond$ *on* $[0, 1]$ *such that for all*
$x = (x_1, x_2),\ y = (y_1, y_2) \in L^*,$
$\Gamma(x, y) = (x_1 * y_1, x_2 \diamond x_2)$
We now define a sequence Γ^n *recursively by* $\Gamma^1 = \Gamma$ *and*

$\Gamma^n(x^{(1)}, x^{(2)}, \cdots, x^{(n+1)}) = \Gamma(\Gamma^{(n-1)}(x^{(1)}, x^{(2)} \cdots, x^{(n)}), x^{(n+1)}),$
$\forall n \geq 2,\ x^{(i)} \in L^*.$

Definition 2.5. *Let* μ *and* ν *be membership and non-membership degree of an intuitionistic fuzzy set from* $X \times (0, \infty)$ *to* $[0, 1]$ *such that* $0 \leq \mu_x(t) + \nu_x(t) \leq 1$ *for all* $x \in X$ *and* $t > 0$*. The triple* $(X, P_{\mu,\nu}, T)$ *is said to be an intuitionistic fuzzy normed space (briefly IFN-space) if* X *is a vector space,* T *is a continuous t-representable and* $P_{\mu,\nu}$ *is a mapping* $X \times (0, \infty) \to L^*$ *satisfying the following conditions :*
for all $x, y \in X$ *and* $t, s > 0$,

(i) $P_{\mu,\nu}(x, 0) = 0_{L^*}$;

(ii) $P_{\mu,\nu}(x, t) = 1_{L^*}$ *if and only if* $x = 0$;

(iii) $P_{\mu,\nu}(\alpha x, t) = P_{\mu,\nu}\left(x, \frac{t}{|\alpha|}\right)$ *for all* $\alpha \neq 0$;

(iv) $P_{\mu,\nu}(x + y, t + s) \geq_{L^*} \Gamma(P_{\mu,\nu}(x, t), P_{\mu,\nu}(y, s))$.

In this case, $P_{\mu,\nu}$ *is called an intuitionistic fuzzy norm. Here,*

$P_{\mu,\nu}(x, t) = (\mu_x(t), \nu_x(t)) = (\mu(x, t), \nu(x, t)).$

Example 2.6. *Let* $(X, \|.\|)$ *be a normed linear space. Let*

$$M(a, b) = (min\{a_1, b_1\}, max\{a_2, b_2\})$$

for all $a = (a_1, a_2),\ b = (b_1, b_2) \in L^*$ *and for* $a, b \in [0, 1]$ *and* μ, ν *be the membership and the non-membership degree of intuitionistic fuzzy set define by*
$P_{\mu,\nu}(x, t) = (\mu_x(t), \nu_x(t)) = \left(\frac{t}{t+k\|x\|}, \frac{k\|x\|}{t+k\|x\|}\right)$*, where* $k > 0$*,*

and for all $x \in R^+$. *Then* $(X, P_{\mu,\nu}, M)$ *is an IFN-space.*

Definition 2.7. (1) *A sequence* $\{x_n\}$ *in an IFN-space* $(X, P_{\mu,\nu}, M)$ *is called a Cauchy sequence if for any* $\varepsilon > 0$ *and* $t > 0$, *there exists* $n_0 \in N$ *such that*

$$P_{\mu,\nu}(x_n - x_m, t) >_{L^*} (1 - \varepsilon, \varepsilon), \forall n, m \geq n_0$$

(2) *The sequence* $\{x_n\}$ *is said to be convergent to a point* $x \in X$ *if* $P_{\mu,\nu}(x_n - x, t) \to 1_{L^*}$ *as* $n \to \infty$ *for every* $t > 0$.
(3) *An IFN-space* $(X, P_{\mu,\nu}, M)$ *is said to be complete if every Cauchy sequence in* X *is convergent to a point* $x \in X$.

