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## A BAYESIAN APPROACH PROPOSAL FOR INVENTORY COST AND DEMAND FORECASTING

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### Abstract

Technology's perpetual vicissitude and product models' distinction in industrial market have a crucial effect on forecasting demand for spare components. In order to set forth the future demand rates for products, inventory managers repetitively update their prognostications. Bayesian model is utilizing a prior probability distribution for the injunctive authorization rate which was habituated in order to get optimum levels of account over a number of periods. However, under sundry demand rates like intermittent demand, Bayesian Model's performance has not been analyzed. With the help of a research question, the study investigates that circumstance.

Keywords: Bayesian model, forecasting, inventory, probability distribution

Jel Code: C11, C16, C53

## ENVANTER MALİYETİ VE TALEP TAHMİNİ İÇİN BAYES YAKLAŞIMI ÖNERİSİ

### Özet

Endüstriyel pazardaki teknolojinin kalıcı değişikliğinin ve ürün modellerinin farklılığının, yedek parçalar için yapılan talep tahmini üzerinde önemli bir etkisi vardır. Ürünlerin gelecekteki talep oranlarını ortaya koymak amacıyla envanter yöneticileri kendi tahminlerini sürekli güncellemektedir. Bayes modeli, önsel olasılık dağılımı kullanarak kabul edilebilir oranı birkaç dönem üzerinden optimum hesap yapmak için kullanılmaktadır. Ancak, aralıklı talep gibi muhtelif talep oranlarının altında, Bayes Modelinin performansı analiz edilmemiştir. Bir araştırma sorusu yardımıyla, bu çalışma bu durum incelenmektedir.

Anahtar Kelimeler : Bayes modeli, envanter, olasılık dağılımı, tahminleme

Jel Kodu : C11, C16, C53

### 1. INTRODUCTION

The process of presage future amount which is asked by consumers is called inventory demand forecasting. It is the substructure for the tenaciousness of the engenderment level of the business. The interpretation of the ordained dictation forecasts are done by which product in which demand for the particular products' spare part would date on the probability for the authoritative ordinance's actualization. Spare market does not only become the contact's principal among customers and business in

nowadays economic life, but additionally, it is an important investment in the eyes of numerous organizations. The market value is going to be 230 billion Euros in 2020 in Europe as said by some latest sector of automotive (Yılmaz, 2012). Because of the market's magnitude; opportune management like engenderment approximations adverts is the ultimate arduous of the primitives to assess. Similar engenderment estimations' adverts' opportune management is the most arduous of the primitives in order to evaluate.

Spare part amount demand forecasting provides the

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basic inputs for production planning and control system with other functions. These estimations made functions to transform raw-material, spare parts, machinery, by-products, manpower, programming, and other decisions. Estimation method with non-scientific or completely subjective forecasting indicates many system and component failures that emerge suddenly. A complicated issue might be occurred when control and management of spare parts come into existence. General statistical applications lose their empiric solutions because of the rapid developments and extraordinary facts in the time of operation. Forecasting demands are in need of historical demands that are not available or not acceptable for brand new consuming components in similar circumstances (Gohdrati et al., 2007).

Inventory serves a useful purpose in the organizations. At the retail level, its main function is to allow the firm to meet expected customer demand and prevent shortages (Li and Kuo, 2008). However, holding inventory always is a challenge and presents real costs to companies for managers who are under pressure to reduce inventories and associated expenses. To ensure top performance, managers have spent millions of dollars on forecasting and planning systems over the years.

The arduousness in forecasting demand for high-end challenge for automobile components is altercated by Snyder (2002). Spare component's ordained dictation forecast has three essential obstacles; although ordained dictation's precise forecasting is supreme in inventory control. The first obstacle is, spare part's requirement is generally irregular and the forecasting is arduous because of nature of demand (Ghobbar and Friend, 2003; Viswanathan et al. 2008). The second one is about the inhibition recorded data of spare part demand. As for the third one, inventory level of spare components is considered as mainly the function which is the maintenance and utilization of the equipment.