3 Stability Result

Theorem 3.1. *Let* X *be a linear space,* $(Z, P'_{\mu,\nu}, M)$ *be a IFN-space,* $\phi : X \times X \to Z$ *be a function such that for some* $0 < \alpha < 4$,

$$P'_{\mu,\nu}(\phi(2x, 0), t) \geq {}_{L^*} P'_{\mu,\nu}(\alpha\phi(x, 0), t) \tag{3}$$

$(x \in X, t > 0)$
and $\lim_{n\to\infty} P'_{\mu,\nu}(\phi(2^n x, 2^n), 4^n t) = 1_{L^*}$ *for all* $x, y \in X$ *and* $t > 0$. *Let* $(Y, P_{\mu,\nu}, M)$ *be a complete IFN-space. If* $f : X \to Y$ *is a mapping such that*
$P_{\mu,\nu}(f(2x + y) - f(x + 2y) - 3f(x) + 3f(y), t)$

$$\geq_{L^*} P'_{\mu,\nu}(\phi(x, y), t) \tag{4}$$

$(x \in X, t > 0)$
and $f(0) = 0$. *Then there exists a unique quadratic mapping* $Q : X \to Y$ *define by* $Q(x) := P_{\mu,\nu} - \lim_{n\to\infty} \frac{f(2^n x)}{4^n}$ *for all* $x \in X$ *satisfying*

$$P_{\mu,\nu}(f(x) - Q(x), t) \geq_{L^*} P'_{\mu,\nu}(\phi(x, 0), (4 - \alpha)t) \tag{5}$$

Proof: Putting $y = 0$ in (4) we get
$P_{\mu,\nu}(f(2x) - 4f(x), t) \geq_{L^*} P'_{\mu,\nu}(\phi(x, 0), t)$

$$\text{or, } P_{\mu,\nu}(\frac{f(2x)}{4} - f(x), t) \geq_{L^*} P'_{\mu,\nu}(\phi(x, 0), 4t) \tag{6}$$

Replacing x by $2^n x$ in (6)

$$\begin{aligned} P_{\mu,\nu}\left(\frac{f(2^{n+1}x)}{4^{n+1}} - \frac{f(2^n x)}{4^n}, t\right) &\geq_{L^*} P'_{\mu,\nu}(\phi(2^n x, 0), 4 \times 4^n t) \\ &\geq_{L^*} P'_{\mu,\nu}(\alpha^n \phi(x, 0), 4 \times 4^n t), \text{using (3)} \\ &\geq_{L^*} P'_{\mu,\nu}(\phi(x, 0), \frac{4 \times 4^n}{\alpha^n} t) \end{aligned} \tag{7}$$

Since $\frac{f(2^n x)}{4^n} - f(x) = \sum_{k=0}^{n-1} \left(\frac{f(2^{k+1}x)}{4^{k+1}} - \frac{f(2^k x)}{4^k}\right)$ we have

$$P_{\mu,\nu}\left(\frac{f(2^n x)}{4^n} - f(x), t\sum_{k=0}^{n-1}\frac{\alpha^k}{4\times 4^k}\right) = P_{\mu,\nu}\left(\sum_{k=0}^{n-1}\left(\frac{f(2^{k+1}x)}{4^{k+1}} - \frac{f(2^k x)}{4^k}\right), t\sum_{k=0}^{n-1}\frac{\alpha^k}{4\times 4^k}\right)$$

$$= P_{\mu,\nu}\left(\frac{f(2x)}{4} - f(x) + \sum_{k=1}^{n-1}\left(\frac{f(2^{k+1}x)}{4^{k+1}} - \frac{f(2^k x)}{4^k}\right), \frac{t}{4} + t\sum_{k=1}^{n-1}\frac{\alpha^k}{4\times 4^k}\right)$$