This academic work explains proposed implements and circumstances of proposed inventory managements' achievement in order to understand the implements' dependability and utility during inventory decision making. The particular analysis concentrates on the status quo that can be required for a manager to guess the expected demand rate of products.

Flowingly, it is possible to see the study's formation. In the second chapter, the analysis which is related with moving inventory forecast techniques are carried. The investigation that brings the methodology on board is studied. Fourth chapter shows the key finding and academic argumentations. As for the last chapter, endings and drawbacks are accounted for and the spaces for the future research are depicted.

## 2. FORECASTING METHODS FOR INVENTORY DEMAND

It is assumed by the several contemporary inventory management and control software packages that a mundane distribution comes before the further demand. Consequently, analytic models which have agreeable forecasting precision are included (Verecke and Verstraeten, 1994). Generally periodic or perpetual reviews are preferred in almost all randomly inventory models (Stevenson, 2007). In order to add up the order quantity needed to bring it back to level which is aimed and popular with minute retailers; the amount of inventory at intermittent intervals is decided by the periodic review approach. Vice versa, the perpetual system approach updates inventory instantly and submits an order for a fine-tuned quantity in order to diminish the absolute cost at any time it reaches a calculated minimum level.

Models for managing spare components are advised by Strijbosch et al. (2000), Razi and Tarn (2003), and Dolgui and Pashkevich (2007). They are consequential in ascertaining adequate items' accessibility, along these lines providing organizations with a competitive advantage. Syntetos and Boylan (2006) make a comparison between simple moving average, single exponential smoothing, Croston's method, and a modified Croston method in a practical way that proposed by Syntetos and Boylan (2010) on spare components from the automotive industry with a fixate on irregular demands.

Hua et al. (2007) propose using explanatory variables to forecast slow-moving spare parts and compare these forecasts to those from exponential smoothing, Croston's method, and bootstrapping. In order to adopt an appropriate method for forecasting slow-moving inventory, Syntetos et al. (2005) propose using the average mean time between demand and the squared coefficient of variation of demand size to categorize demand. They suggest that determining the category of demand is the first step for adopting the most appropriate forecasting methodology. In order to make demands forecasting real and make a comparison with a standard ARMA model Chen et al. (2010) formed a updated Regression Bayesian-BBNN based model. It is seen that this updated model has higher certainty and strength in a superior way.

### 2.1. Forecasting methods for spare part demand

Leven, (2004) adduced an inventory management system for slow-paced and expeditious-moving products. Leven's proposal is predicated on the Croston method and the demand which is random is postulated to have an Erlang allocation utilizing a Bayesian approach for spare components. Standard testing methodology which is habituated by Shalen et al. (2008) is presented that surmounts the complications resulting from the essentiality of sampling from time series data. According

to their research, It is understood that Poisson process symbolizes the frequency of orders well enough. While considering the aggregate data series, Forecasting methods' top-down and bottom-up performance are assessed by Viswanathan et al. (2008) at the same time, the sub-aggregate time series components are irregular and for forecasting the total demand utilizing top-down forecasting, the simple exponential smoothing technique outperformed Croston's method in a case.

In order to roughly calculate intermittent demand that was assessed to outperform some of the methodologies discussed such as, Willemain et al., (2004) built up a distribution seperated bootstrap, Syntetos et al. (2005) used the mean square error as a criterion to compare different prediction methods and classify demand patterns regarding both the average inter-demand interval and the variation's coefficient. Four categories result from their study: erratic but not very intermittent, lumpy, smooth, and intermittent but not very erratic. Appropriate forecasting techniques have been proposed for each of them. Boylan and Syntetos (2010) made a review of forecasting research and extensions about spare part management and mentioned some work on the value of judgmental arrangement of statistical forecast.

2.2. Bayesian approach to demand modelling

A Bayesian approach is formed by Popovic (1987) in order to make inventory decision. This also let the predictions of the parameters of a priori distribution of demand rate to be renewed. As an example, if two time periods of recording sales have passed, then the a posteriori distribution  $\lambda$  of is gamma with parameters  $\alpha > 0$  and  $\beta > 0$  given that values of the demands  $X_1$  and  $X_2$ , namely,  $\lambda | X_1, X_2 \sim \Gamma(\alpha + X_1 + X_2, \beta + 2)$ . Popovic, (1987) stated that the optimal inventory levels should be determined by using the a posteriori distribution and knowledge of the surplus cost per unit of time,  $C_1$  as well as the shortage cost per unit of time,  $C_2$ .

Silver, (1965) applied the Bayesian method to select the reorder point for an inventory model. De Wit, (1983) proposed a Bayesian approach to forecasting slow-moving items; however, slow moving was defined as 10 or fewer demands per unit of time, higher than the demand rate used in this research. Furthermore, the proposed method failed to perform well when demand was extremely low. Price and Haynsworth, (1986) suggested that the Bayesian approach is better suited to predict the sales of products with slow demand than exponential smoothing although its actual performance may depend on the distribution of the demand. Lee, (2014) applied an averaging Bayesian model to get uncertainty of the demand signals in the first order outcome process to generalize the inventory model that include other auto-regressive lags.

3. METHODOLOGY

In order to make a comparison between the optimal levels of inventory with reverence to the costs for shortage and surpluses to a traditional approach predicated on the Poisson distribution, a study is carried through utilizing Bayesian technique. A utilizer-supplied prior distribution for the injunctive authorization rate is needed to apply the alternative method. This injunctive authorization rate is acted with an arbitrary variable having a gamma distribution.

The validity of the reliability of both one-sided and two-sided prognostication intervals are estimated by this analysis for the future demand rate of products without historical sale background and the ones with no more than one sale as well. Through utilizing intermittent sales data, Bayesian approach's performance is analysed.

The Bayesian method utilizes a prior distribution for the demand rate  $\lambda$ . Because of this reason, It is a far cry from the classical Poisson point of view. Popovic, (1987) proposed a Bayesian model in which the demand has a Poisson distribution, but an a priori gamma distribution  $\Gamma(\alpha, \beta)$  with the probability density function in (1) is assigned / assumed since  $\lambda$  is unknown.

$$f(\lambda; \alpha, \beta) = \frac{\beta^\alpha \lambda^{\alpha-1} e^{-\beta\lambda}}{\Gamma(\alpha)}, \alpha > 0, \beta > 0, \lambda > 0 \quad (1)$$

Assuming  $X_t$  is the random variable representing demand during the time interval  $[0, t]$ , demand's the unconditional distribution is displayed in (2).

$$P\{X_t = k\} = \int_0^\infty \frac{(\lambda t)^k}{k!} e^{-\lambda t} f(\lambda) d\lambda = \binom{\alpha+k-1}{k} \left[ \frac{\beta}{\beta+t} \right]^\alpha \left[ \frac{t}{\beta+t} \right]^k, \quad (2)$$

$\alpha > 0, \beta > 0, t > 0$

where  $k = 0, 1, \dots$  and the notation  $\binom{n}{x}$  is the number of combinations of taking  $x$  items from  $n$  distinct items at a time. Note that the combination  $\binom{\alpha+k-1}{k}$  in (2) can also be written equivalently as  $\binom{\alpha+k-1}{\alpha-1}$ . Thus  $X_t$  has a negative binomial distribution NB  $\left( \alpha, \frac{\beta}{\beta+t} \right)$ . Popovic [22] shows that denoting  $P\{X_t = k\}$  by  $p_k$ .