$$\geq_{L^*} M\left(P_{\mu,\nu}\left(\frac{f(2x)}{4} - f(x), \frac{t}{4}\right), P_{\mu,\nu}\left(\sum_{k=1}^{n-1}\left(\frac{f(2^{k+1}x)}{4^{k+1}} - \frac{f(2^k x)}{4^k}\right), t\sum_{k=1}^{n-1}\frac{\alpha^k}{4\times 4^k}\right)\right)$$

$$\geq_{L^*} M^{n-1}\left\{P_{\mu,\nu}\left(\frac{f(2x)}{4} - f(x), t\frac{\alpha^0}{4\times 4^0}\right), P_{\mu,\nu}\left(\frac{f(2^2 x)}{4^2} - \frac{f(2x)}{4}, t\frac{\alpha^1}{4\times 4^1}\right),\right.$$
$$\left. P_{\mu,\nu}\left(\frac{f(2^3 x)}{4^3} - \frac{f(2^2 x)}{4^2}, t\frac{\alpha^2}{4\times 4^2}\right), \ldots, P_{\mu,\nu}\left(\frac{f(2^n x)}{4^n} - \frac{f(2^{n-1}x)}{4^{n-1}}, t\frac{\alpha^{n-1}}{4\times 4^{n-1}}\right)\right\}$$

$$\geq_{L^*} M^{n-1}\left\{P'_{\mu,\nu}\left(\phi(x,0), \frac{4\times 4^0}{\alpha^0}\times t\frac{\alpha^0}{4\times 4^0}\right), P'_{\mu,\nu}\left(\phi(x,0), \frac{4\times 4^1}{\alpha^1}\times t\frac{\alpha^1}{4\times 4^1}\right),\right.$$
$$\left. P'_{\mu,\nu}\left(\phi(x,0), \frac{4\times 4^2}{\alpha^2}\times t\frac{\alpha^2}{4\times 4^2}\right), \ldots, P'_{\mu,\nu}\left(\phi(x,0), \frac{4\times 4^{n-1}}{\alpha^{n-1}}\times t\frac{\alpha^{n-1}}{4\times 4^{n-1}}\right)\right\}, \text{ by(7)}$$

$$= {}_{L^*}P'_{\mu,\nu}(\phi(x,0), t)$$

i.e., $P_{\mu,\nu}\left(\frac{f(2^n x)}{4^n} - f(x), t\sum_{k=0}^{n-1}\frac{\alpha^k}{4\times 4^k}\right) \geq {}_{L^*}P'_{\mu,\nu}(\phi(x,0), t)$

$$\text{or, } P_{\mu,\nu}\left(\frac{f(2^n x)}{4^n} - f(x), t\right) \geq {}_{L^*}P'_{\mu,\nu}\left(\phi(x,0), \frac{t}{\sum_{k=0}^{n-1}\frac{\alpha^k}{4\times 4^k}}\right) \tag{8}$$

Replacing x by $2^m x$ in (8) we have

$$P_{\mu,\nu}\left(\frac{f(2^{n+m}x)}{4^{n+m}} - \frac{f(2^m x)}{4^m}, t\right) \geq {}_{L^*}P'_{\mu,\nu}\left(\phi(x,0), \frac{t}{\sum_{k=m}^{n+m-1}\frac{\alpha^k}{4\times 4^k}}\right) \tag{9}$$

Thus $\left\{\frac{f(2^n x)}{4^n}\right\}$ is a Cauchy sequence in $(Y, P_{\mu,\nu}, M)$. Since $(Y, P_{\mu,\nu}, M)$ is a complete IFN-space, the sequence converges to some point $Q(x) \in Y$. So we can define a mapping $Q : X \to Y$ by $Q(x) := P_{\mu,\nu} - \lim_{n\to\infty}\frac{f(2^n x)}{4^n}$ for all $n \in N$.
Fix $x \in X$ and put $m = 0$ in (9) we get