Where  $k = 0, 1, \dots$ , we have

$$p_0 = \left[ \frac{\beta}{\beta + 1} \right]^\alpha, \tag{3}$$

$$p_k = \left[ \frac{(\alpha + k - 1)}{k(\beta + 1)} \right] P_{k-1}$$

By applying Bayes' rule for the first period, we have:

$$f(\lambda | X_1) = \frac{P(X_1 | \lambda) f(\lambda)}{\int_0^\infty P(X_1 | \lambda) f(\lambda) d(\lambda)} \tag{4}$$

The posteriori distribution of  $\lambda$  over the first unit time interval  $I_1 = [0,1]$  is:

$$f(\lambda | X_1) = \frac{(\beta + 1)^{\alpha + X_1} \lambda^{(\alpha + X_1 - 1)} e^{-(\beta + 1)\lambda}}{\Gamma(\alpha + X_1)} \tag{5}$$

Then a posteriori distribution is also illustrated for the next time to show a pattern in the form of the distribution and is displayed in (6) for time interval  $I_2 = [1,2]$ .

$$P\{X_2 = k\} = \int_0^\infty \frac{\lambda^k e^{-\lambda}}{k!} f(\lambda | X_1) d\lambda = \left[ \frac{\Gamma(\alpha + k - 1)}{k!} \Gamma(\alpha + X_1) \right] \left[ \frac{\beta + 1}{\beta + 2} \right]^{\alpha + X_1} \left[ \frac{1}{\beta + 2} \right]^k \tag{6}$$

Thus  $X_2 \sim \text{NB}\left(\alpha + X_1, \frac{\beta + 1}{\beta + 2}\right)$ . It then follows that the a posteriori distribution of  $\lambda$  for the second time interval  $I_2$  after demand  $X_2$  occurs will be:

$$\lambda | X_1, X_2 \sim \Gamma(\alpha + X_1 + X_2, \beta + 2) \tag{7}$$

Furthermore, it can be shown that the general a posteriori distribution of  $\lambda$  is

$$\lambda | X_1, X_2, \dots, X_n \sim \Gamma\left(\alpha + \sum_{i=1}^n X_i, \beta + n\right) \tag{8}$$

After observing  $X_1, X_2, \dots, X_n$ , one can prove that (9) is the distribution of demand at interval  $I_{n+1}$ .

$$X_{n+1} \sim \text{NB}\left(\alpha + \sum_{i=1}^n X_i, \frac{\beta + n}{\beta + n + 1}\right) \tag{9}$$

Popovic, (1987) expresses the optimal inventory level of the  $I$ th unit time interval  $I_i$  which can be calculated by examining the ratio of the cost of surplus  $C_1$  and the cost of shortage  $C_2$  of an item as well as the demand found by the a posteriori distribution. The more extra knowledge is accumulated; the more posteriori distribution of demand can be updated to improve the accuracy of the parameter estimates. The optimal inventory level  $r_i^*$  for product  $i$  will be such that it satisfies one of the inequalities in (10).

*Bayesian Model Approach:*

$$\sum_{k=0}^{r_i^*-1} \binom{\alpha + k - 1}{k} \left(\frac{\beta}{\beta + 1}\right)^\alpha \left(\frac{1}{\beta + 1}\right)^k < \frac{C_2}{C_1 + C_2} \leq \sum_{k=0}^{r_i^*} \binom{\alpha + k - 1}{k} \left(\frac{\beta}{\beta + 1}\right)^\alpha \left(\frac{1}{\beta + 1}\right)^k \tag{10}$$

*Poisson Model Approach:*

$$\sum_{k=0}^{r_i^*-1} e^{-\lambda t} \frac{(\lambda t)^k}{k!} < \frac{C_2}{C_1 + C_2} \leq \sum_{k=0}^{r_i^*} e^{-\lambda t} \frac{(\lambda t)^k}{k!} \tag{11}$$

We use the ratio of  $C_1$  and  $C_2$  in order to assess optimal inventory levels that contain style goods and perishable items. Since setting relevant costs for shortage and surplus goods for inventory with intermittent demand may not always be empirical, it could be a potential limitation to the particular methodology. Popovic, (1987) depicts a solution of this point of view to the ordering of spare parts from a warehouse at the beginning of each month and describes how the equalities in (10) can be utilized to order optimal stock levels. Undetermined demand rate for moving spare part inventory, an effectors regulation might be hard to build. Managers may assess the planned order amount of a party of non-selling or slow moving products again at the point of a determined period. That can be common disconnect products whose projected order party is smaller than a limit value. The decision is considered to liquidate, if the more elevated endpoint of a unilateral estimation interval for the future demand of outcome allowance is beneath the limit (Lindsey and Pavur, 2009). Thus we assess the robustness of the reliability of a one-sided prediction interval for the future demand rate across a variety of parameters. The unilateral estimation interval is the same as the proposed two-sided interval discussed previously except  $\alpha$  level (Type I error) is not divided by 2 and the lower endpoint is not estimated. The proposed one-sided prediction interval is:

$$\left[ 0, \frac{M_1(t)}{t} + Z_\alpha \sqrt{\frac{M_1(t) + 2M_2(t)}{t^2}} \right] \tag{12}$$

100 time units were selected as the time frame to collect data on sales of slow-moving products. In order to determine OSPIs dependability for making a comparison with a limit value for giving all-important adjudication about a subset of non-selling products, a simulation (Monte-Carlo) with 1,000 copies of the demand for a product groups over 100 units of time are carried out. Consequently, practical Type I errors are determined beyond a different circumstances for the number of

products and the outcome rates.

#### 4. RESEARCH APPLICATION

There is going to be a comparison between a classical maximum likelihood approach and the Bayesian model, referring to the Poisson model, in which the rate parameter of a Poisson distribution is estimated from the earlier data. The effectiveness of each of these two methods's is assessed by the performance of Monte Carlo simulation of authoritative ordinance rates over ten time periods. In this paper, "moderate-demand products" means the products which are not discontinuing. Let us accept the concealed unidentified product's demand rate as  $\lambda$ . The approximation for this particular product might be zero with no demand or without a registered demand history. Accurately culled prior distribution for the ordinant dictation rate may provide a more plausible forecast than a forecast of zero because of the observed period of time that may not be long enough to sanction for a precise prognostication of future demand. For instance, The Bayesian model may utilize a prior demand distribution for a product which is predicated on demand's distribution for all products. As a result of utilizing the entire pool of products, the prior distribution's parameters are guessed according to this point of view. On account of arbitrariness in customer buying patterns, some products may be expeditious and others may be gradually when It comes to selling. In order to mitigate the effects of insufficient data in presaging future demand rates, the use of this approach is preferred.

Inequality's form of the cost expression (10) needs that only the ratio of the surplus cost to shortage cost be kenneed. That is, cost deficiency of \$5 and a cost of excess of \$1 will yield identically tantamount results as a cost deficiency of \$50 and cost of excess of \$10 respectively. For the comparison of the costs predicated on the Bayesian model and the Poisson model, a pool of 100 products is utilized in the study of simulation. The pool of products' demand rates are engendered from a gamma distribution with a culled value of the mean identically tantamount to 3, which sanction some products to be slow moving provided the standard deviation is not too diminutive. The culled four standard deviations as follows: 5.5 (high), 1.7 (moderately high), 0.55 (low), and 0.17 (very low). The following values are assessed by two parameters of the gamma distribution:  $\alpha$ , the shape parameter, and  $\beta$ , the scale parameter. The mean for the gamma distribution is identically tantamount to  $\alpha\beta$  and the variance is equipollent to  $\alpha\beta$ . The Standard deviations come about the written four pairs of  $\alpha$  and  $\beta$  values:  $\alpha = 0.3$  and  $\beta = 0.1$ ,  $\alpha = 3$  and  $\beta = 1$ ,  $\alpha = 30$  and  $\beta = 10$ , and  $\alpha = 300$  and  $\beta = 100$ . Genuine product sales are engendered utilizing a Poisson process, whenever the authoritative demand  $\lambda$  is culled. As a result of symbolising sales over a period of 10 time units, the number of units sold for each product is replicated 10 times. This process is simulated 20 times and the average

costs per time unit are listed. The time units might be days, weeks, months etc. After utilizing the underlying distribution of injuctive authorization for that product, assumed costs from keeping this amount of list of goods can be assessed. In order to understand which model has the lower expected cost, the performances of the two models were criticized in Figure 1.