$$P_{\mu,\nu}\left(\frac{f(2^n x)}{4^n} - f(x), t\right) \geq {}_{L^*}P'_{\mu,\nu}\left(\phi(x,0), \frac{t}{\sum_{k=0}^{n-1}\frac{\alpha^k}{4\times 4^k}}\right) \tag{10}$$

and so for every $\delta > 0$ we have
$P_{\mu,\nu}(Q(x) - f(x), t + \delta)$
$\geq {}_{L^*} M\left(P_{\mu,\nu}\left(Q(x) - \frac{f(2^n x)}{4^n}, \delta\right), P_{\mu,\nu}\left(f(x) - \frac{f(2^n x)}{4^n}, t\right)\right)$

$$\geq_{L^*} M\left(P_{\mu,\nu}\left(Q(x)-\frac{f(2^n x)}{4^n},\delta\right),\,P'_{\mu,\nu}\left(\phi(x,0),\frac{t}{\sum_{k=0}^{n-1}\frac{\alpha^k}{4\times 4^k}}\right)\right) \tag{11}$$

take $n\to\infty$ in (11) we get

$$P_{\mu,\nu}(Q(x)-f(x),t+\delta)\geq_{L^*} P'_{\mu,\nu}\left(\phi(x,0),\frac{4\times t}{\frac{1}{1-\frac{\alpha}{4}}}\right),\ 0<\alpha<4$$

$$\geq_{L^*} P'_{\mu,\nu}(\phi(x,0),t(4-\alpha)) \tag{12}$$

Since δ is an arbitrary, taking $\delta\to 0$ in (12) we get

$P_{\mu,\nu}(Q(x)-f(x),t)\geq_{L^*} P'_{\mu,\nu}(\phi(x,0),t(4-\alpha))$

Now we show that $Q(x)$ satisfies (1)

From (4) we have

$P_{\mu,\nu}(f(2x+y)-f(x+2y)-3f(x)+3f(y),t)\geq_{L*} P'_{\mu,\nu}(\phi(x,y),t)$

Replacing x, y by $2^n x$ and $2^n y$ respectively we get

$$P_{\mu,\nu}(f(2.2^n x+2^n y)-f(2^n x+2.2^n y)-3f(2^n x)+3f(2^n y),t)$$

$$\geq_{L*} P'_{\mu,\nu}(\phi(2^n x,2^n y),t)$$

$$\text{or, } P_{\mu,\nu}\left(\frac{f(2^n(2x+y))}{4^n}-\frac{f(2^n(x+2y))}{4^n}-\frac{3f(2^n x)}{4^n}+\frac{3f(2^n y)}{4^n},\frac{t}{4^n}\right)$$

$$\geq_{L*} P'_{\mu,\nu}(\phi(2^n x,2^n y),t)$$

$$\text{or, } P_{\mu,\nu}\left(\frac{f(2^n(2x+y))}{4^n}-\frac{f(2^n(x+2y))}{4^n}-\frac{3f(2^n x)}{4^n}+\frac{3f(2^n y)}{4^n},t\right)$$

$$\geq_{L*} P'_{\mu,\nu}(\phi(2^n x,2^n y),4^n t)$$

for all $x,y\in X$ and $t>0$.

Since $\lim_{n\to\infty} P'_{\mu,\nu}(\phi(2^n x,2^n y),4^n t)=1_{L^*}$, therefore taking limit $n\to\infty$ we have

$P_{\mu,\nu}(Q(2x+y)-Q(x+2y)-3Q(x)+3Q(y),t)\geq_{L^*} 1_{L^*}$

i.e., $Q(2x+y)-Q(x+2y)=3Q(x)-3Q(y)$.

Hence Q satisfies (1), i. e. , Q is quadratic.

Uniqueness : Let $T:X\to Y$ be an another quadratic mapping which satisfies (5). Fix $x\in X$ and using $Q(2^n x)=4^n Q(x)$ and $T(2^n x)=4^n T(x)$ for all $x\in X$.

Now we have by using (5)

$$P_{\mu,\nu}(Q(x)-T(x),t)=P_{\mu,\nu}\left(\frac{Q(2^n x)}{4^n}-\frac{T(2^n x)}{4^n},t\right)$$

$$\geq_{L^*} M\left(P_{\mu,\nu}\left(\frac{Q(2^n x)}{4^n}-\frac{f(2^n x)}{4^n},\frac{t}{2}\right),P_{\mu,\nu}\left(\frac{T(2^n x)}{4^n}-\frac{f(2^n x)}{4^n},\frac{t}{2}\right)\right)$$

$$\geq_{L^*} P'_{\mu,\nu}\left(\phi(2^n x,0),4^n(4-\alpha)\frac{t}{2}\right)$$

$$\geq_{L^*} P'_{\mu,\nu}\left(\phi(x,0), \frac{4^n(4-\alpha)}{\alpha^n}\frac{t}{2}\right)$$

Since, $0 < \alpha < 4$, so $\lim_{n\to\infty} \frac{4^n(4-\alpha)}{2\alpha^n} = \infty$, we get

$\lim_{n\to\infty} P'_{\mu,\nu}\left(\phi(x,0), \frac{4^n(4-\alpha)}{\alpha^n}\cdot\frac{t}{2}\right) = 1_{L^*}$

Therefore $P_{\mu,\nu}(Q(x) - T(x), t) = 1_{L^*}$ for all $t > 0$ and hence $Q(x) = T(x)$.

Corollary 3.2. *Let $\theta \geq 0$ and p be a non-negative real number and X be linear space,*

$(Z, P'_{\mu,\nu}, M)$ be an IFN-space, $(Y, P'_{\mu,\nu}, M)$ be a complete IFS-space. If $f : X \to Y$ is a mapping such that

$$P_{\mu,\nu}(f(2x+y) - f(x+2y) - 3f(x) + 3f(y), t) \geq_{L*} P'_{\mu,\nu}(\theta(\|x\|^p + \|y\|^p), t)$$

$$(x, y \in X, t > 0, \theta \in Z)$$

with $f(0) = 0$, then $Q(x) := P_{\mu,\nu} - \lim_{n\to\infty} \frac{f(2^n x)}{4^n}$ exists for each $x \in X$ and define a unique quadratic mapping $Q : X \to Y$ such that

$P_{\mu,\nu}(f(x) - Q(x), t) \geq_{L^*} P'_{\mu,\nu}(\theta\|x\|^p, (4-2^p)t)$ for all $x \in X$ and $t > 0$.

Proof : Define $\phi(x, y) = \theta(\|x\|^p + \|y\|^p)$ and it can be proved by similar way as theorem 3.1 by $\alpha = 2^p$

Corollary 3.3. *Let $\theta \geq 0$, $\epsilon > 0$ and X be linear space, $(Z, P'_{\mu,\nu}, M)$ be an IFN-space, $(Y, P'_{\mu,\nu}, M)$ be a complete IFS-space. If $f : X \to Y$ is a mapping such that*

$$P_{\mu,\nu}(f(2x+y) - f(x+2y) - 3f(x) + 3f(y), t) \geq_{L*} P'_{\mu,\nu}(\epsilon\theta, t)$$

$$(x, y \in X, t > 0, \theta \in Z)$$

with $f(0) = 0$, then $Q(x) := P_{\mu,\nu} - \lim_{n\to\infty} \frac{f(2^n x)}{4^n}$ exists for each $x \in X$ and define a unique quadratic mapping $Q : X \to Y$ such that

$P_{\mu,\nu}(f(x) - Q(x), t) \geq_{L^*} P'_{\mu,\nu}(\epsilon\theta, t)$ for all $x \in X$ and $t > 0$.

Proof : Let $\phi : X \times X \to Z$ be define by $\phi(x, y) = \epsilon\theta$, then the proof is followed by Theorem 3.1 by $\alpha = 1$

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