A flamboyantly blatant gap stands among the total costs for the Bayesian model and the Poisson model for the first two time periods. The total cost per period utilizing either model amends dramatically after only three periods like indicated antecedent in the study. If the product demand rates' variance is averagely high, It can be said that an advantage is there to utilize the Bayesian model particularly for the first couple of time periods. Although the surplus cost's ratio has effect on the graphs' scale, there is not an extreme transmutation in the model's relevant performance.

Table 1. Prediction intervals

Units	Two sided Zero and One sale	
	Prediction Intervals	
	90%	99%
50	0.103	0.009
100	0.107	0.009
150	0.104	0.009
500	0.109	0.015
1000	0.100	0.025

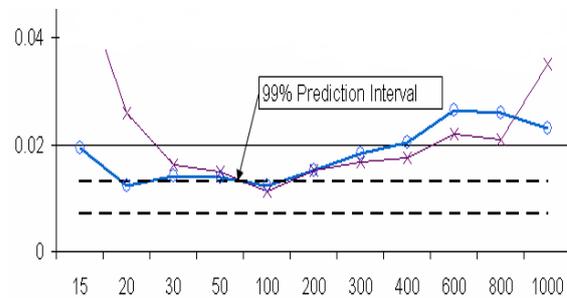


Figure1. Time between demands

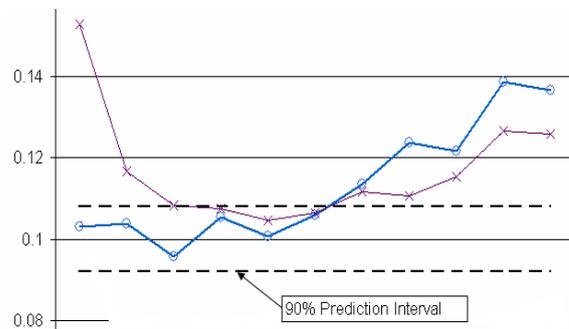


Figure 2. Time between demands

There would be an one explanation possible that when the demand is high, less products have not any sales, and more products will have periods with a demand of one unit. With that circumstance, the Zero and One Sale prediction intervals are reliable, see in Table 1. It looks like sensible to anticipate that prediction interval would be reliable for products with a higher demand rate.

## 5. RESULTS AND DISCUSSION

The Bayesian approach allowed for updates using historical data over specific time intervals. In this approach, the manager must assume a prior probability distribution for the demand rate of the products. This prior distribution may be based on a manager's experience with similar products.

The Bayesian approach in this study used a gamma distribution as a prior distribution of the demand rate. This approach is more involved than using a Poisson approach, but may be easily automated. In the simulation study, the expected cost of inventory for the Bayesian model and Poisson model were compared by varying the mean and standard deviation of the of the demand rates of 100 products, which may be functionally dissimilar. In addition, the cost ratio of shortage and surplus inventory varied between 1:5, 1:1, and 5:1. The mean demand was fixed at 3 and the standard deviations were varied from 0.17 to 5.5. The resulting total cost of the inventory using these approaches typically declined quickly over the first four or five periods and as time approached 10 periods, the last time period for which updates were computed, both

approaches merged.

## 6. CONCLUSIONS

Managers of inventory try to store products and semi-products that customers in need of purchasing. This study wants to highlight this challenge's one form: predicting the future demand rate of products. By the time a demand rates's prior distribution is available and surplus and shortage costs are known, the Bayesian methodology might be accepted as relevant. In addition to this, the standard deviation of the products' demand rate will determine if the Bayesian or the Poisson model would be better suited.

Every product's demand is considered free of the demand from the rest of the products. In reality, product demand is correlated with the demand for other products.

Both Bayesian and Poisson models which are examined in this research conduct certain assumptions, which may not be applicable in practice. In order to indicate the demand of goods, a Poisson distribution was used. That process is mostly found in the literature.

Inventory methodology for optimizing inventory levels which is alternative multi-period should be explored. In order to account for inventory which has a limited life span or makes the replenishment possible in the middle of the single period, the Bayesian approach may be extended to more complex inventory management problems.

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